Hadronic Vacuum Polarisation measurements at Belle II

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On behalf of the Belle II Collaboration

Outline

- Introduction
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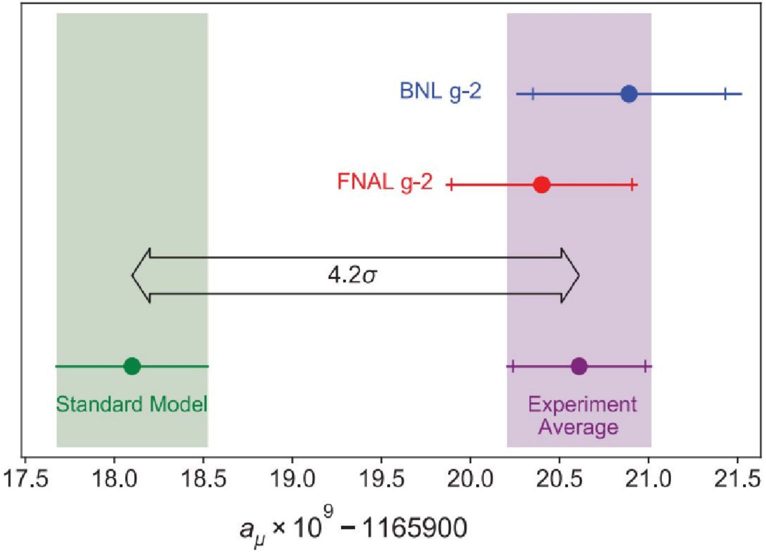
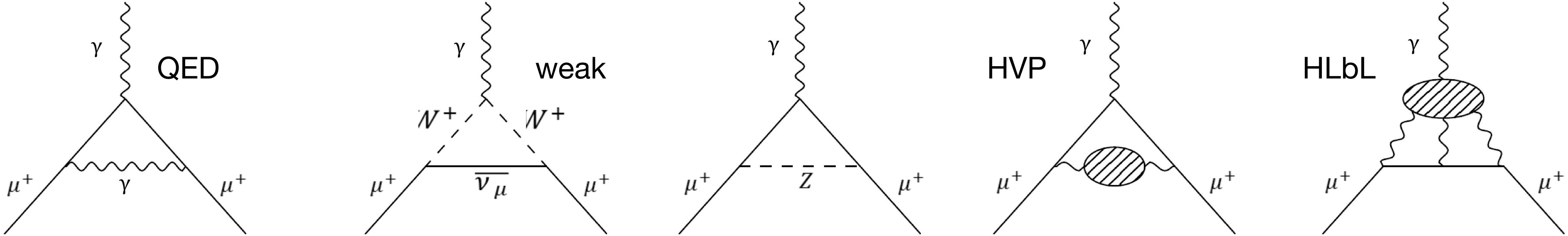
- Radiative return method
- HVP measurements
  - $e^+e^- \to \pi^+\pi^-$
  - $e^+e^- \to \pi^+\pi^-\pi^0$
    - Background rejection
    - Corrections
    - a_μ extraction
    - Systematics

Muon g - 2



$$a_\mu^{\mathrm{SM}} = \frac{g-2}{2} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$

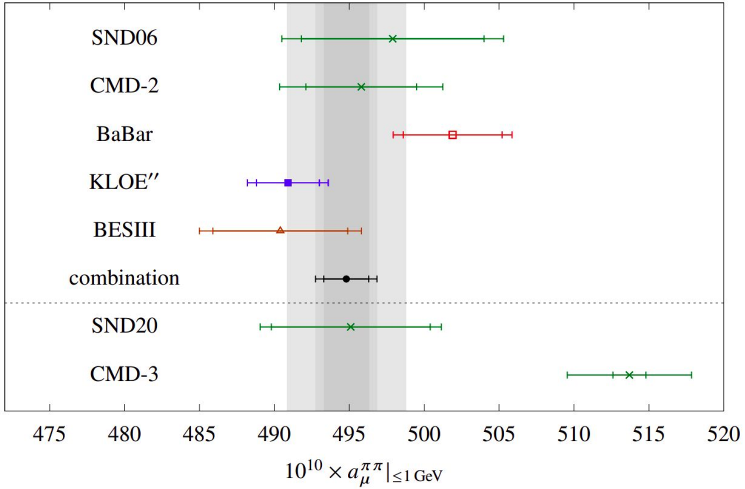


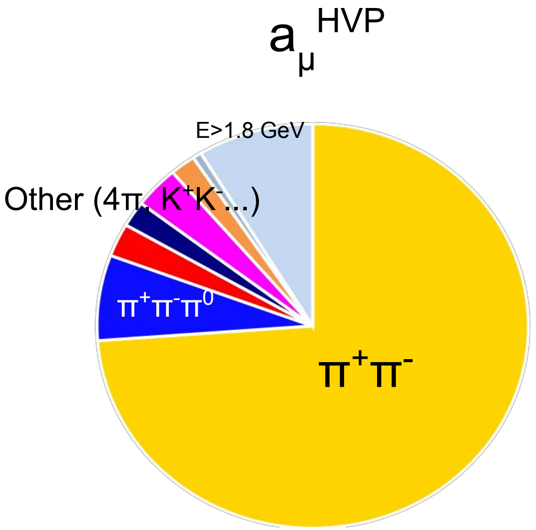
Introduction

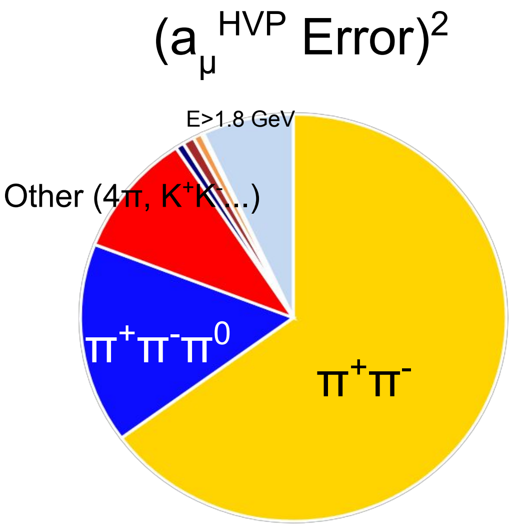
$$a_\mu^{\text{HVP, LO}} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)}{s}R(s)ds$$

- HVP: **largest uncertainty** for a_μ
- **Belle II**
  - Measurement of various **$e^+e^- \to$hadrons cross-sections**

