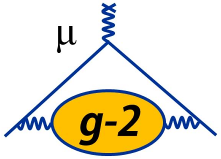
Bi-weekly meeting of the Liverpool Muon Group

29th June 2023

Introduction from Theory: g-2 & MUonE

Thomas Teubner

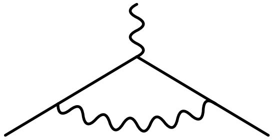


- Introduction, overview & status
- Data-driven HVP: basics, main features & puzzles
- The most important 2π channel, other channels, total HVP
- Lattice
- Pathways to solving the puzzles, MUonE and Liverpool plans

Introduction: it all started with the electron…

- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with g = 2.00229 ± 0.00008

- 1948: Schwinger calculates the famous radiative correction:



➠ g = 2 (1+a), with the **anomaly**

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi} \approx 0.001161$$

``If you can't join 'em, beat 'em"

This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

- In terms of an effective Lagrangian, the anomaly is from the Pauli term:

$$\delta\mathcal{L}_{\rm eff}^{\rm amm} = -\frac{Qe}{4m}\, a\, \bar\psi_L \sigma^{\mu\nu}\psi_R F_{\mu\nu} \ + \ ({\rm L} \leftrightarrow {\rm R})$$

- Similarly, an EDM comes from a term $\delta\mathcal{L}_{\rm eff}^{\rm EDM} = -\frac{d}{2}\bar\psi(x)\, i\, \sigma^{\mu\nu}\gamma_5\psi(x)F_{\mu\nu}(x)$

(At least) dimension 5 operators, non-renormalisable and hence not part of the fundamental (QED) Lagrangian. But can occur through radiative corrections, calculable in perturbation theory in (B)SM.

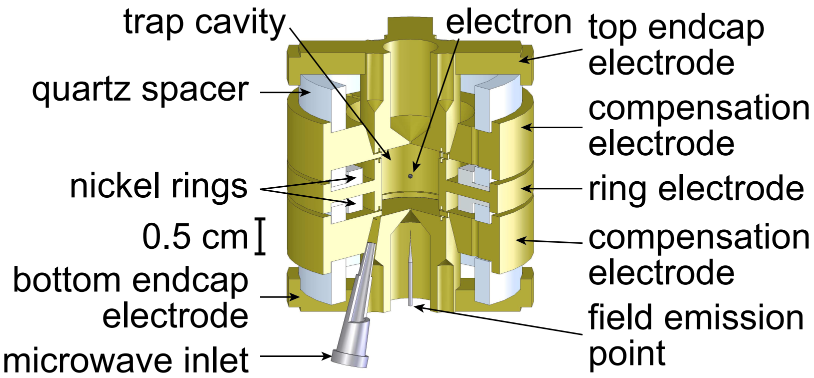
a_e vs. a_μ : why we want to study the muon

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]

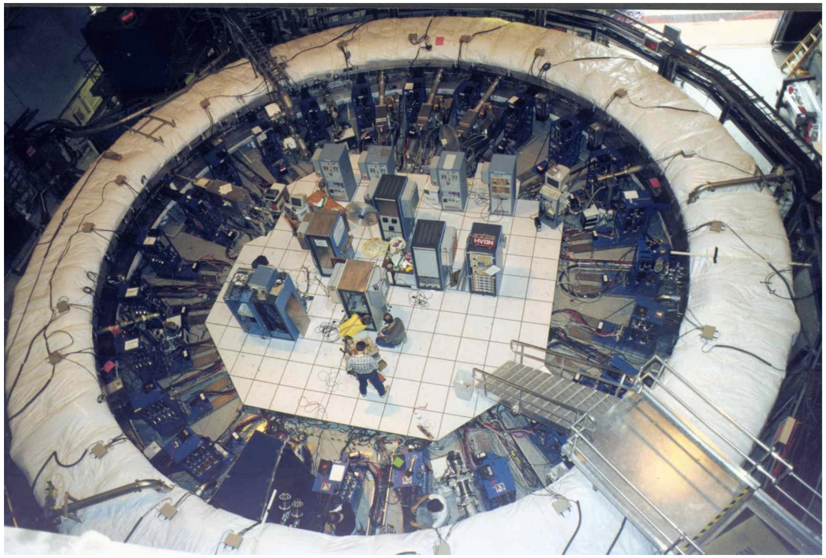
Hanneke et al., PRL 100(2008)120801 @ Harvard

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]

Bennet et al., PRD 73(2006)072003 @ BNL



one-electron quantum cyclotron

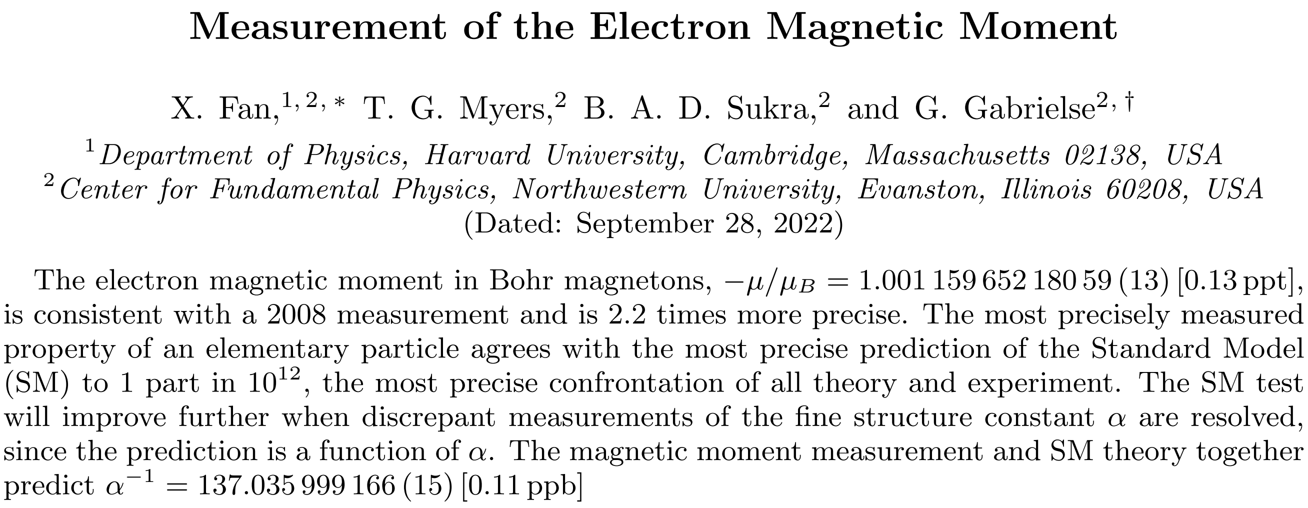


- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales
- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{\rm NP} \sim \mathcal{C}\, m_\ell^2/\Lambda_{\rm NP}^2$

→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, a_e `determines' α, tests QED & low scales

[Note: τ too short-lived for storage-rings]

a_e latest status (exp @ Northwestern): *PRL* 130 (2023) 7, 071801



Measurement of the Electron Magnetic Moment

X. Fan,[1,2,*] T. G. Myers,[2] B. A. D. Sukra,[2] and G. Gabrielse[2,†]
[1] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2] *Center for Fundamental Physics, Northwestern University, Evanston, Illinois 60208, USA*
(Dated: September 28, 2022)

The electron magnetic moment in Bohr magnetons, $-\mu/\mu_B = 1.001\,159\,652\,180\,59\,(13)$ [0.13 ppt], is consistent with a 2008 measurement and is 2.2 times more precise. The most precisely measured property of an elementary particle agrees with the most precise prediction of the Standard Model (SM) to 1 part in 10^{12}, the most precise confrontation of all theory and experiment. The SM test will improve further when discrepant measurements of the fine structure constant α are resolved, since the prediction is a function of α. The magnetic moment measurement and SM theory together predict $\alpha^{-1} = 137.035\,999\,166\,(15)$ [0.11 ppb]

[arXiv:2209.13084]

SM theory prediction depends on **α, but measurements with Cs and Rb disagree by 5.4σ :**

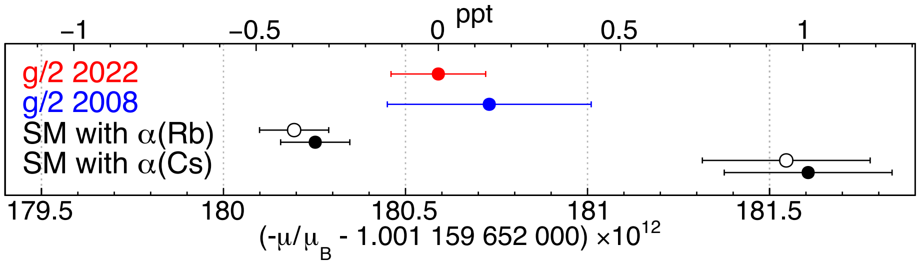
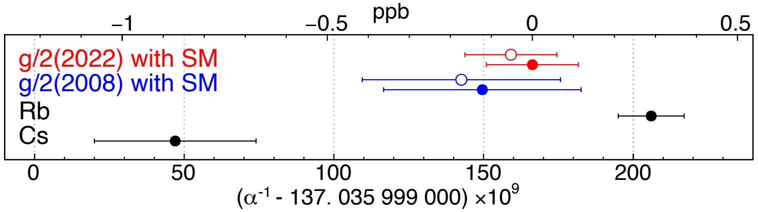


