The BabaYaga@NLO event generator

F. Piccinini

MPP2023: II Workshop on Muon Precision Physics 2023

Leverhulme Trust, The Spine, Liverpool, 7-10 November 2023

in collaboration with
C.M. Carloni Calame, G. Montagna, O. Nicrosini

Outline

- ⋆ References
- ⋆ QED processes & radiative corrections
- ⋆ The BabaYaga and BabaYaga@NLO event generators
  - theoretical framework
  - improving theoretical accuracy:
    QED Parton Shower and matching with NLO corrections
- ⋆ Results, tuned comparisons, theoretical accuracy
- ⋆ Conclusions

Relevant references

⋆ Webpage

http://www.pv.infn.it/hepcomplex/babayaga.html

(or better ask the authors!)

⋆ **BabaYaga** core references:

- Barzè et al., Eur. Phys. J. C **71** (2011) 1680 — BabaYaga with dark photon
- Balossini et al., Phys. Lett. **663** (2008) 209 — BabaYaga@NLO for $e^+e^- \to \gamma\gamma$
- Balossini et al., Nucl. Phys. **B758** (2006) 227 — BabaYaga@NLO for Bhabha
- C.M. Carloni Calame et al., Nucl. Phys. Proc. Suppl. **131** (2004) 48 — BabaYaga@NLO
- C.M. Carloni Calame, Phys. Lett. B **520** (2001) 16 — improved PS BabaYaga
- C.M. Carloni Calame et al., Nucl. Phys. B **584** (2000) 459 — BabaYaga

⋆ Related work:

- S. Actis et al.
  "Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data", Eur. Phys. J. C **66** (2010) 585
  Report of the Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies
- C.M. Carloni Calame et al., JHEP **1107** (2011) 126
  NNLO massive pair corrections

Reference processes for luminosity

- Instead of getting the luminosity from machine parameters, it's more effective to exploit the relation

$$\sigma = \frac{N}{L} \quad \to \quad L = \frac{N_{\rm ref}}{\sigma_{\rm theory}} \qquad \frac{\delta L}{L} = \frac{\delta N_{\rm ref}}{N_{\rm ref}} \oplus \frac{\delta \sigma_{\rm theory}}{\sigma_{\rm theory}}$$

- Reference (normalization) processes are required to have a clean topology, high statistics and be calculable with high theoretical accuracy

⋆ Large-angle QED processes as $e^+e^- \to e^+e^-$ (Bhabha), $e^+e^- \to \gamma\gamma$, $e^+e^- \to \mu^+\mu^-$ are golden processes at flavour factories to achieve a typical precision at the level of $1 \div 0.1\%$

$\hookrightarrow$ QED radiative corrections are mandatory

$\mapsto$ **BabaYaga** has been developed for high-precision simulation of QED processes at flavour factories (primarily for luminosity determination)

Theory of QED corrections into Monte Carlo generators

- ⋆ Typical QED MC generators include exact $\mathcal{O}(\alpha)$ (NLO) photonic corrections matched with higher-order leading logarithmic contributions [multiple photon corrections]

  [+ vacuum polarization, using a data driven routine for the calculation of the non-perturbative $\Delta\alpha^{(5)}_{\rm had}(q^2)$ hadronic contribution]

- ⋆ Common methods used to account for multiple photon corrections are the analytical collinear QED Structure Functions (SF), YFS exponentiation and QED Parton Shower (PS)

- The QED PS [implemented in BabaYaga/BabaYaga@NLO] is an exact MC solution of the QED DGLAP equation for the non-singlet electron SF $D(x,Q^2)$

$$Q^2 \frac{\partial}{\partial Q^2} D(x,Q^2) = \frac{\alpha}{2\pi} \int_x^1 \frac{dt}{t} P_+(t) D(\frac{x}{t},Q^2)$$