$$R(s) = \frac{\sigma_0(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

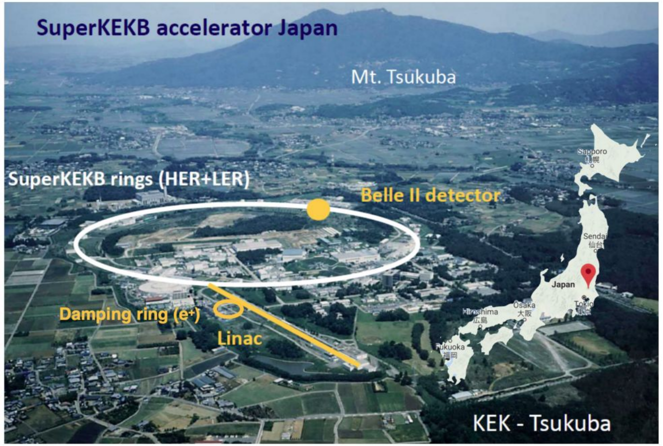


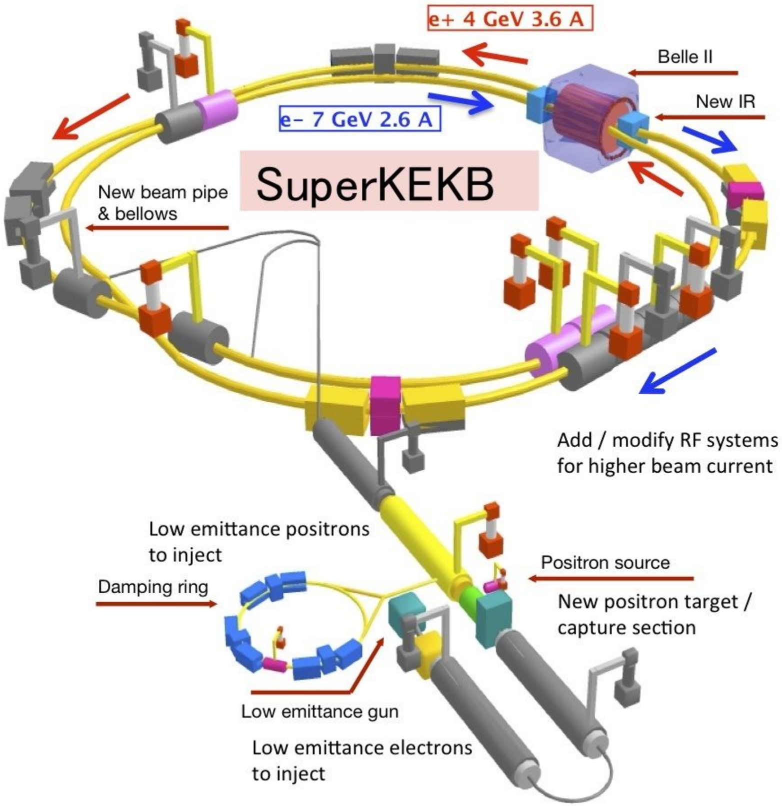




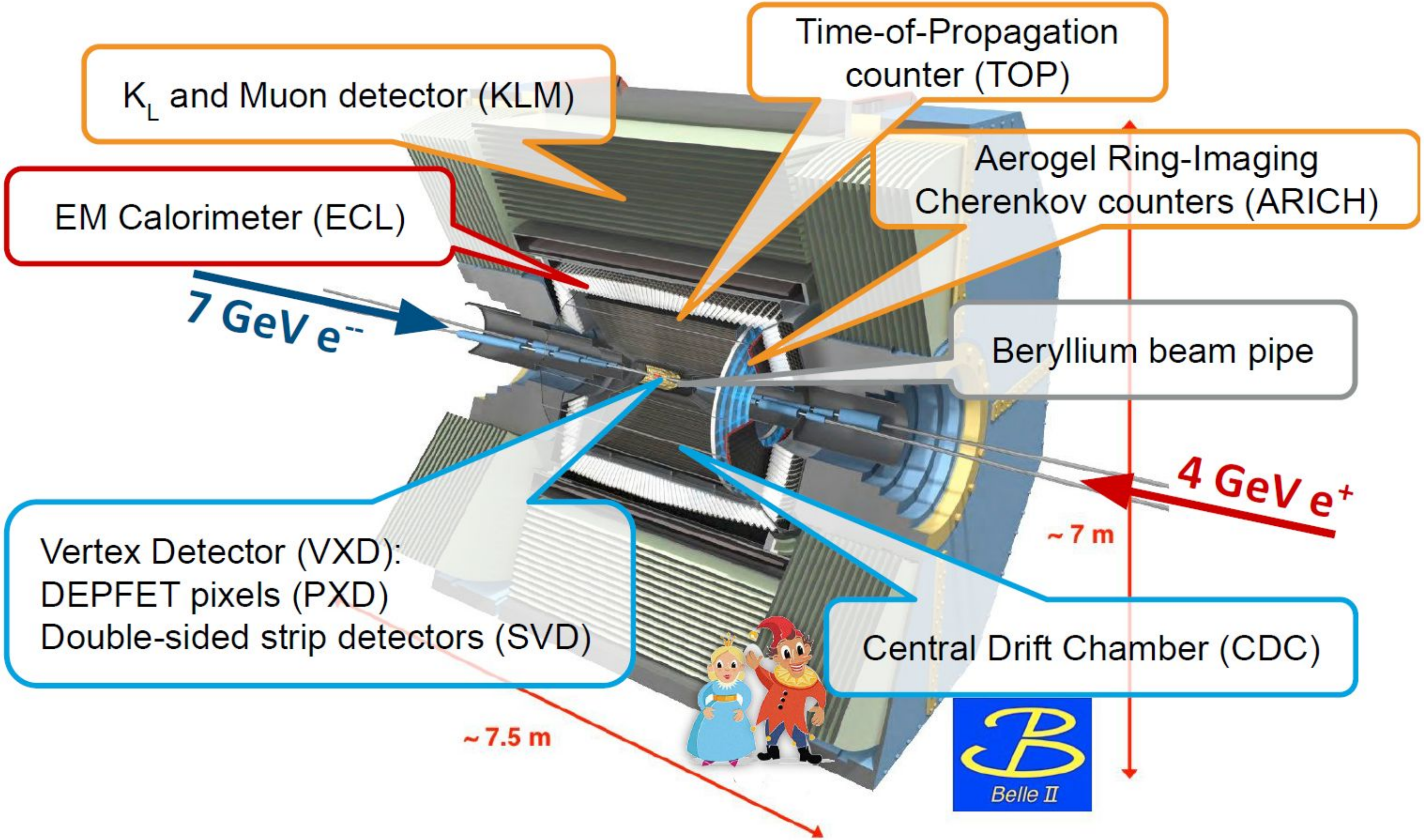
SuperKEKB

- Energy asymmetric e^+e^- collider at c.m energy = 10.58 GeV/c^2
  - Clean environment to study $e^+e^- \rightarrow q\bar{q}$
- World record luminosity: $4.7 \cdot 10^{34}$ cm^{-2} s^{-1}
- 424 fb^{-1} collected so far
- Currently in Long Shutdown 1





Belle II



K_L and Muon detector (KLM)

EM Calorimeter (ECL)

Time-of-Propagation counter (TOP)

Aerogel Ring-Imaging Cherenkov counters (ARICH)

Beryllium beam pipe

Vertex Detector (VXD):
DEPFET pixels (PXD)
Double-sided strip detectors (SVD)

Central Drift Chamber (CDC)

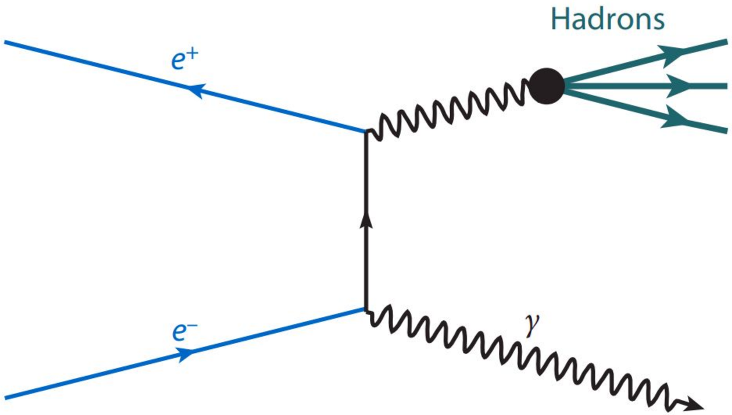
PXD upgrade coming after LS1

Radiative return method

- Use **ISR** photon to effectively **scan the c.m. energy**
    - Scan hadronic system invariant mass → **energy dependent cross-section** measurement

$$\frac{dN_{\mathrm{signal}}}{dM} \propto \sigma^0_{e^+e^-\to\mathrm{hadrons}}(\sqrt{s'})$$

- Used in the past in BaBar, KLOE, BESIII



Measurements

- **New low-multiplicity triggers**
    - Two independent triggers: tracker and calorimeter
    - ISR processes separated from other radiative effects
    - → **~100% efficiency** for energetic ISR photons (> 2 GeV/c^2, $20^o < \theta < 160^o$)

- Two channels currently under study
    - $e^+e^- \rightarrow \pi^+\pi^-$
    - $e^+e^- \rightarrow \pi^+\pi^-\pi^0$