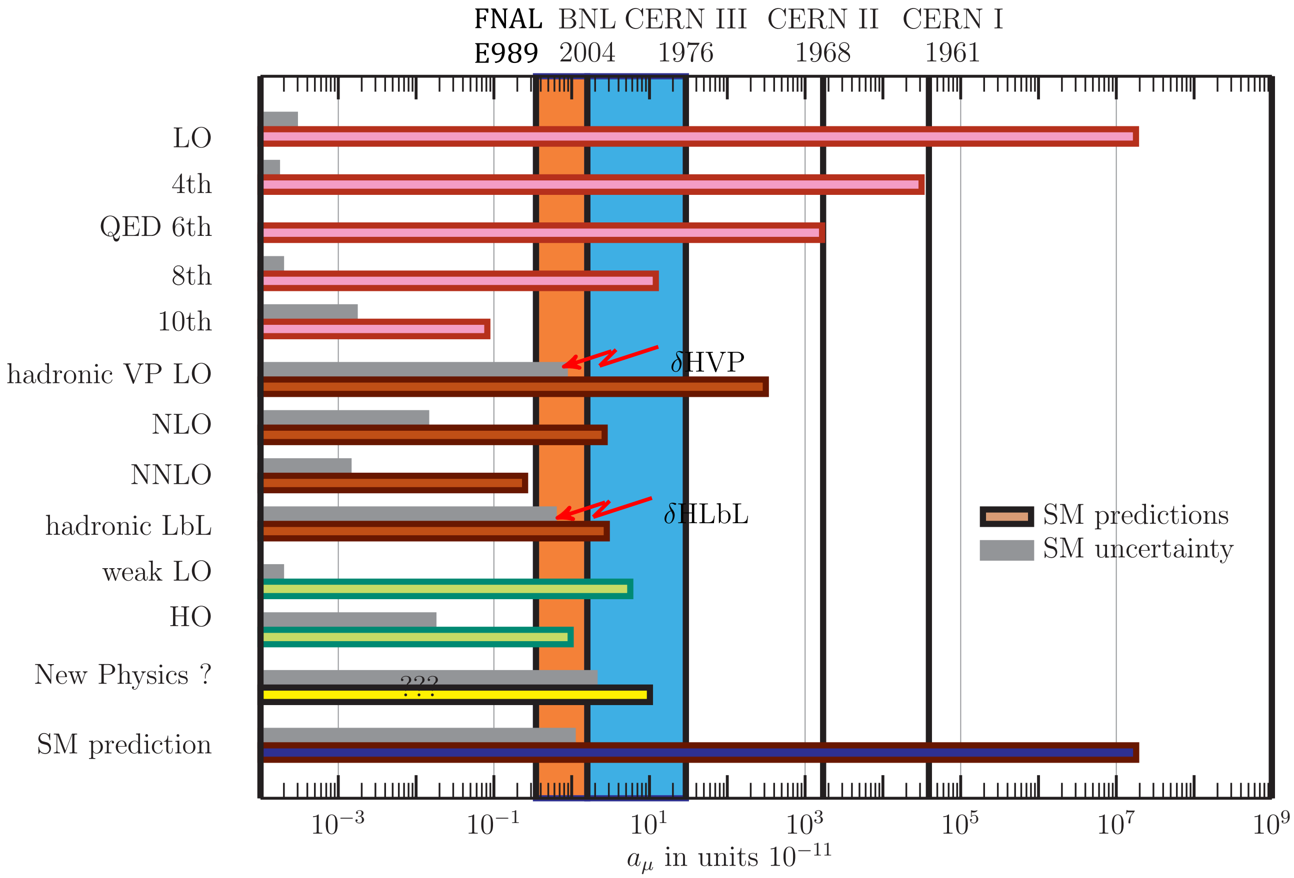
FIG. 1. This Northwestern measurement (red) and our 2008 Harvard measurement (blue) [26]. SM predictions (solid and open black points for slightly differing C_{10} [27, 28]) are functions of discrepant α measurements [29, 30]. A ppt is 10^{-12}.



⟵ Translation to derived value of **α**

Muon g-2: exp. vs theory - sensitivity chart

Plot from Fred Jegerlehner



- ➤ Need to control the hadronic contributions

$$a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}}$$

Muon g-2 Theory Initiative est. 2017

https://muon-gm2-theory.illinois.edu

``… map out strategies for obtaining the best theoretical predictions for these hadronic corrections in advance of the experimental result.''

- Organised 8 int. workshops in 2017-2022, last plenary workshop 5-9.9.2022 @ Higgscentre in Edinburgh
- Next workshop 4-8.9.2023 in Bern
- **White Paper** posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

``The anomalous magnetic moment of the muon in the Standard Model''

[T. Aoyama et al., arXiv:2006.04822, *Phys. Rept.* 887 (2020) 1-166 1000 cites today]

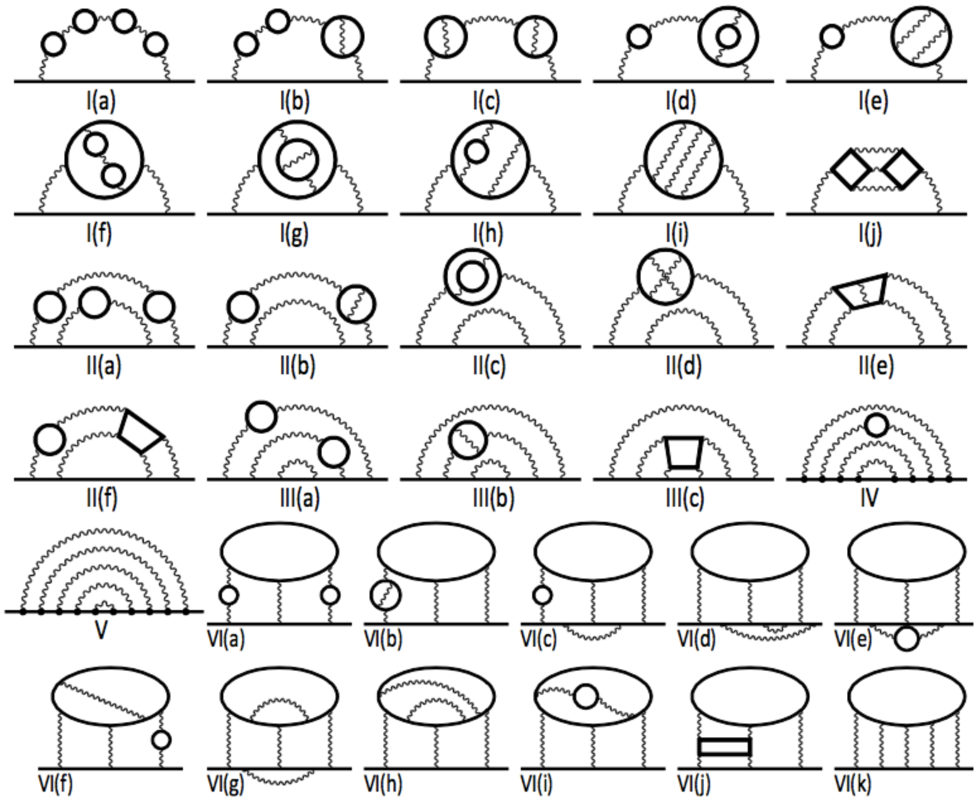
Group photo from the Seattle workshop in September 2019

a_μ^{QED} & a_μ^{weak} : a triumph for perturbative QFT

QED: Kinoshita et al. + many tests

- g-2 @ 1, 2, 3, 4 & 5 loops
- Subset of 12672 5-loop diagrams:
- code-generating code, including
- renormalisation
- multi-dim. numerical integrations

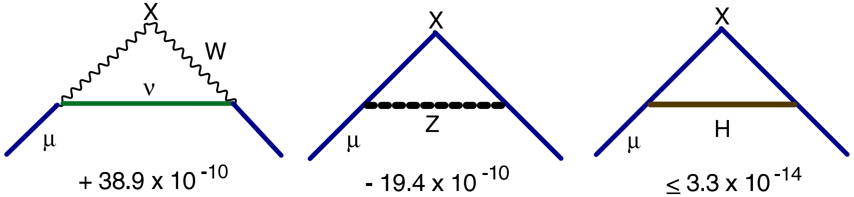
a_μ^{QED} **= 116 584 718.9 (1) × 10^{-11}** ✓



Weak: (several groups agree)

- done to 2-loop order, 1650 diagrams
- the first full 2-loop weak calculation

a_μ^{weak} **= 153.6 (1.0) × 10^{-11}** ✓



SM weak 1-loop diagrams

SM prediction from Theory Initiative vs. Experiment

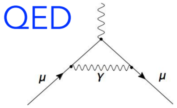
$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

[1347 cites to date]

White **P**aper [T. Aoyama et al., *Phys. Rept.* 887 (2020) 1-166]

[1000 cites to date] (0.37 ppm)

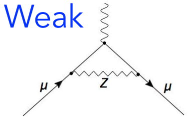
QED



+…

$116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak



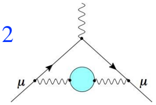
+…

$153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

α^2



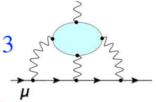
+…

$6845\,(40) \times 10^{-11}$ 0.34 ppm

[0.6%]