- The PS solution can be cast into the form

$$D(x,Q^2) \; = \; \Pi(Q^2) \sum_{n=0}^{\infty} \; \int \frac{\delta(x-x_1\cdots x_n)}{n!} \; \prod_{i=0}^{n} \left[\frac{\alpha}{2\pi} P(x_i) \; L \; dx_i\right]$$

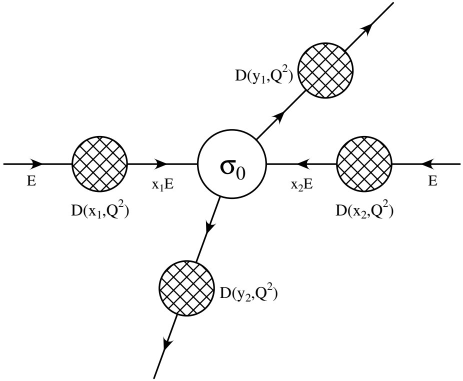
  → $\Pi(Q^2) \equiv e^{-\frac{\alpha}{2\pi} L I_+}$ Sudakov form factor, $I_+ \equiv \int_0^{1-\epsilon} P(x)dx$, $L \equiv \ln Q^2/m^2$ collinear log, ϵ soft–hard separator and Q^2 virtuality scale

  → the kinematics of the photon emissions can be recovered → exclusive photons generation

- The accuracy is improved by matching exact NLO with higher-order leading log corrections
  - ⋆ theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO) QED corrections, for all QED channels [Bhabha, $\gamma\gamma$ and $\mu^+\mu^-$]

The Structure Function (SF) approach

$$\sigma_{corrected} = \int dx_- dx_+ dy_- dy_+ \int d\Omega D^{ISR}(x_-, Q^2) D^{ISR}(x_+, Q^2)$$
$$\times D^{FSR}(y_-, Q^2) D^{FSR}(y_+, Q^2) \frac{d\sigma_0}{d\Omega}\left(x_- x_+ s, \theta\right) \Theta\left(cuts\right)$$



$\mapsto$ The QED PS algorithm numerically gives the $D^{ISR/FSR}(z, Q^2)$'s

Matching NLO and PS in BabaYaga@NLO

Exact $\mathcal{O}(\alpha)$ (NLO) soft+virtual (SV) corrections and hard-bremsstrahlung (H) matrix elements can be combined with QED PS *via* a matching procedure

- $d\sigma^{\infty}_{PS} = \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$
- $d\sigma^{\alpha}_{PS} = [1 + C_{\alpha,PS}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_{1,PS}|^2 d\Phi_3 \equiv d\sigma^{SV}_{PS}(\varepsilon) + d\sigma^{H}_{PS}(\varepsilon)$
- $d\sigma^{\alpha}_{\rm NLO} = [1 + C_{\alpha}]\,|\mathcal{M}_0|^2 d\Phi_2 + |\mathcal{M}_1|^2 d\Phi_3 \equiv d\sigma^{SV}_{\rm NLO}(\varepsilon) + d\sigma^{H}_{\rm NLO}(\varepsilon)$
- $F_{SV} = 1 + (C_\alpha - C_{\alpha,PS}) \qquad F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,PS}|^2}{|\mathcal{M}_{1,PS}|^2}$

$$d\sigma^{\infty}_{\rm matched} = F_{SV}\ \Pi(Q^2,\varepsilon)\ \sum_{n=0}^{\infty} \frac{1}{n!}\ \left(\prod_{i=0}^{n} F_{H,i}\right)\ |\mathcal{M}_{n,PS}|^2\ d\Phi_n$$

$d\Phi_n$ is the exact phase space for n final-state particles

(2 fermions + an arbitrary number of photons)

