UNIVERSITY OF LIVERPOOL

LEVERHULME TRUST

Extraction of $\Delta\alpha_{had}$ and calculation of a_{μ}^{HLO} in MUonE

Riccardo Nunzio Pilato
University of Liverpool

r.pilato@liverpool.ac.uk

μONe

2nd Workshop on Muon Precision Physics
November 8th 2023

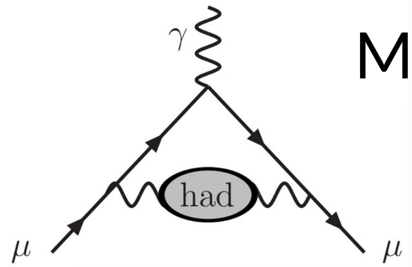
Outline

- Extraction of $\Delta\alpha_{had}$:
  the template fit procedure +
  strategy for the systematics

- a_μ^{HLO}: integral in the space-like region.

- An alternative way to compute a_μ^{HLO} with MUonE data.

The MUonE experiment

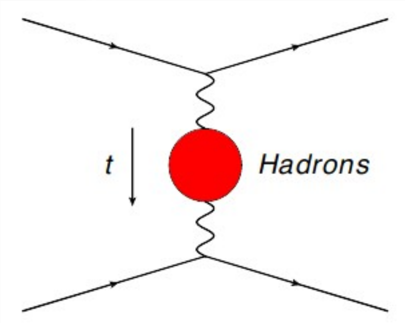


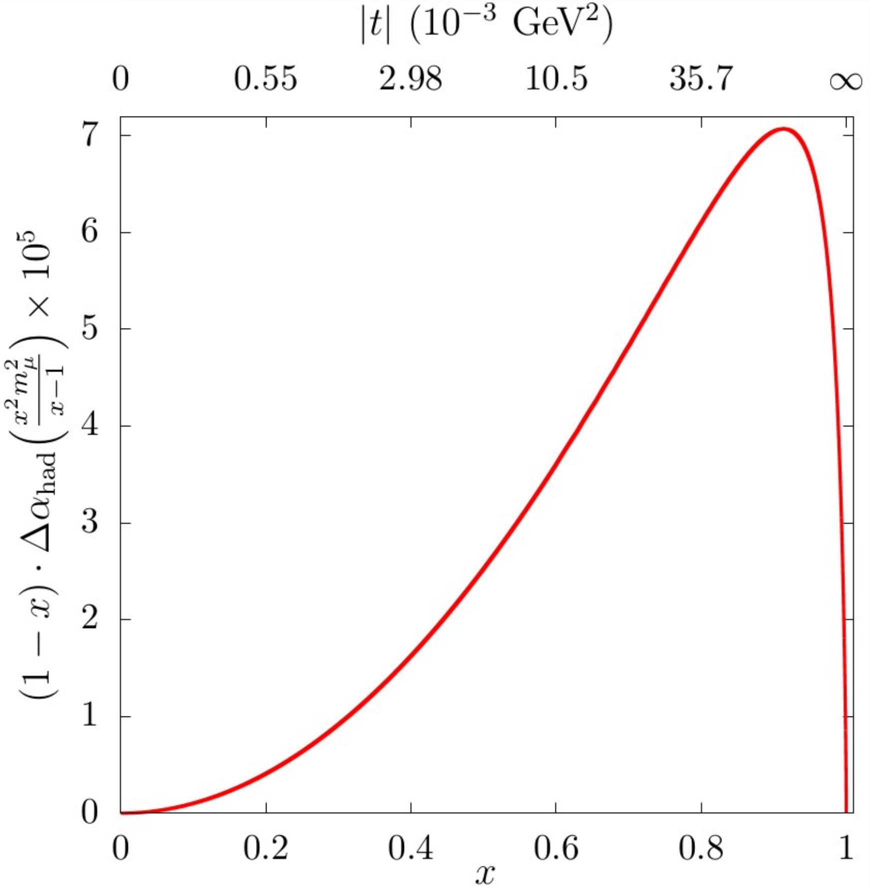
MUonE: a new independent evaluation of a_μ^{HLO}

Phys. Rep. C 3 (1972), 193

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$





Phys. Lett. B 746 (2015), 325

Based on the measurement of $\Delta\alpha_{\mathrm{had}}(t)$: hadronic contribution to the running of the electromagnetic coupling constant.

The MUonE experiment

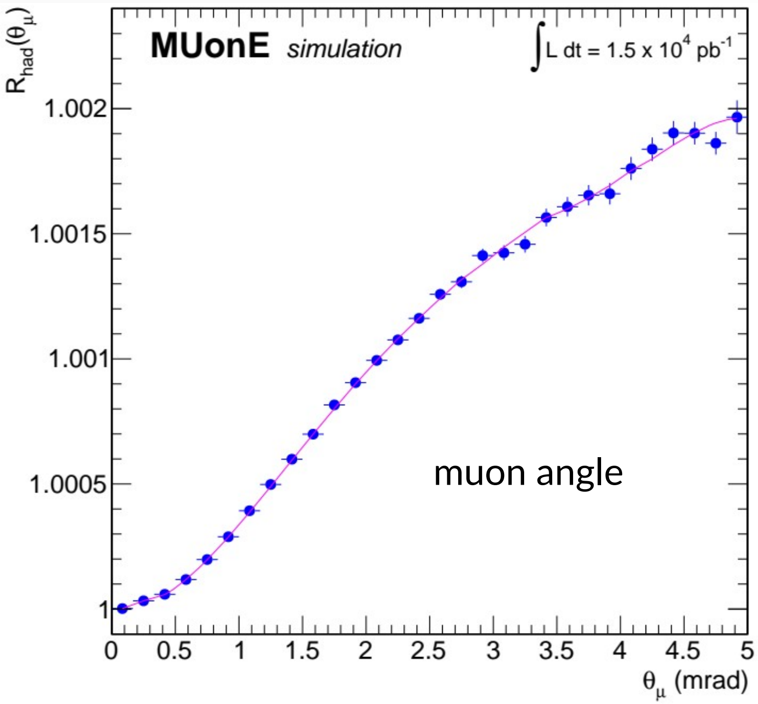
Extraction of $\Delta\alpha_{had}(t)$ from the shape of the $\mu e \to \mu e$ differential cross section

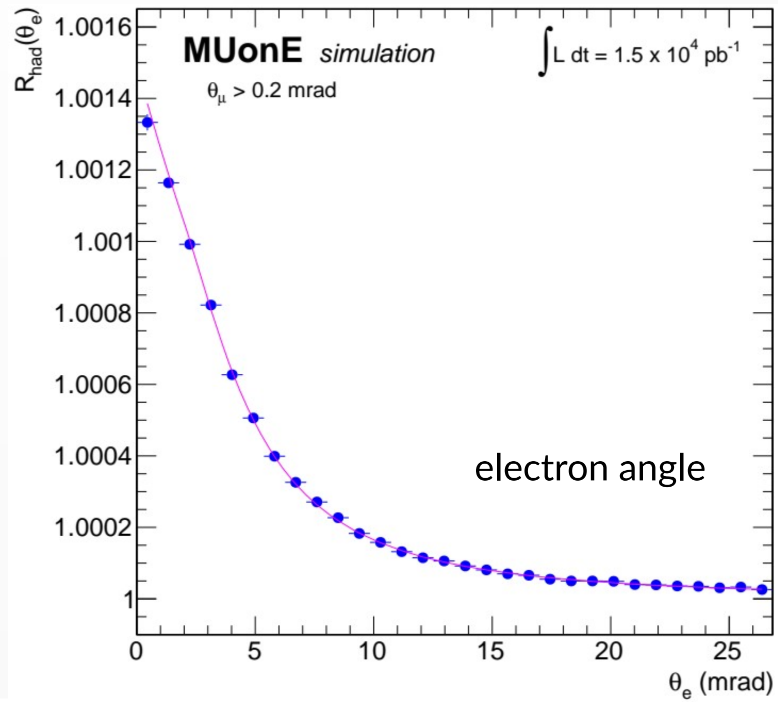
$$R_{\text{had}} = \frac{d\sigma_{\text{data}}(\Delta\alpha_{\text{had}})}{d\sigma_{\text{MC}}(\Delta\alpha_{\text{had}} = 0)} \sim 1 + 2\Delta\alpha_{\text{had}}(t)$$

From theoretical calculation

To be measured

$\Delta\alpha_{had}(t) < 10^{-3}$





$\Delta\alpha_{had}$ parameterization

Inspired from the 1 loop QED contribution of lepton pairs and top quark at $t < 0$

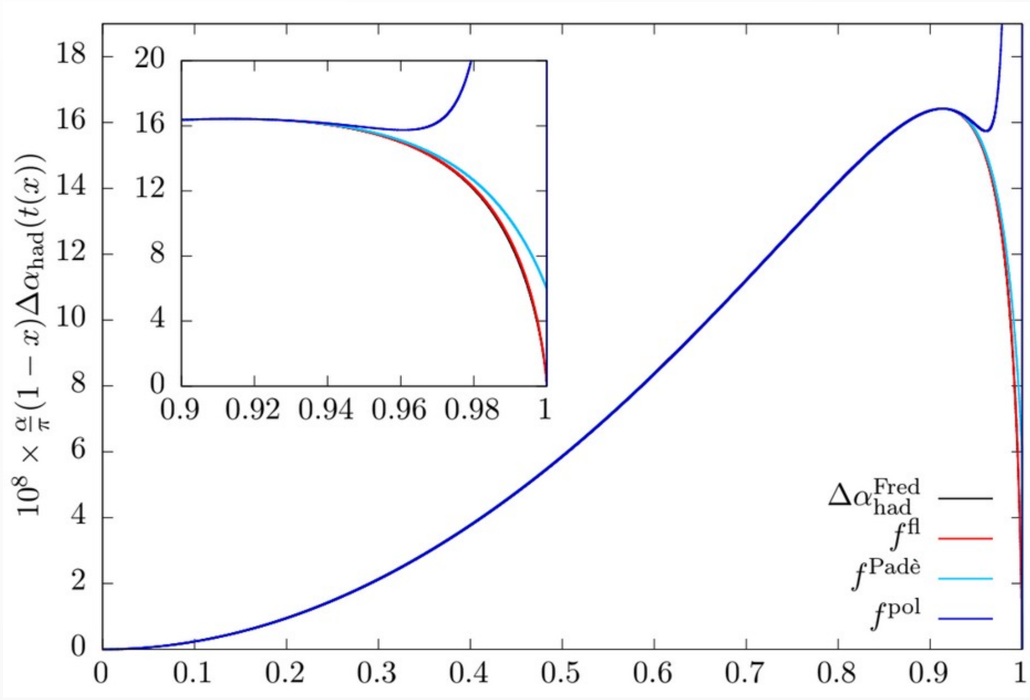
$$\Delta\alpha_{had}(t) = KM\left\{-\frac{5}{9} - \frac{4}{3}\frac{M}{t} + \left(\frac{4}{3}\frac{M^2}{t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\ln\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

2 parameters: K, M

Allows to calculate the full value of a_μ^{HLO}

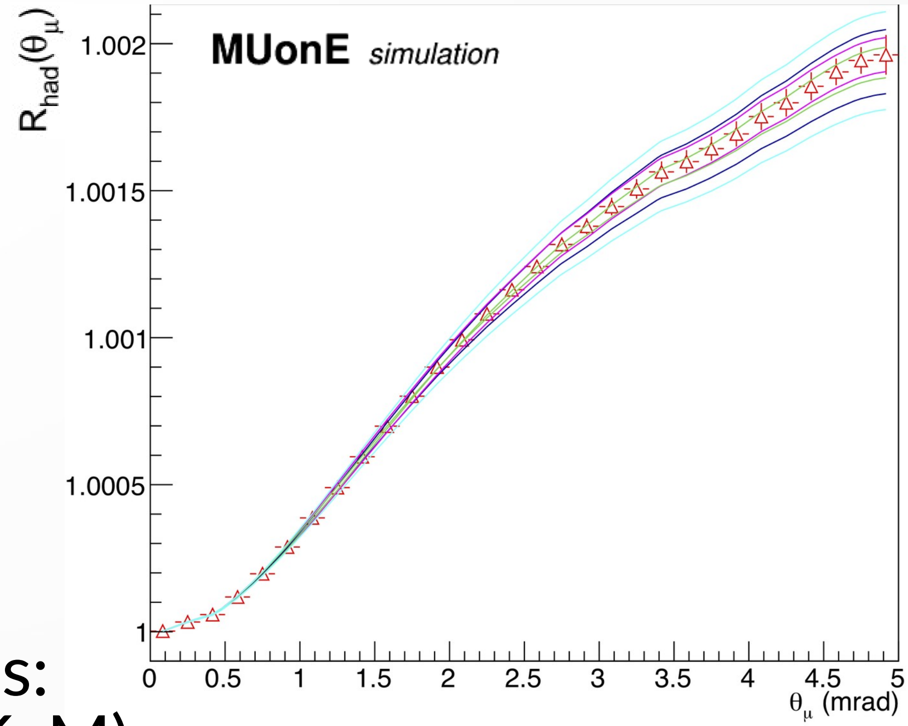
Dominant behaviour in the MUonE kinematic region:

$$\Delta\alpha_{had}(t) \simeq -\frac{1}{15}Kt$$



Template fit procedure

1. Grid of points (K, M)
   in the parameters space:
   cover a region ±5σ around
   the expected values
   (σ = expected uncertainty).



2. Ensemble of template distributions:
   each template is a different pair (K, M).

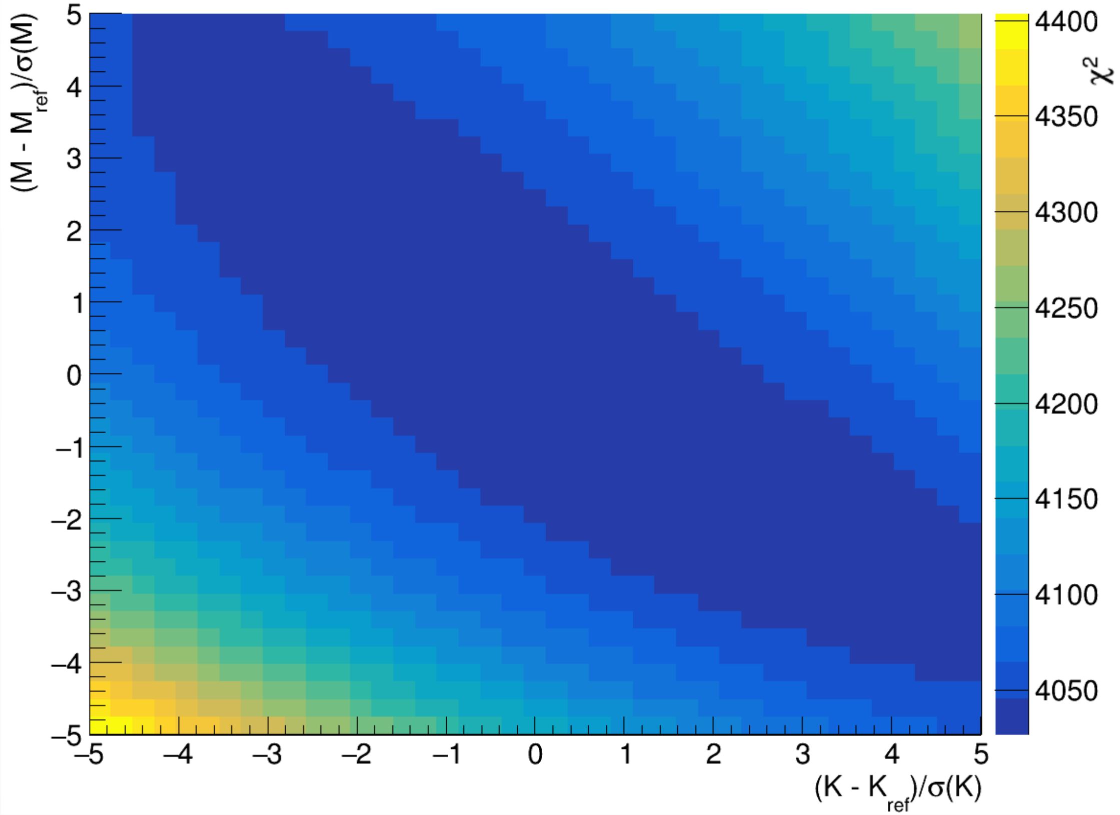
   Generate a unique distribution,
   then build the different templates by reweighting
   the events for any given pair (K, M).