…Light-by-Light (HLbL)

α^3



+…

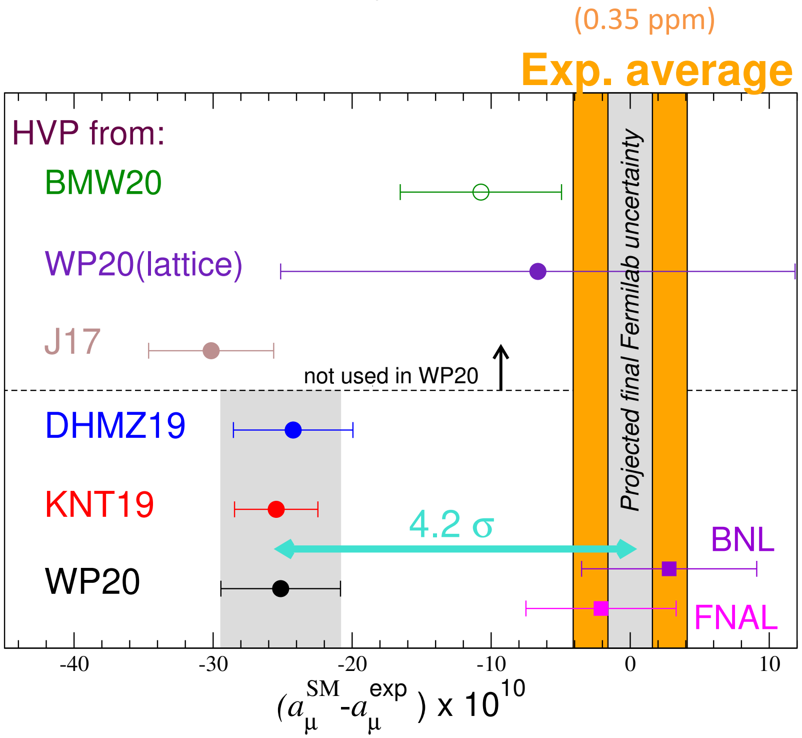
$92\,(18) \times 10^{-11}$ 0.15 ppm

[20%]

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

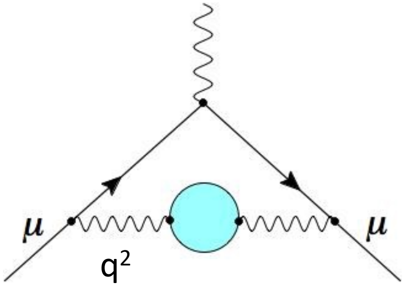
[*Phys. Rev. Lett.* 126 (2021) 14, 141801]

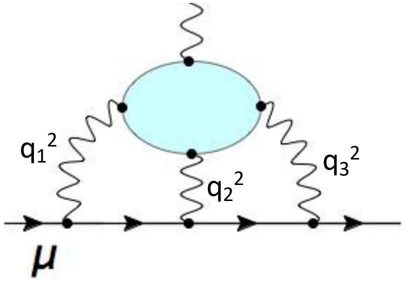
(➢ so far Run-1 only!)



➤ SM uncertainty dominated by hadronic contributions, now with **δ HVP > δ HLbL**

a_μ^{hadronic} : non-perturbative, the limiting factor of the SM prediction





- **Q:** What's in the hadronic (**Vacuum Polarisation** & **Light-by-Light scattering**) blobs?

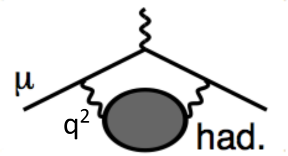
  A: Anything `**hadronic**' the virtual photons couple to, i.e. **quarks + gluons** + photons

  But: low q^2 photons dominate loop integral(s) ➠ cannot calculate blobs with perturbation theory

- **Two very different** (model independent) **strategies:**

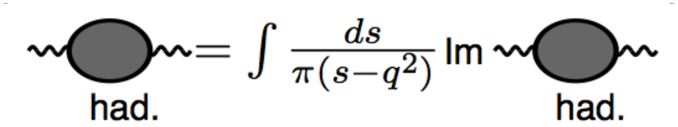
  1. use wealth of hadronic data, `**data-driven dispersive methods**':
     - data combination from many experiments, **radiative corrections** required
  2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `**lattice QCD**':
     - finite size, finite lattice spacing, artifacts from lattice actions, **QCD + QED** needed
     - numerical Monte Carlo methods require large computer resources

a_μ^{HVP} : Basic principles of **dispersive** data-driven method



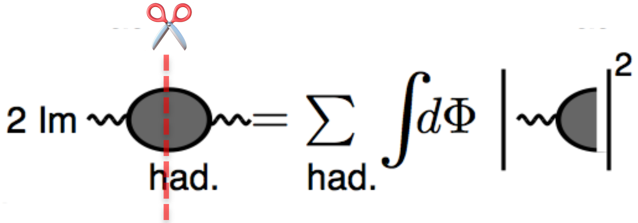
One-loop diagram with hadronic blob =

integral over q^2 of virtual photon, 1 HVP insertion



Causality ➠ analyticity ➠ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➠ Optical Theorem:

imaginary part (`cut diagram') =

sum over |cut diagram|², i.e.

∝ sum over all total hadronic cross sections

$$\boxed{a_\mu^{\rm had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\rm th}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\rm had}(s)}$$

- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

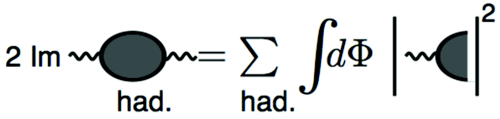
$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\rm had,LO}$

- Total hadronic cross section $\boldsymbol{\sigma_{\rm had}}$ from > 100 data sets for **e^+e^- ➞ hadrons** in > 35 final states
- Uncertainty of a_μ^{HVP} prediction from statistical & systematic uncertainties of input data
- pQCD only at large s, **no modelling** of $\sigma_{\rm had}(s)$, direct data integration

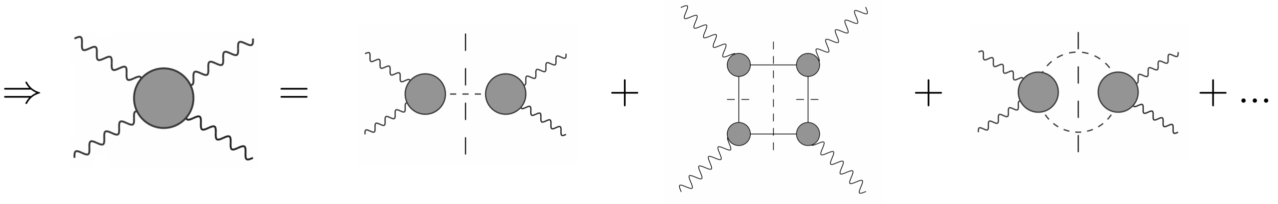
a_μ^{HLbL} : Hadronic Light-by-Light: Dispersive approach

For HVP ⇒



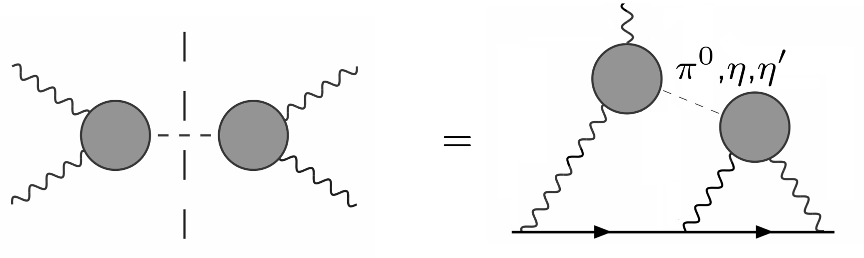
$$\Rightarrow \quad \mathrm{Im}\Pi_{\mathrm{had}}(s) = \left(\frac{s}{4\pi\alpha}\right)\sigma_{\mathrm{had}}(s)$$

For HLbL $\Rightarrow \quad \Pi_{\mu\nu\lambda\sigma} = \Pi^{\mathrm{pole}}_{\mu\nu\lambda\sigma} + \Pi^{\mathrm{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \ldots$



