Discussion on e^+e^- data and MC generators for a_μ^{HLO}

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Helmholtz-Zentrum Dresden-Rossendorf

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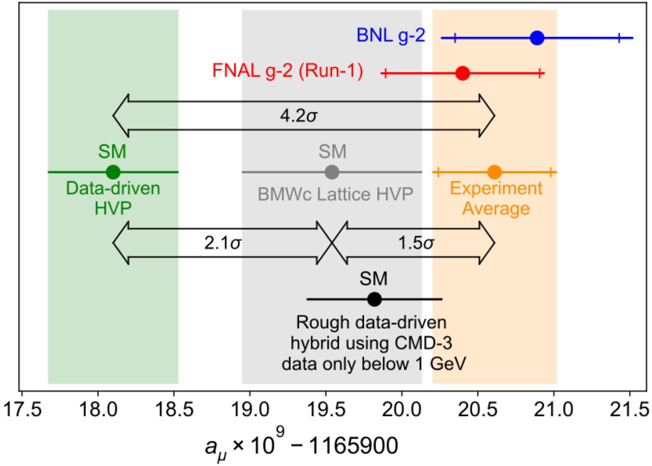
November 8, 2023

DRESDEN concept

HZDR HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Current situation Nov. 2023

From Alex Keshavarzi's presentation at Lattice 2023:



IMPORTANT: THIS PLOT IS VERY ROUGH!

Situation March 2004...

From S. Müller's presentation at DPG 2004 in Cologne:

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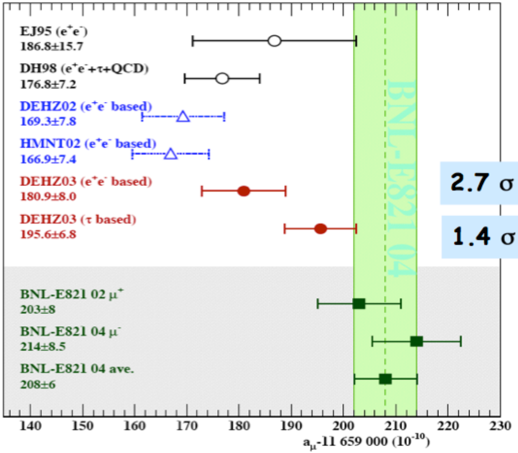
Davier, Eidelman, Höcker, Zhang: hep-ph/0308213,
E821: hep-ex/0401008



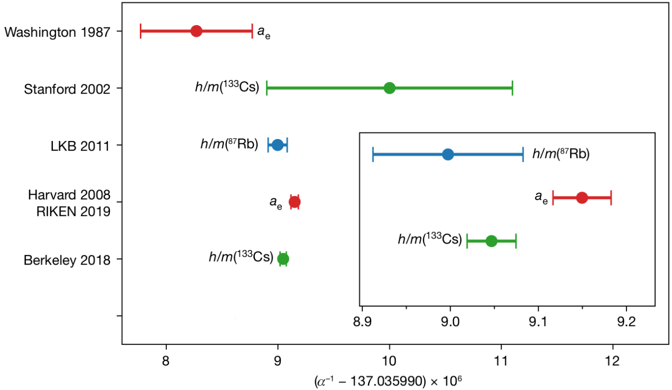
2.7 σ

1.4 σ

Disagreement between e^+e^- based and τ based evaluations

Electron anomalous magnetic moment and α_{em}

Determination of fine structure constant α_{em} from measurement of a_e and direct α_{em} determination from atomic recoil frequency in matter-wave interferometers:



Nature vol. 588, pages 61-65 (2020)

$$a_e(\alpha_{2018}(Cs)) = 1\,159\,652\,181.61(23) \times 10^{-12}$$

PRL 130, 071801 (2023): $$a_e(\exp) = 1\,159\,652\,180.59(13) \times 10^{-12}$$

$\}\ +3.9\sigma$

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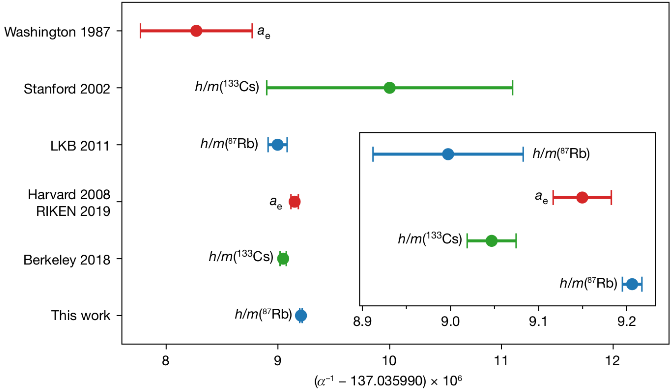

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$$a_e(\alpha_{2020}(\mathrm{Rb})) = 1\,159\,652\,180.252(95) \times 10^{-12}$$

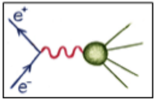
$+3.9\sigma$ (Cs vs. exp)

-2.1σ (exp vs. Rb)

Hadronic cross section measurements

Three ways to obtain hadronic cross sections:

- **Energy scan:** Change beam energy changed to desired value.
  - e.g. at VEPP-2M/VEPP2000 colliders in Novosibirsk (SND/CMD-experiments)



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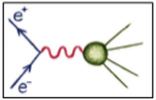
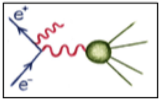


- **Initial State radiation:** Run at fixed energy, use initial state radiation process to access lower lying energies or resonances
  - e.g. at DAΦNE, PEP-II, BEPC and KEKB meson factories (KLOE, BaBar, BES, BELLE -experiments).



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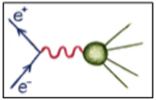

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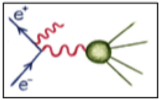
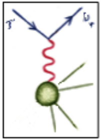


- **Tau data:** Derive cross sections from τ spectral functions using CVC theorem.
  - e.g. at LEP, CESR and KEKB colliders (ALEPH, L3, OPAL, CLEO, BELLE -experiments).



Radiative corrections and Monte Carlo generators

Precise measurements of hadronic cross sections to be used in the dispersion integral for $\mathbf{a}_\mu^{\mathrm{HLO}}$ need reliable estimates of the radiative corrections involved:

- Corrections for initial state radiation effects (radiator function)
- Treatment of final state radiation
- Normalization with Bhabha or muon events
- Effects of Vacuum Polarization

Generators used in the analyses (Alex Keshavarzi's presentation at Lattice 2023):

MC generators for exclusive channels (exact NLO + Higher Order terms in some approx)

MC generator	Channel	Precision	Comment
MCGPJ (VEPP-2M, VEPP-2000)	$e^+e^- \to e^+e^-, \mu^+\mu^-, \pi^+\pi^-, \ldots$	0.2%	photon jets along all particles (collinear Structure function) with exact NLO matrix elements
BabaYaga@NLO (KLOE, BaBar, BESIII)	$e^+e^- \to e^+e^-, \mu^+\mu^-, \gamma\gamma$	0.1%	QED Parton Shower approach with exact NLO matrix elements

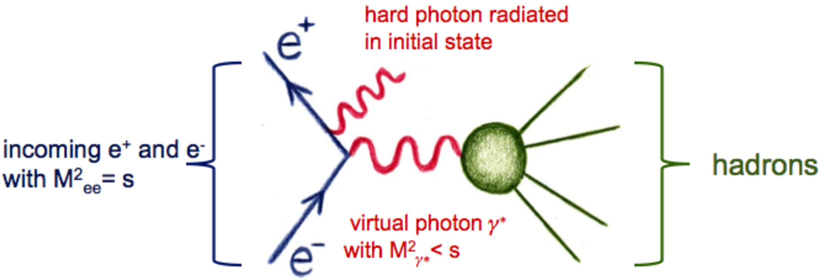
MC generators for ISR (from approximate to exact NLO)