$e^+e^- \rightarrow \pi^+\pi^-$ at Belle II

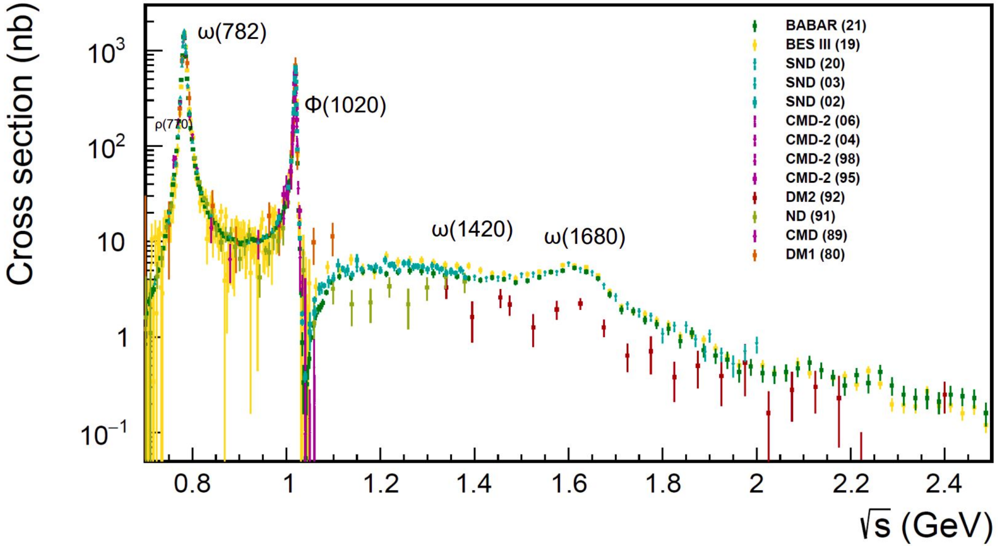
Overview

- The largest contribution to a_μ^{HVP} ~ 73 %
- Target 0.5% precision using 363 fb^{-1} data
- Following BaBar method as a baseline: Phys. Rev. D 104, 112003 (2021)
    - Based on measurement of ππ/μμ ratio with ISR
- Systematics dominated measurement
- Implementation of kinematic fitting tools
    - Useful for reduction of background and correction of tracking efficiency
- MC-level study ongoing
- Design of data-driven efficiency corrections for tracking, trigger and π/μ/K ID ongoing

$e^+e^- \rightarrow \pi^+\pi^-\pi^0$

Overview

- Latest measurement from BaBar: Phys. Rev. D 104, 112003 (2021)
- In Belle II
    - $\pi^+\pi^-\pi^0$ mass range : [0, 1.05], [1.05, 2.0], [2.0, 3.5] GeV/c^2
    - First region contains ω and φ resonances
    - Measurement of $a_\mu(3\pi)$ in range [0.62, 2.0] GeV/c^2
    - Dataset : 190 fb^{-1}

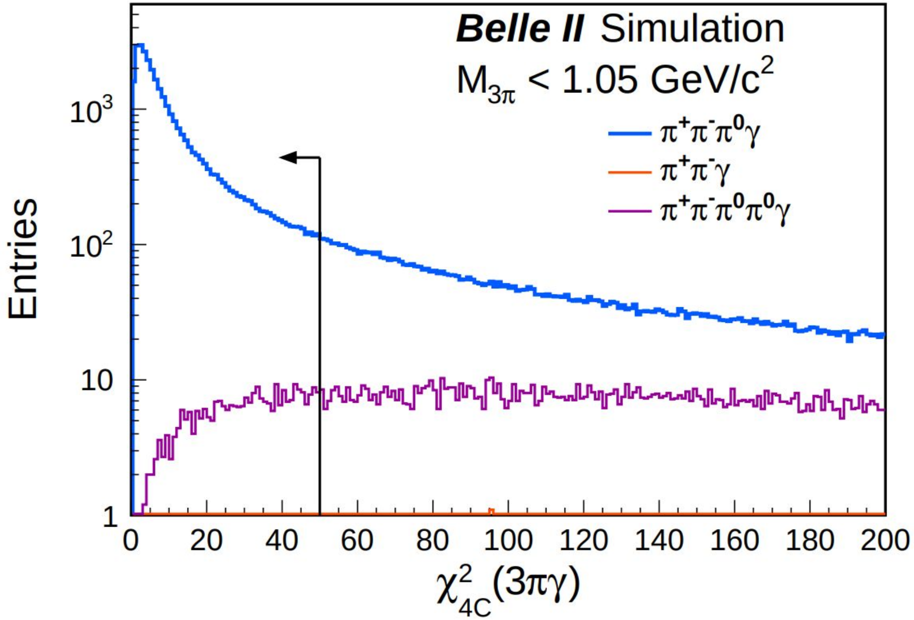


Event selection

- **Two tracks + three photons** : $e^+e^- \to \pi^+ \pi^- \pi^0 (\to \gamma\gamma)\ \gamma_{ISR}$
  - **Tracks** : ensure that track originates close to the interaction point
  - **Photons** : at least one photon must be energetic ISR ($E_{CMS} > 2$ GeV/c^2 in barrel ECL)

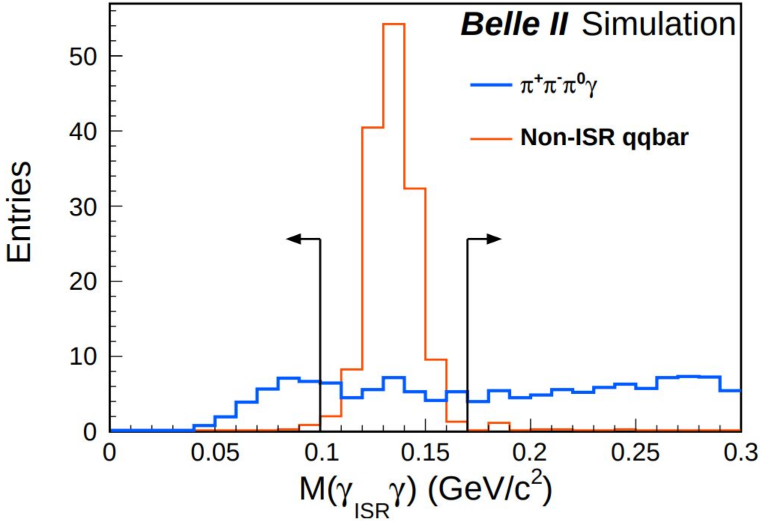
- **π^0 reconstruction**
  - Invariant mass of **two photons** within **0.123 - 0.147 GeV/c^2**

- Select events using **χ^2 kinematic fit** with **4-momentum conservation constraint** under $e^+e^- \to \pi^+ \pi^- \pi^0 \gamma$ hypothesis



Event selection

- **Cuts to reduce remaining backgrounds**
    - Background containing **no real π^0**: $e^+e^- \rightarrow e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$
    - **Charged kaon** : $e^+e^- \rightarrow K^+K^-\pi^0\gamma$
    - **$e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0\gamma$**
    - Background containing **no real ISR photon**: non-ISR $q\bar{q}$ and $\tau^+\tau^-$
        - High-momentum $\rho \rightarrow \pi^+\pi^0$
        - ISR candidate from π^0 decay
        - ISR-like photon from merged π^0 photon pair



Background corrections

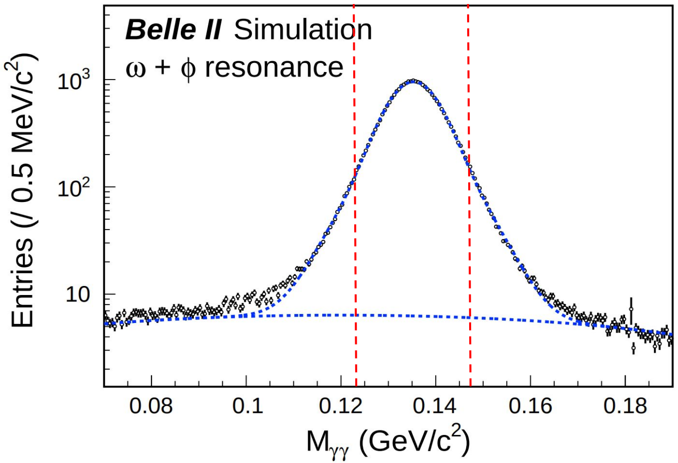
- **Control samples**: invert background rejection cuts
- **Data-driven corrections to M(3π)**

