HVP Dispersive (e^+e^-)

Thomas Teubner

- Data-driven HVP: status & issues
- Theory Initiative
- Pathways to solving the puzzles
  - Strong 2020 activities
  - Liverpool$^+$ efforts

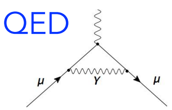
SM prediction from Theory Initiative vs. **Experiment**

$$a_\mu = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{weak}} + a_\mu^{\mathrm{hadronic}} + a_\mu^{\mathrm{NP?}}$$

White **P**aper [T. Aoyama et al., *Phys. Rept.* 887 (2020) 1-166]

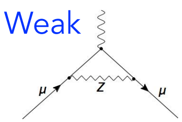
0.37 ppm

QED



$+\ldots$ $\quad 116\,584\,718.9\,(1) \times 10^{-11}$ $\quad$ 0.001 ppm

Weak

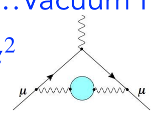


$+\ldots$ $\quad 153.6\,(1.0) \times 10^{-11}$ $\quad$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

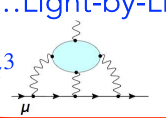
α^2



$+\ldots$ $\quad 6845\,(40) \times 10^{-11}$ [0.6%] $\quad$ 0.34 ppm

…Light-by-Light (HLbL)

α^3

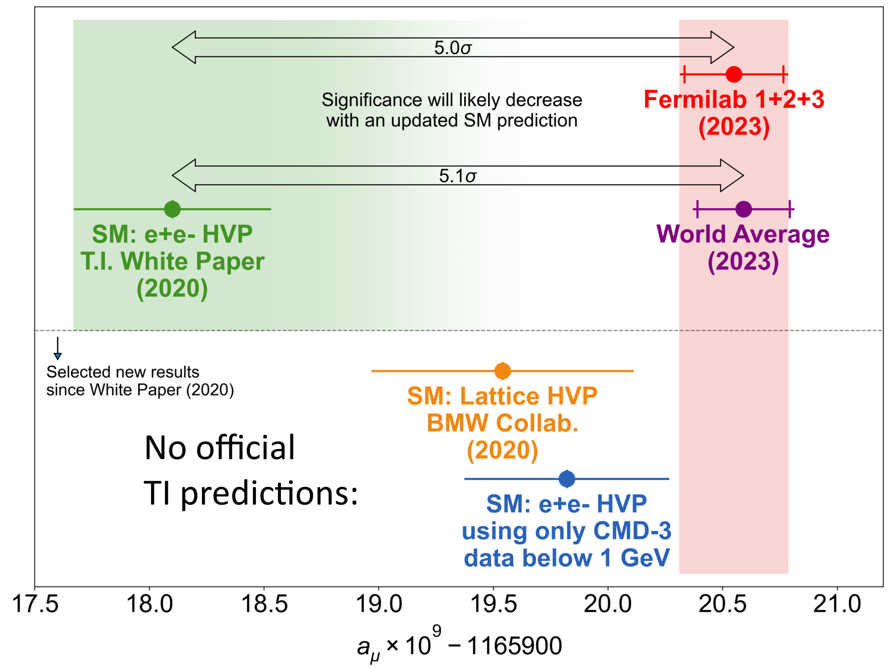


$+\ldots$ $\quad 92\,(18) \times 10^{-11}$ [20%] $\quad$ 0.15 ppm

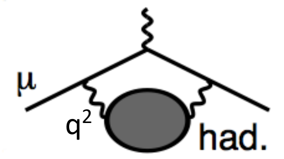
- SM uncertainty dominated by hadronic contributions, now with δ HVP > δ HLbL

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm [*Phys. Rev. Lett.* 126 (2021) 14, 141801]

… **to 0.20 ppm** [PRL, 2308.06230]

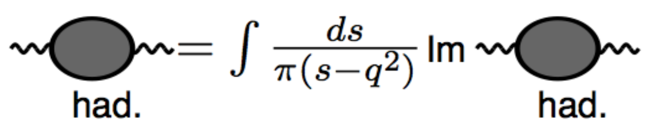


a_μ^{HVP} : Basic principles of **dispersive** data-driven method



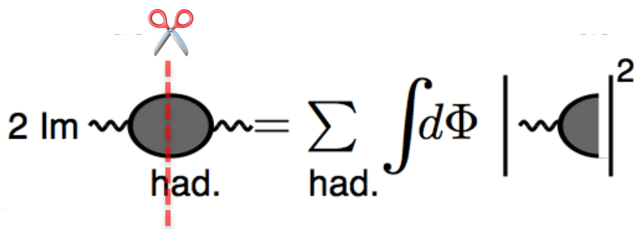
One-loop diagram with hadronic blob =

integral over q^2 of virtual photon, 1 HVP insertion



Causality ➠ analyticity ➠ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➠ Optical Theorem:

imaginary part (\`cut diagram') =

sum over |cut diagram|^2, i.e.

∝ sum over all total hadronic cross sections

$$a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)$$

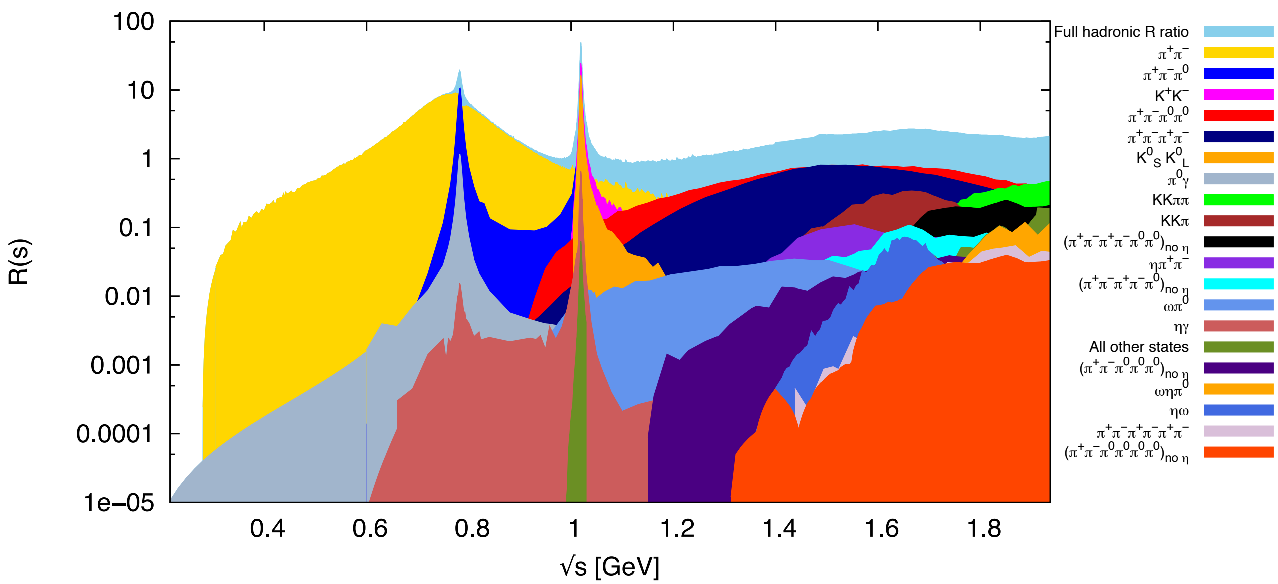
- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$ ♣