MC generator	Channel	Precision	Comment
EVA (KLOE)	$e^+e^- \to \pi^+\pi^-\gamma$	O(%)	Tagged photon ISR at LO + Structure Function FSR: point-like pions
AFKQED (BaBar)	$e^+e^- \to \pi^+\pi^-\gamma, \ldots$	depends on the event selection (can be as good as Phokhara)	ISR at LO +Structure Function
PHOKHARA (KLOE, BaBar BESIII)	$e^+e^- \to \pi^+\pi^-\gamma, \mu^+\mu^-\gamma, 4\pi\gamma, \ldots$	0.5%	ISR and FSR(sQED+Form Factor) at NLO

... and more!

Radiative corrections and Monte Carlo generators

Initial state radiation measurements are using the Radiative Return to energies below the collider energy $\sqrt{s}$.

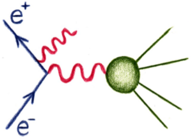


Emission of hard γ in the bremsstrahlung process reduces available energy to produce hadronic system.

Radiative corrections and Monte Carlo generators

Relate measured differential cross section $d\,\sigma_{\text{had}+\gamma}/d\,M^2_{\text{had}}$ to hadronic cross section σ_{had} using radiator function $H(s, M^2_{\text{had}})$:

$$\frac{d\,\sigma(e^+e^- \to \text{had} + \gamma)}{d\,M^2_{\text{had}}} = \frac{\sigma(e^+e^- \to \text{had}, M^2_{\text{had}})}{s} \times H(s, M^2_{\text{had}})$$



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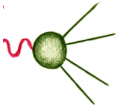

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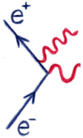


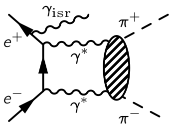
measured cross section — resulting cross section — radiator function

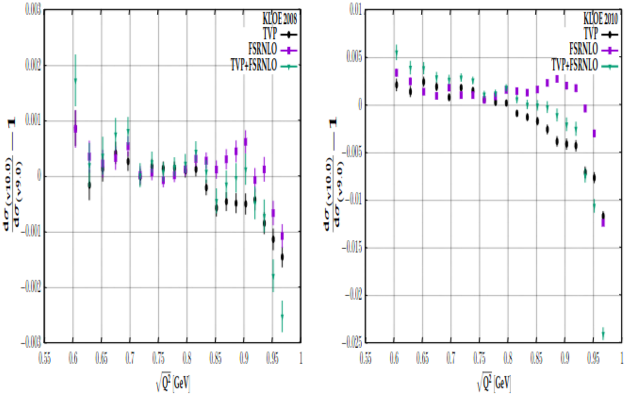
Uses precise calculation of radiator function $H(s, M^2_{\text{had}})$, e.g. from PHOKHARA Monte Carlo event generator.

- Assumes factorization of ISR part
- Final State Radiation (FSR) needs dedicated treatment

Radiative corrections and Monte Carlo generators

Presence of “Two-Virtual-Photon” (TVP) contribution spoils the ISR factorization - needs to be treated as a correction.





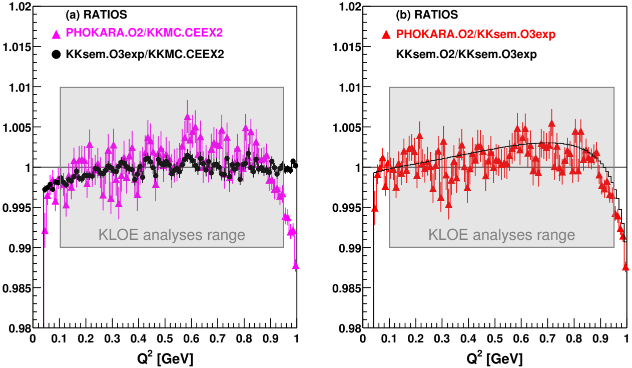
H. Czyż PHOKHARA , 10

KLOE10 range extends down to 0.32 GeV...

