Theory review of Lepton Flavour Violation

Paride Paradisi

University of Padova and INFN

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Plan of the talk

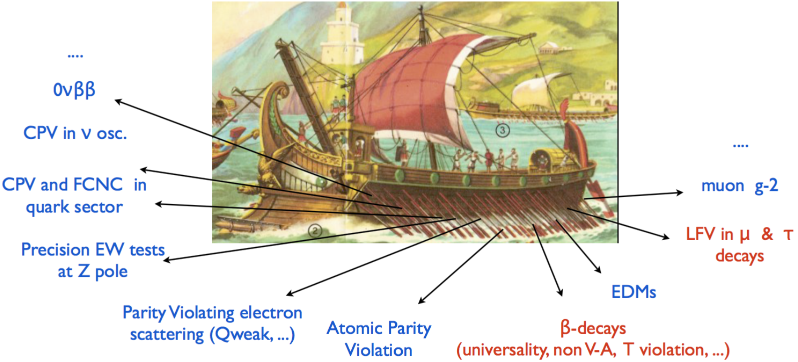
1. **Strategies to look for New Physics at low-energy**
2. **Current status of LFV**
3. **EDMs, g-2 and cLFV interrelationship**
4. **Conclusions and future prospects**

Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
  - LFV processes ($\mu \to e\gamma$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $\tau \to 3\mu$, $\cdots$)
  - CPV effects in the leptonic (e, μ) and neutron EDMs
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes
- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_\mu$: $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (2.51 \pm 0.59) \times 10^{-9}$ (4.2σ discrepancy!)
  - LFUV in $M \to \ell\nu$ (with $M = \pi, K, B$), $B \to D^{(*)}\ell\nu$, $B \to K\ell\ell'$, τ and Z decays
- **High-intensity frontier: A collective effort to determine the NP symmetries**
- **High-energy frontier: A unique effort to determine the NP scale**

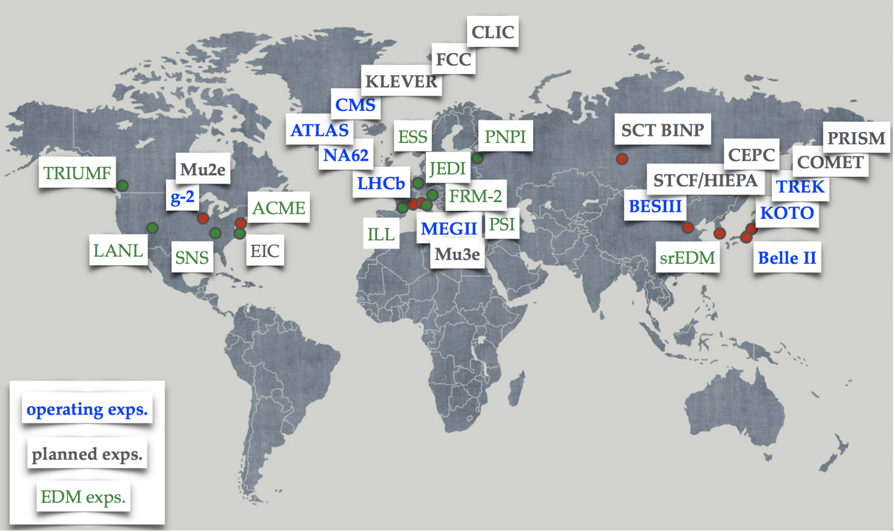
Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
- **Processes predicted with high precision in the SM**



High-intensity frontier: A collective effort to determine the NP dynamics

Experimental status



Experimental bounds

Process	Present	Experiment	Future	Experiment
$\mu \to e\gamma$	4.2×10^{-13}	MEG	$\approx 6 \times 10^{-14}$	MEG II
$\mu \to 3e$	1.0×10^{-12}	SINDRUM	$\approx 10^{-16}$	Mu3e
μ^- Au $\to e^-$ Au	7.0×10^{-13}	SINDRUM II	?	
μ^- Ti $\to e^-$ Ti	4.3×10^{-12}	SINDRUM II	?	
μ^- Al $\to e^-$ Al	$-$		$\approx 10^{-16}$	COMET, MU2e
$\tau \to e\gamma$	3.3×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to \mu\gamma$	4.4×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to 3e$	2.7×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$\tau \to 3\mu$	2.1×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$d_e(\mathrm{e\ cm})$	1.1×10^{-29}	ACME	$\sim 3 \times 10^{-31}$	ACME III
$d_\mu(\mathrm{e\ cm})$	1.8×10^{-19}	Muon (g-2)	$\sim 10^{-22}$	PSI

Table: Present and future experimental sensitivities for relevant low-energy observables.