Any approximation is confined into matrix elements

Matching NLO and PS in BabaYaga@NLO

- F_{SV} and $F_{H,i}$ are infrared/collinear safe and account for missing $\mathcal{O}(\alpha)$ non-logs, avoiding double counting of leading-logs
- $\left[\sigma^{\infty}_{matched}\right]_{\mathcal{O}(\alpha)} = \sigma^{\alpha}_{\mathrm{NLO}}$
- resummation of higher orders LL (PS) contributions is preserved
- the cross section is still fully differential in the momenta of the final state particles (e^+, e^- and $n\gamma$)
  (F's correction factors are applied on an event-by-event basis)
- as a by-product, part of photonic $\alpha^2 L$ included by means of terms of the type $F_{SV\ |\ H,i} \otimes$ [leading-logs]

G. Montagna et al., **PLB** 385 (1996)

- the theoretical error is shifted to $\mathcal{O}(\alpha^2)$ (NNLO, 2 loop) not infrared, singly collinear terms: very naively and roughly (for photonic corrections)

$$\frac{1}{2}\alpha^2 L \equiv \frac{1}{2}\alpha^2 \log\frac{s}{m_e^2} \sim 5 \times 10^{-4}$$

Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\ldots$

Blue: Leading-Log PS, Leading-Log YFS, SF

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

LO	α^0		
NLO	αL	α	
NNLO	$\frac{1}{2}\alpha^2 L^2$	$\frac{1}{2}\alpha^2 L$	$\frac{1}{2}\alpha^2$
h.o.	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^n$	$\sum_{n=3}^{\infty} \frac{\alpha^n}{n!} L^{n-1}$	$\cdots$

Red: matched PS, YFS, SF + NLO

Summary of QED (photonic) radiative corrections

Loosely and schematically, the corrections to the LO cross section can be arranged as (collinear log $L \equiv \log \frac{s}{m_e^2}$)

LO	90%		
NLO	10%	0.5%	
NNLO	0.5%	0.05%	0.01%
h.o.	0.01%	$\cdots$	$\cdots$

Tipically at flavour factories (on integrated Bhabha σ)

- to show the typical size of RC, the following setups and definitions are used (for Bhabha)
  - a) $\sqrt{s} = 1.02 \text{ GeV}, \ E_{min} = 0.408 \text{ GeV}, \ 20^\circ < \theta_\pm < 160^\circ, \ \xi_{max} = 10^\circ$
  - b) $\sqrt{s} = 1.02 \text{ GeV}, \ E_{min} = 0.408 \text{ GeV}, \ 55^\circ < \theta_\pm < 125^\circ, \ \xi_{max} = 10^\circ$
  - c) $\sqrt{s} = 10 \text{ GeV}, \ E_{min} = 4 \text{ GeV}, \ 20^\circ < \theta_\pm < 160^\circ, \ \xi_{max} = 10^\circ$
  - d) $\sqrt{s} = 10 \text{ GeV}, \ E_{min} = 4 \text{ GeV}, \ 55^\circ < \theta_\pm < 125^\circ, \ \xi_{max} = 10^\circ$

$$\delta_{VP} \equiv \frac{\sigma_{0,VP} - \sigma_0}{\sigma_0} \qquad \delta_\alpha \equiv \frac{\sigma_\alpha^{NLO} - \sigma_0}{\sigma_0}$$

$$\delta_{HO} \equiv \frac{\sigma_{matched}^{PS} - \sigma_\alpha^{NLO}}{\sigma_0} \qquad \delta_{HO}^{PS} \equiv \frac{\sigma^{PS} - \sigma_\alpha^{PS}}{\sigma_0}$$

$$\delta_\alpha^{non\text{-}log} \equiv \frac{\sigma_\alpha^{NLO} - \sigma_\alpha^{PS}}{\sigma_0} \qquad \delta_\infty^{non\text{-}log} \equiv \frac{\sigma_{matched}^{PS} - \sigma^{PS}}{\sigma_0}$$

