II Workshop on Muon Precision Physics

Towards NNLO for low-energy $e^+ e^-$ into leptons and hadrons

Adrian Signer

Paul Scherrer Institut / Universität Zürich

MPP23 / Liverpool / 07 Nov 2023

- Spring 2022: start of a community effort
- theoretical description for $e^+ e^- \to$ hadrons at low energies $\sqrt{s} \lesssim 1 - 2\,\text{GeV}$
- main processes

$$e^+ e^- \to \mu^+ \mu^- + \gamma \; \{+\gamma\}$$
$$e^+ e^- \to e^+ e^- + \gamma \; \{+\gamma\}$$
$$e^+ e^- \to \pi^+ \pi^- + \gamma \; \{+\gamma\}$$

- more processes

$$e^+ e^- \to \gamma \gamma \; \{+\gamma\}$$
$$e^+ e^- \to \pi^+ \pi^- \pi^0$$
$$e^+ e^- \to \pi^+ \pi^- \pi^0 \pi^0$$
$$e^+ e^- \to \pi^+ \pi^- \pi^+ \pi^-$$

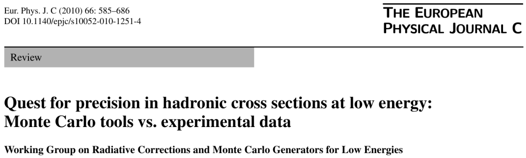
- there are more processes and $(e^+ e^-)$ in final state

outline

- two parts of my talk
- part 1: 'politics'
- part 2: science $\rightarrow$ later contributions

part 1

- Strong2020: Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions WorkStop/ThinkStart in Zurich
  https://indico.psi.ch/event/13707/
- inspired by [0912.0749]


Eur. Phys. J. C (2010) 66: 585–686
DOI 10.1140/epjc/s10052-010-1251-4

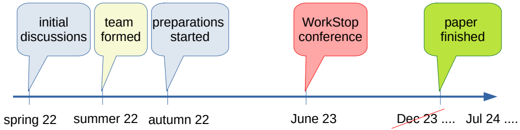
THE EUROPEAN PHYSICAL JOURNAL C

Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

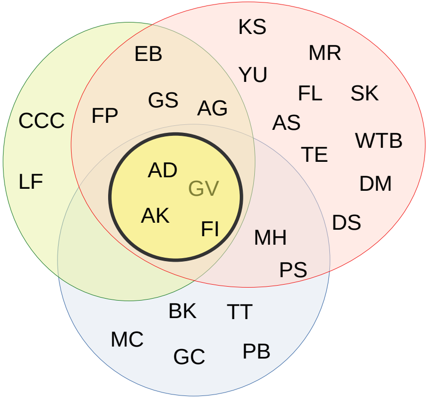
Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

- consolidate and implement the progress since 2010



Team: P. Beltrame, E. Budassi, C. Carloni Calame, G. Colangelo, M. Cottini, A. Driutti, T. Engel, L. Flower, A. Gurgone, M. Hoferichter, F. Ignatov, S. Kollatzsch, B. Kubis, A. Kupsc, F. Lange, D. Moreno, F. Piccinini, M. Rocco, K. Schönwald, A. Signer, G. Stagnitto, D. Stöckinger, P. Stoffer, T. Teubner, W. Torres Bobadilla, Y. Ulrich, G. Venanzoni + ???

WP1:	QED for leptons at NNLO
WP2:	Form factor contributions at N^3LO
WP3:	Processes with hadrons
WP4:	Parton showers
WP5:	Experimental input



part 1

WP1: QED for leptons at NNLO

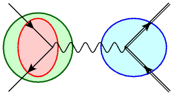
WP2: Form factor contributions at N^{3}LO