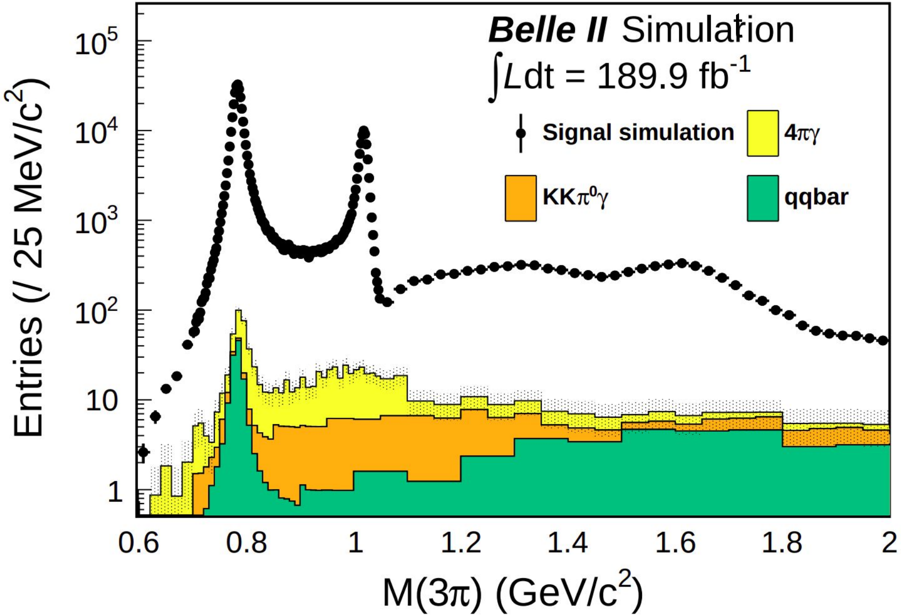
- **Final State Radiation**
  - Photon emitted from **final state charged pions**: **negligible** contribution
  - Photon **emitted from quarks**
    - Dominated by production of C = +1 resonances decaying to $\pi^+ \pi^- \pi^0$: ηɣ, a_1(1260)ɣ, a_2(1320)ɣ…
    - Contribution estimated from perturbative QCD and sum of Breit-Wigner widths
    - Large uncertainties but overall small contribution

Signal extraction

- **$M(\pi^+\pi^-\pi^0)$** is split in ~200 bins in [0, 3.5] GeV/c^2 range
- **Signal is extracted from M(ɣɣ) distribution** in each M(3π) bin
  - Integration in range (123, 147) MeV/c^2 → **number of events in M(3π) bin**
  - Combinatorial background is rejected
- **Obtain M(3π) distribution**

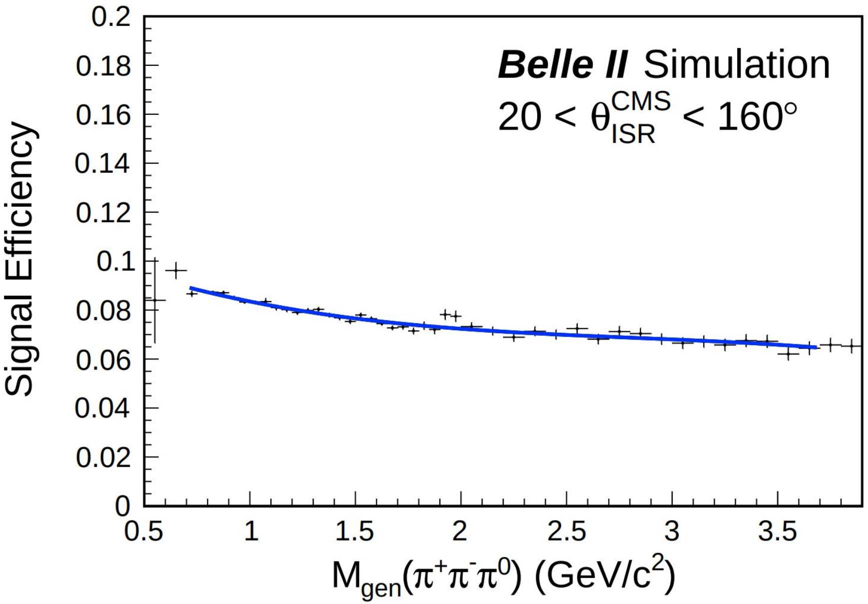
M(3π) < 1.05 GeV/c^2





Efficiency corrections

- **Detection efficiency estimated using MC**
  - selected / generated # of events
- **Data-MC corrections** applied
  - Trigger efficiency
  - High energy photon detection efficiency
  - Tracking efficiency
  - π^0 efficiency
  - χ^2 selection
  - Background reduction cut efficiency
- **Systematic uncertainties** derived together with corrections

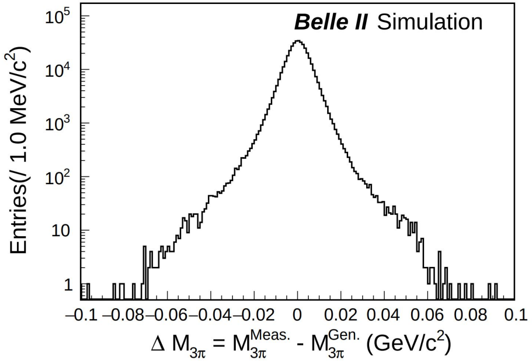


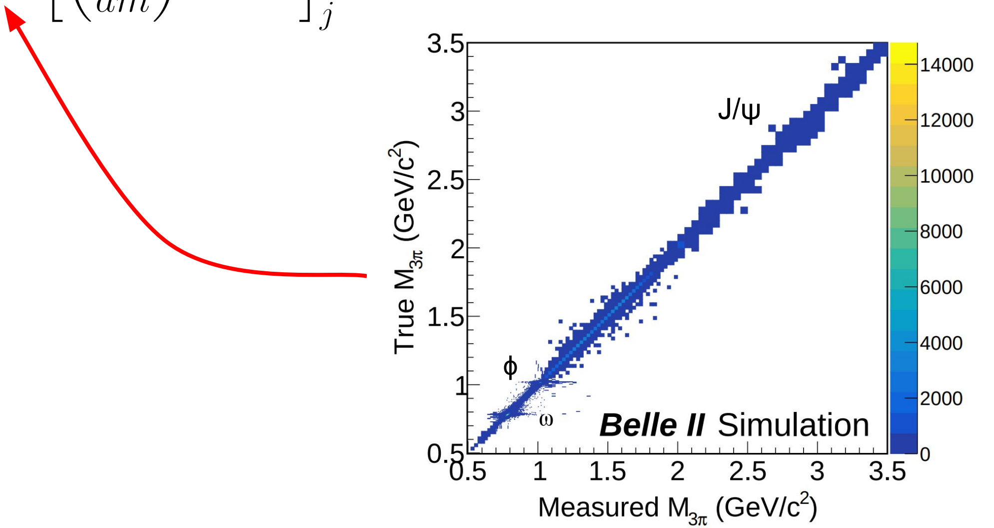
Unfolding

- Unfolding is applied to background subtracted spectrum
  - **Mitigate detector response** and FSR
  - Kinematic fit constrained by beam 4-momentum and π^0 mass
- **Gaussian convolution fit** to ω, ϕ and J/ψ resonances

$$\left(\frac{dN}{dm}\right)_i^{\text{meas.}} = \sum_j A_{ij}\left[\left(\frac{dN}{dm}\right)^{\text{gen.}} * G\right]_j$$