⇒ Dominated by pole (pseudoscalar exchange) contributions

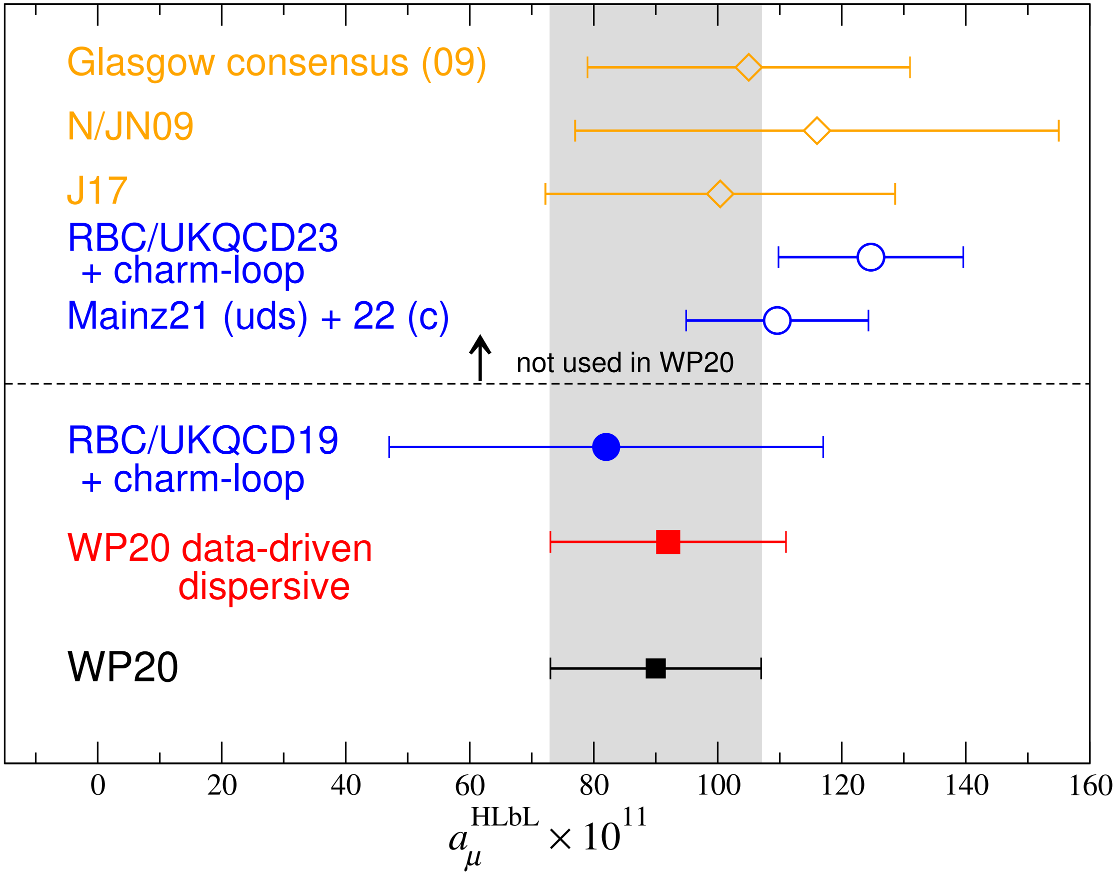
$\Pi^{\mathrm{pole}}_{\mu\nu\lambda\sigma} =$



⇒ Sum all possible diagrams to get a_μ^{HLbL}

- With new results & progress, L-by-L now more reliably predicted

a_μ^{HLbL} : WP Status/Summary of Hadronic Light-by-Light contributions



hadronic models + pQCD

lattice QCD + QED (after WP)

lattice QCD + QED

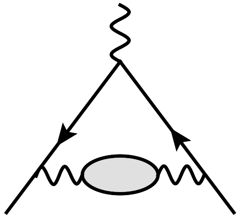
data-driven

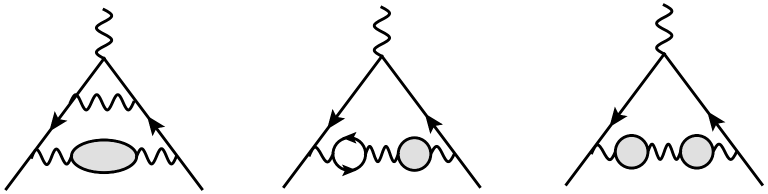
TI White Paper 2020 value:

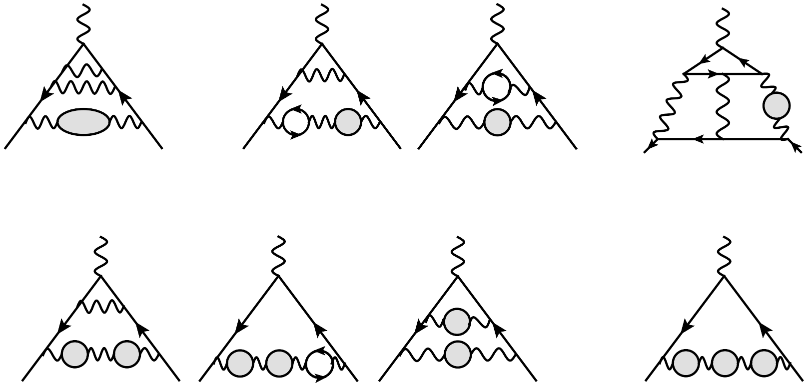
a_μ^{HLbL} **= 92 (18) × 10^{-11}** ✓

- data-driven dispersive & lattice results have confirmed the earlier model-based predictions
- **uncertainty better under control** and at 0.15ppm already **sub-leading compared to HVP**
- lattice predictions now competitive, good prospects for further error reduction needed for final expected FNAL g-2 precision

a_μ^{HVP} : Higher orders & power counting; WP20 values in $\mathbf{10^{-11}}$







➤ All hadronic blobs also contain photons,
i.e. real + virtual corrections in $\sigma_{had}(s)$

- **LO:** **6931(40)**

- **NLO:** **- 98.3(7)**

  from three classes of graphs:
  - 207.7(7) + 105.9(4) + 3.4(1) [KNT19]
  (photonic, extra e-loop, 2 had-loops)

- **NNLO:** **12.4(1)** [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]

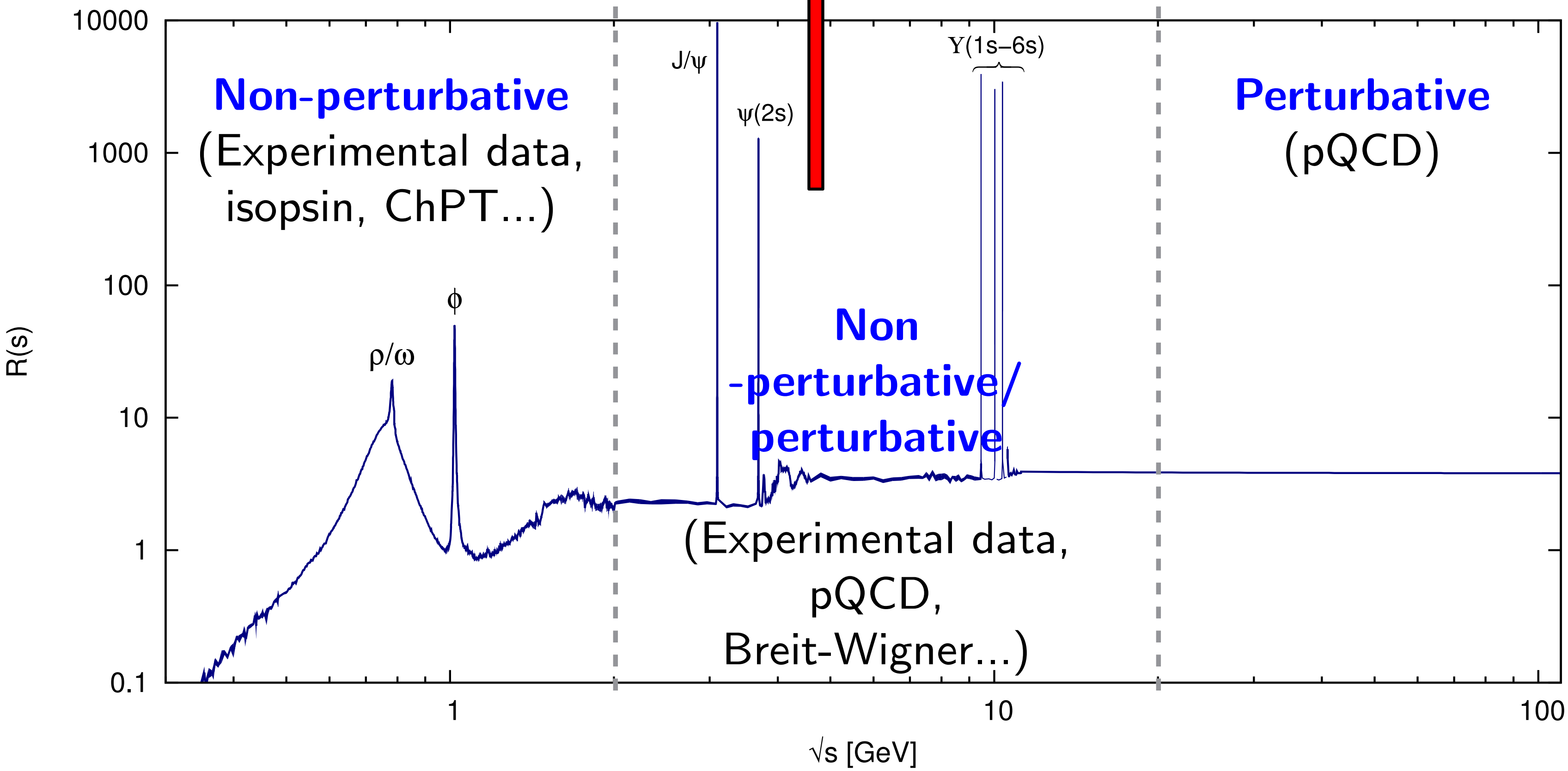
  from five classes of graphs:
  8.0 - 4.1 + 9.1 - 0.6 + 0.005

➥ good convergence,
iterations of hadronic blobs _very_ small

➠ `double-bubbles' very small

HVP disp.: cross section (in terms of R-ratio) input

$$a_\mu^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}R(s)K(s), \text{ where } R(s) = \frac{\sigma^0_{\text{had},\gamma}(s)}{4\pi\alpha^2/3s}$$