KKMC-PHOKHARA comparison

Comparison of ISR effect in the process $e^+e^- \rightarrow \mu^+\mu^-\gamma$ with **PHOKHARA** and the **KKMC** generator (S. Jadach, arXiv:hep-ph/0506180):

- **KKMC** uses second order CEEX matrix element (complete NLL and NNLL contributions)
- also analytical formula is used ("KKsem")



A 0.25% difference was found in central region (larger deviation at high Q^2 due to missing soft photon resummation in **PHOKHARA**).

Charge asymmetry observable

The forward-backward asymmetry $\mathcal{A}_{FB}(Q^2) = \frac{N(\theta_{\pi^+} > 90^\circ) - N(\theta_{\pi^+} < 90^\circ)}{N(\theta_{\pi^+} > 90^\circ) + N(\theta_{\pi^+} < 90^\circ)}(Q^2)$ can be used to test the validity of the description of the various mechanisms of the $\pi^+\pi^-$ final state photon emission:

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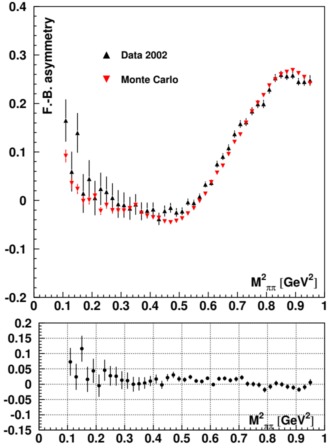

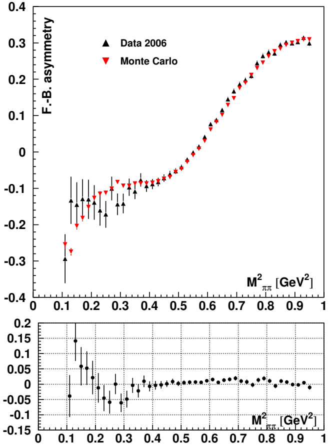

KLOE data vs. PHOKHARA 6.1, in Eur. Phys. J. C 66, 585-686 (2010)

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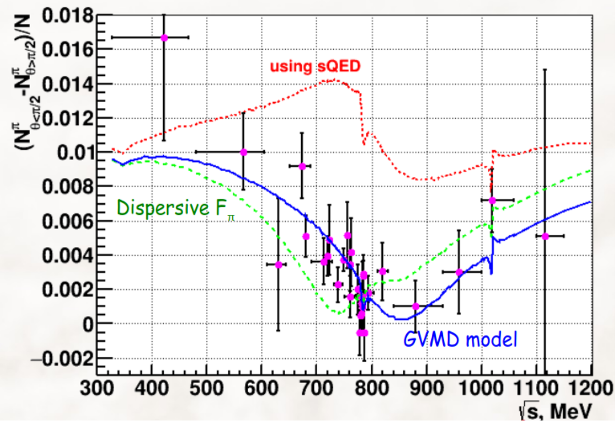
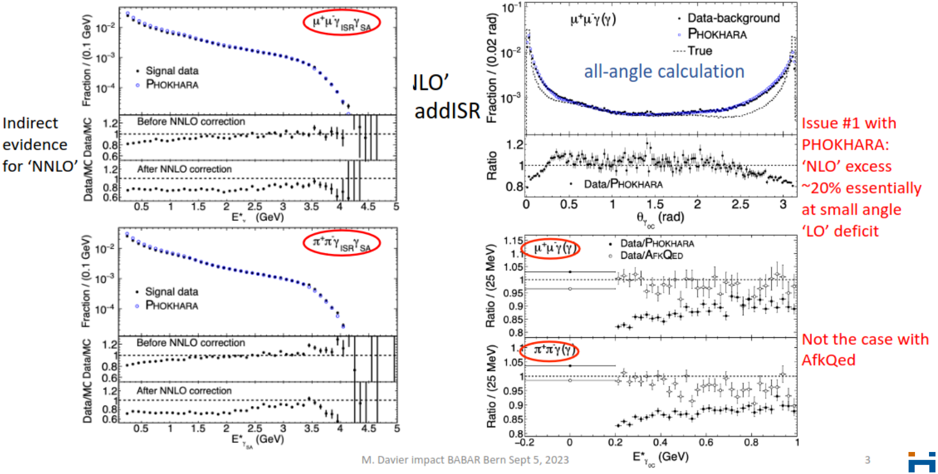