- So far, only upper bounds. Still excellent prospects for exp. improvements.
- We can expect a NP signal in all above observables below the current bounds.

Charged LFV in the SM

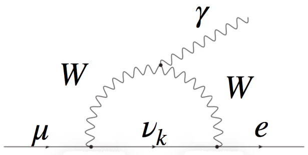
- GIM mechanism very effective in LFV transitions
- amplitude proportional to $A(\mu \to e\gamma) \propto m_\nu^2$

Very small !!!

$$\mathrm{BR}(\mu \to e\gamma) \simeq \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U^*_{ek} m^2_{\nu_k}}{M_W^2} \right|^2 .$$

$$\mathrm{BR}(\mu \to e\gamma) = 10^{-55} \div 10^{-54}$$

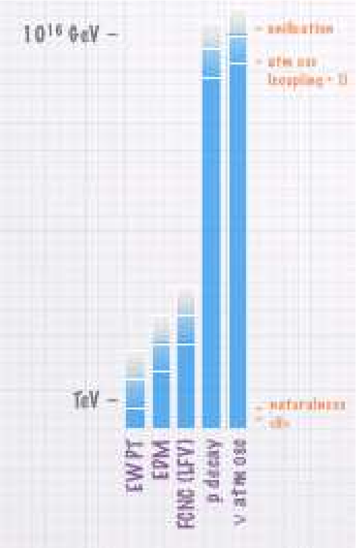
- similar suppressions for $\mu \to 3e, \tau \to 3\mu, \mu \to e, \ldots$



Why flavor violation is visible in neutrino oscillation while it's not in charged LFV?
The uncertainty principle sets the oscillation time for $\mu \to e\gamma$ to be $t \sim h/M_W$!

Why do we need New Physics (NP)?

- **Gravity** $\Longrightarrow$ $\Lambda_{\text{Planck}} \sim 10^{18-19}$ GeV
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}$ GeV
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim$ TeV
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\text{see-saw}} \lesssim 10^{15}$ GeV
- **Dark Matter (WIMP)** $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim$ TeV
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{\text{NP}} \lesssim$ TeV



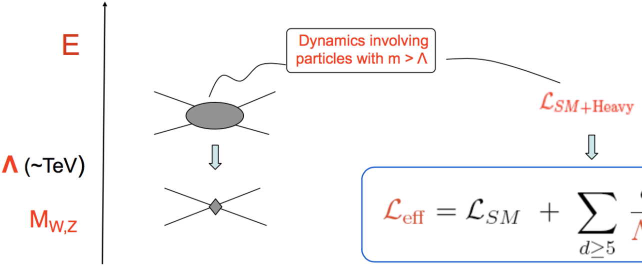
SM = effective theory at the EW scale

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}} O_{ij}^{(d)}$$

- $\mathcal{L}_{\text{eff}}^{d=5} = \frac{y_\nu^{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi\phi$,
- $\mathcal{L}_{\text{eff}}^{d=6}$ generates FCNC operators

EFT approach to NP

- **Dynamics below the scale Λ [$\sim$ mass of new particles] is described by $L_{\rm eff}$**



$$\mathcal{L}_{\rm eff} = \mathcal{L}_{SM} + \sum_{d\geq5} \frac{c_n^{(d)}}{\Lambda^{d-4}} \hat{O}_n^{(d)}[\phi_{SM}]$$

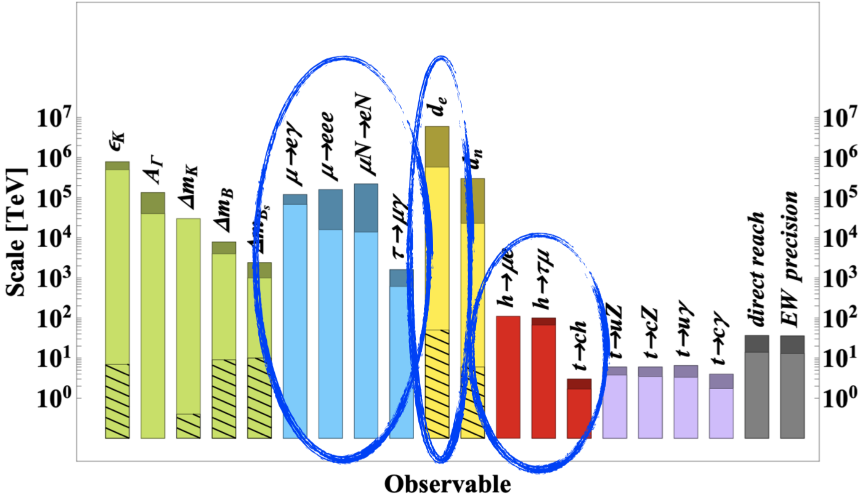
  - $L_{\rm eff}$ is built out of relevant low-energy degrees of freedom (SM fields)
  - $L_{\rm eff}$ respects the SM gauge symmetries $G_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
  - $L_{\rm eff}$ is organized in inverse powers of Λ (amplitudes suppressed by powers of E/Λ)