Template fit procedure

3. Make a χ² (or likelihood) comparison between the data and each template distribution.

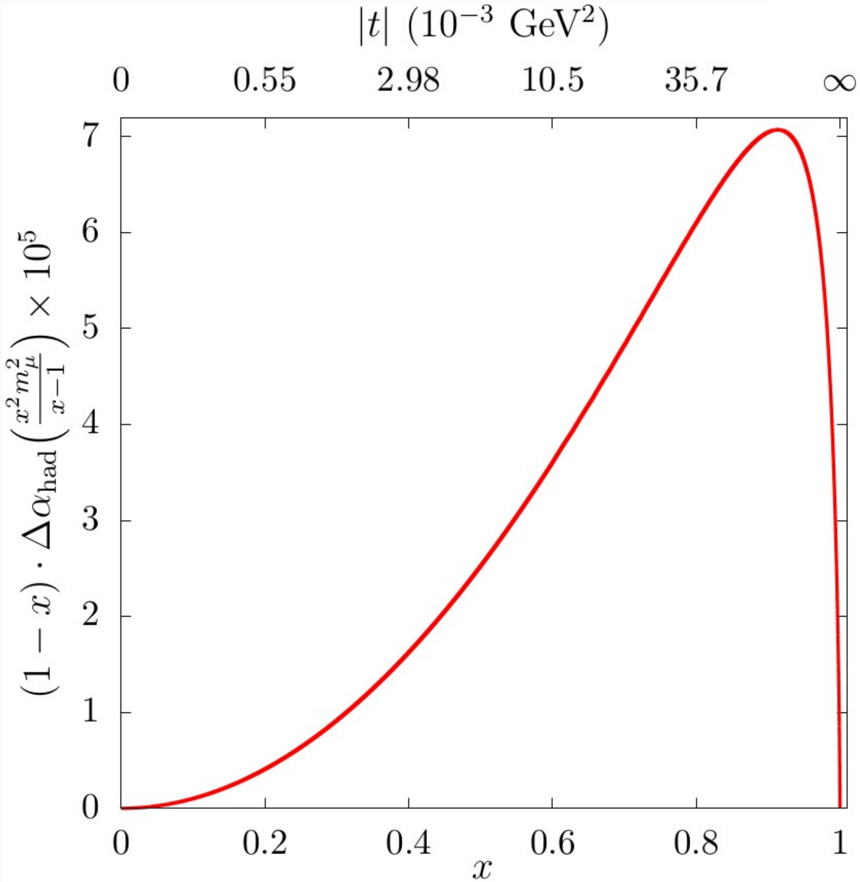
4. Perform a parabolic interpolation across the grid points to get the best fit parameters (K, M).

$$\chi^2 = \sum_i^{\text{bins}} \left(\frac{\text{data}_i - \text{templ}(K, M)_i}{\sigma_i^{\text{data}}} \right)^2$$



Compute a_μ^{HLO}

5. Input the best fit parameters (K, M) in the MUonE master integral



(K_{best}, M_{best})

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi}\int_0^1 dx(1-x)\underline{\Delta\alpha_{had}[t(x)]}$$

Results from a simulation with the expected final statistics (4×10^{12} elastic events):

$a_\mu^{HLO} = (688.8 \pm 2.4) \times 10^{-10}$

(0.35% accuracy)

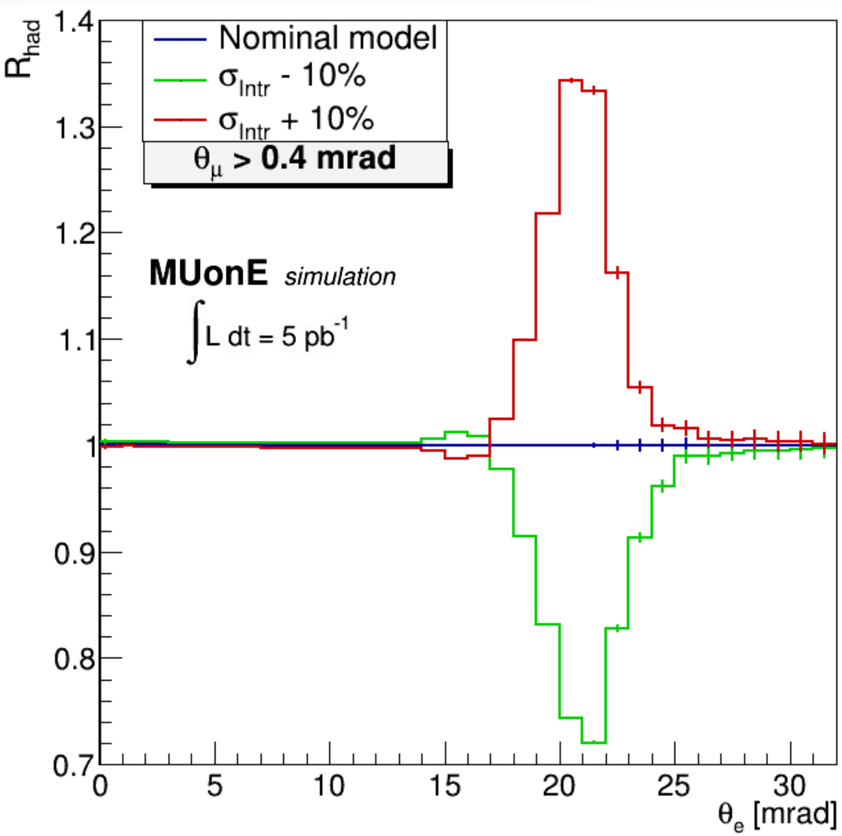
Input value

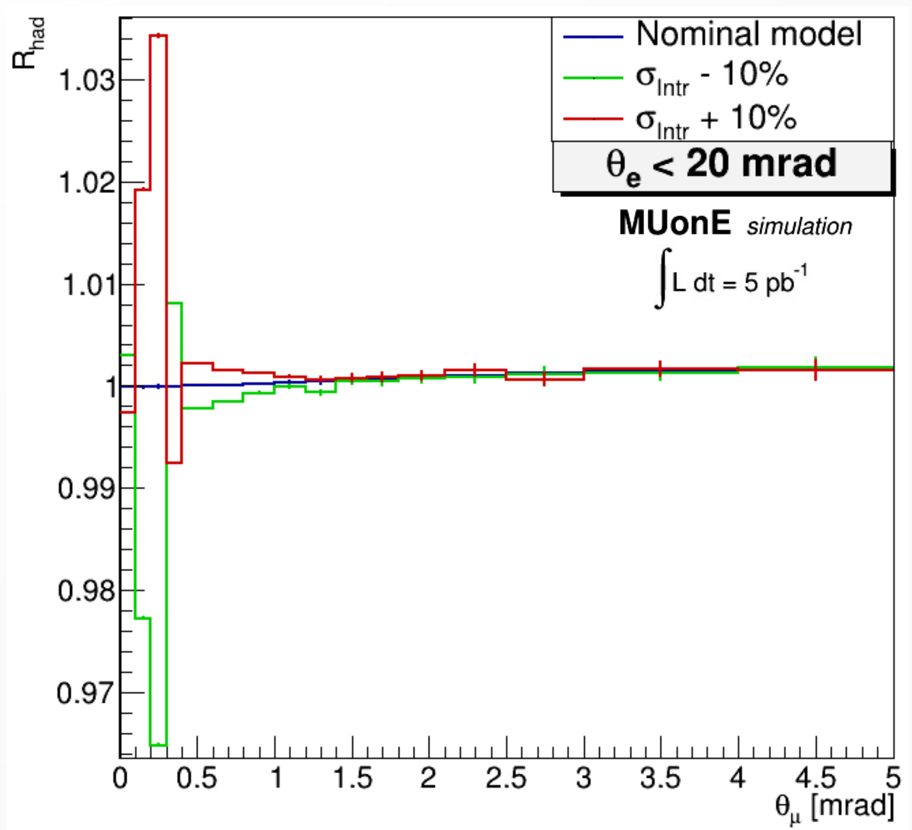
$a_\mu^{HLO} = 688.6 \times 10^{-10}$

The need of including systematic effects in the analysis

Some systematic effects can produce huge distortions in the shape of the elastic scattering cross section.

Example: ±10% error on the angular intrinsic resolution.





The need of including systematic effects in the analysis

What if systematic effects are not included in the template fit?

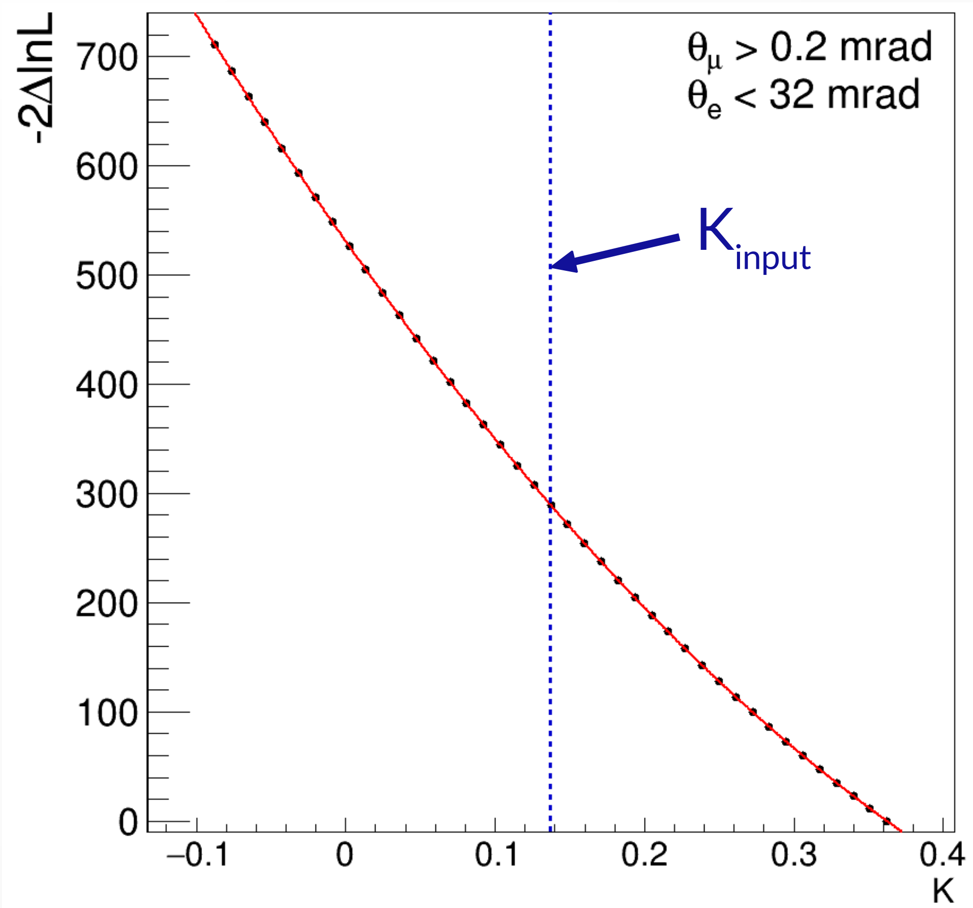
Simplified situation:

- 1 fit parameter (K).

$$\Delta\alpha_{had}(t) \simeq -\frac{1}{15}Kt$$

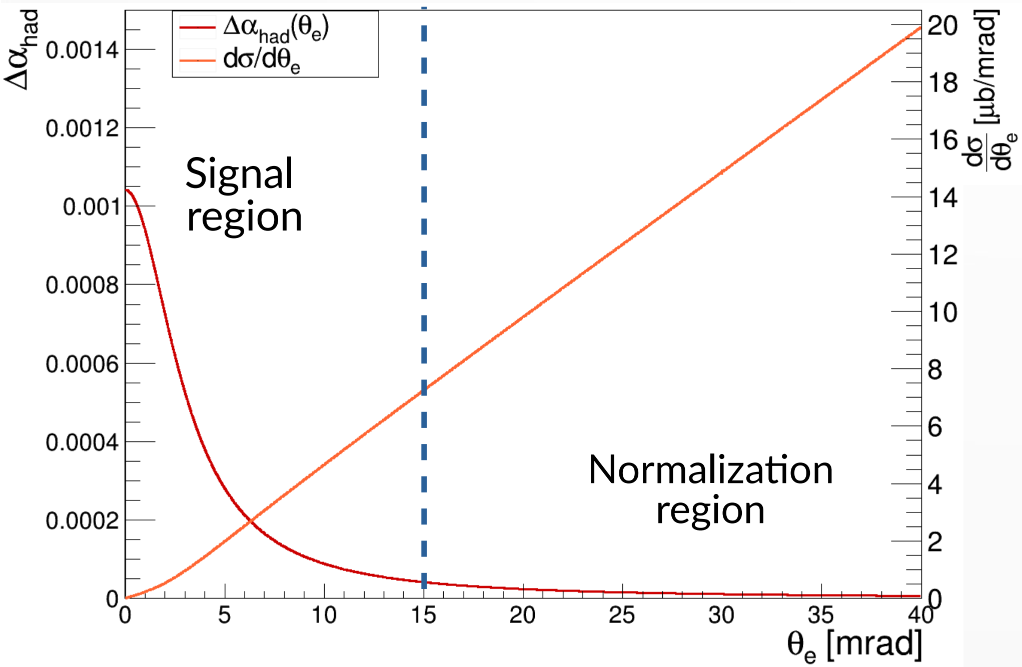
- L = 5 pb^{-1}.
  ~10^9 elastic events
  (~4000 times less than the final statistics)

- Shift in the pseudo-data sample:
  $\sigma_{Intr} \rightarrow \sigma_{Intr} + 5\%$.



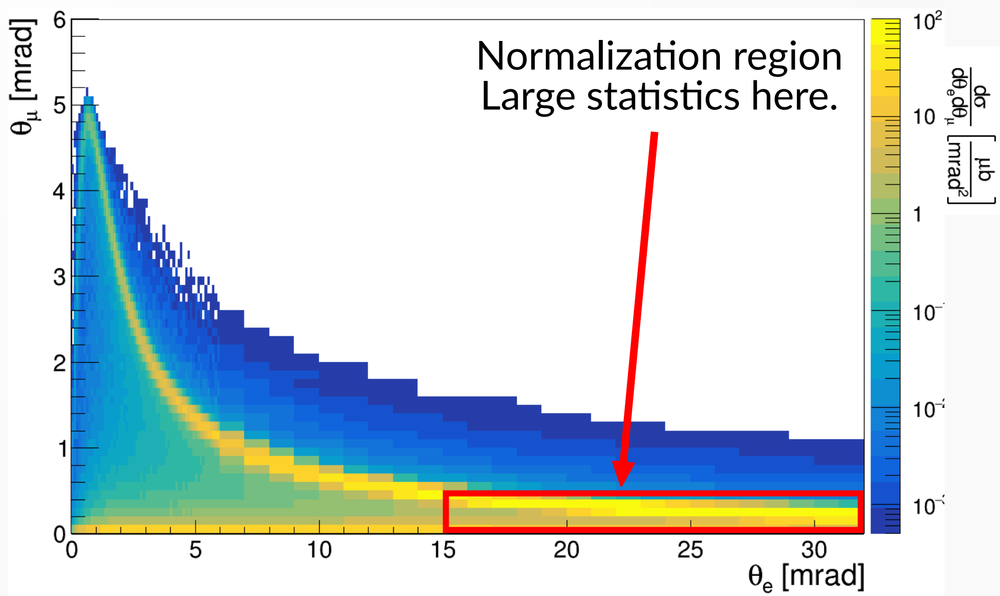
Strategy for the systematic effects

Main systematics have large effects in the normalization region. (no sensitivity to $\Delta\alpha_{\text{had}}$ here)



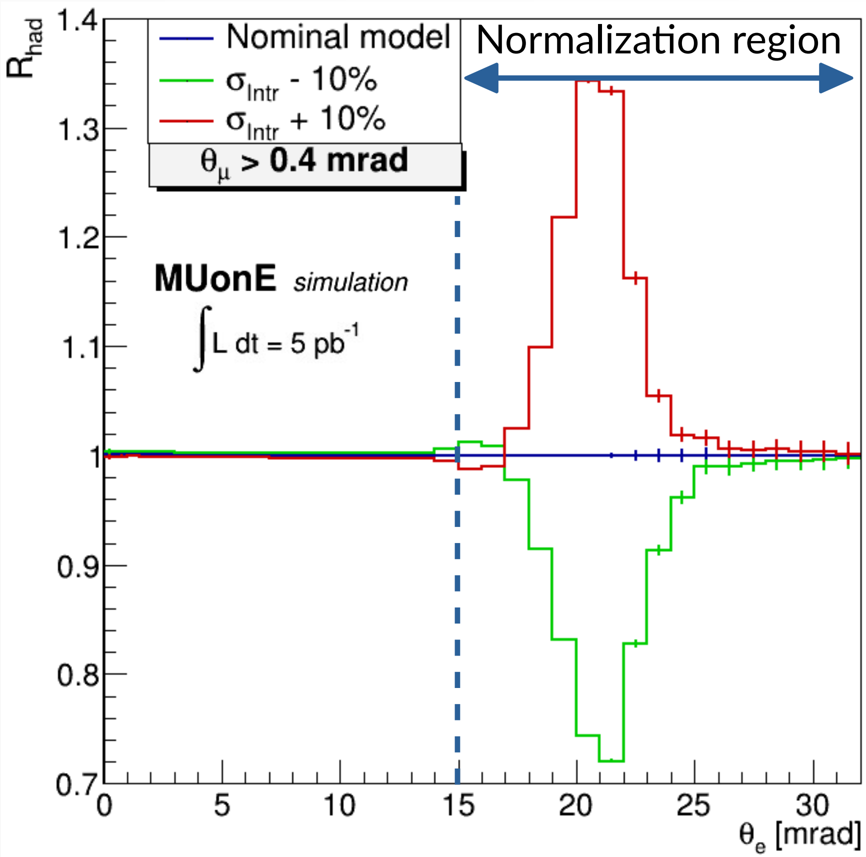
Promising strategy:

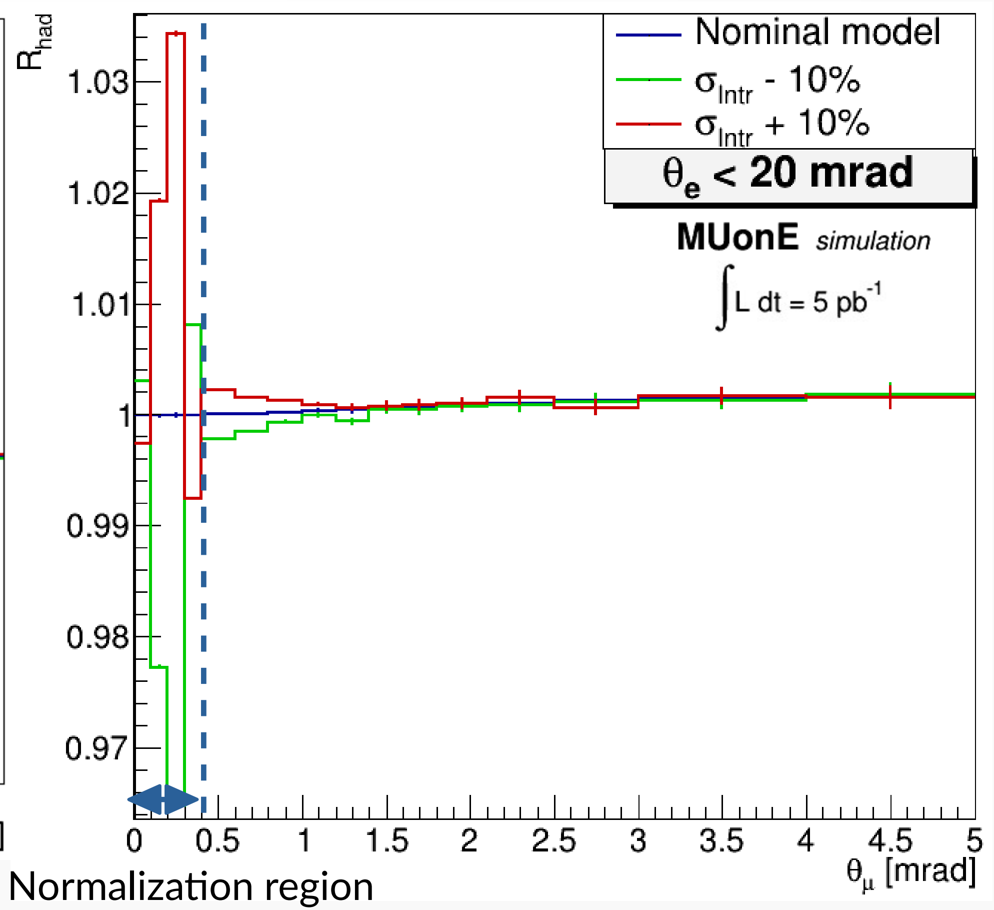
- Study the main systematics in the normalization region.
- Include residual systematics as nuisance parameters in a combined fit with signal.



Systematic error on the angular intrinsic resolution

±10% error on the angular intrinsic resolution.

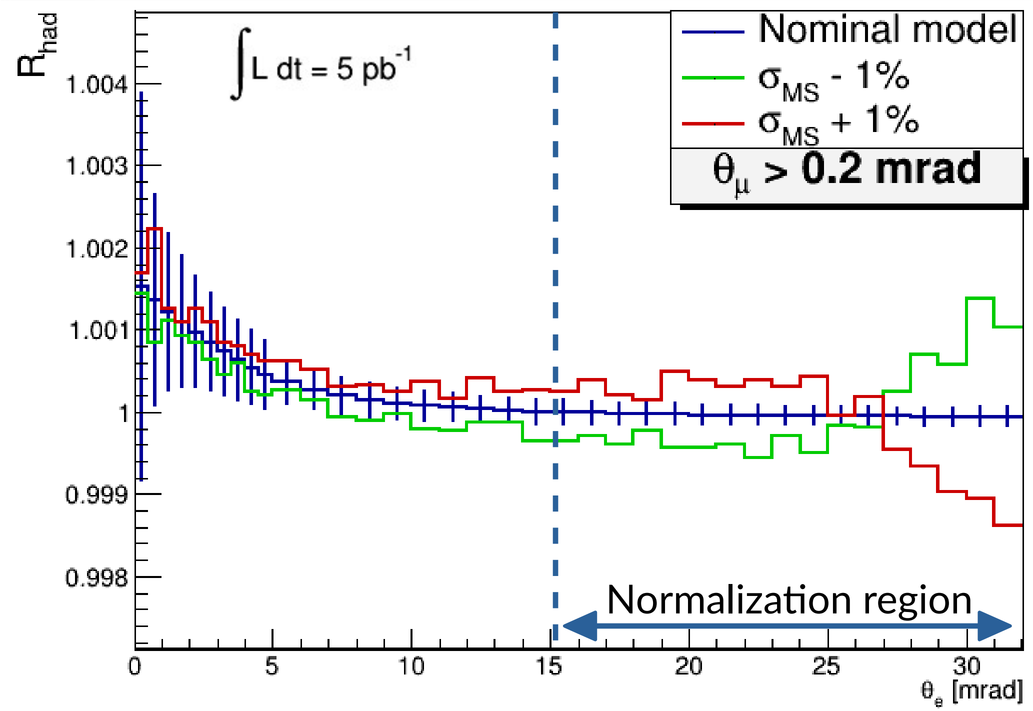


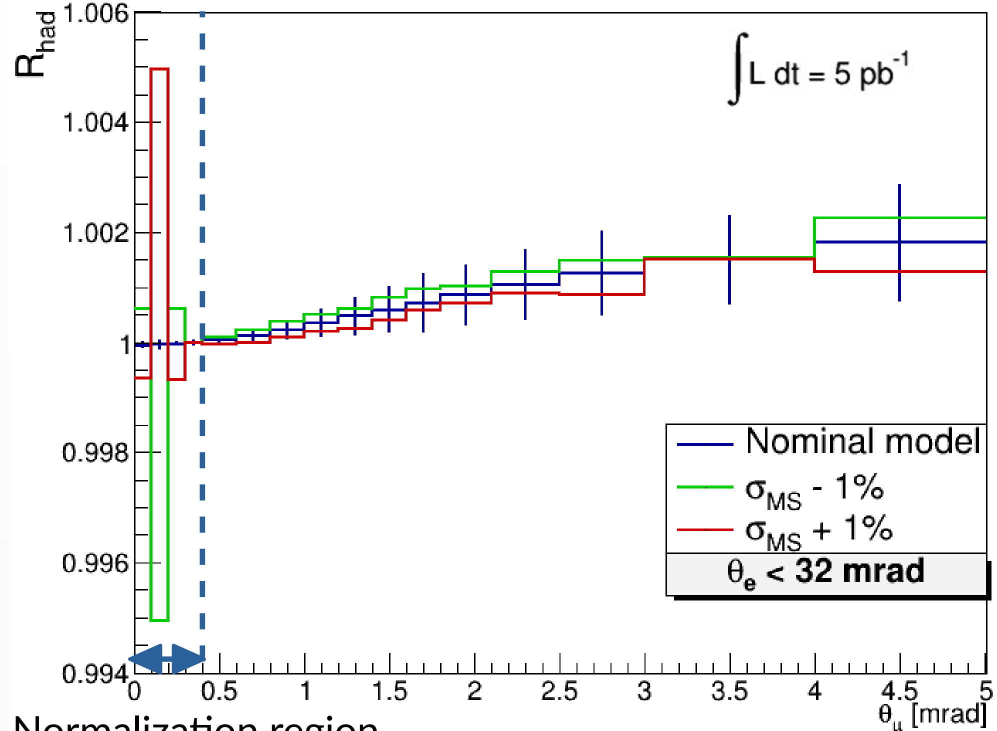


Systematic error on the multiple scattering

Expected precision on the multiple scattering model: ± 1%

G. Abbiendi et al JINST (2020) 15 P01017





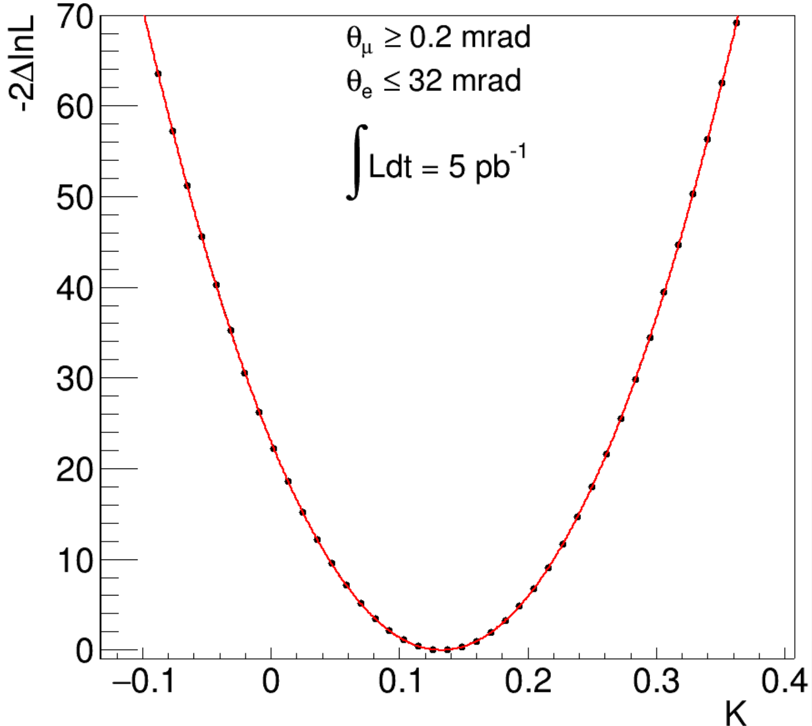
Normalization region

Combined fit
signal + systematics

- Include residual systematics as nuisance parameters in the fit.
- Simultaneous likelihood fit to K and systematics using the Combine tool.

- K_{ref} = 0.137
- shift MS: +0.5%
- shift intr. res: +5%
- shift E_{beam}: +6 MeV

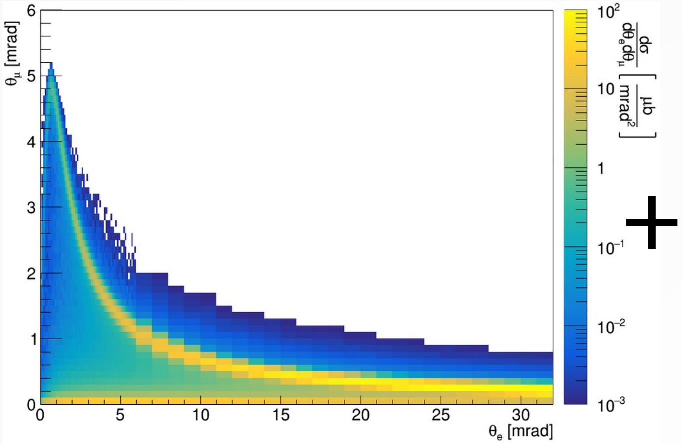
Selection cuts	Fit results
$\theta_e \leq 32\,\mathrm{mrad}$ $\theta_\mu \geq 0.2\,\mathrm{mrad}$	$K = 0.133 \pm 0.028$ $\mu_{\mathrm{MS}} = (0.47 \pm 0.03)\%$ $\mu_{\mathrm{Intr}} = (5.02 \pm 0.02)\%$ $\mu_{\mathrm{E_{Beam}}} = (6.5 \pm 0.5)\ \mathrm{MeV}$ $\nu = -0.001 \pm 0.003$



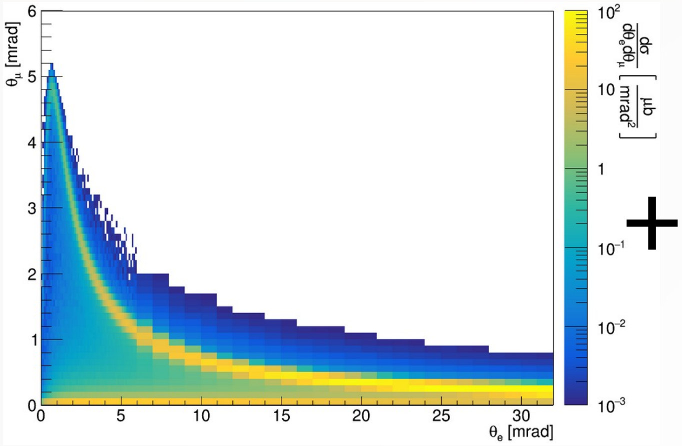
Similar results also for different selection cuts.

Next steps:

- Test the procedure for the MuonE design statistics.
- Improve the modelization of systematic effects.