Typically good resolution ~ 7-10 MeV





Cross-section and a_μ

As a function of the invariant mass of the hadronic system

Cross-section without ISR

$$\sigma_{e^+e^-\to 3\pi}(\sqrt{s'}) = \frac{(dN_{\mathrm{signal}}/d\sqrt{s'})_{\mathrm{unfolded}}}{\varepsilon \cdot (d\mathcal{L}_{\mathrm{eff}}/d\sqrt{s'}) \cdot R \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)}$$

Cross-section and a_μ

As a function of the invariant mass of the hadronic system

Signal distribution

Cross-section without ISR

$$\sigma_{e^+e^-\to 3\pi}(\sqrt{s'}) = \frac{(dN_{\text{signal}}/d\sqrt{s'})_{\text{unfolded}}}{\varepsilon \cdot (d\mathcal{L}_{\text{eff}}/d\sqrt{s'}) \cdot R \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)}$$

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Detection efficiency

Cross-section and a_μ

As a function of the invariant mass of the hadronic system

Signal distribution

Cross-section without ISR

$$\sigma_{e^+e^-\to 3\pi}(\sqrt{s'}) = \frac{(dN_{\text{signal}}/d\sqrt{s'})_{\text{unfolded}}}{\varepsilon \cdot (d\mathcal{L}_{\text{eff}}/d\sqrt{s'}) \cdot R \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)}$$

Detection efficiency

Effective ISR luminosity

Cross-section and a_μ

As a function of the invariant mass of the hadronic system

Cross-section without ISR

$$\sigma_{e^+e^-\to 3\pi}(\sqrt{s'}) = \frac{(dN_{\text{signal}}/d\sqrt{s'})_{\text{unfolded}}}{\varepsilon \cdot (d\mathcal{L}_{\text{eff}}/d\sqrt{s'}) \cdot R \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)}$$

Signal distribution

Detection efficiency

Effective ISR luminosity

Radiative correction factor (multiple ISR)

Cross-section and a_μ

As a function of the invariant mass of the hadronic system

Signal distribution

Cross-section without ISR

$$\sigma_{e^+e^-\to 3\pi}(\sqrt{s'}) = \frac{(dN_{\mathrm{signal}}/d\sqrt{s'})_{\mathrm{unfolded}}}{\varepsilon \cdot (d\mathcal{L}_{\mathrm{eff}}/d\sqrt{s'}) \cdot R \cdot \mathcal{B}(\pi^0 \to \gamma\gamma)}$$

Detection efficiency

Effective ISR luminosity

Radiative correction factor (multiple ISR)

Bare cross-section without vacuum polarisation

$$\sigma_0 = \sigma_{\mathrm{dressed}} \frac{1}{|1+\Pi(s)|^2} = \sigma_{\mathrm{dressed}} |1-P(s)|^2$$

$$a_\mu(3\pi) = \frac{\alpha^2}{3\pi^2} \int_{m_{\pi^2}}^{\infty} \frac{K(s)}{s} R(s)ds$$

Systematics

- **Result** in range: **[0.62, 2.0] GeV/c^2**
- Major systematic uncertainties
  - **π^0** and **tracking efficiencies**
  - In M(3π) > 1.05 GeV/c^2, the uncertainty of **selection efficiency** is dominant
- For $a_\mu(3\pi)$, the total uncertainty is expected to be **~2%** including stat. uncertainty of **0.5%**
- The results will be released within a few months

Source	Systematic uncertainty (%)	
	M < 1.05 GeV/c^2	M > 1.05 GeV/c^2
Trigger	0.2	0.2
ISR photon detection	0.7	0.7
Tracking	0.8	0.8
π^0 reconstruction	1.0	1.0
χ^2 distribution	0.3	0.3
Selection	0.2	1.9*
Integrated luminosity	0.7	0.7
Radiative correction	0.5	0.5
Total systematics	1.8	2.6
Total statistics	0.5	
Total	1.9	

*Statistical error dominant

Summary

- Belle II has collected **424 fb^{-1} of data**
  - **Data taking restarting in February 2024** after Long Shutdown 1

- **Two active HVP measurements**
  - **$e^+e^- \rightarrow \pi^+\pi^-$**: target 0.5% precision
  - **$e^+e^- \rightarrow \pi^+\pi^-\pi^0$**: ~2% precision with 190 fb^{-1} of data

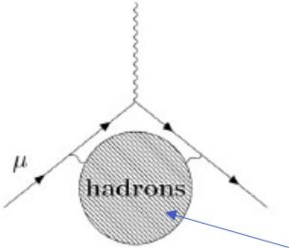
- Expect **systematic uncertainties** to **decrease** as we understand our detector better and analyses are refined

THANK YOU FOR YOUR ATTENTION

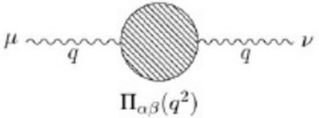
BACKUP

Dispersive method

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

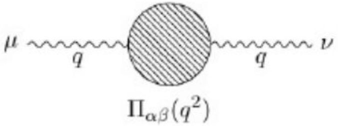


1) Feynman rules for HVP insertion to photon propagator:



$$= \frac{-ig^{\mu\alpha}}{(q^2-i\varepsilon)}(-ie)i\Pi_{\alpha\beta}(q^2)(-ie)\frac{-ig^{\beta\nu}}{(q^2-i\varepsilon)}$$

2) Employ analyticity:



$$= \frac{ie^2 g_{\mu\nu}}{(q^2-i\varepsilon)^2}\frac{q^4}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}s\,\frac{\mathrm{Im}\,\Pi(s)}{s(s-q^2-i\varepsilon)}$$

3) Insert to vertex correction, solve for a_μ: $a_\mu^{\text{had, LO VP}} = \frac{\alpha}{\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\,\mathrm{Im}\,\Pi_{\text{had}}(s)K(s)$

4) Utilise optical theorem:

5) Arrive at equation for $a_\mu^{\text{had, LO VP}}$:

$$a_\mu^{\text{had, LO VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} \underline{\mathrm{d}s\,\sigma^0_{\text{had},\gamma}(s)}\,\underline{K(s)}$$

$\sigma^0_{\text{had},\gamma}$ = bare cross section, FSR included

Any and all permitted hadrons

Strongly weighted at low-energy (non-perturbative regime)

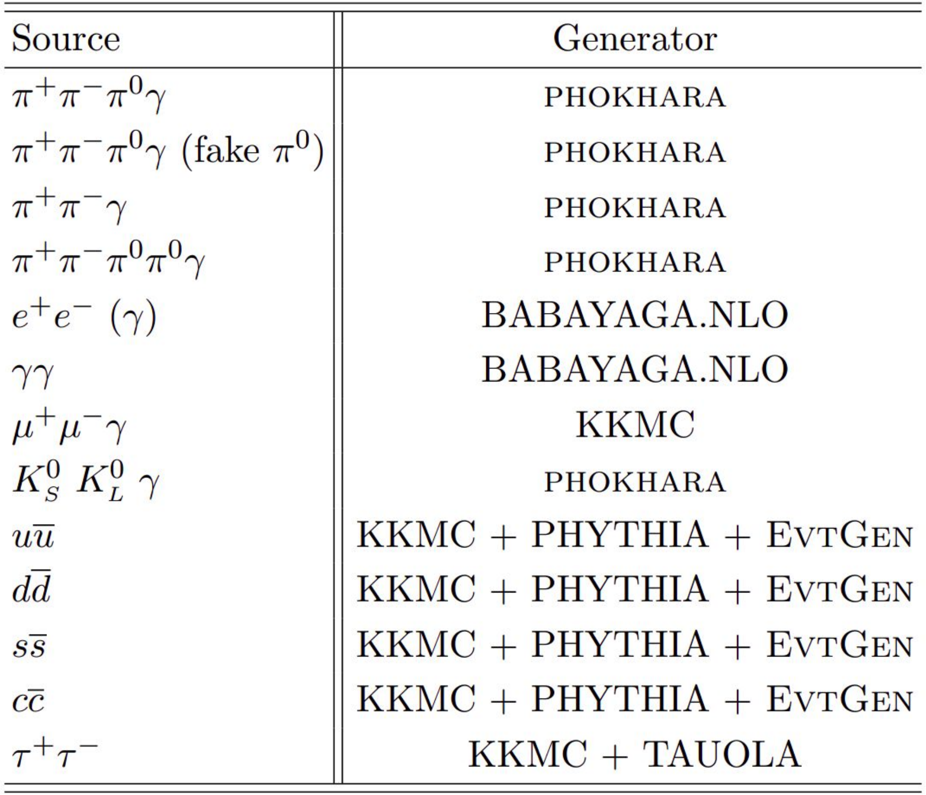
⇒ Similar dispersion integrals for NLO and NNLO HVP