- Total hadronic cross section σ_{had} from > 100 data sets for **$e^+e^- \to$ hadrons** in > 35 final states
- Uncertainty of a_μ^{HVP} prediction from statistical & systematic uncertainties of input data
- pQCD only at large s, **no modelling** of $\sigma_{had}(s)$, direct data integration

HVP disp: **Landscape** of $\sigma_{had}(s)$ data. **Most important $\pi^+\pi^-$ channel**



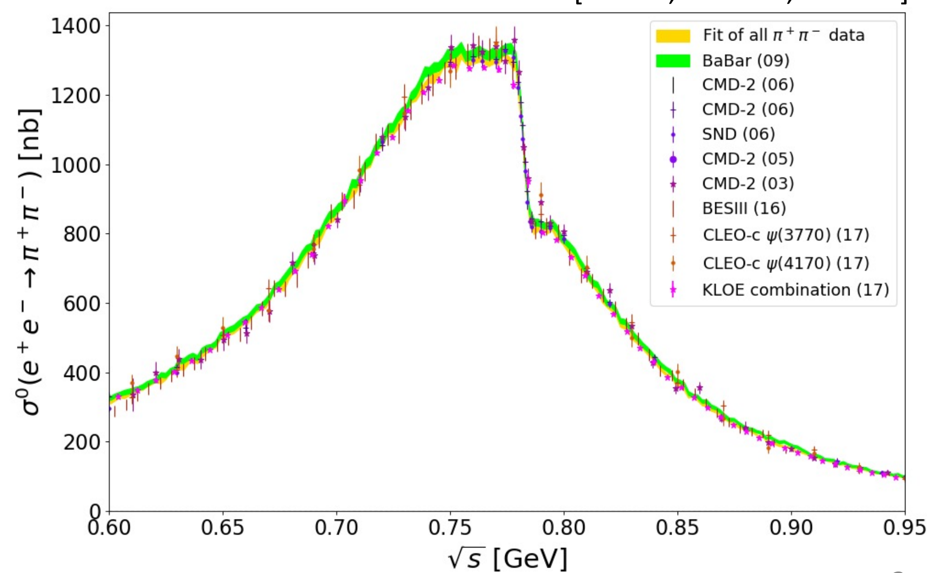
[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of 2π

$\pi^+\pi^-$:

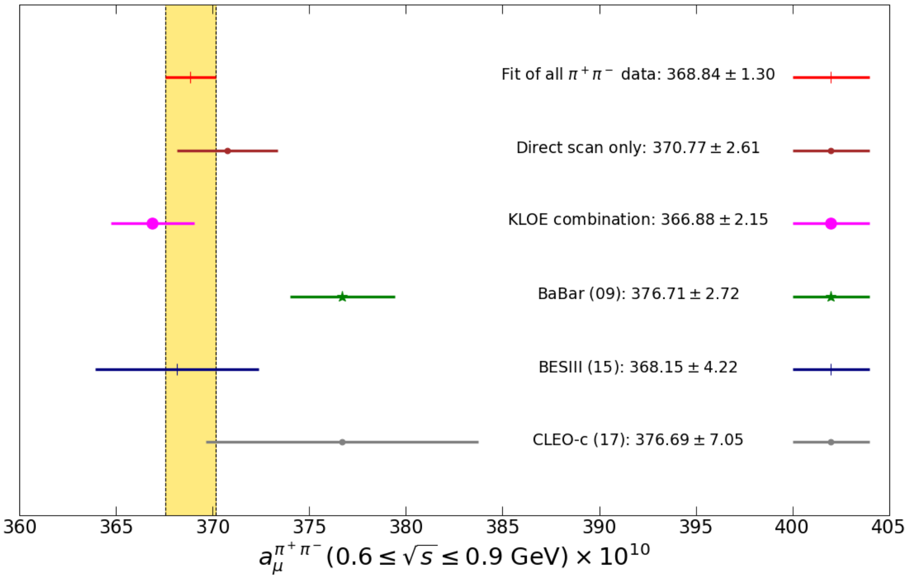
- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from **6 independent experiments** with different systematics and different radiative corrections
- Data sets from Radiative Return dominate, **until now…**

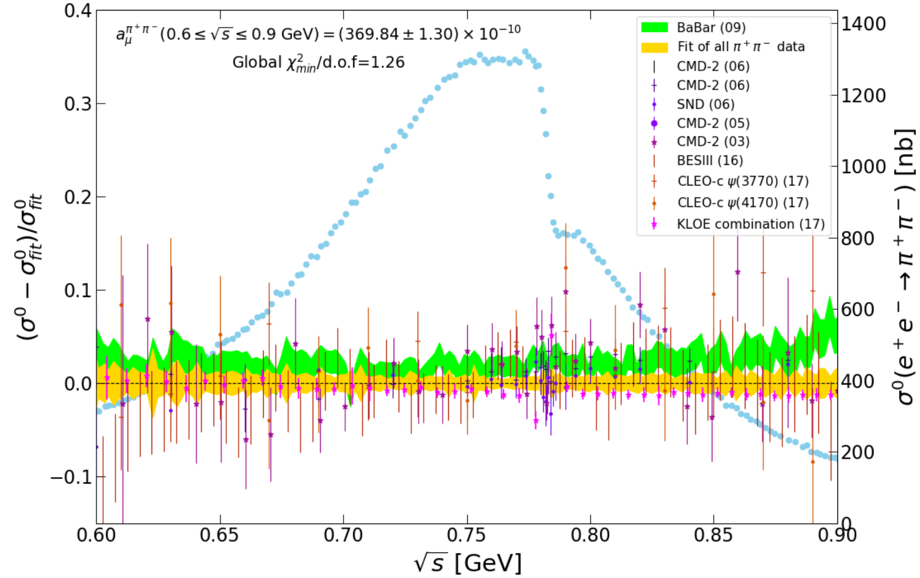
[KNT19, PRD101, 014029]