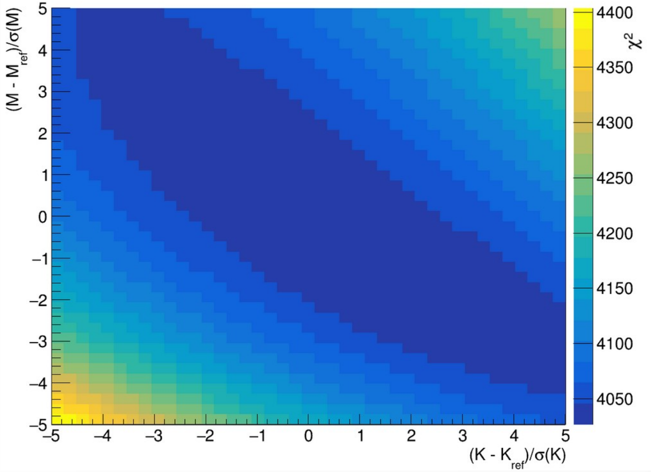


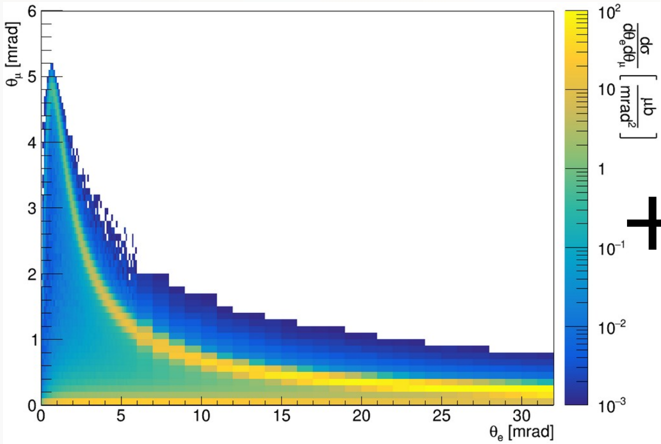
$+\ \Delta\alpha_{had}(t; K, M)$



+ $\Delta\alpha_{had}(t; K, M)$

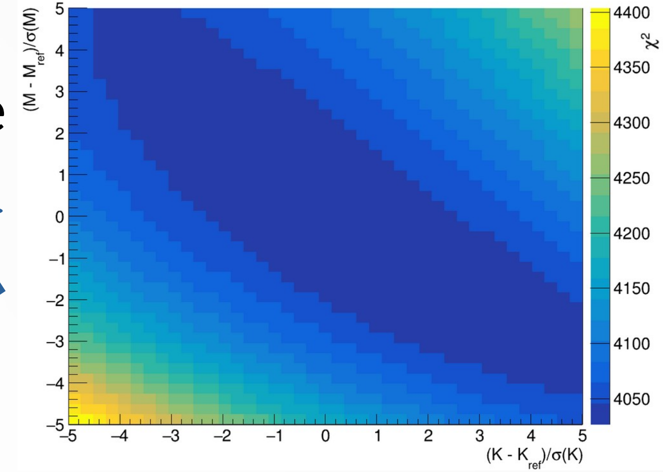
Template fit



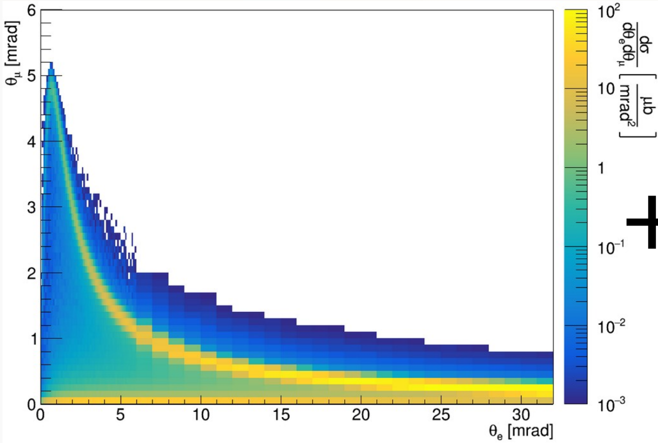


$+\ \Delta\alpha_{had}(t;\, K,\, M)$

Template fit

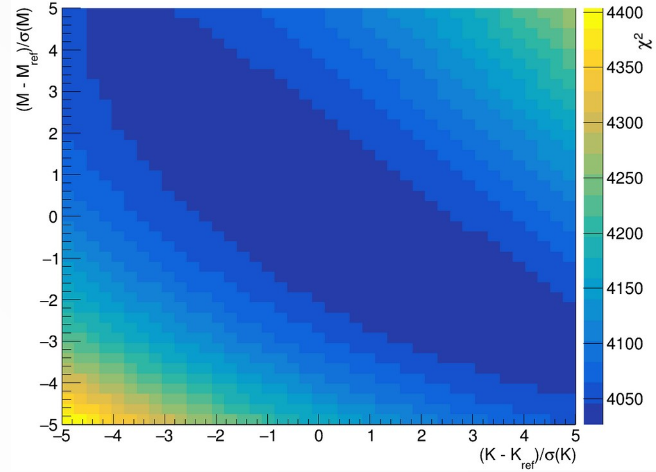


$\Delta\alpha_{had}(t;\, K_{best},\, M_{best})$



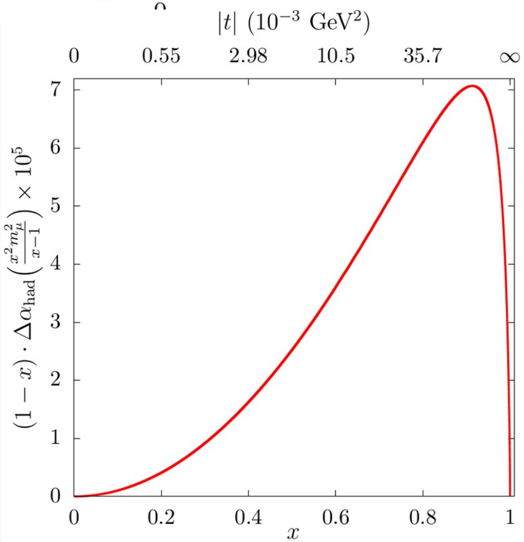
+ $\Delta\alpha_{had}(t; K, M)$

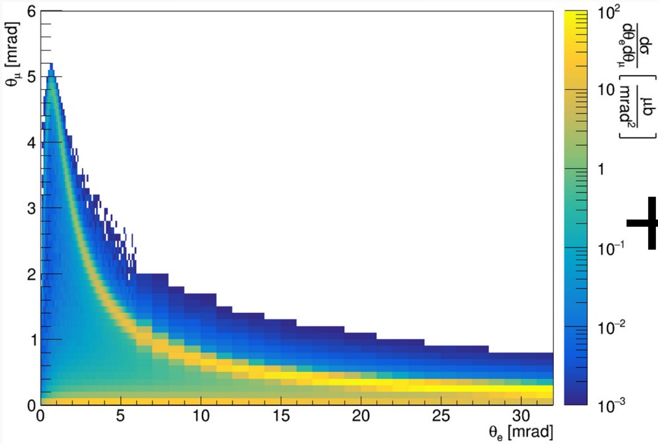
Template fit



$\Delta\alpha_{had}(t; K_{best}, M_{best})$

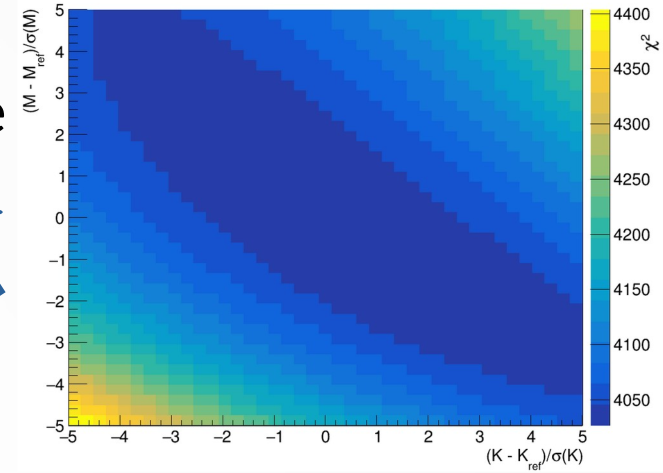
$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$





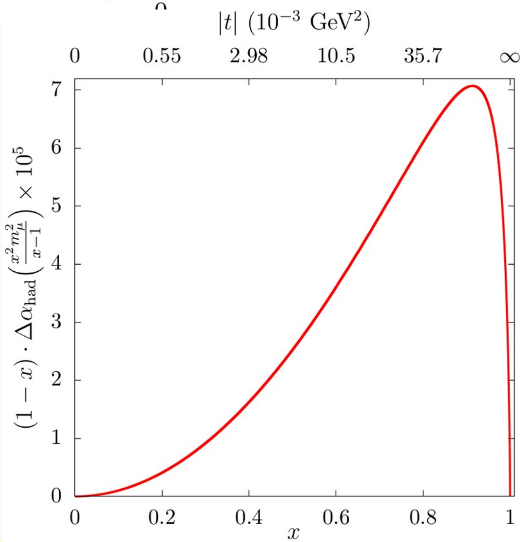
+ $\Delta\alpha_{had}(t; K, M)$

Template fit



$\Delta\alpha_{had}(t; K_{best}, M_{best})$

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{had}[t(x)]$$



?

Can we compute a_μ^{HLO} in a different way using MUonE data?

arXiv:2309.14205 [hep-ph]

submitted to PLB

An alternative evaluation of the leading-order hadronic contribution to the muon $g-2$ with MUonE

Fedor Ignatov[a], Riccardo Nunzio Pilato[a], Thomas Teubner[a], Graziano Venanzoni[a,b]

[a] *University of Liverpool, Foundation Building, Brownlow Hill, L69 3BX, Liverpool, United Kingdom*
[b] *INFN Sezione di Pisa, Largo Bruno Pontecorvo 3, 56127, Pisa, Italy*

An alternative method to compute a_μ^{HLO} with MUonE

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$$s_{\mathrm{th}} = m_{\pi^0}^2$$

An alternative method to compute a_μ^{HLO} with MUonE

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$$s_{\mathrm{th}} = m_{\pi^0}^2$$

$$s_0 \gtrsim (2\,\mathrm{GeV})^2$$

$$\frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}K(s)R(s) \quad + \quad \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}K(s)R(s)$$

pQCD

$$-\mathrm{Im}\Pi_{had}(s) = \frac{\alpha}{3}R(s)$$

Low energy integral

S. Bodenstein et al, Phys. Rev. D 85 (2012)
C.A. Dominguez et al, Phys. Rev. D 96 (2017)

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K(s) \frac{\mathrm{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{\mathrm{Im}\Pi_{had}(s)}{\pi} + \int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{\mathrm{Im}\Pi_{had}(s)}{\pi}$$

Low energy integral

S. Bodenstein et al, Phys. Rev. D 85 (2012)
C. A. Dominguez et al, Phys. Rev. D 96 (2017)

$$\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K(s) \frac{\text{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] \frac{\text{Im}\Pi_{had}(s)}{\pi} + \boxed{\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} K_1(s) \frac{\text{Im}\Pi_{had}(s)}{\pi}}$$

$$K_1(s) = a_0 s + \sum_{n=1}^{3} \frac{a_n}{s^n}$$

$K_1(s)$ approximates $K(s)$ for $s < s_0$.
Meromorphic function:
no cuts, poles in $s = 0$.

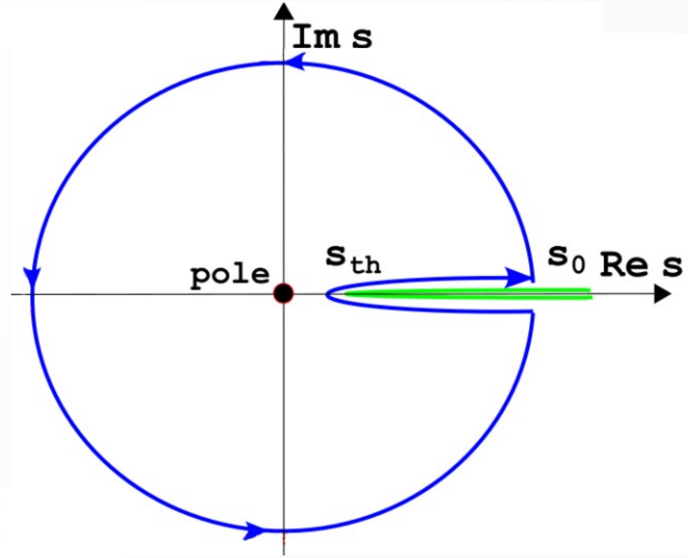
Two different techniques to get $K_1(s)$:

1) Least squares minimization

2) Minimize $\int_{s_{\text{th}}}^{s_0} \frac{ds}{s} |K(s) - K_1(s)| R(s)$

Low energy integral

S. Bodenstein et al, Phys. Rev. D 85 (2012)
C.A. Dominguez et al, Phys. Rev. D 96 (2017)



Use Cauchy's theorem

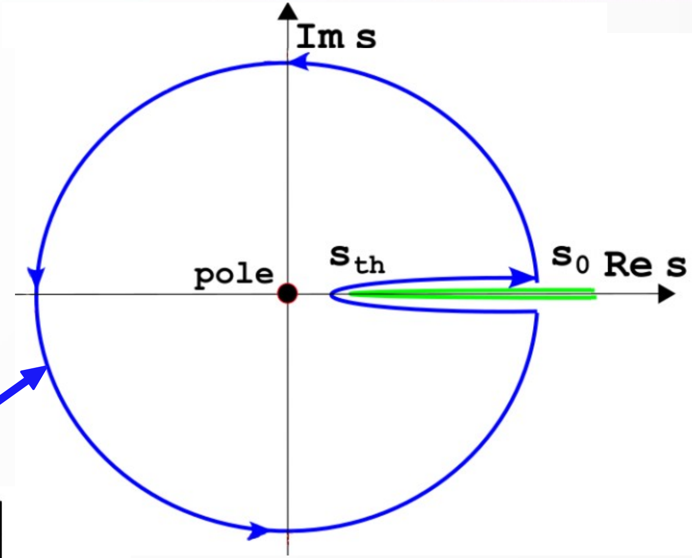
$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{{\rm Im}\Pi_{had}(s)}{\pi} =$$

$$\text{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} - \frac{1}{2\pi i}\oint_{|s|=s_0} \frac{ds}{s} K_1(s)\Pi_{had}(s)\Big|_{\rm pQCD}$$

Low energy integral

S. Bodenstein et al, Phys. Rev. D 85 (2012)
C.A. Dominguez et al, Phys. Rev. D 96 (2017)

Use Cauchy's theorem



$$\int_{s_{\rm th}}^{s_0} \frac{ds}{s} K_1(s) \frac{\mathrm{Im}\Pi_{had}(s)}{\pi} =$$

$$\mathrm{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} - \frac{1}{2\pi i}\oint_{|s|=s_0} \frac{ds}{s} K_1(s)\Pi_{had}(s)\Big|_{\rm pQCD}$$

$$\mathrm{Res}\left[\Pi_{had}(s)\frac{K_1(s)}{s}\right]_{s=0} = \sum_{n=1}^{3} \frac{c_n}{n!}\frac{d^{(n)}}{ds^n}\Pi_{had}(s)\Big|_{s=0} = \sum_{n=1}^{3} \frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

From MUonE

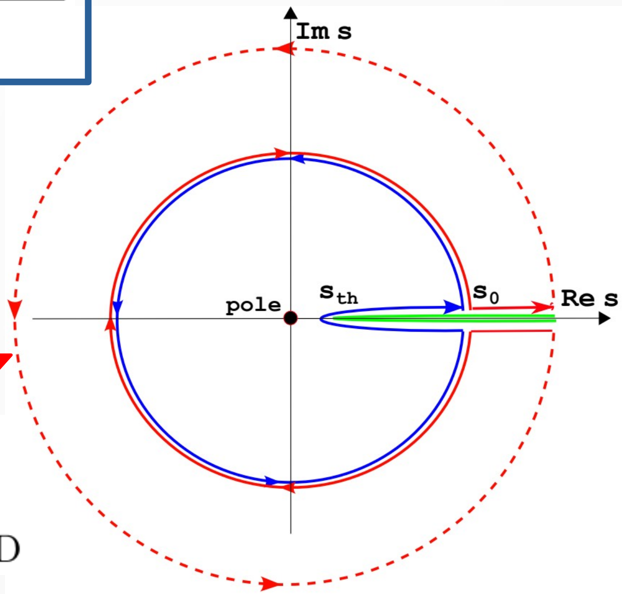
High energy integral

Similar strategy for the high energy part

$$\int_{s_0}^{\infty} \frac{ds}{s} K(s) \frac{\mathrm{Im}\Pi_{had}(s)}{\pi} =$$

$$\int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] \frac{\mathrm{Im}\Pi_{had}(s)}{\pi} + \boxed{\int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\mathrm{Im}\Pi_{had}(s)}{\pi}}$$

$$\tilde{K}_1(s) = K_1(s) - c_0 s$$



$$\int_{s_0}^{\infty} \frac{ds}{s} \tilde{K}_1(s) \frac{\mathrm{Im}\Pi_{had}(s)}{\pi} = \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} \tilde{K}_1(s) \Pi_{had}(s) \Big|_{\mathrm{pQCD}}$$

Compute a_μ^{HLO}

Rearranging the previous equations...

$$a_\mu^{\mathrm{HLO}} = a_\mu^{\mathrm{HLO\ (I)}} + a_\mu^{\mathrm{HLO\ (II)}} + a_\mu^{\mathrm{HLO\ (III)}} + a_\mu^{\mathrm{HLO\ (IV)}}$$

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi} \sum_{n=1}^{3} \frac{c_n}{n!} \frac{d^{(n)}}{dt^n} \Delta\alpha_{had}(t)\Big|_{t=0}$$

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi} \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s} c_0 s\ \Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2} \int_{s_{\mathrm{th}}}^{s_0} \frac{ds}{s} [K(s) - K_1(s)] R(s)$$

$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2} \int_{s_0}^{\infty} \frac{ds}{s} [K(s) - \tilde{K}_1(s)] R(s)$$

Compute a_μ^{HLO}

Rearranging the previous equations…

$$a_\mu^{\mathrm{HLO}} = a_\mu^{\mathrm{HLO\ (I)}} + a_\mu^{\mathrm{HLO\ (II)}} + a_\mu^{\mathrm{HLO\ (III)}} + a_\mu^{\mathrm{HLO\ (IV)}}$$

99%

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

MUonE

$$a_\mu^{\mathrm{HLO\ (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\ \Pi_{had}(s)\Big|_{\mathrm{pQCD}}$$

$$a_\mu^{\mathrm{HLO\ (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\mathrm{th}}}^{s_0}\frac{ds}{s}[K(s) - K_1(s)]R(s)$$

$$a_\mu^{\mathrm{HLO\ (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s) - \tilde{K}_1(s)]R(s)$$

Compute $a_\mu^{\rm HLO}$

Rearranging the previous equations...

$$a_\mu^{\rm HLO} = a_\mu^{\rm HLO\ (I)} + a_\mu^{\rm HLO\ (II)} + a_\mu^{\rm HLO\ (III)} + a_\mu^{\rm HLO\ (IV)}$$

99%

$$a_\mu^{\rm HLO\ (I)} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

MUonE

1%

$$a_\mu^{\rm HLO\ (II)} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\ \Pi_{had}(s)\Big|_{\rm pQCD}$$

$$a_\mu^{\rm HLO\ (III)} = \frac{\alpha^2}{3\pi^2}\int_{s_{\rm th}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]R(s)$$

$$a_\mu^{\rm HLO\ (IV)} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]R(s)$$

Time-like data
+
pQCD

$a_\mu^{\mathrm{HLO\ (I)}}$ from MUonE data

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

The relevant quantities are the derivatives of $\Delta\alpha_{had}(t)$ at $t = 0$.

$a_\mu^{\mathrm{HLO\ (I)}}$ from MUonE data

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

The relevant quantities are the derivatives of $\Delta\alpha_{had}(t)$ at t = 0.