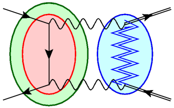
WP3: Processes with hadrons

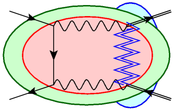
WP4: Parton showers / YFS

WP5: Experimental input

$e^+ e^- \to \pi^+ \pi^-$







Working Group Strong2020

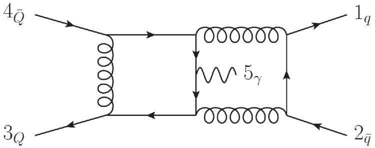
- preserve and further develop some well established codes for low-energy $e^+ e^-$
- cross fertilisation from huge effort made for LHC
- get new (preferably young) people to join with new ideas/approaches
- open science approach: what is in which generator, where can I get it
- open minded assessment of importance (or not) of particular contributions
- a community effort (we really try/ied)
- it is not yet too late to join (but soon it will be)

  contact Andrzej Kupsc, AS, Yannick Ulrich, Graziano Venanzoni
- it worked marvellously for the MUonE theory initiative

connections

QCD@LHC

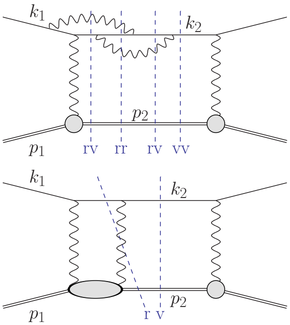
- e.g. [2304.06682] NNLO cross section for $p\,p \to j\,j\,\gamma$ (i.e. $2 \to 3$ massless)



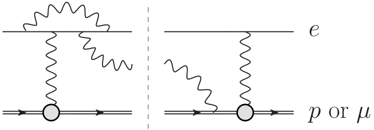
- automated one-loop codes, e.g. OpenLoops
- high-energy event generators e.g. Sherpa
- match NNLO to shower e.g. MiNNLO$_{PS}$

lepton-proton

(MUSE, PRad, Amber, ULQ2 ...)



and, of course, MUonE



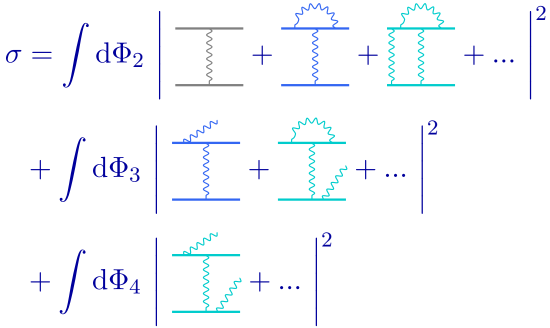
QCD $\longrightarrow$ low-energy

QCD @ LHC	$\Leftrightarrow$	QED @ low & medium energy	
non-abelian	$\gtrsim$	abelian	matrix elements somewhat easier
non-abelian	$\gg$	abelian	IR structure much easier
massless fermions	$\ll$	massive fermions	loop amplitudes much harder
jets	$<$	exclusive w.r.t.	numerics harder $\supset \log(m^2/Q^2) \equiv L$
		collinear radiation	much harder for small masses

stealing from QCD and other fields

- master integrals (reduction and computation)
- automated tools for one-loop amplitudes
- effective field theory methods
- match fixed-order result to parton shower / YFS

physical $(2 \to 2)$ cross section



$+ \dots$

gives all $\alpha^2 L^n$, soft and coll

challenges

- fully differential phase-space integration

⇒ QED huge simplification

- virtual amplitudes with massive particles

⇒ one-loop: "standard"

⇒ two-loop: bottleneck

- numerical instabilities due to pseudo-singularities

⇒ need stabilisation

massive fermions $\rightarrow$ only soft singularities (use $m_\gamma \neq 0$ or $d = 4 - 2\epsilon$)
and collinear logarithms (mathematically ok, but numerics !?)

slicing or subtraction (for $2 \rightarrow 2$), can be extended to NNLO (and beyond)

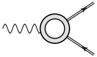
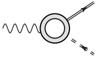
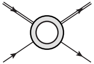
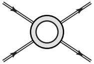
$$d\sigma_{\mathrm{r}}^{(1)}\big|_{\mathsf{sl}} = \int\limits_{E_\gamma<\delta} d\Phi_{3=2+\gamma}\, \mathcal{S}@|\mathcal{A}_\gamma^{(0)}|^2\, S(k_2,p_2) + \int\limits_{E_\gamma\geq\delta} d\Phi_3\, |\mathcal{A}_\gamma^{(0)}|^2\, S(k_2,p_2,k_\gamma)$$

$$d\sigma_{\mathrm{r}}^{(1)}\big|_{\mathsf{su}} = \int d\Phi_{3=2+\gamma}\, \mathcal{S}@|\mathcal{A}_\gamma^{(0)}|^2\, S(k_2,p_2) + \int d\Phi_3 \Big(|\mathcal{A}_\gamma^{(0)}|^2 S(k_2,p_2,k_\gamma) - \mathcal{S}@|\mathcal{A}_\gamma^{(0)}|^2 S(k_2,p_2) \Big)$$

loops with masses

- scales (e.g. masses) are the enemy of loop-integral calculators
- for one-loop amplitudes generic tools e.g. OpenLoops
- but massive two-loop integrals for $2 \to 2$ are not all known