Non-perturbative
(Experimental data, isopsin, ChPT…)

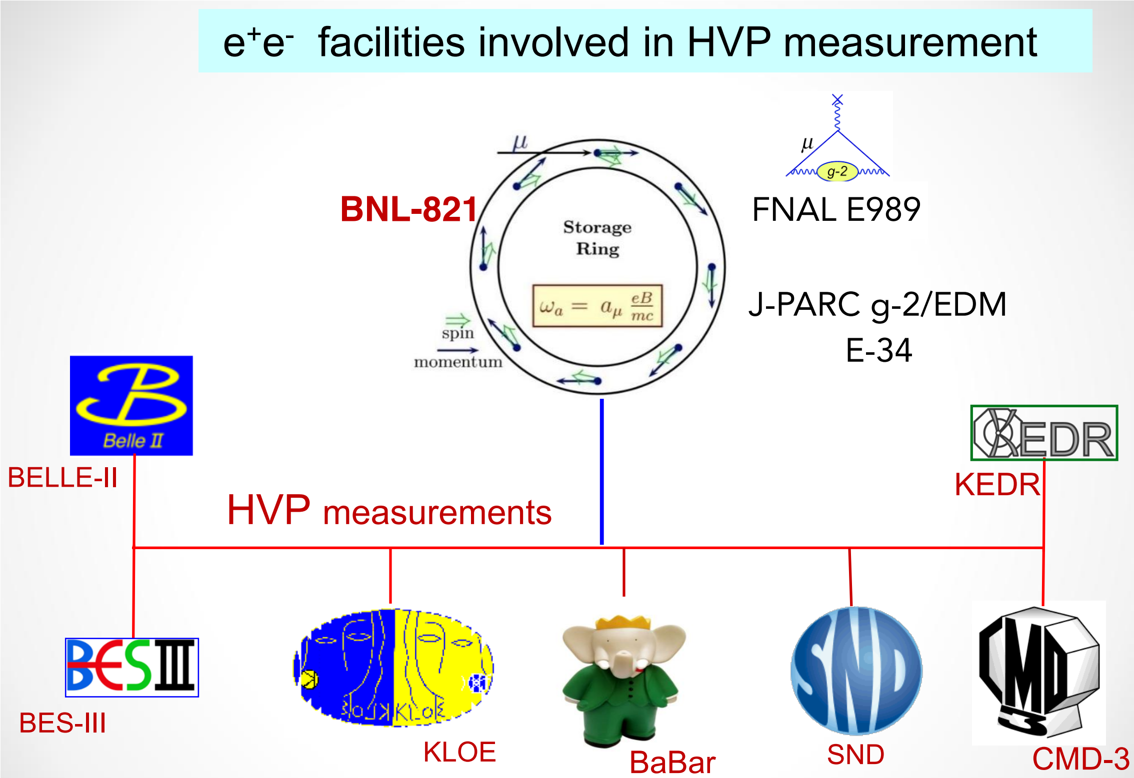
Non -perturbative/ perturbative
(Experimental data, pQCD, Breit-Wigner…)

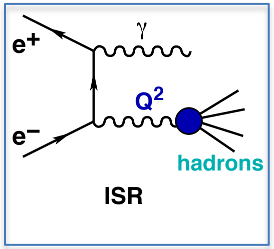
Perturbative
(pQCD)

Must build full hadronic cross section/R-ratio…

HVP: Recent (of >25 years) experiments providing input $\sigma_{had}(s)$ data

S. Serednyakov (for SND) @ HVP KEK workshop





- Different methods: \`**Direct Scan**' (tunable e^+e^- beams) &
  \`**Radiative Return**' (Initial State Radiation scan at fixed cm energy) ↗
- Over last decades detailed studies of **radiative corrections** & **Monte Carlo Generators** for $\sigma_{had}(s)$
  - ➤ **RadioMonteCarLow** Working Group report: *Eur. Phys. J.* C66 (2010) 585-686
  - ➤ full NLO radiative corrections in ISR MC ***Phokhara***: Campanario et al, PRD 100(2019)7,076004

HVP dispersive: cross section compilation

How to get the most precise σ^0_{had}? Use of $e^+e^- \rightarrow$ hadrons ($+\gamma$) data:

- **Low energies: sum ~35 exclusive channels**, 2π, 3π, 4π, 5π, 6π, KK, KKπ, KKππ, ηπ, …, [now very limited use iso-spin relations for missing channels]
- **Above √s ~1.8 GeV:** use of **inclusive data or pQCD** (away from flavour thresholds), supplemented by narrow resonances (J/Ψ, Υ)
- Challenge of **data combination** (locally in √s, with **error inflation if tensions**):
  - many experiments, different energy ranges and bins,
  - **statistical + systematic errors** from many different sources, use of **correlations**

➤ Significant differences between **DHMZ** and **KNT** in use of correlated errors:
  - KNT allow non-local correlations to influence mean values,
  - DHMZ restrict this but retain correlations for errors, also estimate cross channel corrs.

- σ^0_{had} means the \`bare’ cross section, i.e. **excluding** \`running coupling’ (**VP**) effects, but **including** Final State (γ) Radiation:

☛ data need radiative corrections, compilations estimate additional uncertainty,

e.g. in KNT: $\delta a_\mu^{had,\,VP} = 2.1 \times 10^{-11}$, and $\delta a_\mu^{had,\,FSR} = 7.0 \times 10^{-11}$

Rad Corrs: ISR. Scan vs ISR method. Phokara

- ISR is always there, also for `**direct scan**' measurements, well understood theoretically and routinely taken into account in the experimental analyses
  (deconvolution of measured hadrons (+γ) cross section to get the cross section w/out ISR)

- In `**Radiative Return**' analyses, ISR emission defines already the lowest order process, hence higher orders, including FSR, are crucial

- The origin of additional photons can not be determined on an event-by-even basis

- Making use of high luminosities at meson factories, large event numbers can still be achieved with the ISR method, despite the parametric $\boldsymbol{\alpha/\pi}$ suppression

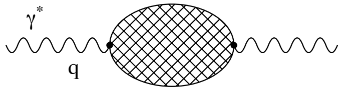
- Different variants: w. or w/out γ detection (large/small angle), luminosity from Bhabha or $\boldsymbol{\mu^+\mu^-}$

- Crucial Monte Carlo generator: *Phokhara*
  - -- now with complete NLO corrections for $e^+e^- \rightarrow \boldsymbol{\mu^+\mu^-\gamma}, \boldsymbol{\pi^+\pi^-\gamma}$
  - -- was not available for the earlier KLOE & BaBar analyses; study of higher orders using the latest version of *Phokhara* indicate that (missing) higher order corrections are not the source of the KLOE vs BaBar discrepancy (see below)

Rad. Corrs.: HVP for running α(q²). Undressing

- Dyson summation of Real part of one-particle irreducible blobs $\mathbf{\Pi}$ into the effective, real running coupling $\boldsymbol{\alpha}_{\mathbf{QED}}$:

$\Pi =$



Full photon propagator $\sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \ldots$

$$\rightsquigarrow \qquad \alpha(q^2) = \frac{\alpha}{1 - \mathrm{Re}\Pi(q^2)} = \alpha \,/\, \left(1 - \Delta\alpha_{\mathrm{lep}}(q^2) - \Delta\alpha_{\mathrm{had}}(q^2)\right)$$

- The Real part of the VP, $\mathrm{Re}\Pi$, is obtained from the Imaginary part, which via the *Optical Theorem* is directly related to the cross section, $\mathrm{Im}\Pi \sim \sigma(e^+e^- \to hadrons)$:

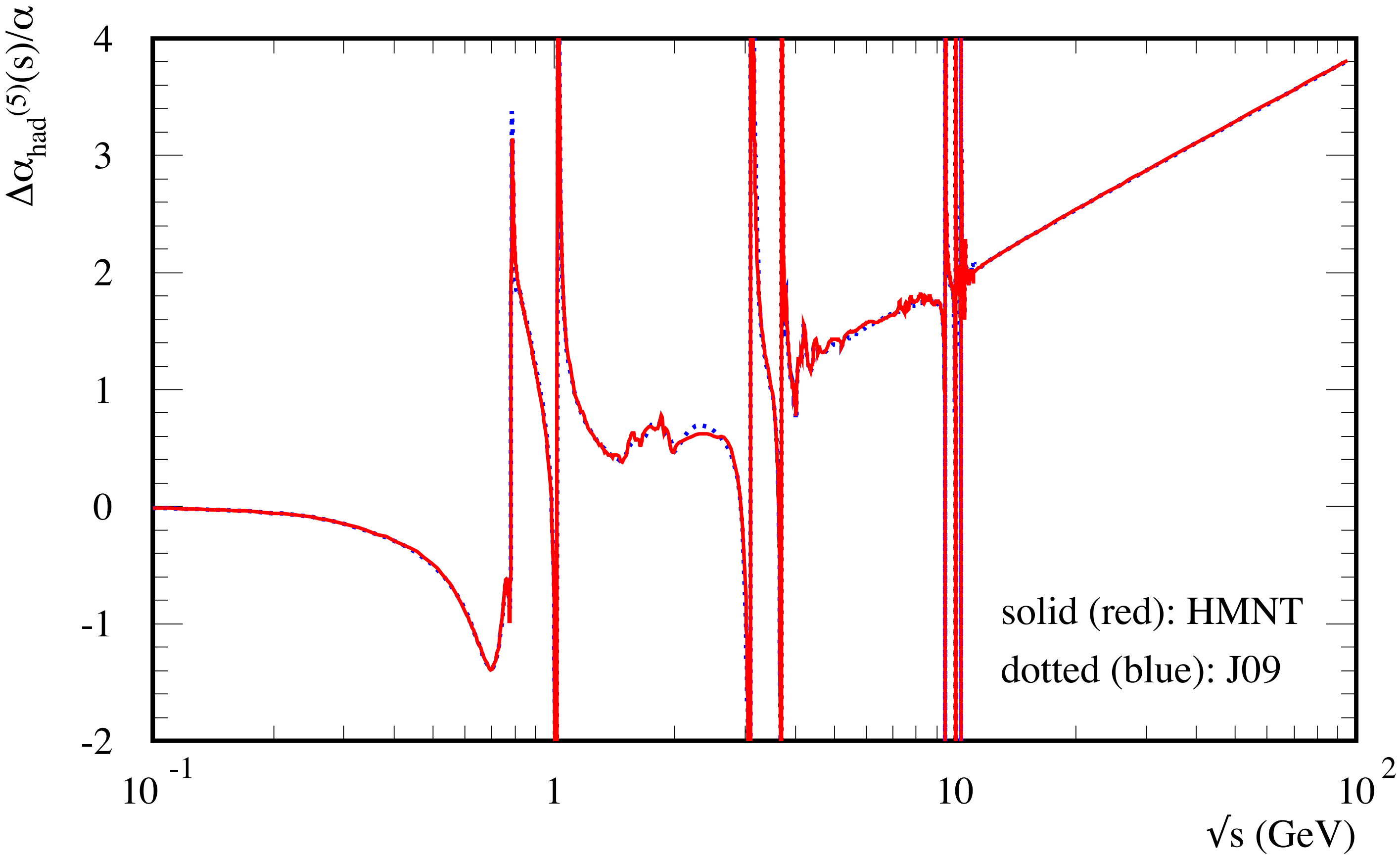
$$\Delta\alpha^{(5)}_{\mathrm{had}}(q^2) = -\frac{q^2}{4\pi^2\alpha} \mathrm{P} \int_{m_\pi^2}^{\infty} \frac{\sigma^0_{\mathrm{had}}(s)\,\mathrm{d}s}{s - q^2}\,, \quad \sigma_{\mathrm{had}}(s) = \frac{\sigma^0_{\mathrm{had}}(s)}{|1 - \Pi|^2}$$

[$\rightarrow \sigma^0$ requires 'undressing', e.g. via $\cdot(\alpha/\alpha(s))^2$ $\rightsquigarrow$ iteration needed]

- Observable cross sections σ_{had} contain the |full photon propagator|^2, i.e. |infinite sum|^2.
  → To include the subleading Imaginary part, use dressing factor $\frac{1}{|1-\Pi|^2}$.

Rad. Corrs.: HVP for running α(q^2). Undressing

- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner's new routines



For demonstration only, results >10 years old!

Different groups use their own HVP routines:

- Fred Jegerlehner,
- DHMZ,
- KNT,
- Novosibirsk
  (Fedor Ignatov)

→ with new version big differences (with 2003 version) gone

– smaller differences remain and reflect different choices, smoothing etc.

Rad. Corrs.: Final State γ Radiation

- Real + virtual , <u>must be included</u> in $\boldsymbol{\sigma^0}_{\mathbf{had}}$ as part of the (hadronic) dynamics
- In measured cross sections, virtual and soft/collinear photons are always included,
- but some events with hard real radiation are cut-off by experimental analyses (through event selection/classification, cuts, acceptances):

  -- limited phase space for hard radiation at low energies in scan mode

  -- no problem if γ missed but the event counted, but

  -- possibly important effect in radiative return (ISR) mode, depending on energy

- Experiments account for this and add (back missed) FSR in their data analyses

  -- using **MC generators** with **corrections based on scalar QED** for **π**s and **K**s

  (checked to work ok at low energies when hadronic substructure hardly resolved)

  -- for analyses based on Radiative Return (in particular for the 2***π*** channel),

  ISR and FSR are an integral part of the MCs used (*EVA*, *Phokhara*)

  -- **possible limitations for accuracy discussed at recent WorkStop/ThinkStart,**

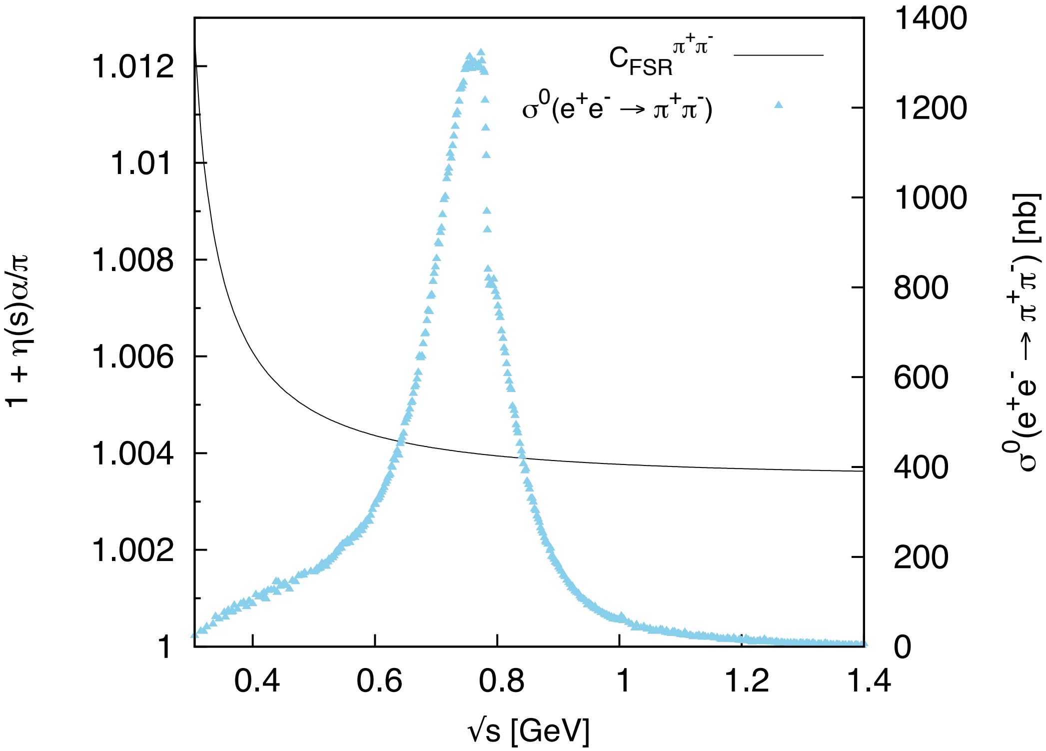
  work planned for higher order corrections & MC implementation

Rad. Corrs.: inclusive Final State γ Radiation in sQED

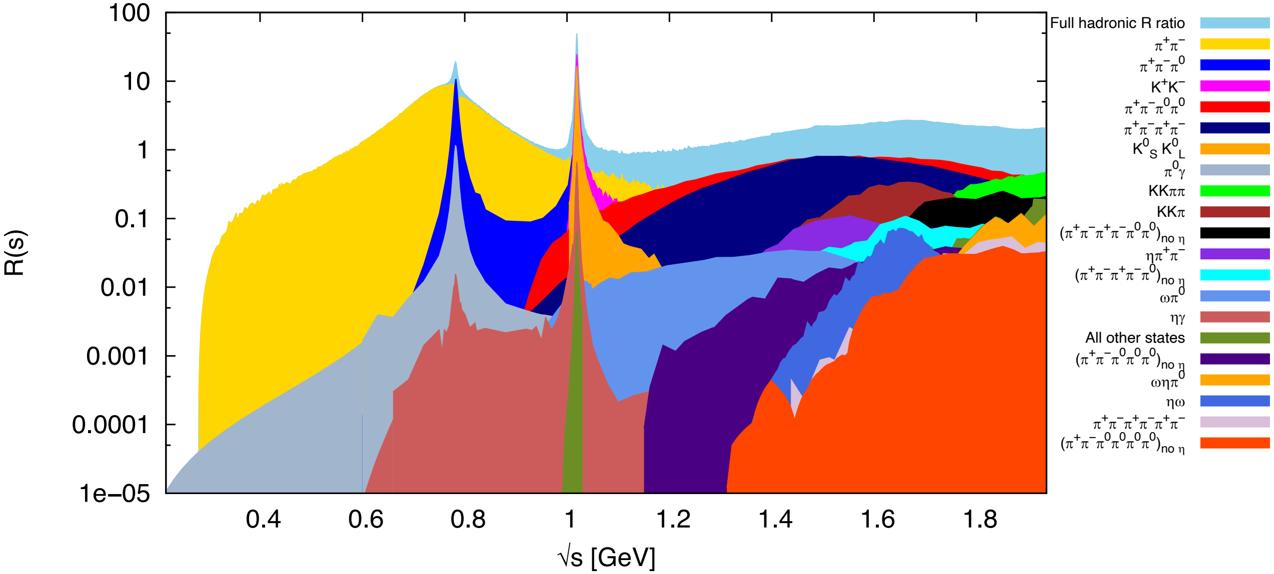
- \`Schwinger' formula for inclusive (r+v) FSR: $\sigma^0_{\mathrm{had},(\gamma)}(s) = \sigma^0_{\mathrm{had}}(s)\left(1 + \eta(s)\frac{\alpha}{\pi}\right)$