Try different parameterizations to fit MUonE data

(max 3 fit parameters, due to the statistics collected by MUonE)

$$\Delta\alpha_{had}(t) = KM\left\{-\frac{5}{9}-\frac{4}{3}\frac{M}{t}+\left(\frac{4}{3}\frac{M^2}{t^2}+\frac{M}{3t}-\frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\ln\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

Lepton-like

$$\Delta\alpha_{\mathrm{had}}(t) = P_1 t\frac{1+P_2 t}{1+P_3 t}$$

Padé approximant

$$\Delta\alpha_{\mathrm{had}}(t) = P_1 t + P_2 t^2 + P_3 t^3$$

3° polynomial

$a_\mu^{\mathrm{HLO\ (I)}}$ from MUonE data

Reconstruction approximants

D. Greynat, E. de Rafael, JHEP 2022 (5)

$$\Delta\alpha_{\mathrm{had}}(t) = \sum_{n=1}^{\mathrm{N}} \mathscr{A}(n,\mathrm{L}) \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right)^n + \sum_{p=1}^{\lfloor \frac{\mathrm{L}+1}{2} \rfloor} \mathscr{B}(2p-1)\, \mathrm{Li}_{2p-1} \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right)$$

$$\Delta\alpha_{\mathrm{had}}(t) = A_1 \mathscr{S}_1 + A_2 \mathscr{S}_2 + A_3 \mathscr{S}_3 + B_1 \mathscr{L}_1$$

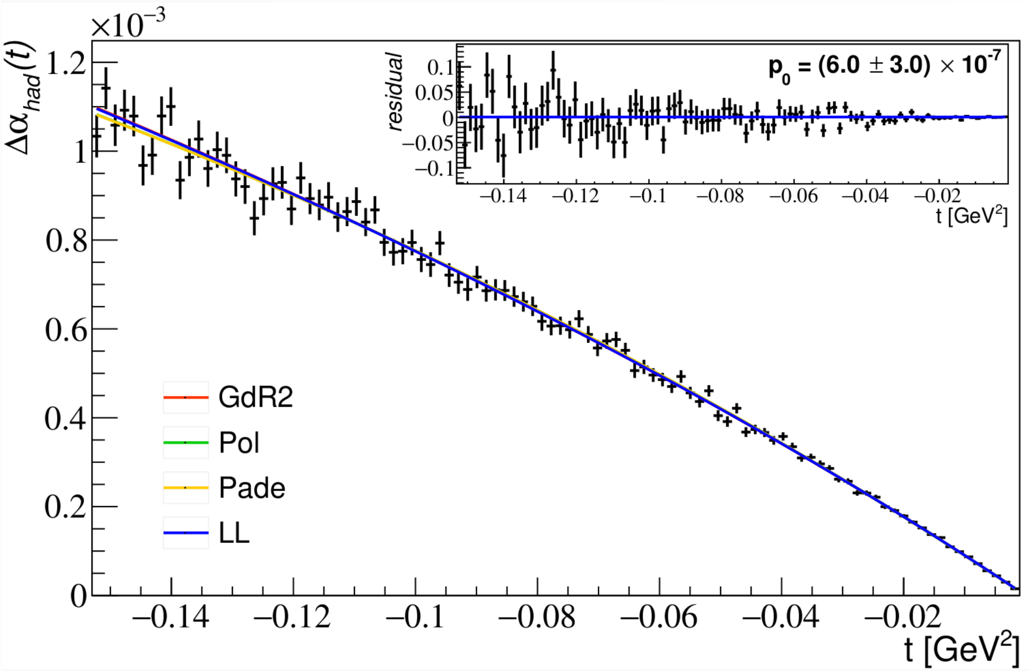
$$\mathscr{S}_i = \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right); \qquad A_i = \mathscr{A}(i,1) \quad i = 1,\ 2,\ 3$$

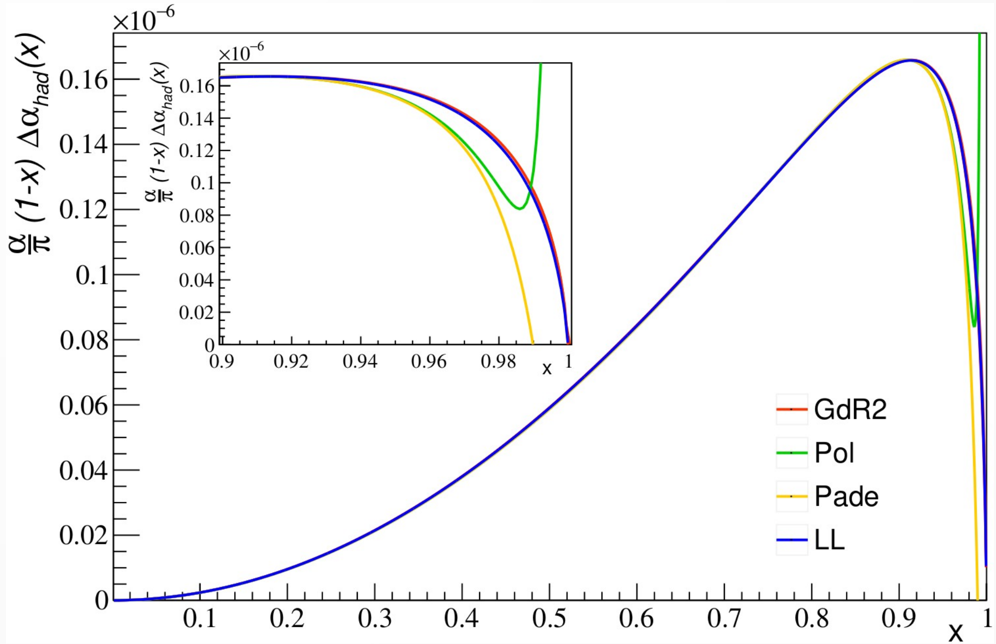
$$\mathscr{L}_1 = \mathrm{Li}_1 \left(\frac{\sqrt{1-\frac{t}{t_0}}-1}{\sqrt{1-\frac{t}{t_0}}+1} \right); \qquad B_1 = \mathscr{B}(1)$$

Tested L = 1, N = 3
Several variants with different number of free parameters

Fit the MUonE data

Simplified fit: simulate the MUonE signal using time-like compilations of $\Delta\alpha_{had}$. Error bars according to the MUonE final statistics.





$a_\mu^{\mathrm{HLO\ (I)}}$: results

$$a_\mu^{\mathrm{HLO\ (I)}} = -\frac{\alpha}{\pi}\sum_{n=1}^{3}\frac{c_n}{n!}\frac{d^{(n)}}{dt^n}\Delta\alpha_{had}(t)\Big|_{t=0}$$

Minimization I	$a_\mu^{\mathrm{HLO\ (I)}}$ (10^{-10})							
s_0 values	LL	Padé	Pol	GdR1	GdR2	GdR3	GdR4	GdR5
$(1.8\ \mathrm{GeV})^2$	688.7±2.2	688.7±2.9	688.9±2.9	688.2±2.2	688.0±2.2	688.0±2.2	687.0±2.3	688.0±2.6
$(2.5\ \mathrm{GeV})^2$	691.7±2.2	691.6±3.0	691.8±3.0	691.0±2.2	690.8±2.2	690.8±2.2	689.8±2.3	690.9±2.9
$(12\ \mathrm{GeV})^2$	696.3±2.2	696.3±3.0	696.3±3.2	695.4±2.2	695.3±2.2	695.2±2.2	694.1±2.3	695.3±3.7
Minimization II	$a_\mu^{\mathrm{HLO\ (I)}}$ (10^{-10})							
s_0 values	LL	Padé	Pol	GdR1	GdR2	GdR3	GdR4	GdR5
$(1.8\ \mathrm{GeV})^2$	688.5±2.2	688.1±4.2	689.8±3.3	688.3±2.1	688.4±2.1	688.6±2.2	687.1±2.1	688.4±5.8
$(2.5\ \mathrm{GeV})^2$	689.5±2.2	689.1±4.2	690.8±3.3	689.3±2.1	689.4±2.1	689.6±2.2	688.1±2.1	689.4±5.7
$(12\ \mathrm{GeV})^2$	690.3±2.1	689.9±4.6	691.6±3.6	689.8±2.1	690.1±2.2	690.2±2.2	688.6±2.1	690.0±5.9

$a_\mu^{\mathrm{HLO\ (I)}}$ ~ 99% of the total value.

(a_μ^{HLO} = 695.1×10^{-10} input from time-like data).

$a_\mu^{\text{HLO (II, III, IV)}}$: results

$$a_\mu^{\text{HLO (II)}} = \frac{\alpha}{\pi}\frac{1}{2\pi i}\oint_{|s|=s_0}\frac{ds}{s}c_0 s\,\Pi_{had}(s)\Big|_{\text{pQCD}}$$

$$a_\mu^{\text{HLO (III)}} = \frac{\alpha^2}{3\pi^2}\int_{s_{\text{th}}}^{s_0}\frac{ds}{s}[K(s)-K_1(s)]R(s)$$

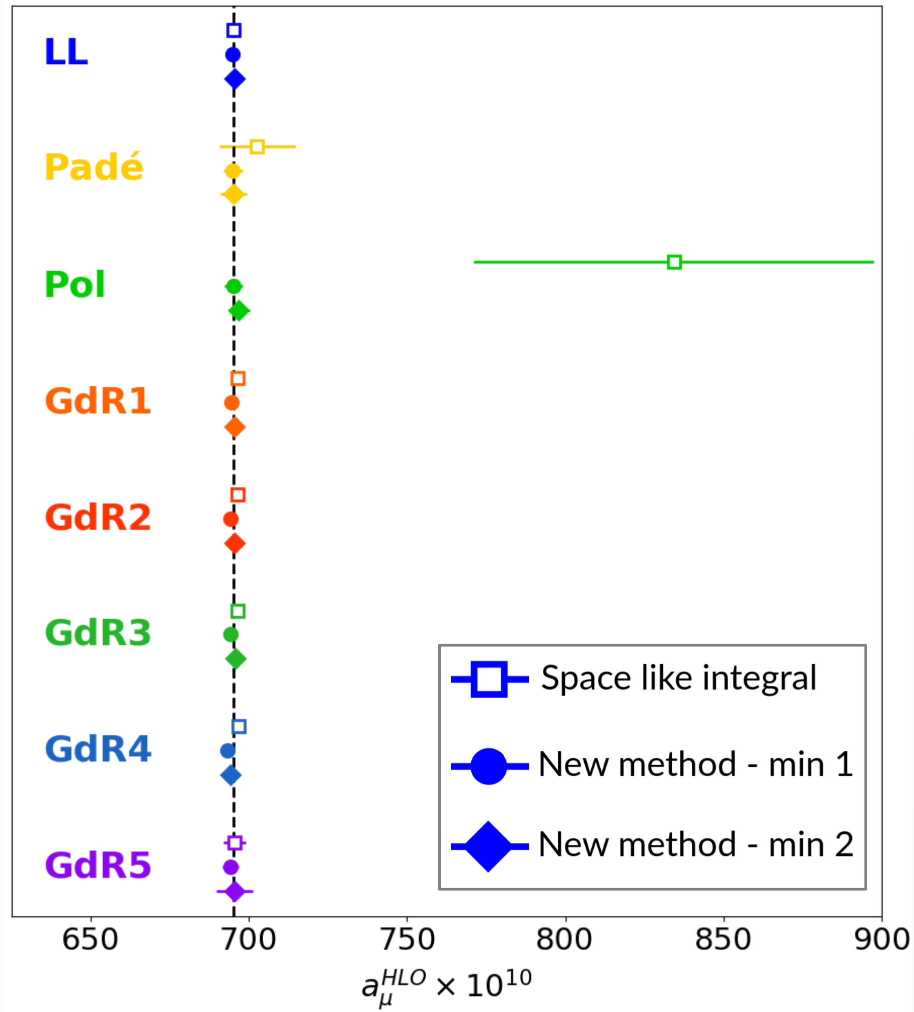
$$a_\mu^{\text{HLO (IV)}} = \frac{\alpha^2}{3\pi^2}\int_{s_0}^{\infty}\frac{ds}{s}[K(s)-\tilde{K}_1(s)]R(s)$$

Minimization I			
s_0 values	$a_\mu^{\text{HLO (II)}}$ (10^{-10})	$a_\mu^{\text{HLO (III)}}$ (10^{-10})	$a_\mu^{\text{HLO (IV)}}$ (10^{-10})
$(1.8\text{ GeV})^2$	2.94 ± 0.04	0.43 ± 0.01	2.95 ± 0.05
$(2.5\text{ GeV})^2$	1.84 ± 0.01	-0.34 ± 0.01	1.79 ± 0.02
$(12\text{ GeV})^2$	0.208 ± 0.001	-1.695 ± 0.035	0.079 ± 0.001
Minimization II			
s_0 values	$a_\mu^{\text{HLO (II)}}$ (10^{-10})	$a_\mu^{\text{HLO (III)}}$ (10^{-10})	$a_\mu^{\text{HLO (IV)}}$ (10^{-10})
$(1.8\text{ GeV})^2$	3.23 ± 0.04	0.91 ± 0.02	3.00 ± 0.05
$(2.5\text{ GeV})^2$	2.54 ± 0.01	1.52 ± 0.02	1.96 ± 0.02
$(12\text{ GeV})^2$	0.360 ± 0.001	4.85 ± 0.05	0.096 ± 0.001

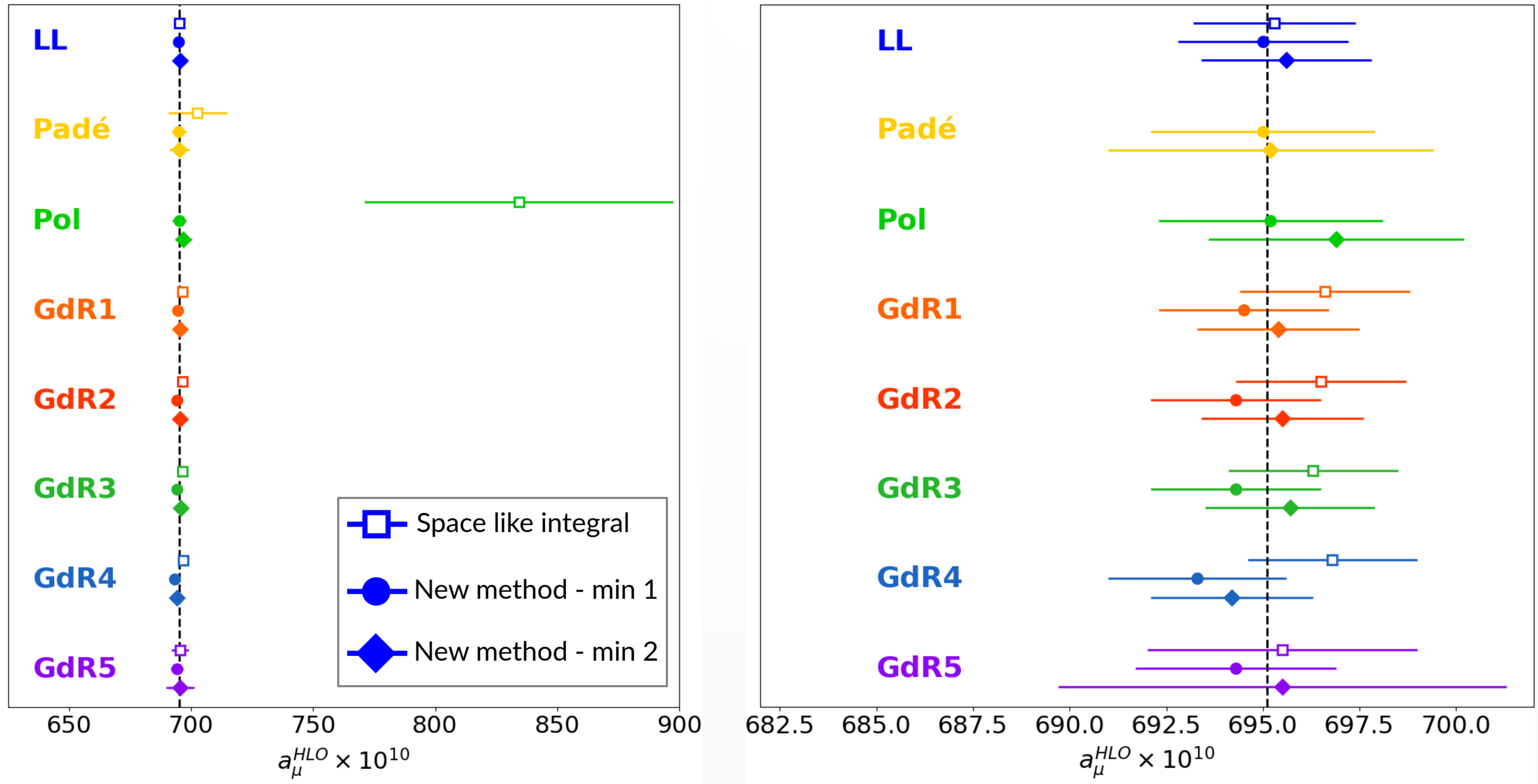
$a_\mu^{\text{HLO (II+III+IV)}}$ ~ 1% of the total value.

($a_\mu^{\text{HLO}} = 695.1\times10^{-10}$ input from time-like data).

Results a_μ^{HLO}



Results a_μ^{HLO}



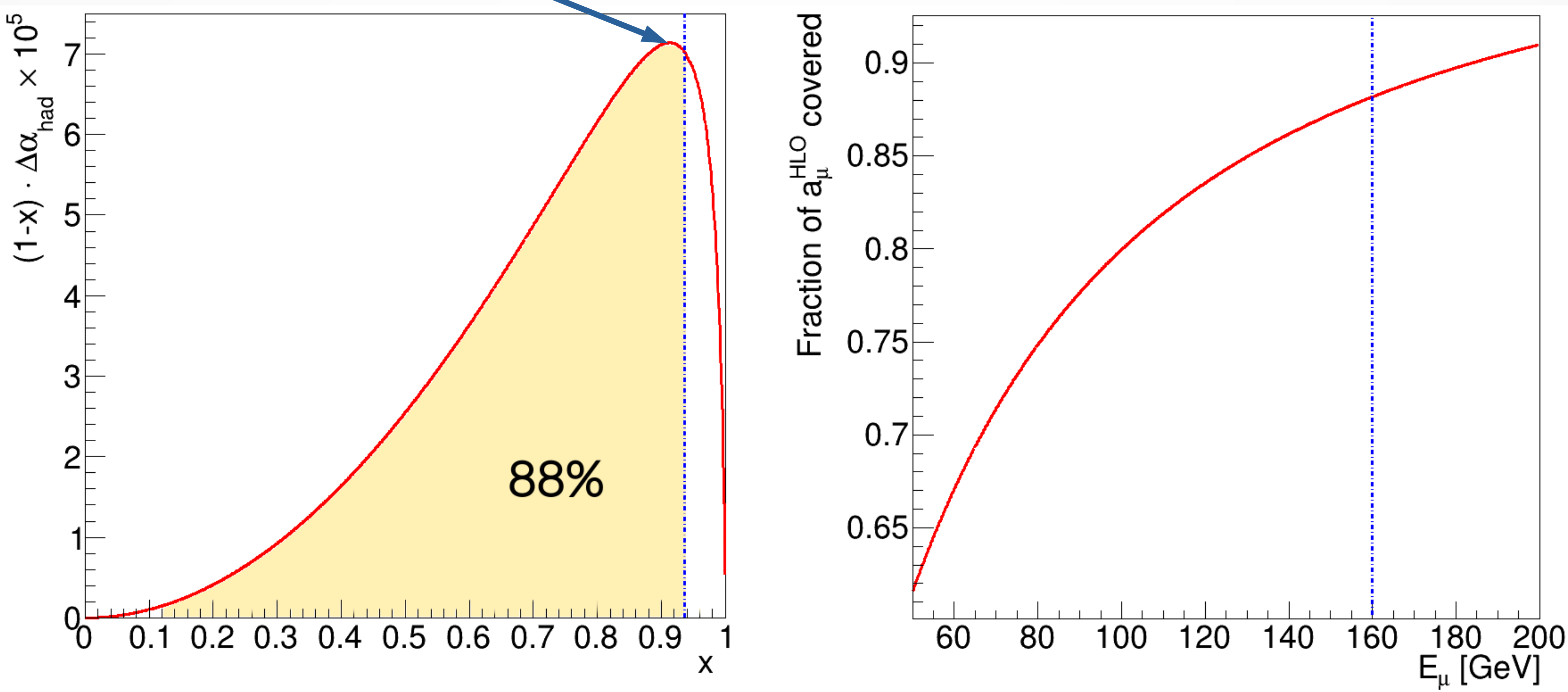
Conclusions

- Extraction of $\Delta\alpha_{\rm had}(t)$ using the distribution of the two scattering angles of μ-e elastic interactions: template fit technique.
- Promising strategy to control the systematic effects:
use the elastic scattering events to determine the main systematics, then perform a combined fit to the signal and the residual systematic effects.
- Space-like integral to calculate $a_\mu^{\rm HLO}$: independent method, competitive with the latest evaluations (~0.35% stat. uncertainty).
- Alternative method to calculate $a_\mu^{\rm HLO}$ with MUonE data:
less sensitive to the parameterization chosen to model $\Delta\alpha_{\rm had}(t)$ in the MUonE kinematic range.
Comparable uncertainty to the space-like integral method.