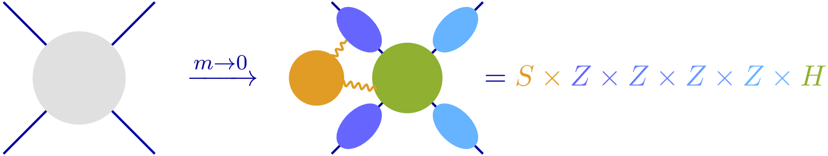
[here should go a list of an army of loop-calculating theoreticians ... sorry]

$\gamma^* \to ee$		✔ full m dependence	harmonic (M)PL	$2004(/23)$
$\mu \to e\nu\bar{\nu}$		✔ full m_1 and m_2	generalised (M)PL	2018
$\mu e \to \mu e$		✔ full m_1, $m_2 = 0$	generalised (M)PL	2021
$ee \to ee$		✗ (so far?) only planar	elliptic MPL	2022

loops with masses

- scales (e.g. masses) are the enemy of loop-integral calculators
- for one-loop amplitudes autimated tools available
- for two-loop can use massification and miss terms $\sim (\alpha/\pi)^2\, m_\ell^2/q^2$

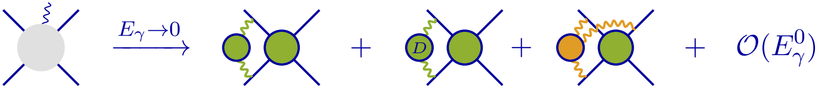
collinear factorization / SCET inspired



$\Rightarrow$ massification, i.e. obtain leading mass effects based on massless loops
assumes m_ℓ is the only small scale, much smaller than all other scales

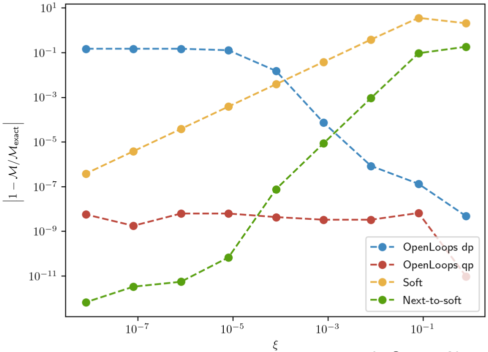
next-to-soft stabilisation

real-virtual corrections trivial in principle, extremely delicate numerically

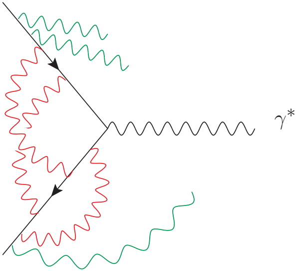


eikonal $1/E_\gamma^2$ — LBK diff. op. $1/E_\gamma$ — soft contribution $1/E_\gamma$

- Bhabha scattering (as example) [2106.07469]
- M_{exact} Mathematica expression
- full M vs next-to-soft limit LBK theorem beyond LO [2112.07570, 2304.11689]

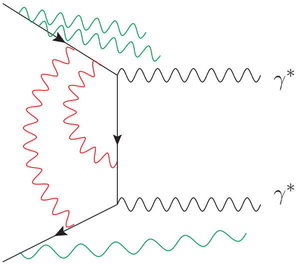


Buliding block $e^+ e^- \to \gamma^*$



- NNLO available (used for $e\,\mu \to e\,\mu$ and $\ell\,p \to \ell\,p$) including real and vitual, no approximation
- moving towards NNNLO, $\exists$ open questions
  - 3-loop known [2202.05276]
  - 2-loop $e^+ e^- \to \gamma^*\gamma$ bottleneck, need $m_e \neq 0$ cannot use massification in full phase space $\exists$ regions with e.g. $(p_\gamma + p_e)^2 \simeq m_e^2$
  - 1-loop $e^+ e^- \to \gamma^*\gamma\gamma$ use tools, numerics?
  - 0-loop $e^+ e^- \to \gamma^*\gamma\gamma\gamma$ trivial but dangerous
- playground for combination with parton showers
- (dominant??) subset of N^{3}LO for MUonE
- can we use next-to-soft LBK to improve YFS ??