  [\`hard' real radiation (above a cutoff) is finite and easy to calculate as part of **η**(s)]

- Example 2***π***: inclusive correction compared to cross section in the **ρ** peak region



HVP: **Landscape** of $\sigma_{had}(s)$ data & most important $\pi^+\pi^-$ channel



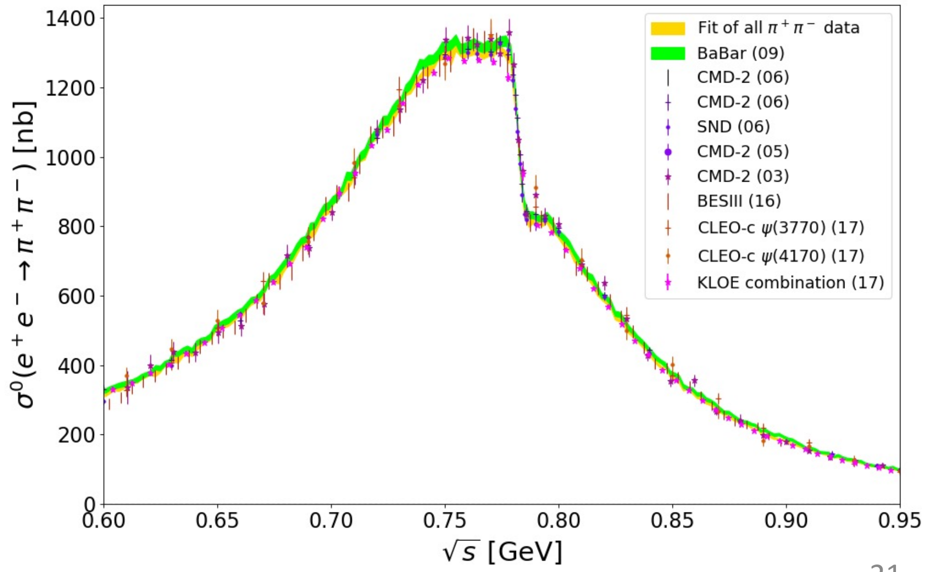
[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of 2π

$\pi^+\pi^-$:

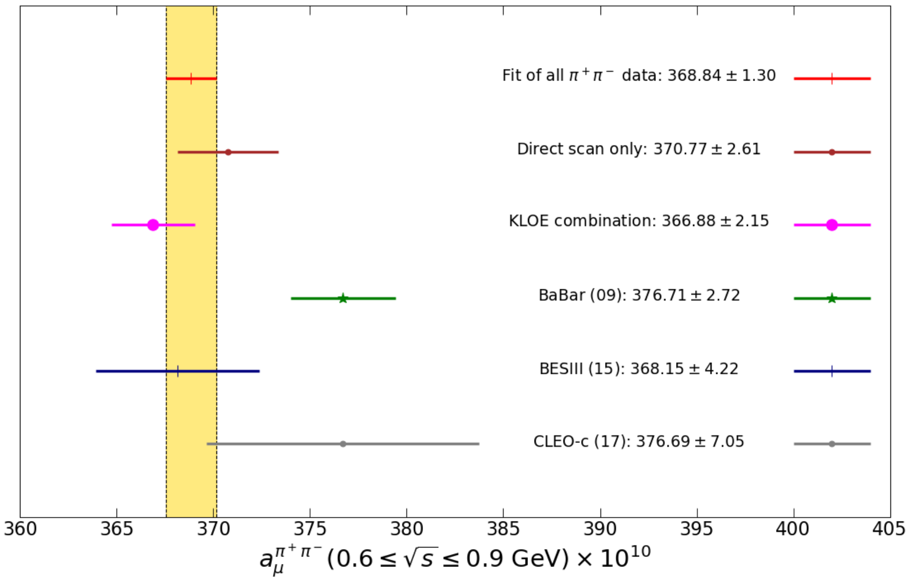
- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from **6 independent experiments** with different systematics and different radiative corrections
- Data sets from Radiative Return dominate,
  until now…

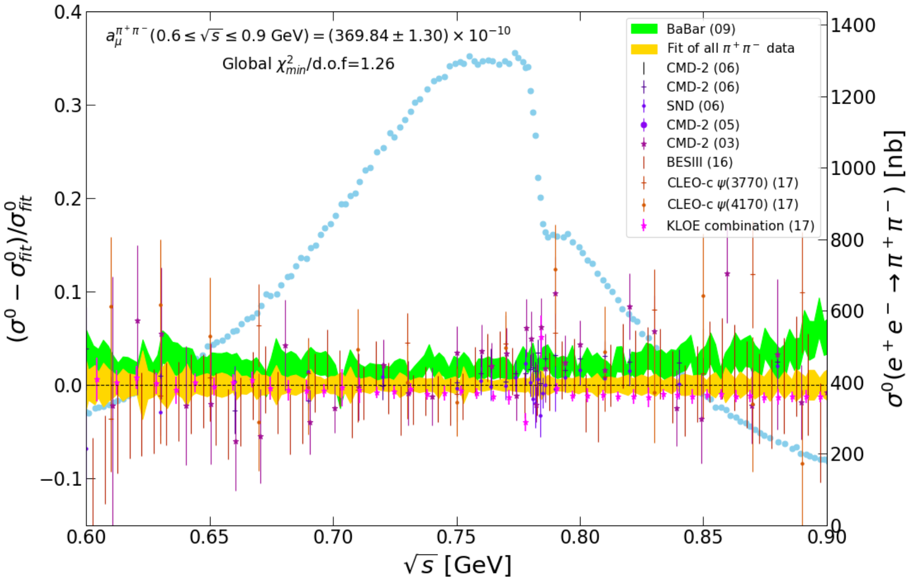
[KNT19, PRD101, 014029]



a_μ^{HVP} : $\pi^+\pi^-$ channel **KLOE vs. Babar puzzle, enlarged WP error**

[Plots from KNT19]





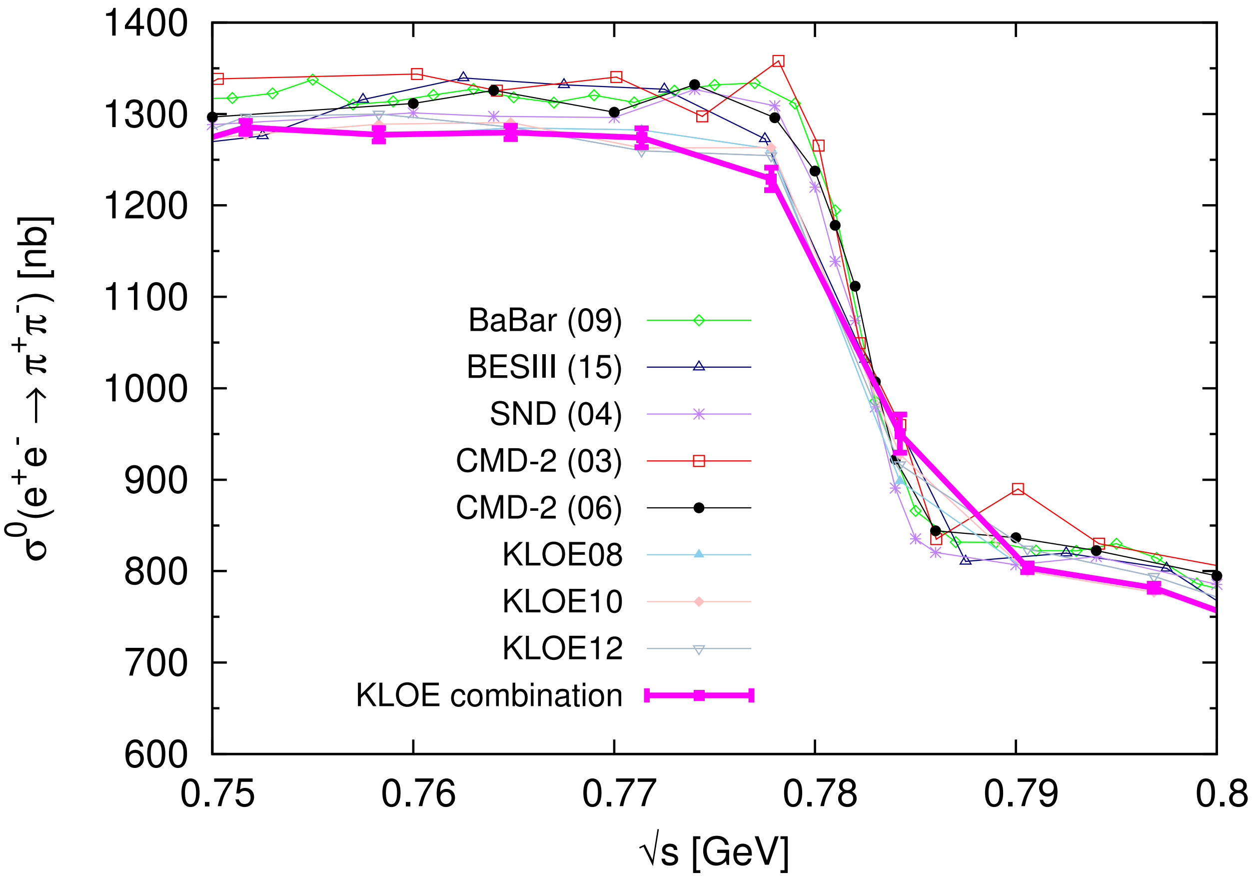
- **Tension** between different sets, especially between the most precise 4 sets from **BaBar** and **KLOE**
- Inflation of error with **local χ^2_{min}** accounts for tensions, leading to a **~14% error inflation**
- Important role of **correlations**; their treatment in the data combination is crucial and can lead to significant differences between different combination methods (KNT vs. DHMZ)