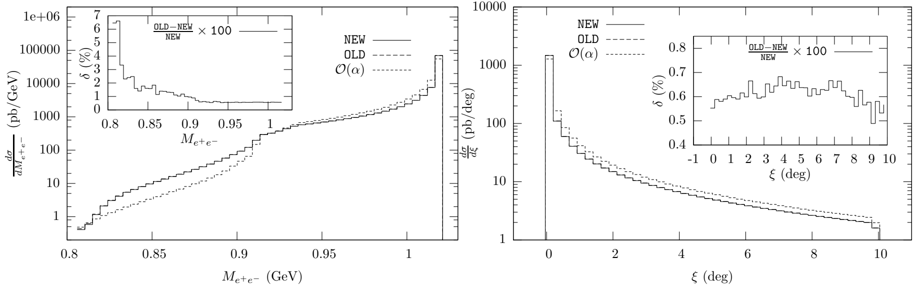
setup	(a)	(b)	(c)	(d)
δ_{VP}	1.76	2.49	4.81	6.41
δ_{α}	-11.61	-14.72	-16.03	-19.57
δ_{HO}	0.39	0.82	0.73	1.44
δ_{HO}^{PS}	0.35	0.74	0.68	1.34
$\delta_{\alpha}^{non\text{-}log}$	-0.34	-0.56	-0.34	-0.56
$\delta_{\infty}^{non\text{-}log}$	-0.30	-0.49	-0.29	-0.46

Table: Relative corrections (in per cent) to the Bhabha cross section for the four setups

⋆ in short, the fact that $\delta_{\alpha}^{non\text{-}log} \simeq \delta_{\infty}^{non\text{-}log}$ and $\delta_{HO} \simeq \delta_{HO}^{PS}$ means that the matching algorithm preserves both the advantages of exact NLO calculation and PS approach:

- ↦ it includes the missing NLO RC to the PS
- ↦ it adds the missing higher-order RC to the NLO

↦ $M_{e^+e^-}$ invariant mass and acollinearity distributions, setup (a)



OLD → pure PS NEW → matched PS with NLO $\mathcal{O}(\alpha)$ → exact NLO

Luminosity/QED generators

Luminosity measured with $0.1 \div 1\%$ precision using **large–angle Bhabha** (and $e^+e^- \to \gamma\gamma$) as reference process, **simulated with two independent generators**

$$\mathcal{L} = \frac{N_{\rm obs}}{\sigma_{\rm theory}}$$

Generator	Processes	Theory	Accuracy
BabaYaga 3.5	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	QED Parton Shower	$\sim 0.5\%$
BabaYaga@NLO	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + QED PS	$\sim 0.1\%$
BHWIDE	e^+e^-	$\mathcal{O}(\alpha)$ YFS	$\sim 0.1\%$
MCGPJ	$e^+e^-, \gamma\gamma, \mu^+\mu^-$	$\mathcal{O}(\alpha)$ + coll. SF	$\sim 0.2\%$
KKMC	$\mu^+\mu^-, \tau^+\tau^-, \ldots$	$\mathcal{O}(\alpha)$ CEEX	$\sim 0.1\%$

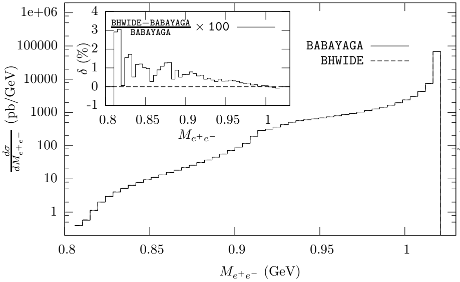
- BabaYaga 3.5/BabaYaga@NLO `http://www2.pv.infn.it/~hepcomplex/babayaga.html`
  Used by BaBar, Belle, BESIII, CLEO, KEDR and KLOE. Carloni Calame *et al.*, 2000 / 2006, 2008
- BHWIDE `http://placzek.web.cern.ch/placzek/bhwide/`
  Used by BaBar, BESIII, KEDR, KLOE and SND. Jadach, Placzek and Ward, 1997
- MCGPJ `http://cmd.inp.nsk.su/~sibid/`
  Used by CMD, Belle and SND. Arbuzov *et al.*, 2005 / Eidelman *et al.*, 2011
- KKMC `http://jadach.web.cern.ch/jadach/KKindex.html`
  Used by BaBar, Belle and BESIII (τ physics, ISR and NP studies). Jadach *et al.*, 2000