Kernel function

$$K(s) = \frac{x^2}{2}(2-x^2) + \frac{(1+x^2)(1+x)^2}{x^2}\left(\log(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x}x^2 \log x$$

$$x = \frac{1-\beta_\mu}{1+\beta_\mu} \qquad \beta_\mu = \sqrt{1 - \frac{4m_\mu^2}{s}}$$

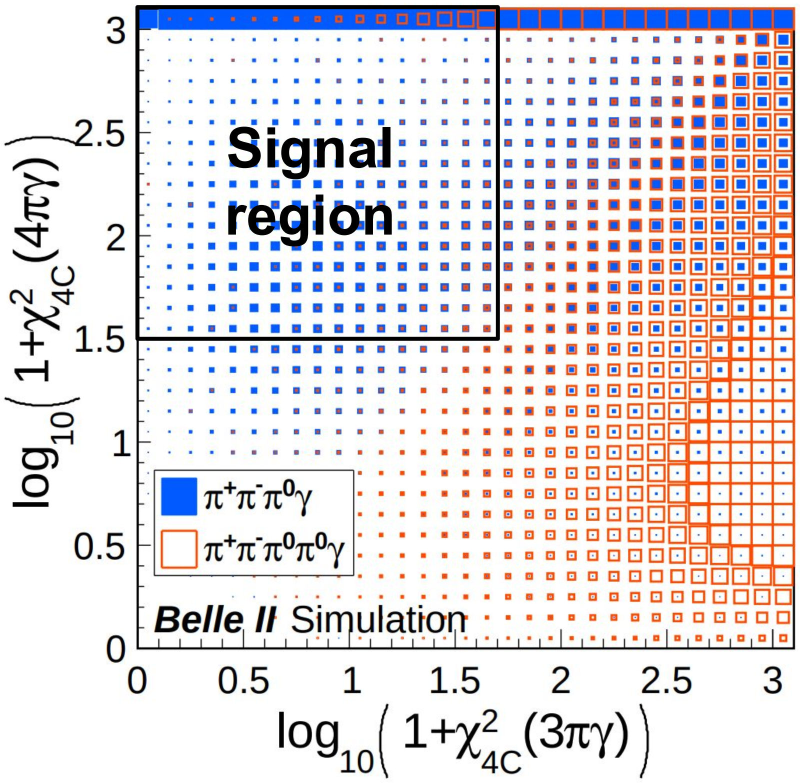
Monte Carlo generators



Source	Generator
$\pi^+\pi^-\pi^0\gamma$	PHOKHARA
$\pi^+\pi^-\pi^0\gamma$ (fake π^0)	PHOKHARA
$\pi^+\pi^-\gamma$	PHOKHARA
$\pi^+\pi^-\pi^0\pi^0\gamma$	PHOKHARA
$e^+e^-\ (\gamma)$	BABAYAGA.NLO
$\gamma\gamma$	BABAYAGA.NLO
$\mu^+\mu^-\gamma$	KKMC
$K_S^0\ K_L^0\ \gamma$	PHOKHARA
$u\bar{u}$	KKMC + PHYTHIA + EvtGen
$d\bar{d}$	KKMC + PHYTHIA + EvtGen
$s\bar{s}$	KKMC + PHYTHIA + EvtGen
$c\bar{c}$	KKMC + PHYTHIA + EvtGen
$\tau^+\tau^-$	KKMC + TAUOLA

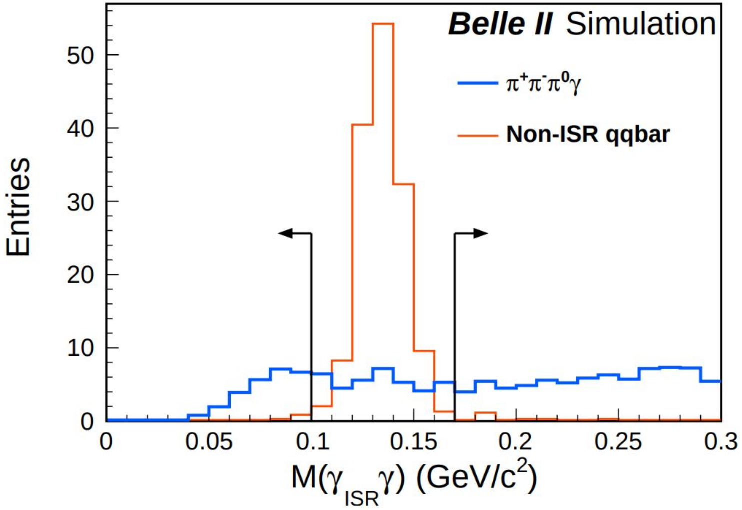
Event selection

- **Cuts to reduce remaining backgrounds**
  - Background containing **no real π^0**:
    $e^+e^- \to e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$
    - Pion/Electron ID: Likelihood $\pi/e > 0.1$
    - $M^2_{\text{recoil}}(\pi^+\pi^-) > 4\ \text{GeV}^2$
  - **Charged kaon** : $e^+e^- \to K^+K^-\pi^0\gamma$
    - Pion/Kaon ID: Likelihood $\pi/K > 0.1$
  - **$e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$**
    - Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$
    - **Four-momentum kinematic fit**
      under $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$ hypothesis



Event selection

- **Cuts to reduce remaining backgrounds**
  - Background containing **no real ISR photon**: non-ISR $q\bar{q}$ and $\tau^+\tau^-$
    - High-momentum $\rho \to \pi^+ \pi^0$: $M(\pi^+\gamma_{ISR}) > 2$ GeV
    - ISR candidate from π^0 decay: $M(\gamma_{ISR}\gamma)$ cut
    - ISR-like photon from merged π^0 photon pair