BACKUP

$x < 0.936$

$t_{peak} \sim -0.108\ \mathrm{GeV}^2$ $x_{peak} \sim 0.92$

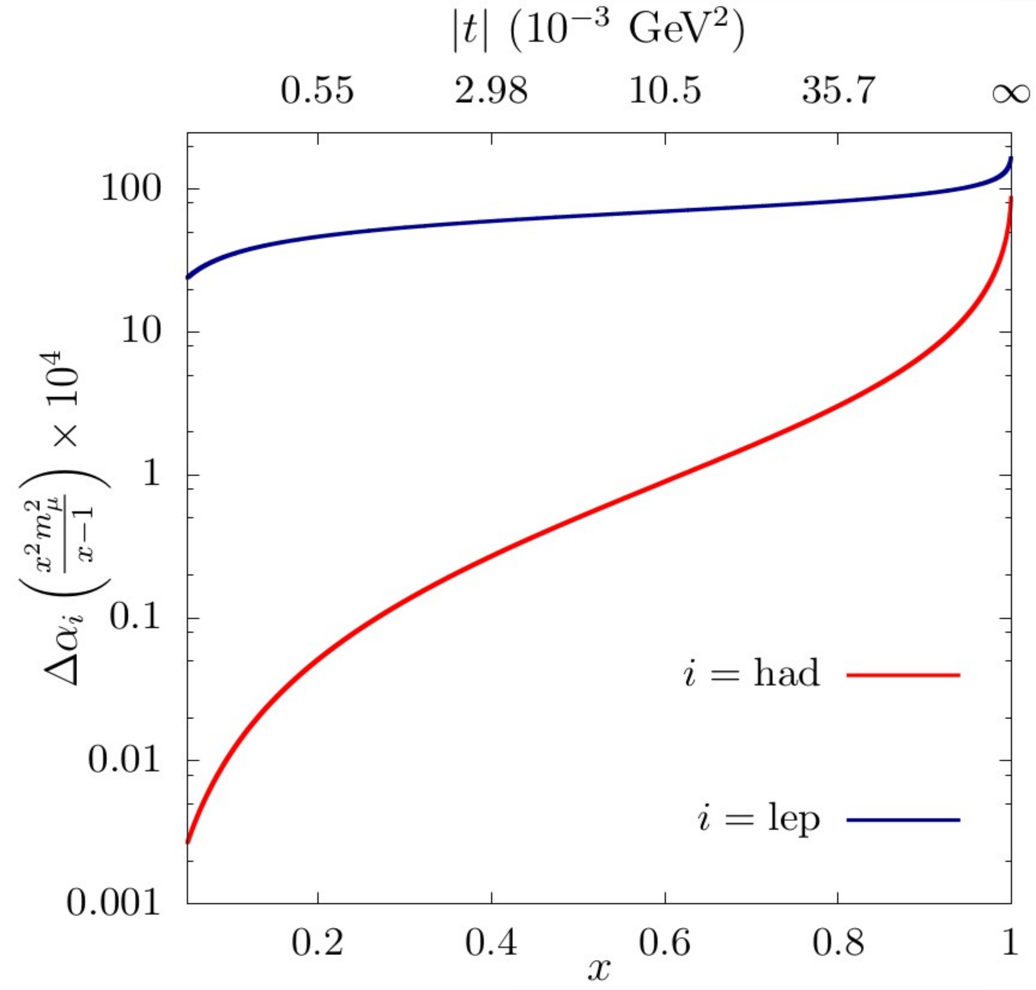


- 160 GeV muon beam on atomic electrons.

$$\sqrt{s} \sim 420\,\mathrm{MeV}$$

$$-0.153\,\mathrm{GeV}^2 < t < 0\,\mathrm{GeV}^2$$

$$\Delta\alpha_{had}(t) \lesssim 10^{-3}$$



Achievable accuracy

40 stations (60 cm Be) + 3 years of data taking ($\sim 4\text{x}10^7$ s) ($I_\mu \sim 10^7\ \mu^+/\text{s}$) $\sim 4\text{x}10^{12}$ events with $E_e > 1$ GeV = ~0.3% statistical accuracy on a_μ^{HLO}

Competitive with the latest theoretical predictions.

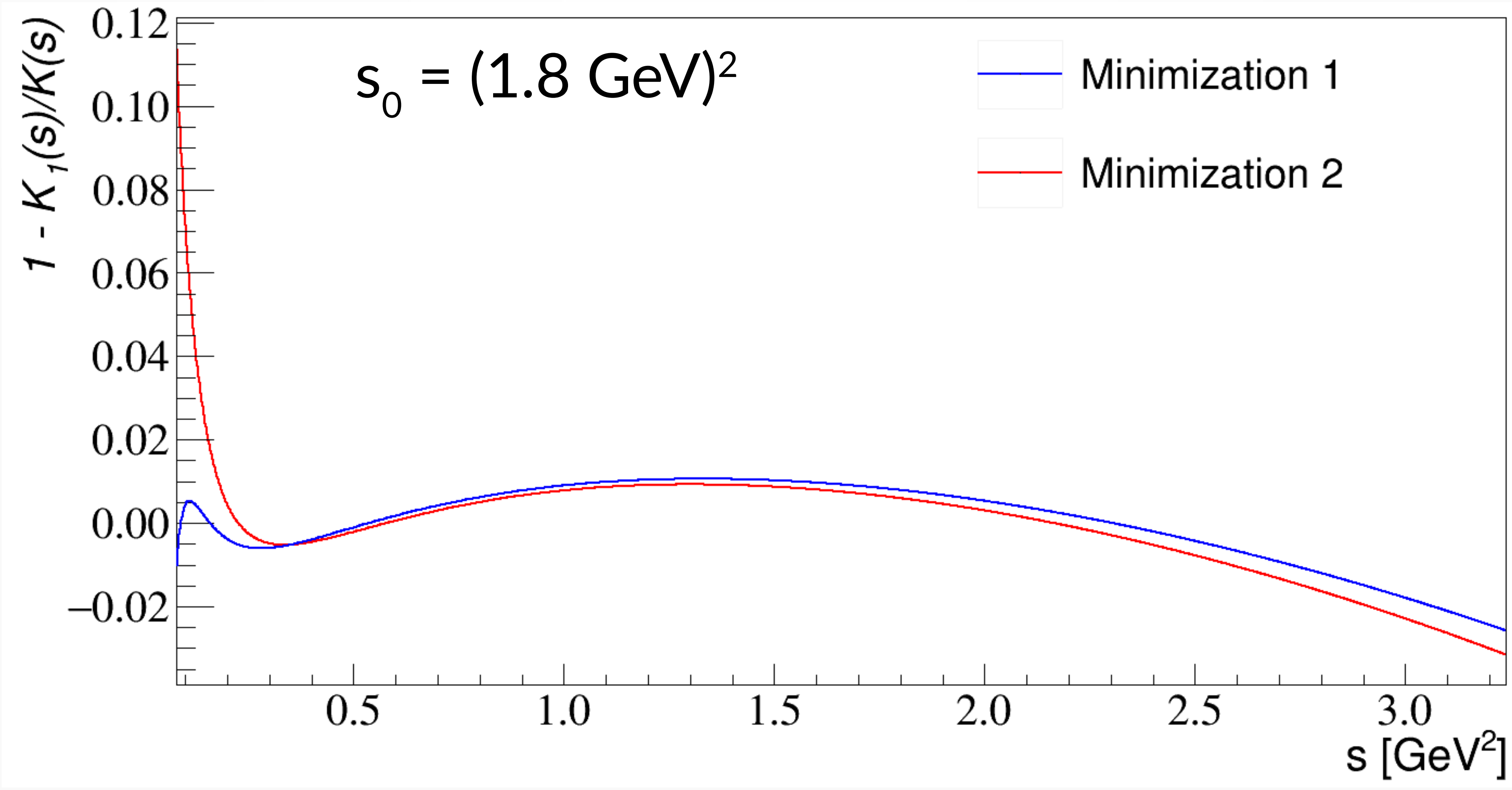
Main challenge:
keep systematic accuracy at the same level of the statistical one

Systematic uncertainty of 10 ppm at the peak of the integrand function (low θ_e, large θ_μ)

Main systematic effects:

- Longitudinal alignment (~10 µm)
- Knowledge of the beam energy (few MeV)
- Multiple scattering (~1%)
- Angular intrinsic resolution (few %)

Difference $K_1(s)$ - $K(s)$

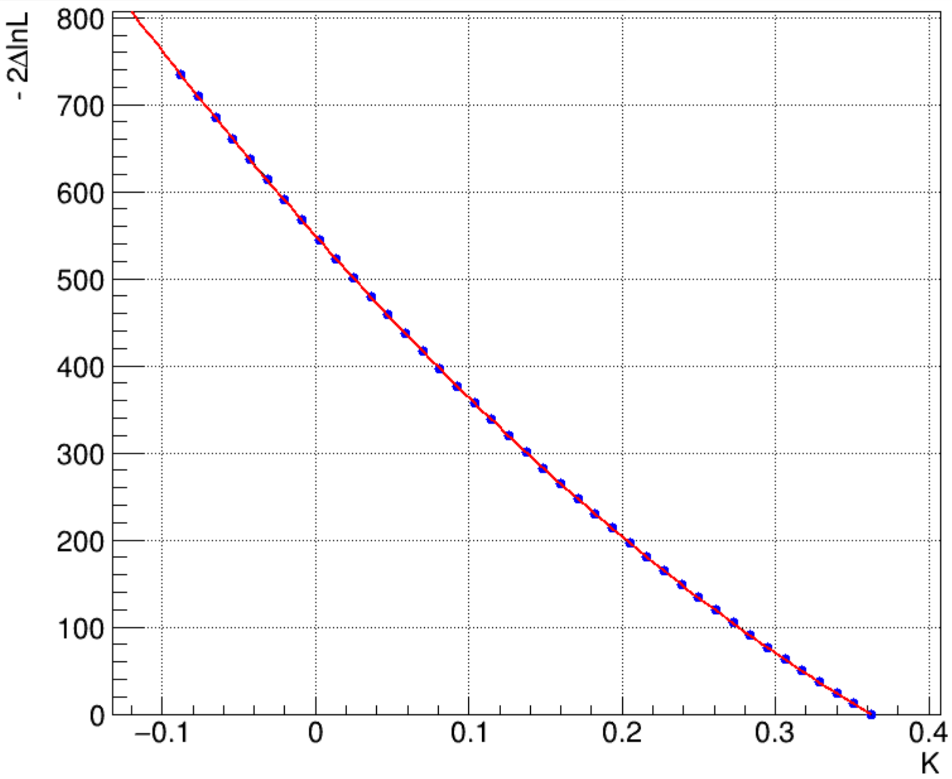


Tools used for the current analysis

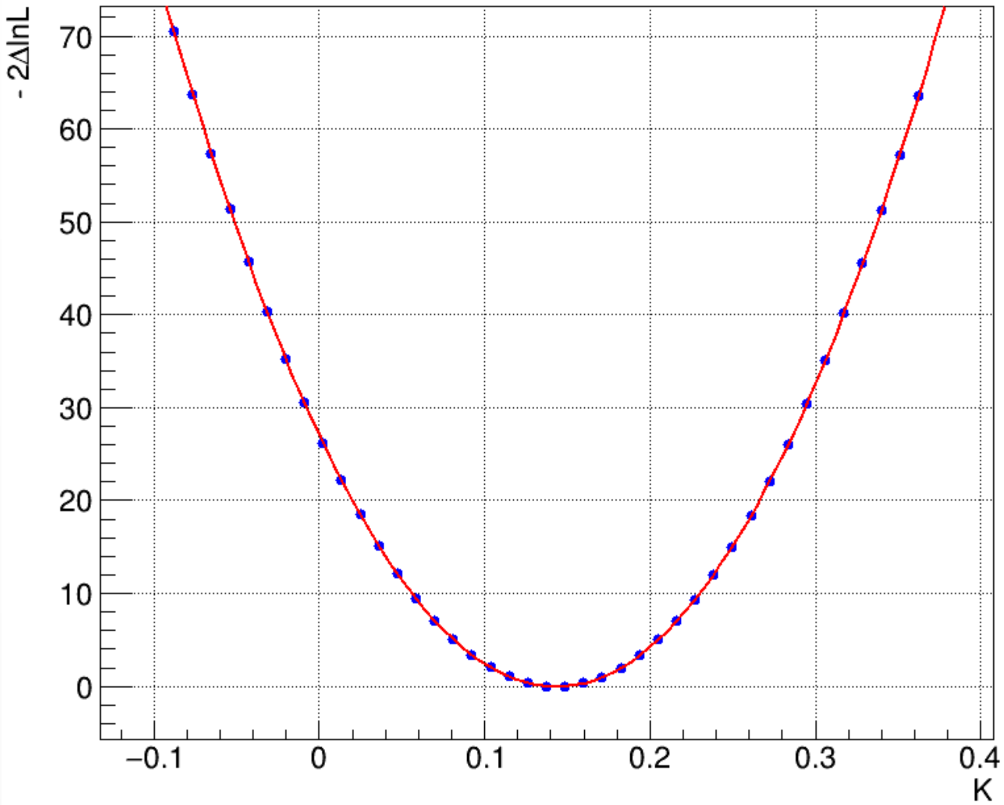
- NLO MonteCarlo generator: MESMER
  - Allows to change the muon beam energy and simulate the beam energy spread.
- C++ fast simulation to include detector effects:
  - Multiple scattering effects in the target.
  - Angular intrinsic resolution.
  - Effects applied to (θ_e, θ_μ) taken from the NLO generator: track reconstruction effects are currently neglected.
- Combine software to include the systematic effects.

Simultaneous fit signal + nuisance parameters @L_{TR}

If the systematics are not taken into account in the fit...



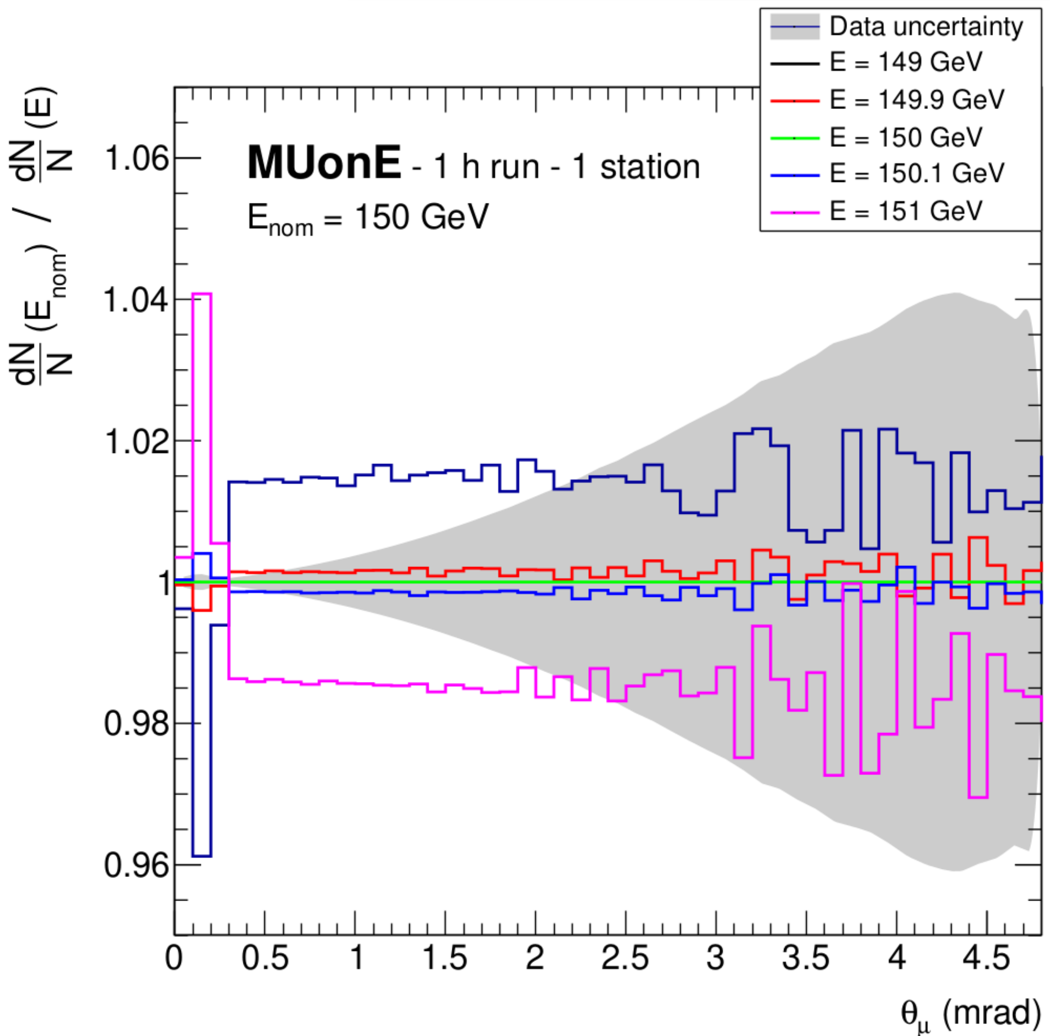
If the nuisance parameters are introduced in the fit procedure...