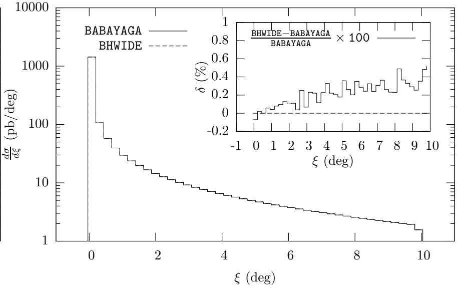
from G. Montagna's talk at FCCP2015

S. Actis et al. Eur. Phys. J. C **66** (2010) 585

"Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data"

- **It is extremely important to compare independent calculations/implementations/codes, in order to**
  - ↦ asses the technical precision, spot bugs (with the same th. ingredients)
  - ↦ estimate the theoretical error when including partial/incomplete higher-order corrections
- E.g. comparison `BabaYaga@NLO` vs. `Bhwide` at KLOE





Without vacuum polarization, to compare QED corrections consistently

At the Φ and τ–charm factories (cross sections in nb)

By BabaYaga group, Ping Wang and A. Sibidanov

setup	BabaYaga@NLO	BHWIDE	MCGPJ	$\delta(\%)$
$\sqrt{s} = 1.02$ GeV, $20^\circ \leq \vartheta_\mp \leq 160^\circ$	6086.6(1)	6086.3(2)	—	0.005
$\sqrt{s} = 1.02$ GeV, $55^\circ \leq \vartheta_\mp \leq 125^\circ$	455.85(1)	455.73(1)	—	0.030
$\sqrt{s} = 3.5$ GeV, $\vert\vartheta_+ + \vartheta_- - \pi\vert \leq 0.25$ rad	35.20(2)	—	35.181(5)	0.050

$\mapsto$ Agreement well below 0.1%

At BaBar (cross sections in nb)

By A. Hafner and A. Denig

angular acceptance cuts	BabaYaga@NLO	BHWIDE	$\delta(\%)$
$15^\circ \div 165^\circ$	119.5(1)	119.53(8)	0.025
$40^\circ \div 140^\circ$	11.67(3)	11.660(8)	0.086
$50^\circ \div 130^\circ$	6.31(3)	6.289(4)	0.332
$60^\circ \div 120^\circ$	3.554(6)	3.549(3)	0.141

$\mapsto$ Agreement at the $\sim 0.1\%$ level

Theoretical accuracy, comparisons with NNLO

- NLO RC being included, the theoretical error starts at $\mathcal{O}(\alpha^2)$ (NNLO)

  $\hookrightarrow$ anyway large NNLO RC already included by exponentiation
  (and by $\mathcal{O}(\alpha)$ PS $\times$ non-log-NLO)

⋆ The full set of NNLO QED corrections to Bhabha scattering are known since ~10 years

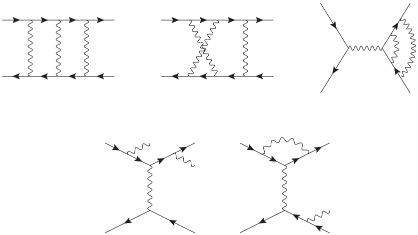
- `BabaYaga@NLO` formulae can be truncated at $\mathcal{O}(\alpha^2)$ to be consistently and systematically compared with all the classes of NNLO corrections

$$\sigma^{\alpha^2} = \sigma^{\alpha^2}_{\rm SV} + \sigma^{\alpha^2}_{\rm SV,H} + \sigma^{\alpha^2}_{\rm HH}$$