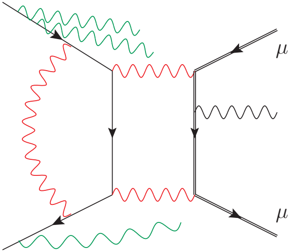
Buliding block $e^+ e^- \to \gamma^* \gamma^*$



- doubly virtual Compton scattering (gauge invariant)
- NLO doable, including real and vitual
- NNLO painful, would it be useful ?
  - if one $\gamma^* \to \gamma$ it is desperately wanted
  - dispersive approach to combine with $\pi^+\pi^-$ final state
  - a (gauge-invariant) subset of N^{3}LO for MUonE
- playground for combination with parton showers

leptonic processes

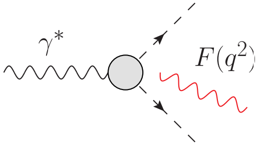
Process $e^+ e^- \to \mu^+ \mu^- \gamma$



- NLO done, with massified e part of NNLO MUonE can 'easily' be done with full mass dependence
- NNLO no show stoppers, but approximations
  - two-loop amplitude use massification, for e and μ
  - one-loop available with full mass dependence
  - phase-space integration tricky but doable
- massification ok'ish for tagged well-isolated photon
- some subsets (OPE) can be done with full mass dependence, very useful for δ_{th} determination
- ??? full m_μ mass dependence for two-loop ???

What about $e^+ e^- \to \pi^+ \pi^- \ (\gamma)$?

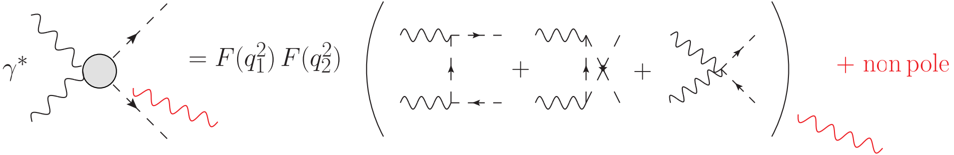
- all this can in principle be done for sQED or even FsQED (scalar QED with form factors)



- but is this useful ??
- $\mathcal{L}_{\text{sQED}} \subset \mathcal{L}_{\chi\text{PT}}\big|_{p^2}$ but nothing beyond (sQED $\to$ FsQED !?! resonances !?!)
- additional (hard) final-state radiation questionable with FsQED
- need dispersive approach for more solid footing

Buliding block $\gamma^*\gamma^* \to \pi^+\pi^-$

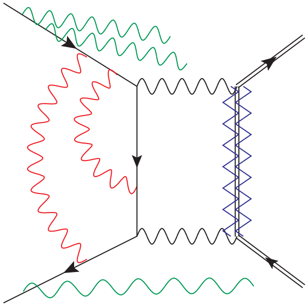
- partial justification of FsQED:



- additional final-state radiation questionable with FsQED
- dispersive approach badly needed !!
- non-pole contributions are also beyond FsQED-LO

sewing together building blocks

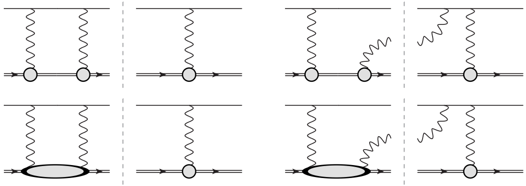
$e^+ e^- \to \ell^+ \ell^-$ and $e^+ e^- \to \pi^+ \pi^-$



cp TPE in $\ell p \to \ell p$

NLO TPE with form factors and resonances

cross check with dispersive approach



importance of beyond FF depends on q^2

what to expect in the next few years

- $e^+ e^- \to \mu^+ \mu^- \gamma$ at NNLO with some approximations $(\alpha/\pi)^2 m_\ell^2/s$
- $e^+ e^- \to \gamma^*$ at NNNLO with some gymnastics for the real-virtual-virtual part

what we hopefully get in the next few years

- further dispersive input for building blocks
- better understanding when FsQED is (how) ok and when it fails completely

what we *will* get in the next few months

- an updated report Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+ e^-$ last call for people to join