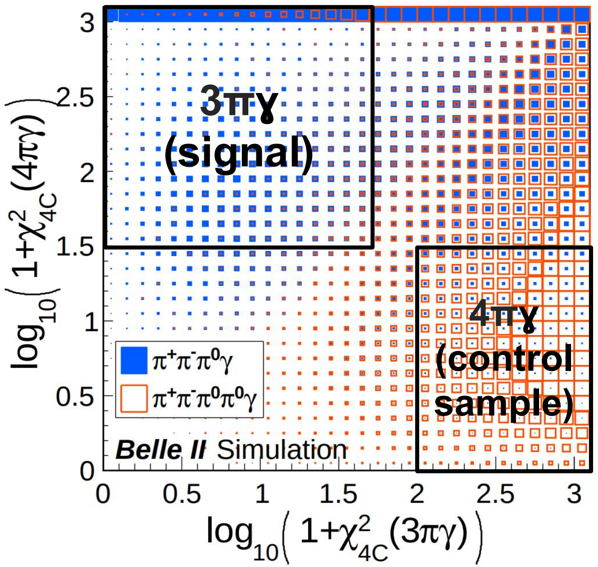


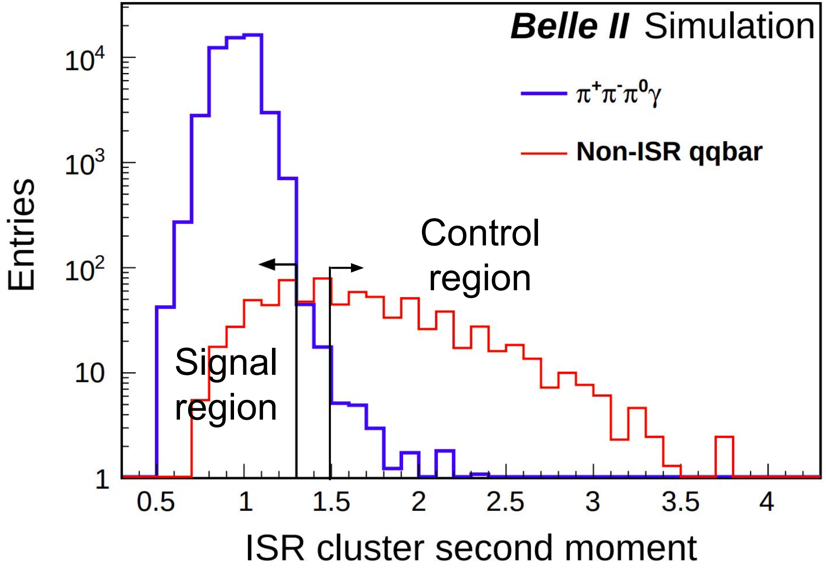
Background corrections

Estimate by determining a **mass-dependent data-MC scale factor using a control sample**

$$N^{\mathrm{MC}}_{i,\mathrm{Signal}} \to N^{\mathrm{MC}}_{i,\mathrm{Signal}} \cdot \frac{N^{\mathrm{data}}_{i,\mathrm{Control}}}{N^{\mathrm{MC}}_{i,\mathrm{Control}}}$$

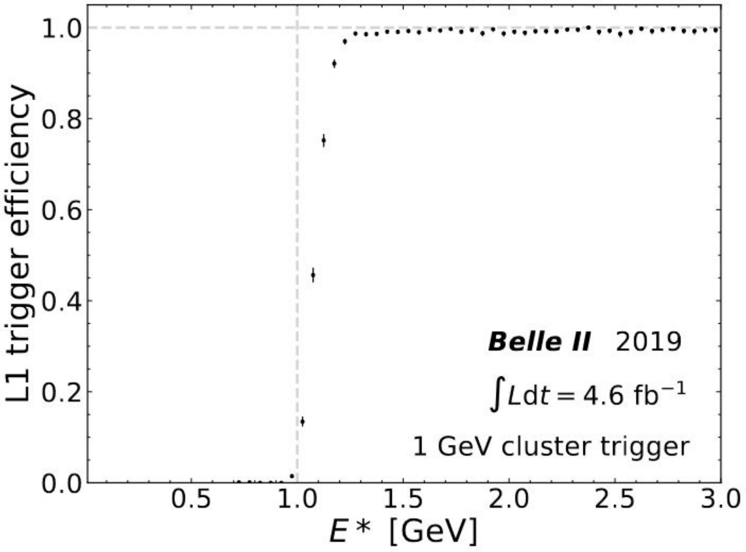
- $e^+e^-\to K^+K^-\pi^0\gamma$: Invert π/K ID : $L(\pi/K) > 0.1 \Rightarrow L(\pi/K) < 0.1$
- $e^+e^-\to \pi^+\pi^-\pi^0\pi^0\gamma$: Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$
- Non-ISR $q\bar{q}$: $0.10 < M(\gamma_{\mathrm{ISR}}\gamma) < 0.17$ GeV / large cluster second moment





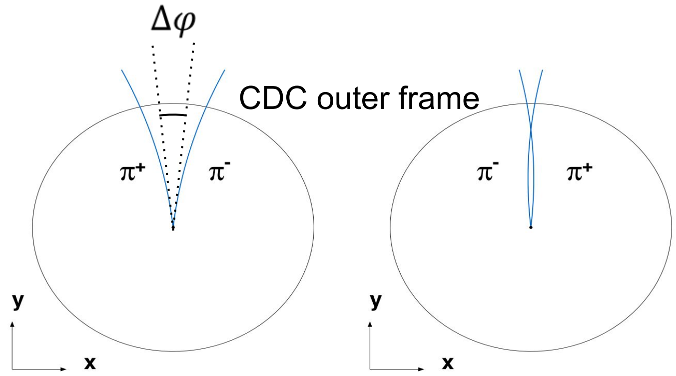
Trigger efficiency

- ISR events are triggered by the energy trigger (E_{total} > 1 GeV) in the calorimeter
- The efficiency can be measured by using the events triggered independently by the track trigger
  - Efficiency for energetic ISR > 99%
- The systematic uncertainty related to trigger is well suppressed, 0.2%.
- The high trigger efficiency for energetic ISR is also beneficial for other ISR processes

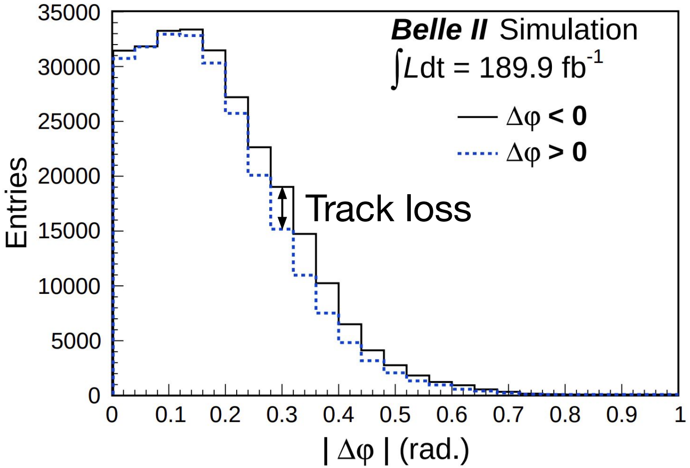


Tracking efficiency

- Tracking efficiency is confirmed by tag-and-probe method using τ pairs.
- Track loss due to shared hits on the drift chamber is confirmed.
  - Evaluate using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ process at the ω resonance
- Define $\Delta\varphi = \varphi(\pi^+) - \varphi(\pi^-)$
- The inefficiency due to track loss is given by $f = \dfrac{N(\Delta\phi < 0) - N(\Delta\phi > 0)}{2N(\Delta\phi < 0)}$
  - The track loss in MC is 4%
- In total, the systematic uncertainty of tracking is 0.8%



$\Delta\varphi < 0$ $\Delta\varphi > 0$

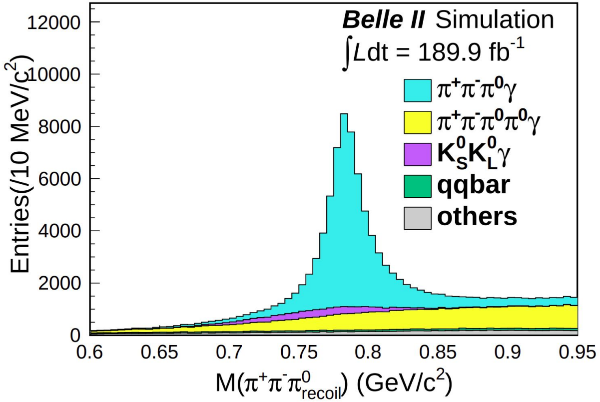


π^0 efficiency correction

- π^0 detection efficiency is 50-60%
- Evaluate efficiency using the $e^+e^- \rightarrow \pi^+\pi^-\pi^0\gamma$ events around ω resonance

$$\pi^0\text{efficiency} = \frac{N(\text{Fullreconstruction}: \pi^+\pi^-\pi^0\gamma_{\text{ISR}})}{N(\text{Partialreconstruction}: \pi^+\pi^-\gamma_{\text{ISR}})}$$