  - $\sigma^{\alpha^2}_{\rm SV}$: soft+virtual photonic corrections up to $\mathcal{O}(\alpha^2)$
    $\mapsto$ compared with the corresponding available NNLO QED calculation
  - $\sigma^{\alpha^2}_{\rm SV,H}$: one–loop soft+virtual corrections to single hard bremsstrahlung
    $\mapsto$ estimated relying on existing (partial) results
  - $\sigma^{\alpha^2}_{\rm HH}$: double hard bremsstrahlung
    $\mapsto$ compared with the exact $e^+e^- \to e^+e^-\gamma\gamma$ cross section, to register really negligible differences (at the 1×10^{-5} level)

NNLO Bhabha calculations

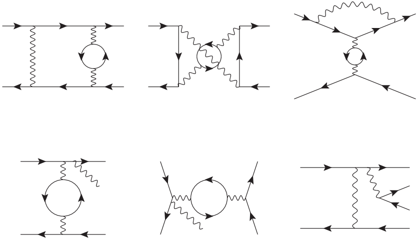
- Photonic corrections A. Penin, PRL **95** (2005) 010408 & Nucl. Phys. **B734** (2006) 185



here real γ is "soft"

- Electron loop corrections

R. Bonciani *et al.*, Nucl. Phys. **B701** (2004) 121 & Nucl. Phys. **B716** (2005) 280
S. Actis, M. Czakon, J. Gluza and T. Riemann, Nucl. Phys. **B786** (2007) 26



here real γ is "soft"

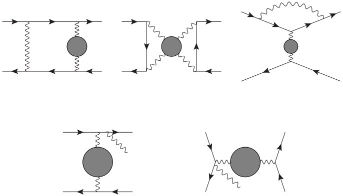
NNLO Bhabha calculations

- Heavy fermion and hadronic loops

R. Bonciani, A. Ferroglia and A. Penin, PRL **100** (2008) 131601

S. Actis, M. Czakon, J. Gluza and T. Riemann, PRL **100** (2008) 131602

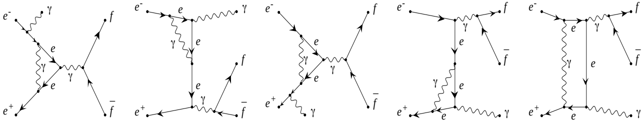
J.H. Kühn and S. Uccirati, Nucl. Phys. **B806** (2009) 300



here real γ is "soft"

- One-loop soft+virtual corrections to single hard bremsstrahlung

S. Actis, P. Mastrolia and G. Ossola, Phys. Lett. **B682** (2010) 419



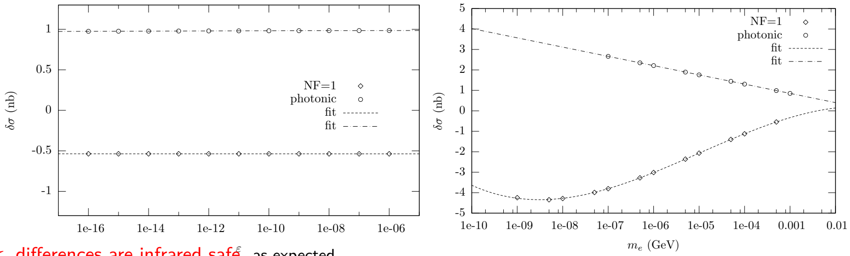
here real γ is "hard"

Comparison with NNLO calculation for $\sigma_{SV}^{\alpha^2}$

Using realistic cuts for luminosity at KLOE

Comparison of $\sigma_{SV}^{\alpha^2}$ calculation of `BabaYaga@NLO` with

- Penin (photonic): function of the logarithm of the soft photon cut–off (left plot) and a fictitious electron mass (right plot)



★ differences are infrared safe$^{\varepsilon}$, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{SV}^{\alpha^2}(\text{Penin}) - \sigma_{SV}^{\alpha^2}(\texttt{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