a_μ^{HVP} : $\pi^+\pi^-$ channel **KLOE vs. Babar puzzle, enlarged WP error**

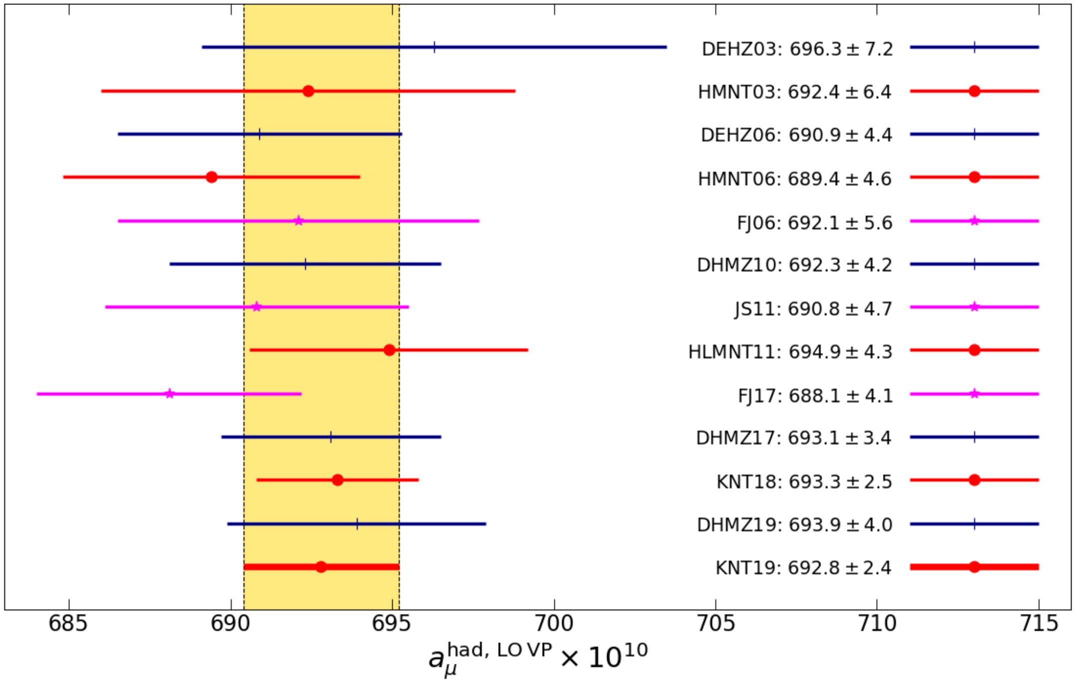
[Plots from KNT19]





- **Tension** between different sets, especially between the most precise 4 sets from **BaBar** and **KLOE**
- Inflation of error with **local** χ^2_{min} accounts for tensions, leading to a **~14% error inflation**
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods (KNT vs. DHMZ)
- Differences in data and methods accounted for in **WP merging procedure,** leading to enlarged error for a_μ^{HVP}. **Procedure not well suited to cover CMD-3**

a_μ^{HVP} : > 20 years of data based predictions, `pies'

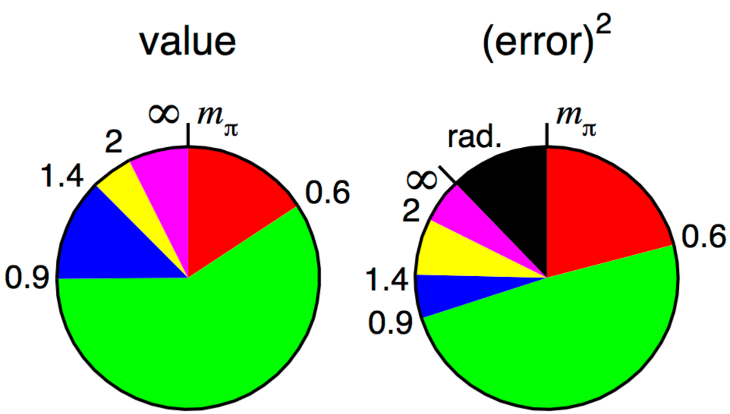


- **Stability** and **consolidation** over two decades thanks to more and better data input and improved compilation procedures

- Compare with **merged** DHMZ & KNT WP20 value:

$$a_\mu^{had, LO VP}(WP20) = 693.1(4.0)\times10^{-10}$$

Pie diagrams for KNT compilation:

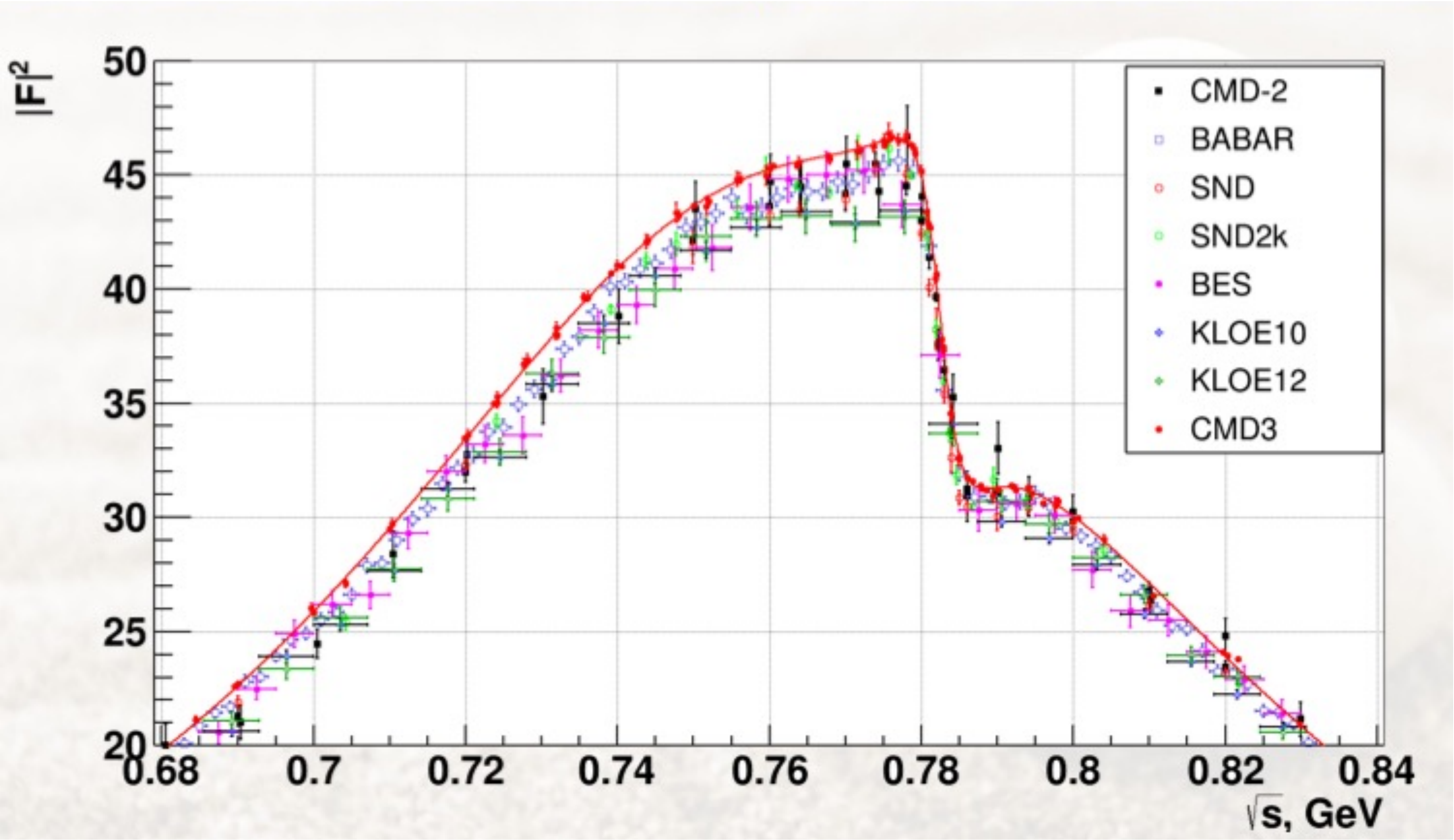


- error still dominated by the two pion channel
- significant contribution to error from additional **uncertainty from radiative corrections**
- **Is all this invalidated by the recent CMD-3 data?**

New CMD-3 $\pi^+\pi^-$ data vs. other experiments

Slide from Fedor Ignatov's TI talk 27.3.2023

arXiv:2302.08834



Muon g-2 Theory Initiative est. 2017

https://muon-gm2-theory.illinois.edu

``… map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental result.''

- Organised 9 int. workshops in 2017-2023, last plenary workshop 4-8.9.2023 in Bern
- **White Paper** posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

  ``**The anomalous magnetic moment of the muon in the Standard Model**''
  [T. Aoyama et al., arXiv:2006.04822, *Phys. Rept.* 887 (2020) 1-166 >1000 cites]

Group photo from the Bern workshop in September 2023

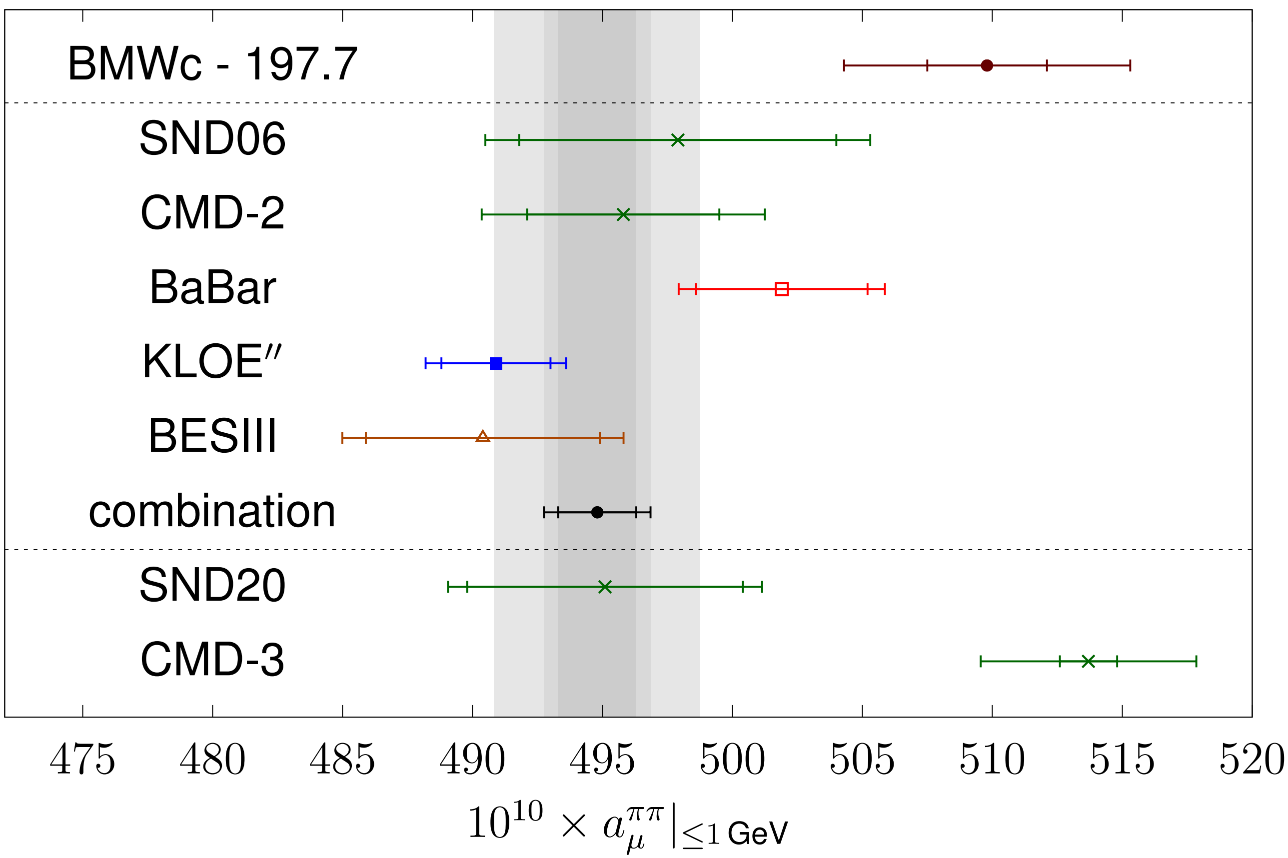
Theory Initiative: Sep. 2023 workshop at Bern

Peter Stoffer: studies of Colangelo et al. with analyticity&unitarity based fits:

(no combination w. CMD-3 yet)

More tensions: CMD-3

→ F. Ignatov et al. (CMD-3), 2302.08834 [hep-ex]



Theory Initiative: Sep. 2023 workshop at Bern

Peter Stoffer: studies of Colangelo et al. with analyticity&unitarity based fits:

Summary

- dispersive fits to **CMD-3** work well, **p-value of 20%**
- new CMD-3 results **disagree** with other e^+e^- results at the **$2-5\sigma$ level**
- further discrepancies in phase of ρ–ω mixing parameter $\arg(\epsilon_\omega)$
- high values for $\arg(\epsilon_\omega)$ from all scan experiments not in line with narrow-width expectation
- **SND20**: only data set that does **not lead to good fit**

Theory Initiative: Sep. 2023 workshop at Bern

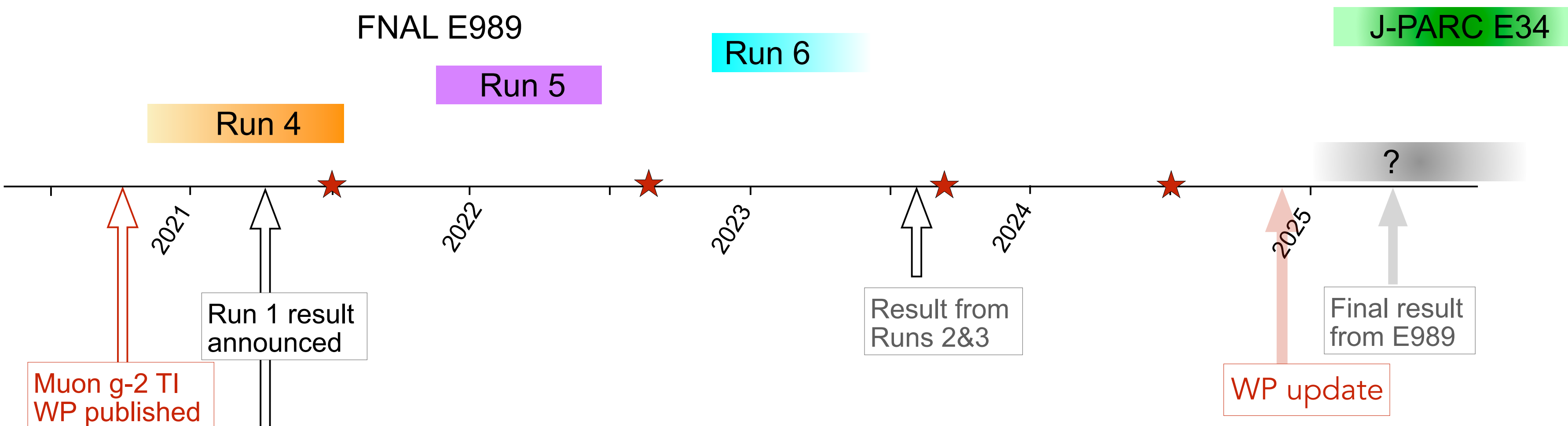
Michel Davier's summary report of the **49 Questions to CMD-3** (all answered by Fedor):

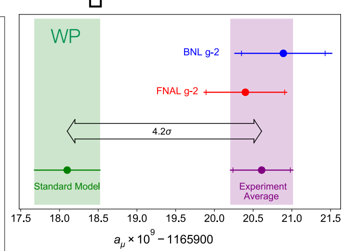
Conclusions

- Difficult exercise: sophisticated analyses are not easy to penetrate without access to the data
- However we got documented answers on detailed questions covering the important aspects of the analysis
- It is fair to say that no major issue significantly impacting the results has been identified
- The strength of the analysis lies in (1) the large statistics accumulated giving the possibility to perform systematic tests with high precision, (2) improved performance of the CMD-3 detector, and (3) the fact that two independent methods were used for channel separation
- Still several points remained unclear to us and /or not enough convincing with the information available
- Possible effects on the results from these minor issues need to be quantified with respect to the claimed accuracy
- Need guidance from CMD-2/3 on how to handle their data

Theory Initiative: Sep. 2023 workshop at Bern

Aida El-Khadra: TI outlook and plans:





Theory Initiative:

- ☆ CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- ☆ 2nd CMD-3 discussion meeting
- ☆ 8/9/2023: Status of Muon g-2 Theory in SM

★ TI workshops:

Jun 2021 @ KEK (virtual)	Sep 2022 @ Higgscentre
Sep 2023 @ Bern	Sep 2024 @ KEK or KMI

Theory Initiative: Sep. 2023 workshop at Bern

Aida El-Khadra: TI outlook and plans:

WP update: proposed timeline

Goal

Obtain the best possible prediction for a_μ **before** the Fermilab g-2 experiment releases their final measurement (based on runs 4,5,6) in 2025.

Considerations

Writing a WP is a major undertaking, we should make sure it's worth the effort.
➠ Timing of WP update informed by availability of new results & information

Summarize the status of SM predictions
➠ Include everything in update to enable detailed comparisons between the different approaches (e.g. lattice/dispersive) for HVP & HLbL and related quantities

➠ Aim WP update for late 2024

Pathways to solving the (HVP) puzzles

- No easy way out! Signs for Beyond the Standard Model physics?

- BSM at high scales? Many explanations for `**4.2σ**' **puzzle**, few seem natural,
  NP smoking guns in the flavour sector weakened

- BSM `faking' low $\boldsymbol{\sigma_{had}}$? Possible but not probable
  [DiLuzio, Masiero, Paradisi, Passera, *Phys.Lett.B* 829 (2022) 137037]

  .. a new Z' [Coyle, Wagner, 2305.02354]

  … or even new hadronic states (like sexa-quarks [Farrar, 2206.13460]) ?