- **Experiments at the precision frontier probe energy scale Λ and symmetries of the new interactions (coeff. & structure of $\hat{O}_n^{(d)}$)**

Complete list of dim-6 LFV operators

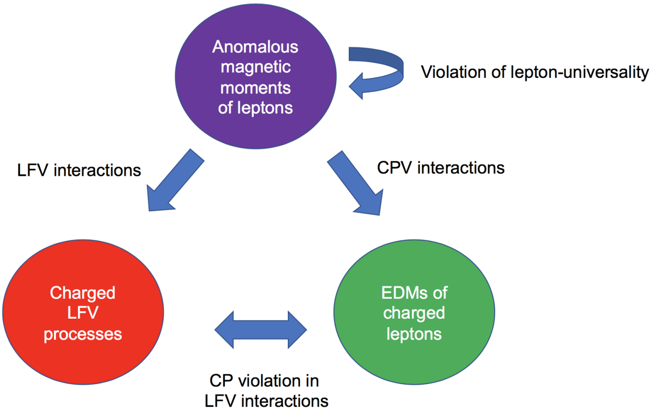
4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{L}_L\gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\tau_I\Phi W^I_{\mu\nu}$
Q_{ee}	$(\bar{e}_R\gamma_\mu e_R)(\bar{e}_R\gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L\sigma^{\mu\nu}e_R)\Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{e}_R\gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q^{(1)}_{\ell q}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{Q}_L\gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{u}_R\gamma^\mu u_R)$
$Q^{(3)}_{\ell q}$	$(\bar{L}_L\gamma_\mu\tau_I L_L)(\bar{Q}_L\gamma^\mu\tau_I Q_L)$	Q_{eu}	$(\bar{e}_R\gamma_\mu e_R)(\bar{u}_R\gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R\gamma^\mu e_R)(\bar{Q}_L\gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}^a_L e_R)(\bar{d}_R Q^a_L)$
$Q_{\ell d}$	$(\bar{L}_L\gamma_\mu L_L)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(1)}_{\ell equ}$	$(\bar{L}^a_L e_R)\epsilon_{ab}(\bar{Q}^b_L u_R)$
Q_{ed}	$(\bar{e}_R\gamma_\mu e_R)(\bar{d}_R\gamma^\mu d_R)$	$Q^{(3)}_{\ell equ}$	$(\bar{L}^a_i\sigma_{\mu\nu}e_R)\epsilon_{ab}(\bar{Q}^b_L\sigma^{\mu\nu}u_R)$
Lepton-Higgs operators			
$Q^{(1)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{L}_L\gamma^\mu L_L)$	$Q^{(3)}_{\Phi\ell}$	$(\Phi^\dagger i\overleftrightarrow{D}^I_\mu\Phi)(\bar{L}_L\tau_I\gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i\overleftrightarrow{D}_\mu\Phi)(\bar{e}_R\gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R\Phi)(\Phi^\dagger\Phi)$

$\mu\to e\gamma$ $\mu\to 3e$ $\mu\to e$



[Physics Briefing Book, 1910.11775]

Probing NP in the leptonic sector



New Physics for the muon $g-2$: at which scale?

- **Δa_μ discrepancy at $\sim 4.2\,\sigma$ level:**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.*
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.

*Favoured by the *hierarchy problem* and by a WIMP DM candidate but disfavoured by the LEP and LHC bounds (supersymmetry being the most prominent example).

On leptonic dipoles: $\ell \to \ell'\gamma$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e\frac{m_\ell}{2}\left(\bar{\ell}_R\sigma_{\mu\nu}A_{\ell\ell'}\ell'_L + \bar{\ell}'_L\sigma_{\mu\nu}A^\star_{\ell\ell'}\ell_R\right)F^{\mu\nu} \qquad \ell,\ell' = e,\mu,\tau\,,$$

  - **Branching ratios of $\ell \to \ell'\gamma$**

$$\frac{\mathrm{BR}(\ell\to\ell'\gamma)}{\mathrm{BR}(\ell\to\ell'\nu_\ell\bar{\nu}_{\ell'})} = \frac{48\pi^3\alpha}{G_F^2}\left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2\right).$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2m_\ell^2\,\mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell\,\mathrm{Im}(A_{\ell\ell})\,.$$