Leptonic and hadronic loops & pairs at NNLO

- The exact NNLO virtual corrections + $2 \to 4$ matrix elements $e^+e^- \to e^+e^-(l^+l^-)$ $[l = e, \mu, \tau]$, $e^+e^- \to e^+e^-(\pi^+\pi^-)$ are available
- Compared to the *approximation* in **BabaYaga@NLO**, using realistic luminosity cuts ($S_i \equiv \sigma_i^{\mathrm{NNLO}}/\sigma_{BY}$)

	$\sqrt{s}$		σ_{BY}	$S_{e^+e^-}$ [‰]	S_{lep} [‰]	S_{had} [‰]	S_{tot} [‰]
KLOE	1.020	NNLO		-3.935(4)	-4.472(4)	1.02(2)	-3.45(2)
		BB@NLO	455.71	-3.445(2)	-4.001(2)	0.876(5)	-3.126(5)
BES	3.650	NNLO		-1.469(9)	-1.913(9)	-1.3(1)	-3.2(1)
		BB@NLO	116.41	-1.521(4)	-1.971(4)	-1.071(4)	-3.042(5)
BaBar	10.56	NNLO		-1.48(2)	-2.17(2)	-1.69(8)	-3.86(8)
		BB@NLO	5.195	-1.40(1)	-2.09(1)	-1.49(1)	-3.58(2)
Belle	10.58	NNLO		-4.93(2)	-6.84(2)	-4.1(1)	-10.9(1)
		BB@NLO	5.501	-4.42(1)	-6.38(1)	-3.86(1)	-10.24(2)

⋆ The uncertainty due to leptonic and hadronic pair NNLO corrections is at the level of a few units in 10^{-4}

Carloni, Czyż, Gluza, Gunia, Montagna, Nicrosini, Piccinini, Riemann *et al.*, JHEP **1107** (2011) 126

Conclusions

- ⋆ In the last ~20 years BabaYaga/BabaYaga@NLO has been developed for high-precision luminometry at flavour factories
- ⋆ It simulates QED processes
  $\hookrightarrow$ $e^+e^- \to e^+e^-$ $(+n\gamma)$
  $\hookrightarrow$ $e^+e^- \to \mu^+\mu^-$ $(+n\gamma)$
  $\hookrightarrow$ $e^+e^- \to \gamma\gamma$ $(+n\gamma)$
  with multiple-photon emission in a QED Parton Shower framework, matched with exact NLO matrix elements
- ⋆ A theoretical precision at the 0.5×10^{-3} level is achieved (at least for Bhabha), with a systematic comparison to independent calculations/codes and assessing the size of missing higher-order corrections

Conclusions

- ⋆ In the last ~20 years BabaYaga/BabaYaga@NLO has been developed for high-precision luminometry at flavour factories
- ⋆ It simulates QED processes
  - ↪ $e^+e^- \to e^+e^- \quad (+n\gamma)$
  - ↪ $e^+e^- \to \mu^+\mu^- \quad (+n\gamma)$
  - ↪ $e^+e^- \to \gamma\gamma \quad (+n\gamma)$

  with multiple-photon emission in a QED Parton Shower framework, matched with exact NLO matrix elements
- ⋆ A theoretical precision at the 0.5×10^{-3} level is achieved (at least for Bhabha), with a systematic comparison to independent calculations/codes and assessing the size of missing higher-order corrections
- ⋆ Looking ahead: future improvements
  - ⋆ inclusion of all finite mass terms in the matrix elements
  - ⋆ addition of pion final states $\pi^+\pi^-(+n\gamma)$ and $\pi^+\pi^-\gamma(+n\gamma)$ (together with $\mu^+\mu^-\gamma(+n\gamma)$)
  - ⋆ going beyond NLOPS
  - ⋆ inclusion of weak corrections (for higher energies)