- Situation now very complicated due to emerged **lattice & CMD-3 puzzles**

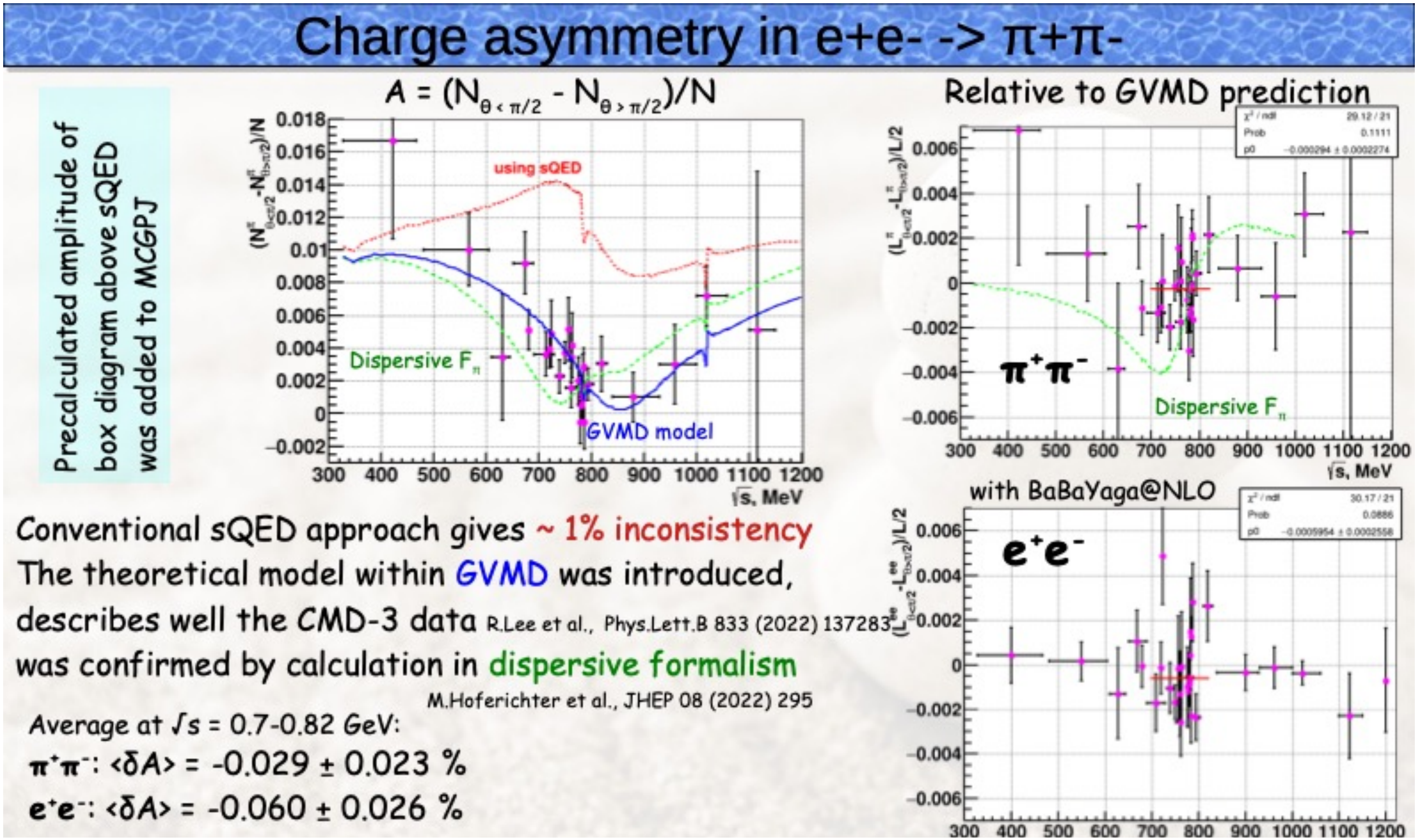
- **More & more precise data are needed (and coming) to clarify data puzzle:**
  BaBar, CMD-3, SND, BES III, Belle II, and **KLOE**

- To avoid any possible bias, **blinded analyses** are now the standard, for both experiments (g-2 and $\boldsymbol{\sigma_{had}}$) and lattice, and also the next KNT+W compilation

- The third way: **MUonE**

Fedor Ignatov's talk on MC generators:

☞ **Need to study FF models**



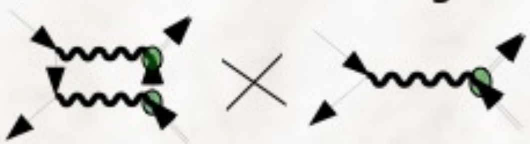
Fedor Ignatov's talk on MC generators:

☞ **Need to study FF models**

How it can affect pion form factor measurements?

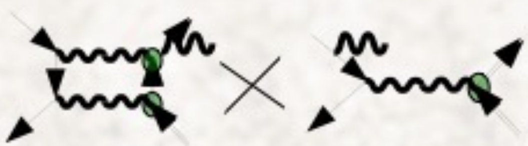
Usually event selections in analyses are charge/angle symmetric

Main effect at lowest order comes from:
Interference of box vs born diagrams



=> only charge-odd contribution
effect is integrated out
in full cross-section

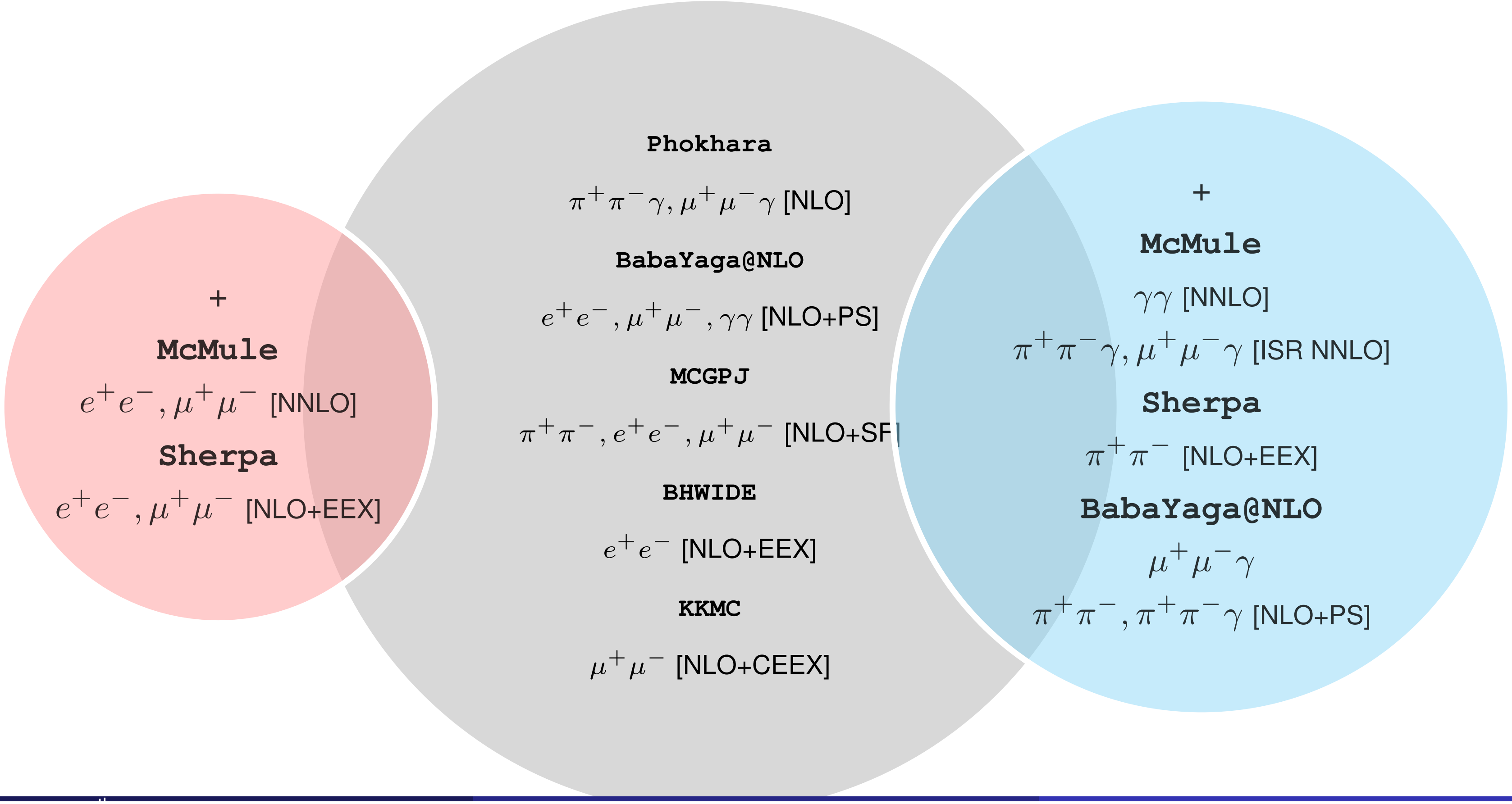
Interference of ISR & box vs FSR (or v.v.)