CMD3 data (arXiv:2302.08834) vs. Phys.Lett.B 833 (2022) 137283

BaBar studies: AFKqed vs. PHOKHARA

Study of BaBar collaboration on events with 2 hard photons claims discrepancies between data and PHOKHARA, while Afkqed shows better agreement (arXiv:2308.05233):

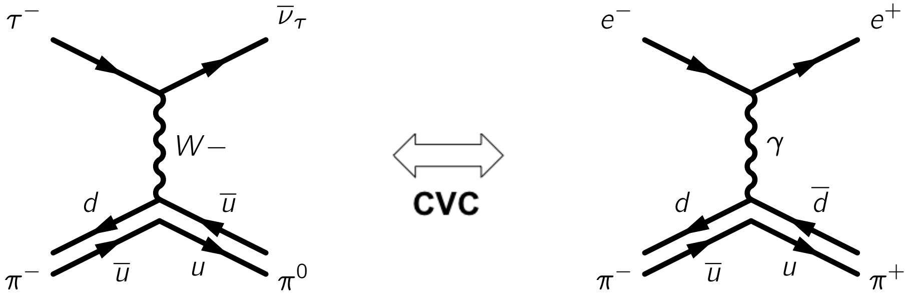


τ data:

Relate data from τ decays to e^+e^- data using CVC theorem:

$\tau^- \rightarrow \nu_\tau \pi^- \pi^0$

$e^+e^- \rightarrow \pi^-\pi^-$



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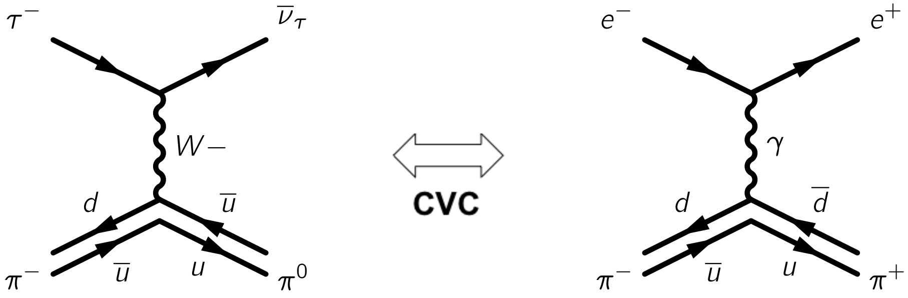

$$\sigma^{I=1}_{e^+e^-\to\pi^+\pi^-}(s) = \frac{4\pi\alpha^2}{s} v_{\pi^-\pi^0}(s) \quad , \quad \sqrt{s} \leq m_\tau$$

Isospin symmetry breaking effects need to be taken into account

Conclusions

- The current situation is not optimal...
  - ...but it could be worse.
- We need Monte Carlo generators to evaluate and estimate radiative corrections
  - Normalization (Bhabha, muons,...)
  - ISR corrections (Radiator function)
  - Treatment of final state radiation
- Many generators available, nevertheless need to work on
  - implementation of higher order corrections
  - efficiency of calculation (CPU cost)
  - cross checks and comparisons between generators (and with data)
- Let's not fully forget data from τ-decays.