- Partial reconstruction $\pi^+\pi^-\gamma$: ISR + two tracks
- The squared 3π mass $M(\pi^+\pi^-\pi^0_{\text{recoil}})$ is defined as $M^2(\pi^+\pi^-\pi^0_{\text{recoil}}) = (p_{\pi^+} + p_{\pi^-} + p_{\text{recoil}})^2$
- Recoil momentum p_{recoil} is determined by kinematic fit to $\pi^+\pi^-\gamma$ with hypothesis that recoil mass equals π^0 mass (1-constraint)
- Fit on $M(\pi^+\pi^-\pi^0_{\text{recoil}})$ distribution around ω resonance to estimate the number of $3\pi\gamma$
    - Count the number of events in ω region

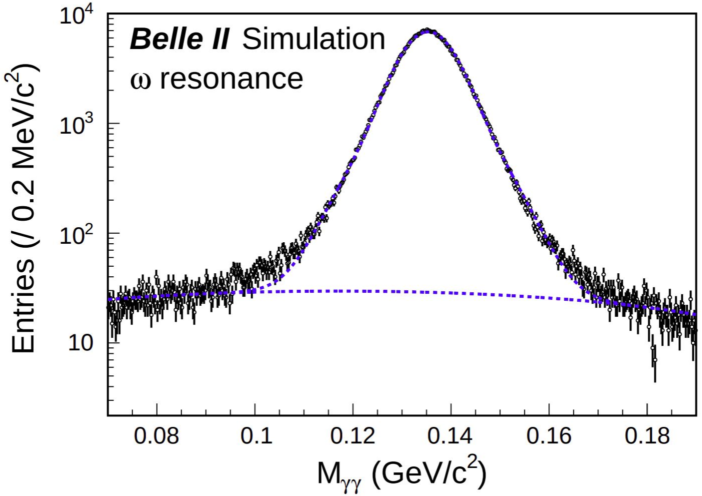


π^0 efficiency correction

- π^0 detection efficiency is 50-60%
- Evaluate efficiency using the $e^+e^- \to \pi^+\pi^-\pi^0\gamma$ events of ω resonance

$$\pi^0\text{efficiency} = \frac{N(\text{Fullreconstruction} : \pi^+\pi^-\pi^0\gamma_{\text{ISR}})}{N(\text{Partialreconstruction} : \pi^+\pi^-\gamma_{\text{ISR}})}$$

- Full reconstruction: partial reconstruction + π^0 selection + $\chi^2 < 50$
- Fit M(γγ) with signal extraction parameters in ω region
  - Signal: Novosibirsk function + Gaussian (fixed parameters)
  - Background: Quadratic function (floating parameters)
- The π^0 efficiency is independently evaluated by data-MC comparison
- The systematic uncertainty related to π^0 is 1.0%
- The uncertainty is evaluated by variations of the M(γγ) signal pdf, background pdfs and selections



Effective ISR Luminosity

$$\frac{d\mathcal{L}_{\mathrm{eff}}}{d\sqrt{s'}}(\sqrt{s'}) = \frac{2\sqrt{s'}}{s}\frac{\alpha}{\pi}\left(\frac{s^2+s'^2}{s(s-s')}\ln\frac{1+\cos\theta_\gamma}{1-\cos\theta_\gamma} - \frac{s-s'}{s}\cos\theta_\gamma\right)\mathcal{L}_{\mathrm{int}}$$

$20^{o} < \theta < 160^{o}$

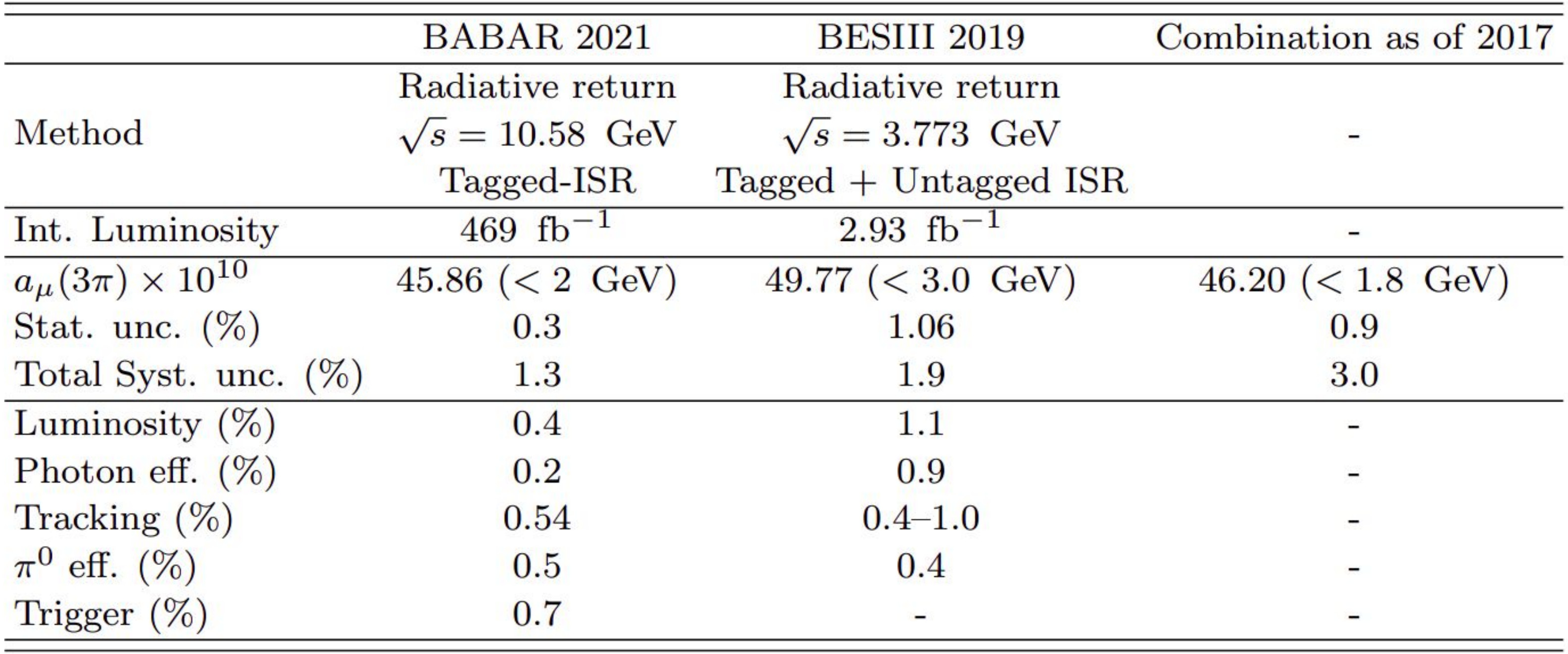
$L_{int} = 189.88 \pm 0.05 \pm 2.85$

Radiative correction

- Take into account multiple ISR photons
- Calculated using PHOKHARA
- μμɣ events simulated with LO+NLO ISR, VP effects, without FSR
- R is very close to 1 and associated uncertainty is relatively small (~0.5%)

$$R = \frac{\sigma(e^+e^- \to \mu^+\mu^- + \text{multiple } \gamma_{\text{ISR}})}{\sigma(e^+e^- \to \mu^+\mu^- + \text{single } \gamma_{\text{ISR}})}$$

Previous measurements



	BABAR 2021	BESIII 2019	Combination as of 2017
Method	Radiative return $\sqrt{s} = 10.58$ GeV Tagged-ISR	Radiative return $\sqrt{s} = 3.773$ GeV Tagged + Untagged ISR	-
Int. Luminosity	469 fb^{-1}	2.93 fb^{-1}	-
$a_\mu(3\pi) \times 10^{10}$	45.86 (< 2 GeV)	49.77 (< 3.0 GeV)	46.20 (< 1.8 GeV)
Stat. unc. (%)	0.3	1.06	0.9
Total Syst. unc. (%)	1.3	1.9	3.0
Luminosity (%)	0.4	1.1	-
Photon eff. (%)	0.2	0.9	-
Tracking (%)	0.54	0.4–1.0	-
π^0 eff. (%)	0.5	0.4	-
Trigger (%)	0.7	-	-