Systematic error on the muon beam energy

Accelerator division provides E_{beam} with O(1%) precision (~ 1 GeV).
It must be controlled by a physical process.

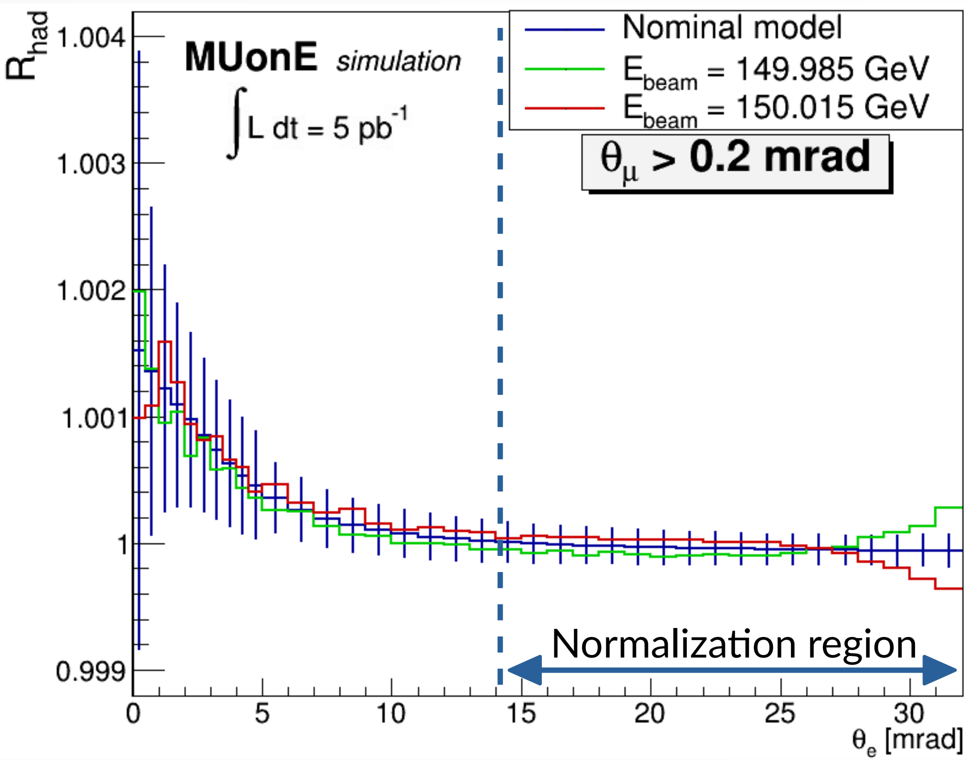
Effects of such shift on E_{beam} can be seen in our data in 1h of data taking per station.

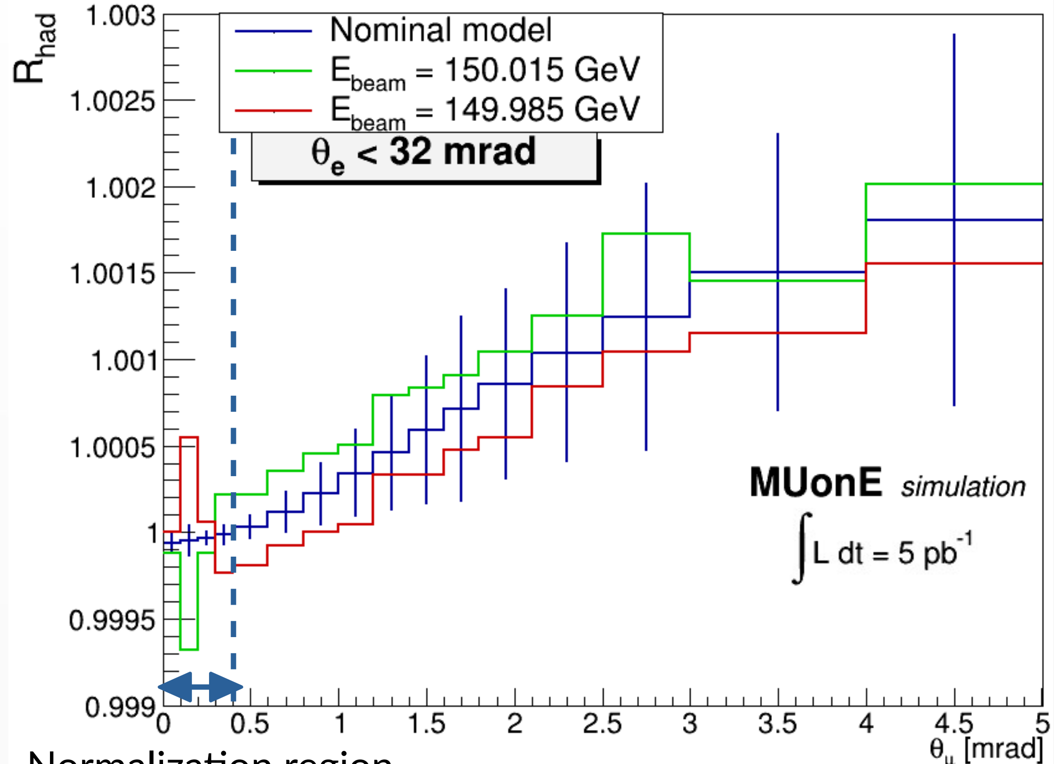


G Abbiendi Phys. Scr. (2022) 97 054007

Systematic error on the beam energy scale

Effect of a ± 15 MeV shift



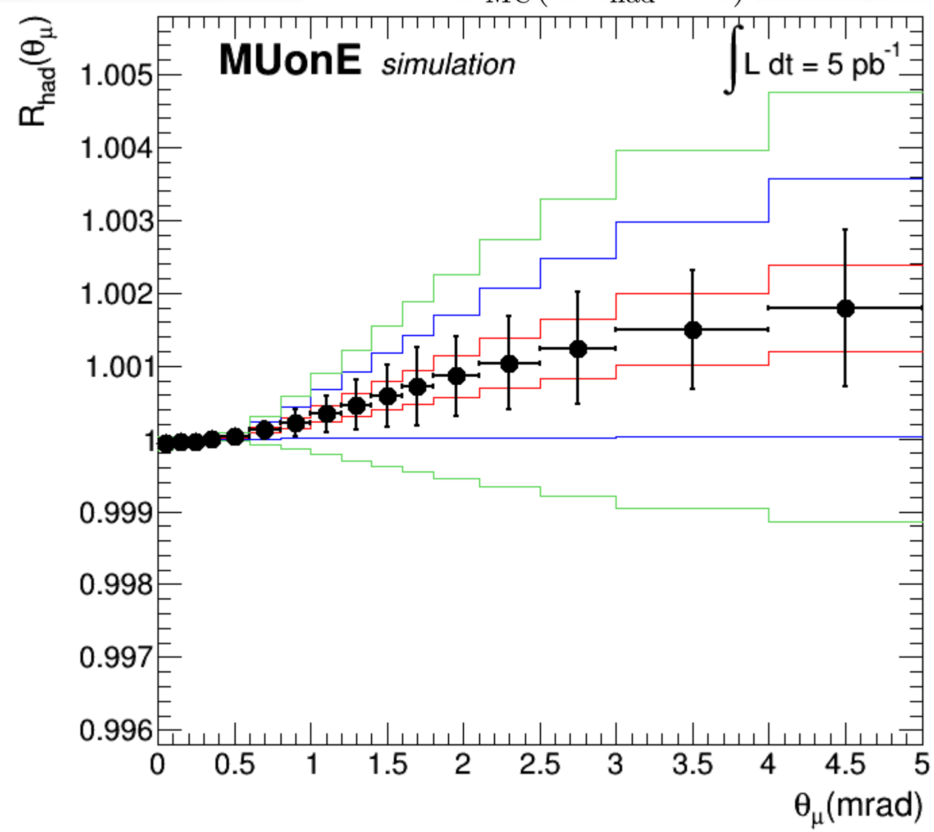


Normalization region

5 pb^{-1} expected sensitivity on $\Delta\alpha_{had}(t)$

Expected luminosity: 5 pb^{-1} ⟷ ~10^9 events with E_e > 1 GeV

$$R_{\rm had} = \frac{d\sigma_{\rm data}(\Delta\alpha_{\rm had})}{d\sigma_{\rm MC}(\Delta\alpha_{\rm had}=0)}$$



Low sensitivity to the hadronic running ($\Delta\alpha_{had}(t) < 10^{-3}$)

$$\Delta\alpha_{had}(t) \simeq -\frac{1}{15}Kt$$

K = 0.137 ± 0.028
(20% stat error)

We will be sensitive to the leptonic running ($\Delta\alpha_{lep}(t) < 10^{-2}$)

Strategy for the systematic effects

The Combine analysis tool is used to include the nuisance parameters in the fit procedure.

2 classes of nuisance parameters currently included:

- Normalization nuisance parameters, ν
- Shape nuisance parameters, μ

Binned likelihood fit:

$$\mathcal{L} = \prod_{i=1}^{N} \frac{n_i^{k_i}}{k_i!} e^{-n_i}$$

k_i = events in the *i-th* bin of data
n_i = events in the *i-th* bin of a given template
N = total number of bins

Nuisance parameters are used to adjust n_i and make it fit to k_i.

$$n_i \rightarrow n_i(\vec{\nu}, \vec{\mu})$$

Normalization nuisance parameters

Used to account for residual shifts in the normalization of template distributions with respect to data.

The expected number of events is modified as follows:

$$n_i \to n_i(\nu) = n_i(1+\varepsilon)^{\nu}$$

ν → Nuisance parameter

ε → Relative uncertainty on the systematic effect

Example: systematic error due to a limited knowledge of the luminosity → $\varepsilon \sim O(1\%)$

Shape nuisance parameters

Used to control effects that change the *shape* of the differential cross section.

The expected number of events in each bin is modified as:

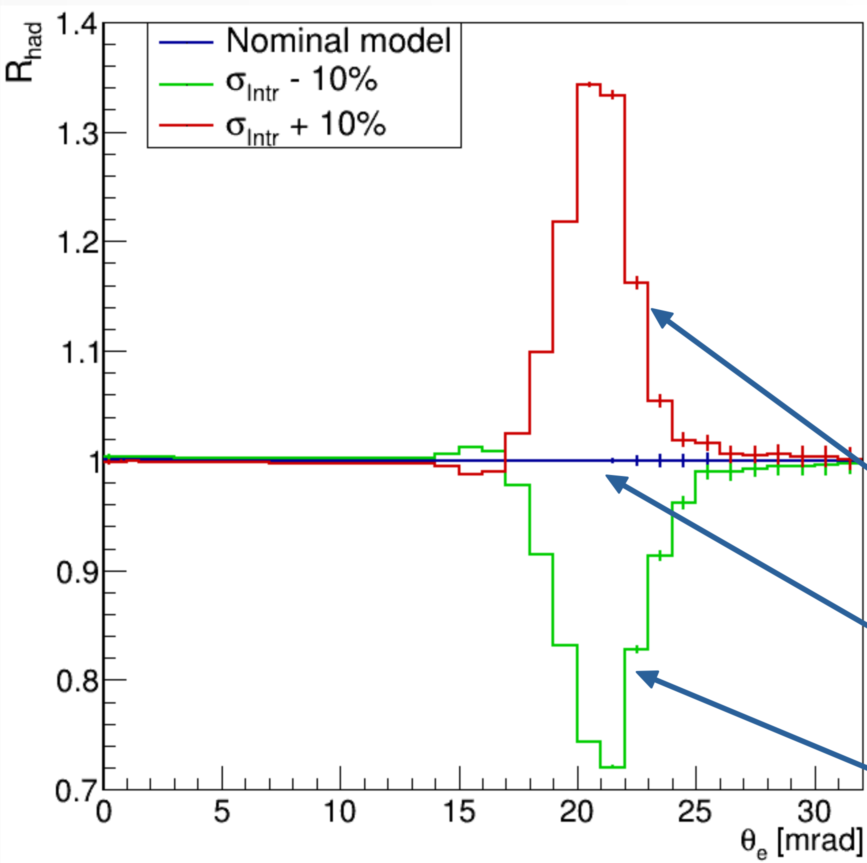
$$n_i \to n_i(\mu) = n_i[1 + s_i(\mu)]$$

Spline ensuring continuity and differentiability of 1st and 2nd derivatives.
Each bin has its own spline.

$$s_i(\mu) = \begin{cases} \frac{1}{2}\left[(\delta_i^+ - \delta_i^-)\mu + \frac{1}{8}(\delta_i^+ + \delta_i^-)(3\mu^6 - 10\mu^4 + 15\mu^2)\right] & |\mu| \leq 1 \\ \delta_i^+ \mu & \mu > 1 \\ -\delta_i^- \mu & \mu < -1 \end{cases}$$

Shape nuisance parameters

$$s_i(\mu) \text{ depends on } \delta_i^{\pm} = \frac{n_i^{\pm} - n_i^0}{n_i^0}$$



Shape nuisance parameters are determined by vertical interpolation of the template histograms.

3 inputs are needed for the interpolation:

n_i^+ Template with systematic effect shifted by +1σ

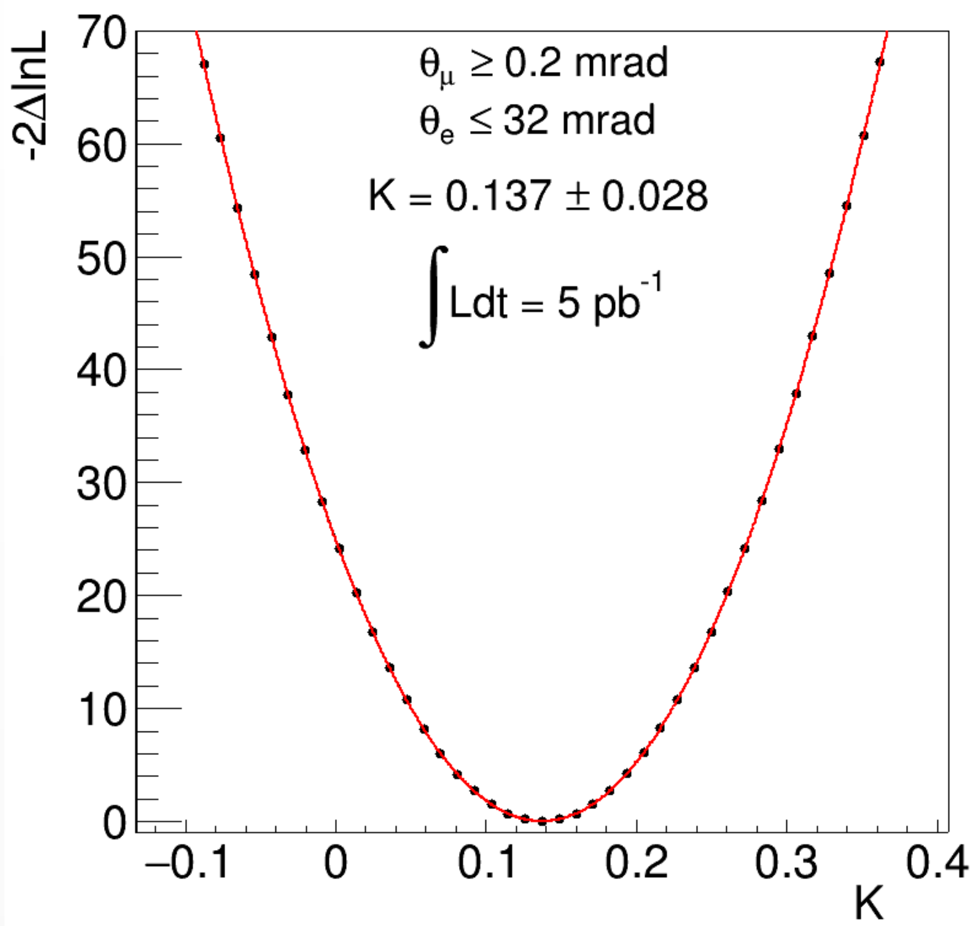
n_i^0 Template with the expected modelization

n_i^- Template with systematic effect shifted by -1σ

Analysis workflow

- Combine performs a likelihood fit to the nuisance parameters for each template.
- Obtain the profile likelihood as a function of K.
- Best fit value of K is determined by parabolic interpolation among the template points.
- Nuisance parameters values for $K = K_{\text{best fit}}$ are obtained by interpolation among the values obtained in the first step.