=> charge-even
can affect integrated cross-section

Carlo Carloni Calame & Marek Schoenherr:

Workstop/Thinkstart outcome for WP4



-- (C)EEX: (Coherent) Exclusive Exponentiation, based on YFS exponentiation, coherent is on amplitude level

-- Sherpa also working to include photon splitting in exponentiation, see Lois Flower's talk

KLOE 2π, RC & MC activities have started

- Challenges and opportunities to get a clearer understanding of the puzzles from data,

  to re-establish a stable SM prediction of g-2 [and the running QED coupling, $\alpha(M_Z^2)$]

- New Liverpool[+] effort to analyse the full statistics KLOE **2π** data (**integrated *L* ~ 1.7 fb^{-1}**)

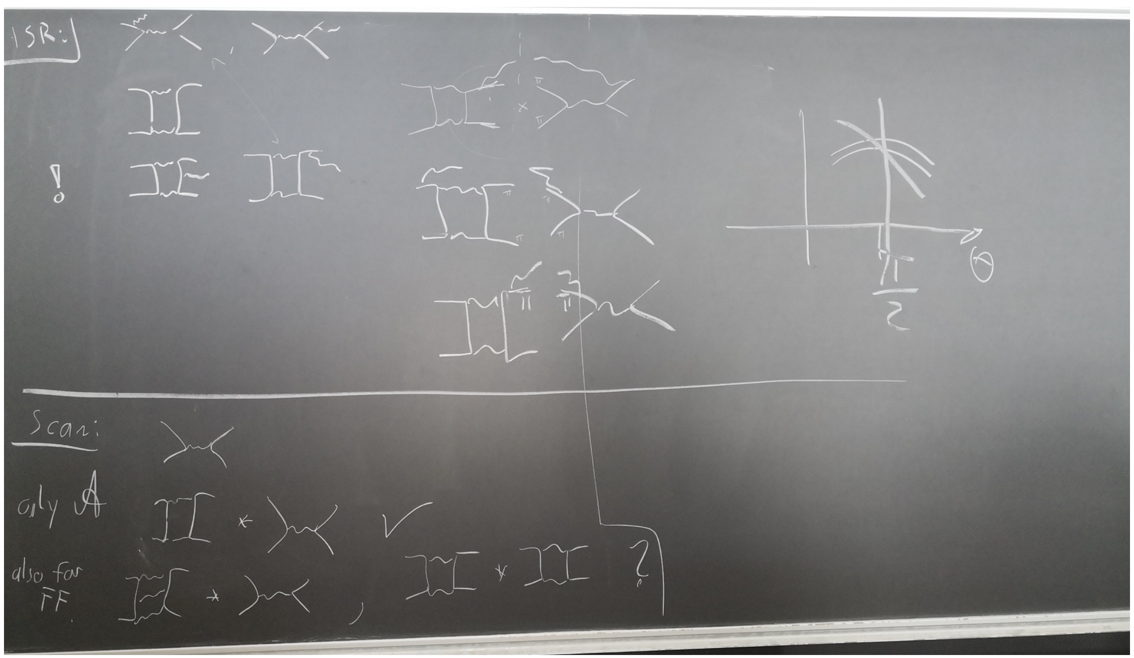
  (details on the new KLOE $\pi\pi\gamma$ analysis in Paolo's talk)

- Goal: sub-percent accuracy for $e^+e^- \to \pi^+\pi^-$,
  and improvement of a factor of ~2 on the total uncertainty => $\Delta a_\mu^{HLO} \lesssim 0.4\%$

- This will require significant involvement from theoretical groups
  - improvement of MC(s) to better describe **ISR and FSR** (`PHOKHARA`, …)
  - main aim is NNLO for ISR and improvement of/consistent FF treatment for FSR
  - other MC groups have agreed to also concentrate on $e^+e^- \to \pi^+\pi^-$, $\mu^+\mu^-$, e^+e^-

    (Babayaga, Sherpa, McMule, KKMC)
  - ongoing activity: 5th WorkStop/ThinkStart: Radiative corrections and MC tools for Strong 2020

Extras/Discussion



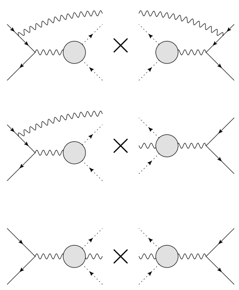
Another loop on the board

We don't need no speculation
We do need H.O. control
No dark sarcasm in the classroom
Teacher, let them kids get on

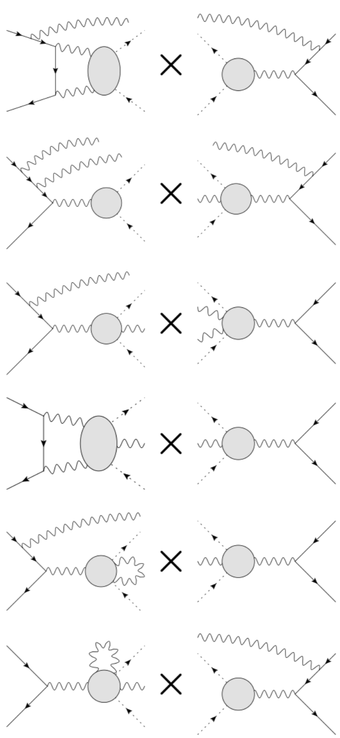
Zurich ThinkStart: diagram classification ISR (P. Stoffer's WP3 summary)