  - **"Naive scaling":** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}\,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j \gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}}\right)^2$$

- **EDMs vs.** $(g-2)_\mu$

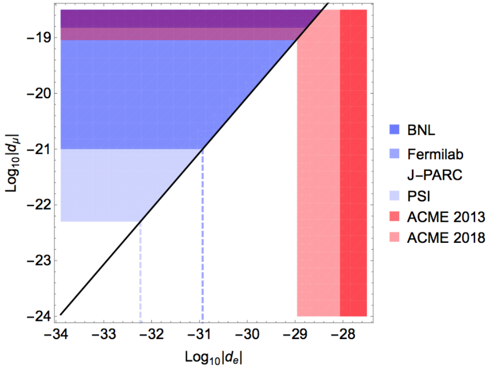
$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-29} \left(\frac{\phi_e^{CPV}}{10^{-5}}\right) \, e\,\mathrm{cm}\,,$$

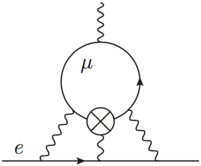
$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 2 \times 10^{-22} \, \phi_\mu^{CPV} \, e\,\mathrm{cm}\,,$$

- **Main messages:**
  - $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\, e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12]

Experimental status of the muon EDM





$$d_\mu \leq 10^{-21}\ e\,\mathrm{cm} \left(\frac{d_e}{10^{-31}\ e\,\mathrm{cm}}\right)$$

[Crivellin, Hoferichter & Schmidt-Wellenburg, '18]

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\ \phi_\mu^{CPV}\ e\,\mathrm{cm}\,,$$

[Giudice, PP & Passera, '12]

Not only $\mu \to e\gamma$...

- **LFV** operators **@ dim-6**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2_{\rm LFV}} \mathcal{O}^{\rm dim-6} + \dots .$$

$$\mathcal{O}^{\rm dim-6} \ni \;\; \bar{\mu}_R\, \sigma^{\mu\nu}\, H\, e_L\, F_{\mu\nu}\,, \;\; (\bar{\mu}_L \gamma^\mu e_L)\left(\bar{f}_L \gamma^\mu f_L\right)\,, \;\; (\bar{\mu}_R e_L)\left(\bar{f}_R f_L\right)\,, \;\; f = e, u, d$$

- $\ell \to \ell'\gamma$ probe ONLY the dipole-operator (at tree level)
- $\ell_i \to \ell_j \bar{\ell}_k \ell_k$ and $\mu \to e$ in Nuclei probe dipole and 4-fermion operators
- When the dipole-operator is dominant:

$$\mathrm{BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k) \approx \alpha \times \mathrm{BR}(\ell_i \to \ell_j \gamma)$$
$$\mathrm{CR}(\mu \to e \text{ in N}) \approx \alpha \times \mathrm{BR}(\mu \to e\gamma)$$

$$\frac{\mathrm{BR}(\mu \to 3e)}{3 \times 10^{-15}} \approx \frac{\mathrm{BR}(\mu \to e\gamma)}{5 \times 10^{-13}} \approx \frac{\mathrm{CR}(\mu \to e \text{ in N})}{3 \times 10^{-15}}$$

- **Ratios like $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$ probe the NP flavor structure**
- **Ratios like $Br(\mu \to e\gamma)/Br(\mu \to eee)$ probe the NP operator at work**

Testing new physics with the electron $g-2$

- **Longstanding muon $g-2$ anomaly**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

- **Testing the muon $g-2$ anomaly through the electron $g-2$**

$$\frac{\Delta a_e}{\Delta a_\mu} = \frac{m_e^2}{m_\mu^2} \qquad \Longleftrightarrow \qquad \Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}$$

  - a_e has never played a role in testing NP effects. From $a_e^{\rm SM}(\alpha) = a_e^{\rm EXP}$, we extract α which was is the most precise value of α up to 2018!
  - The situation has now changed thanks to th. and exp. progresses.
  - α can be extracted from atomic physics and a_e used to perform NP tests!

[Giudice, P.P, & Passera, '12]

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - We can expect any deviation from the SM expectations below the current bounds.
  - LFV processes, leptonic EDMs and LFUV observables do not suffer from theoretical limitations and there are still excellent prospects for experimental improvements.
  - If the muon $g-2$ anomaly will survive, we expect relevant enhancements in leptonic EDMs (especially in the muon EDM) and LFV decays $\mu \to e\gamma$, $\mu \to eee$, ...
  - Testing New Physics effects in the electron $g-2$ at the 10^{-13} is not too far! This will bring a_e to play a pivotal role in probing New Physics in the leptonic sector.

Message: an exciting Physics program is in progress at the Intensity Frontier!