Statistical fit @L_{TR} = 5 pb^{-1}



Analysis workflow

Promising strategy: staged approach.

1. Use a small fraction of data to refine the knowledge of the main sources of systematic error with respect to the initial modelization.

2. Include the residual systematics as nuisance parameters in a combined fit with the signal parameter on the entire dataset.

Currently tested on the Test Run statistics including the main systematic errors.

Testing the procedure

Generate a pseudo-data sample introducing shifts in the main sources of systematic error with respect to the expectations.

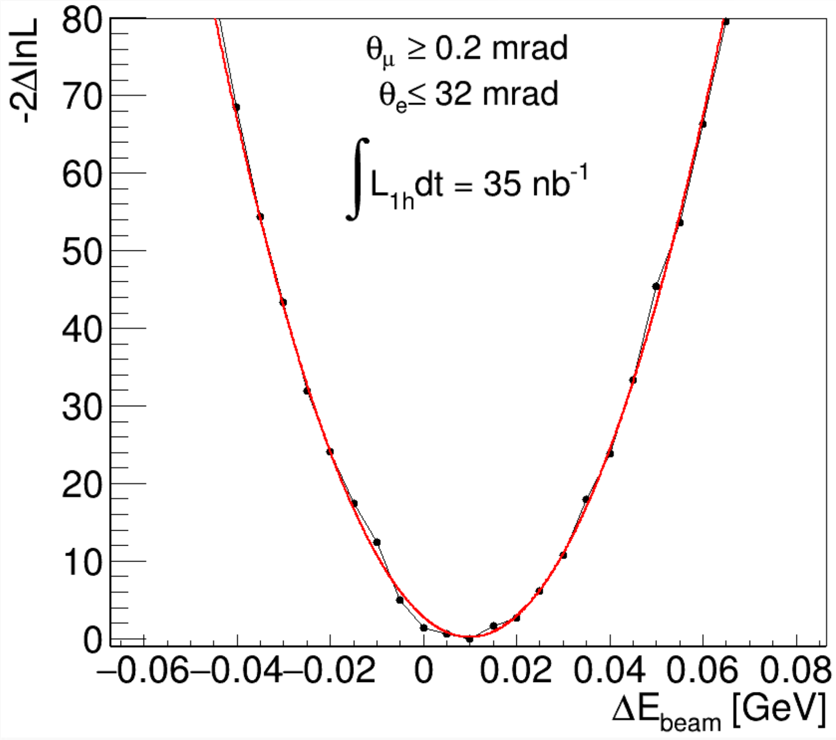
Source of systematics	Shift in the pseudo-data	Expected uncertainty
Beam energy scale	$\mathrm{E_{beam}} \to \mathrm{E_{beam}} + 6\,\mathrm{MeV}$	$\Delta\mathrm{E_{beam}} = \pm 1\,\mathrm{GeV}$
Multiple scattering	$\sigma_{\mathrm{MS}} \to \sigma_{\mathrm{MS}} + 0.5\%$	$\Delta\sigma_{\mathrm{MS}} = \pm 1\%$
Angular intrinsic resolution	$\sigma_{\mathrm{Intr}} \to \sigma_{\mathrm{Intr}} + 5\%$	$\Delta\sigma_{\mathrm{Intr}} = \pm 10\%$
Luminosity		$\varepsilon = 1\%$

Are we able to determine precisely K and the nuisance parameters using this analysis strategy?

Step 1: identify the main systematic effects

1h of data taking per single station.

Allows to assume a fixed model for $\Delta\alpha_{had}$.



- Template fit as a function of E_{beam}.
- μ_{MS}: nuisance parameter for systematics on the multiple scattering.
- μ_{Intr}: nuisance parameter for systematics on the angular intrinsic resolution.
- ν: nuisance parameter for systematics on the normalization.

Selection cuts	Fit results
$\theta_e \leq 32\,\mathrm{mrad}$ $\theta_\mu \geq 0.2\,\mathrm{mrad}$	$\Delta E_{beam} = (0.006 \pm 0.006)$ GeV $\mu_{Intr} = (4.9 \pm 0.1)\%$ $\mu_{MS} = (0.6 \pm 0.1)\%$ $\nu = 0.01 \pm 0.03$

Similar results also for different selection cuts.

Update the knowledge on the sources of systematic error

Exploit results obtained in step 1 to refine the knowledge on the sources of systematic error.

Source of systematics	Expected uncertainty
Beam energy scale	$\Delta E_{\mathrm{beam}} = \pm 1\,\mathrm{GeV}$
Multiple scattering	$\Delta \sigma_{\mathrm{MS}} = \pm 1\%$
Angular intrinsic resolution	$\Delta \sigma_{\mathrm{Intr}} = \pm 10\%$

Update the knowledge on the sources of systematic error

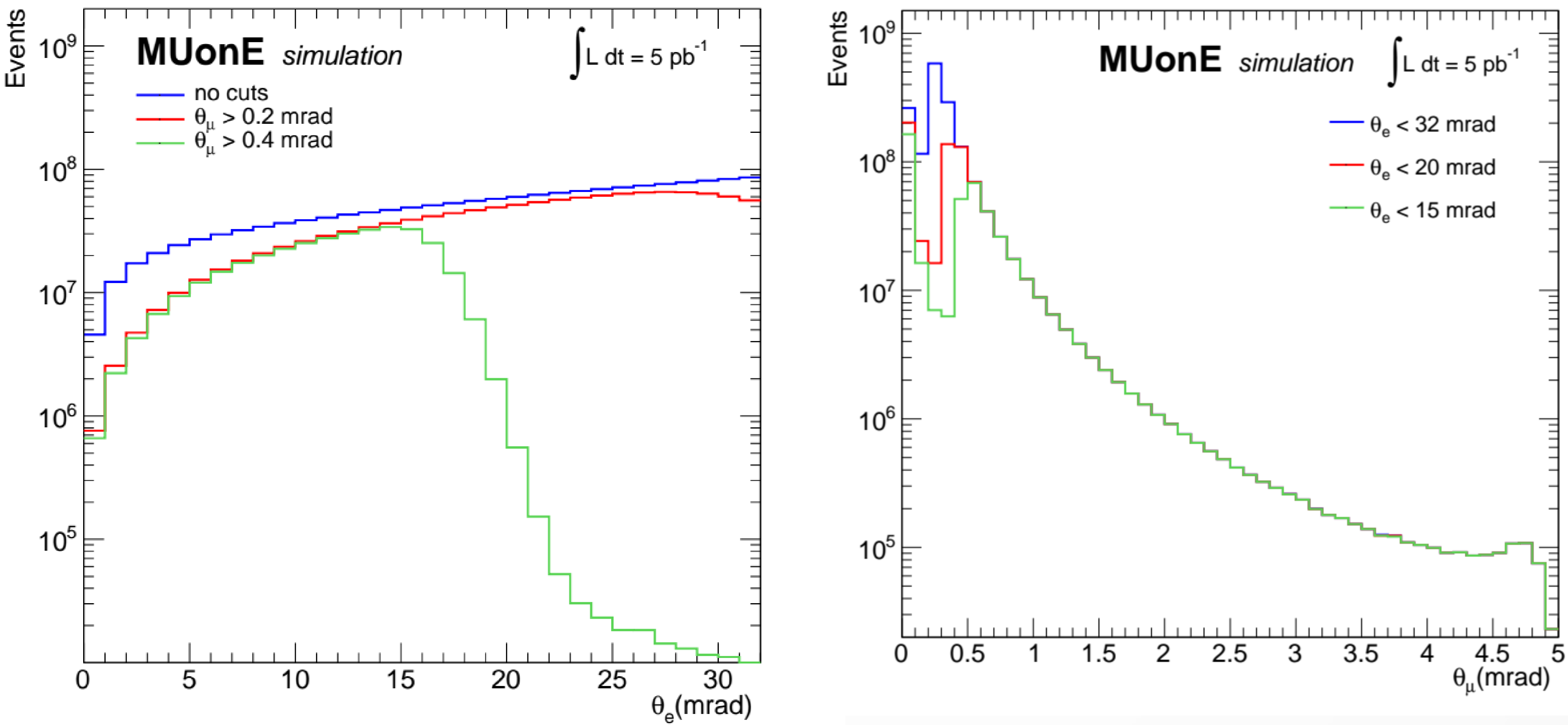
Exploit results obtained in step 1 to refine the knowledge on the sources of systematic error.

Source of systematics	Expected uncertainty
Beam energy scale	$\Delta E_{\text{beam}} = \pm 1\,\text{GeV}$
Multiple scattering	$\Delta\sigma_{\text{MS}} = \pm 1\%$
Angular intrinsic resolution	$\Delta\sigma_{\text{Intr}} = \pm 10\%$

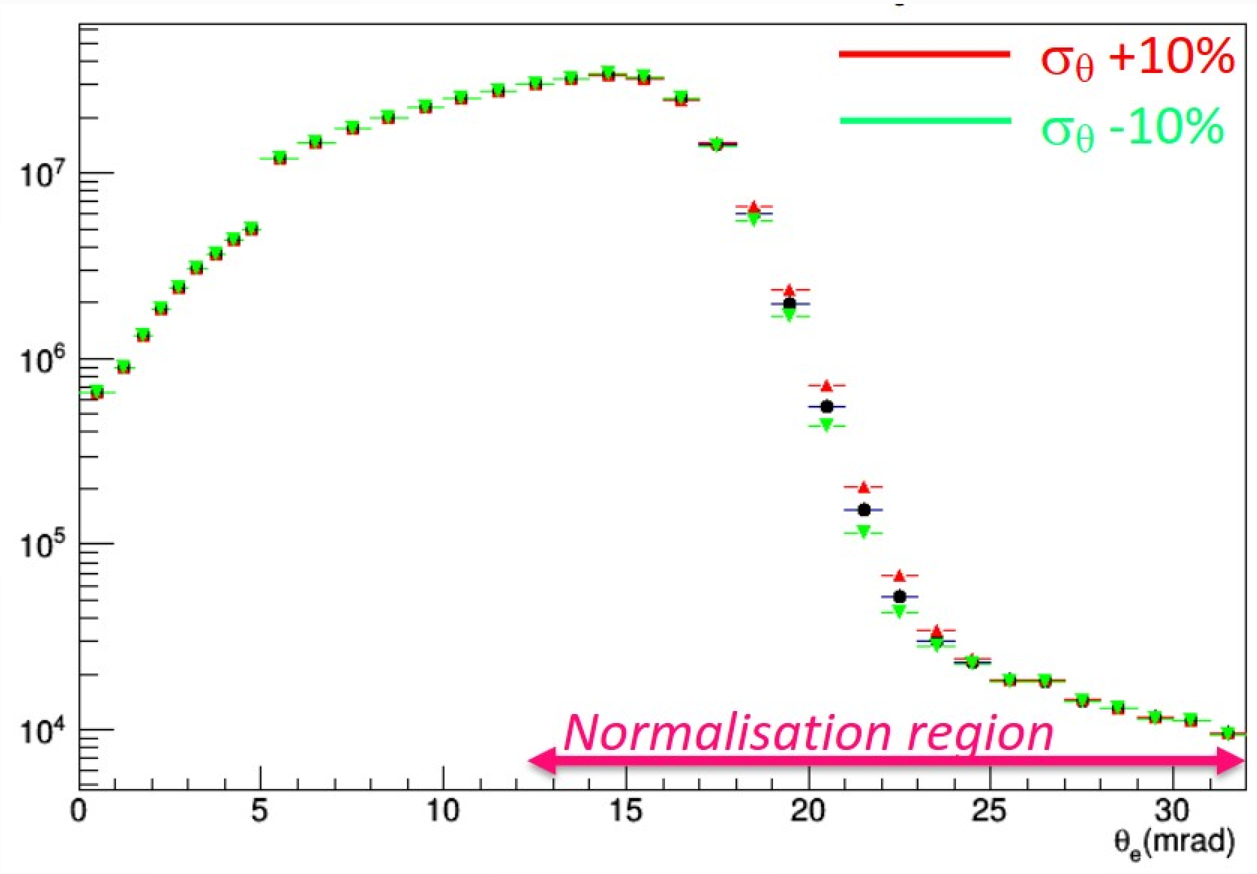
→

Updated model
$\Delta E_{\text{beam}} = \pm 20\,\text{MeV}$
$\sigma_{\text{MS}} \to \sigma_{\text{MS}} + 0.6\%$ $\Delta\sigma_{\text{MS}} = \pm 0.5\%$
$\sigma_{\text{Intr}} \to \sigma_{\text{Intr}} + 5\%$ $\Delta\sigma_{\text{Intr}} = \pm 0.6\%$

Use this improved modelization to perform the combined fit to K and the residual systematics.

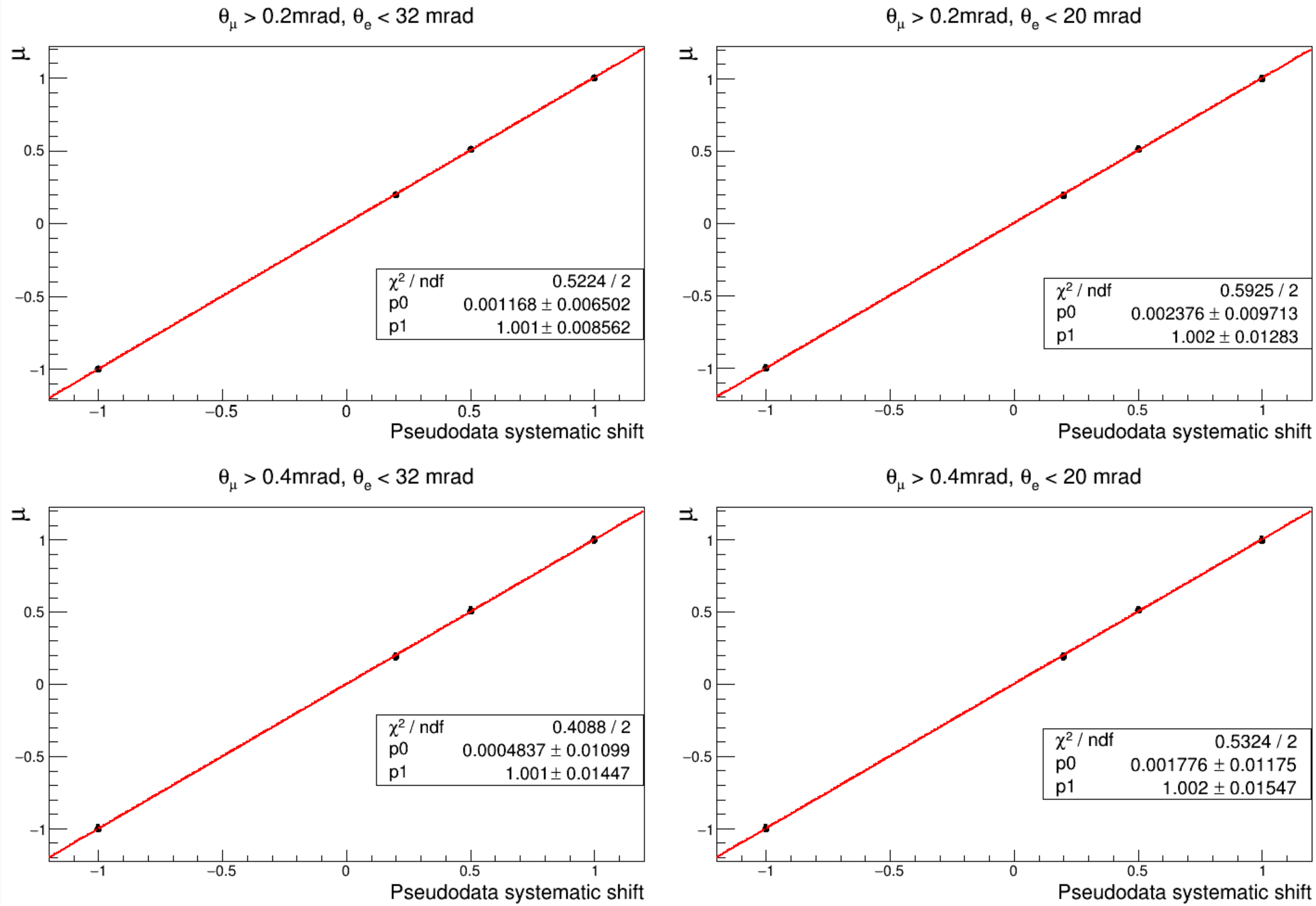


MUonE simulation
∫L dt = 5 pb⁻¹
no cuts
θμ > 0.2 mrad
θμ > 0.4 mrad
Events
θe(mrad)
θe < 32 mrad
θe < 20 mrad
θe < 15 mrad
θμ(mrad)



σθ +10%
σθ -10%
10⁷
10⁶
10⁵
10⁴
Normalisation region
0
5
10
15
20
25
30
θe(mrad)

Fit of MS nuisance using different pseudodata shifts



μ = {-1%, 0.2%, 0.5%, 1%}

Linear relation between fitted value of μ and pseudodata shift

OK!