[From: 5th WorkStop/ThinkStart: Radiative corrections and MC tools for Strong 2020, Zurich, 5-9 June 2023]

ISR experiments: LO

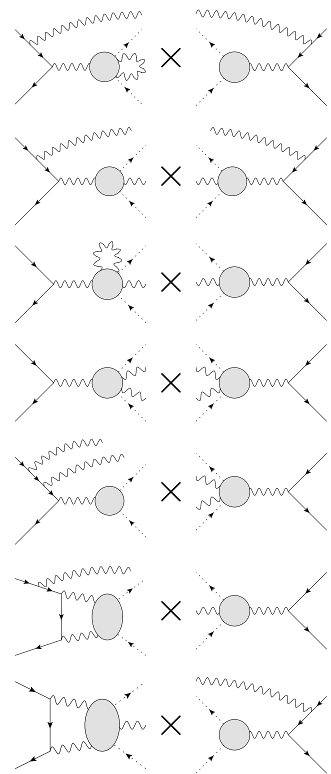


- pure QED
- contributing only to charge asymmetry
- suppressed by cuts?
  PHOKHARA: sQED + resonance approximations
  dispersive approach by Colangelo et al.



contributes only to asymmetry

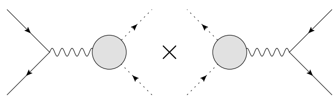
ISR experiments: NLO (omitting pure QED corrections to LO)



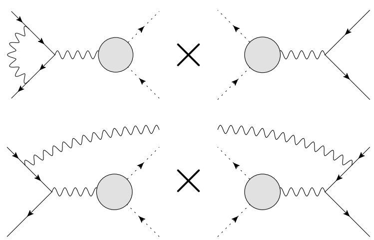
- PHOKHARA: sQED + resonance approximations
  dispersive approach by Colangelo et al.
- contained in PHOKHARA
  pure FSR: sufficiently suppressed by experimental cuts?
- ???
  PHOKHARA: sQED, multiplied by form factors *outside* loop
  ISR–FSR interference
  potential red flag identified during WorkStop

Zurich ThinkStart: diagram classification scan (P. Stoffer's WP3 summary)

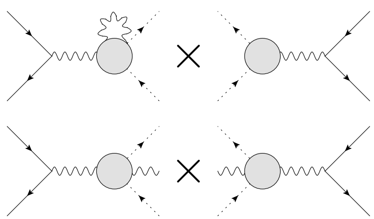
Direct scan experiments: LO



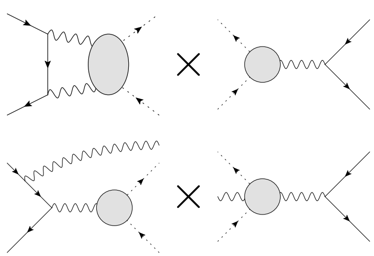
Direct scan experiments: NLO



pure QED



included in generators in terms of sQED
dispersive approach by Colangelo et al.



contributes only to asymmetry;

only pole terms:
$\longrightarrow$ Ignatov, Lee (2022)
$\longrightarrow$ Colangelo, Hoferichter, Monnard, Ruiz de Elvira (2022)

HVP dispersive: cross section compilation

How to get the most precise σ^0_{had}? Use of $e^+e^- \rightarrow$ hadrons (+γ) data:

- **Low energies: sum >35 exclusive channels**, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, …, [now very limited use iso-spin relations for missing channels]

- **Above √s ~1.8 GeV:** use of **inclusive data or pQCD** (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)

- Challenge of **data combination** (locally in √s, with **error inflation if tensions**):
  - many experiments, different energy ranges and bins,
  - **statistical + systematic errors** from many different sources, use of **correlations**

  ➤ Significant differences between **DHMZ** and **KNT** in use of correlated errors:
  - KNT allow non-local correlations to influence mean values,
  - DHMZ restrict this but retain correlations for errors, also estimate cross channel corrs.

- $\boldsymbol{\sigma^0_{had}}$ means the `bare' cross section, i.e. <u>**excluding**</u> `running coupling' (**VP**) effects, but <u>**including**</u> Final State (γ) Radiation:

  ☛ data need radiative corrections, compilations estimate additional uncertainty,

  e.g. in KNT: $\boldsymbol{\delta a_\mu^{\text{had, VP}} = 2.1 \times 10^{-11}}$, and $\boldsymbol{\delta a_\mu^{\text{had, FSR}} = 7.0 \times 10^{-11}}$

HVP: White Paper comparison

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels [CHS 2018, HHKS 2019]

a_μ^{HVP}: Hadronic tau decay data

- Historically, hadronic tau decay data, e.g. $\tau^- \to \pi^0\pi^-\nu_\tau$, were used to improve precision of e^+e^- based evaluations

- However, with the increased precision of the e^+e^- data there is now limited merit in this (there are some conflicting evaluations, DHMZ have dropped it)

- The required iso-spin breaking corrections re-introduce a model-dependence and connected systematic uncertainty (there is, e.g., no ρ–ω mixing in τ decays)

- Quote from the WP, where this approach is discussed in detail:

"Concluding this part, it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Sec. 3.4.2, may provide a solution to this problem."

- New contribution to the discussion by Masjuan, Miranda, Roig: arXiv:2305.20005
  \`τ data-driven evaluation of Euclidean windows for the hadronic vacuum polarization'

a_μ^{HVP}: Hadronic tau decay data

Mattia Bruno: Summary slide from TI talk on tau (Sep. 2023, Bern)

Windows very powerful quantities: intermediate window a_μ^W
- hadronic τ-decays can shed light on tension lattice vs e^+e^-

τ data very competitive on intermediate window
- historic tension w/ ee data and in IB τ effects
- preliminary analysis Aleph $< 0.5\%$ accuracy on a_μ^W
- (old) LQCD IB effects precision $O(1.5) \cdot 10^{-10}$ [MB Edinburgh '22]
  - new EuroHPC allocation, blinding

Work in progress to finalize full formalism [MB et al, in prep]
- W-regularization and short-distance corrections
- (re-)calculation of initial state rad.cor.
- initial-final rad.cor: proof for analytic continuation
- numerical calculation of final state IB corrections
  - relevant also for QED correction to HVP

Thanks for your attention

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