- Differences in data and methods accounted for in **WP merging procedure,** leading to enlarged error for a_μ^{HVP}. **Procedure not well suited to cover CMD-3.**

HVP: $\pi^+\pi^-$ channel

- **Tension** between data sets from KLOE, BaBar, CMD-2, SND and BESIII in the **ρ-ω** interference region
- Note that some differences, possibly due to binning effects, are washed out in the dispersion integral for $a_\mu^{2\pi}$

Figure from KLOE (+KT) combination paper JHEP 03(2018)173

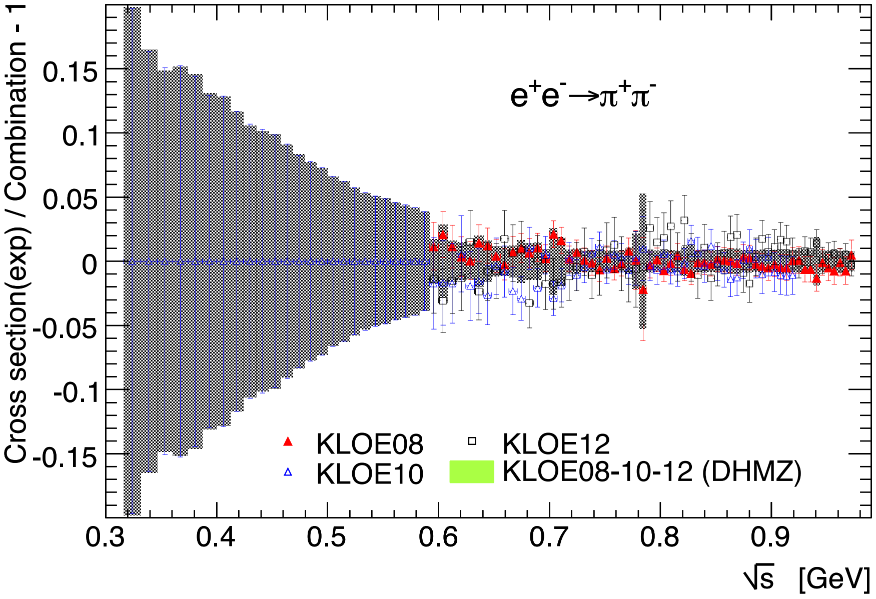


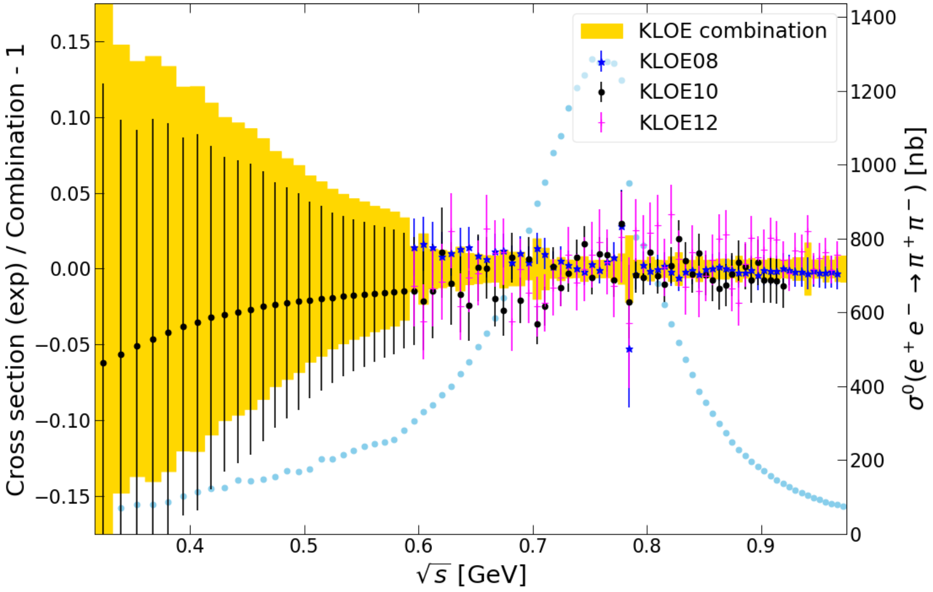
HVP: $\pi^+\pi^-$ channel

- Combination of same three KLOE data sets by DHMZ (left) and KNT (right), leading to

- different results, depending on use of **long-range correlations** through systematic errors;

  -- DHMZ: restricted to error estimate, but not used to determine combination mean values
  -- KNT: full use of correlated errors in fit, allowing change of mean values within errors



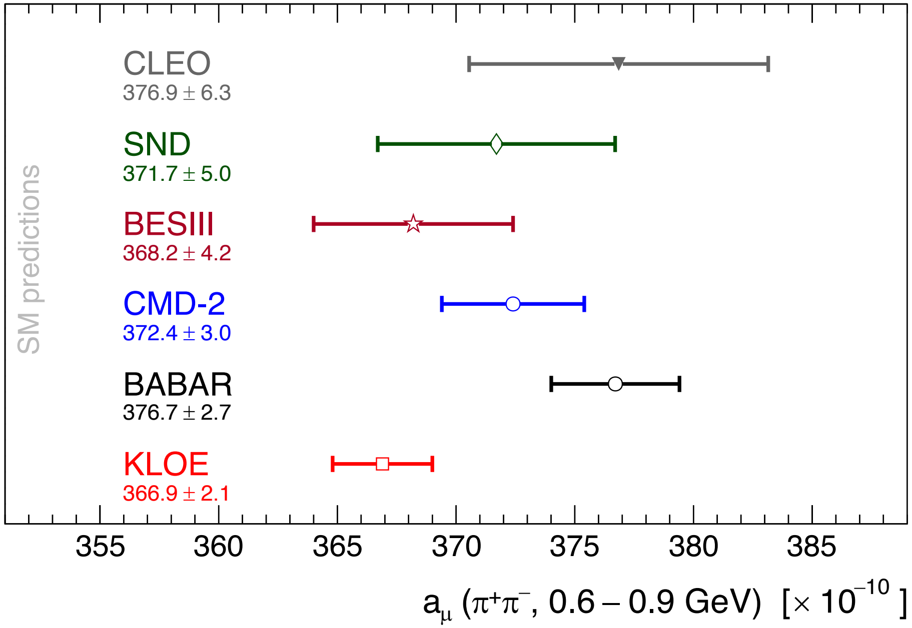


HVP: $\pi^+\pi^-$ channel [DHMZ, *Eur. Phys. J.* C 80(2020)3, 241]

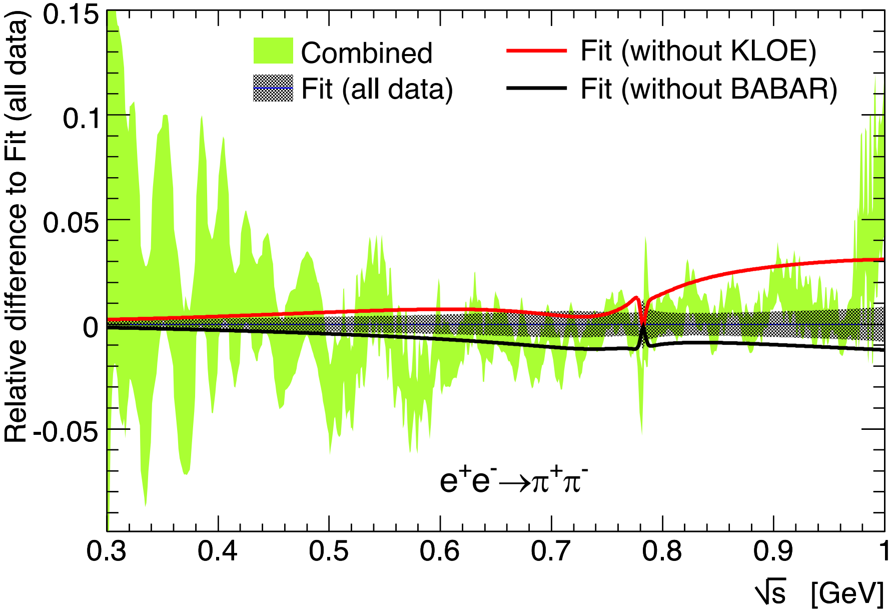
- In addition they employ a fit, based on analyticity + unitarity + crossing symmetery, similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraints/lower errors at low energies

- For 2**π**, based on difference between result for $a_\mu^{\pi\pi}$ w/out KLOE and BaBar, sizeable additional systematic error is applied and mean value adjusted

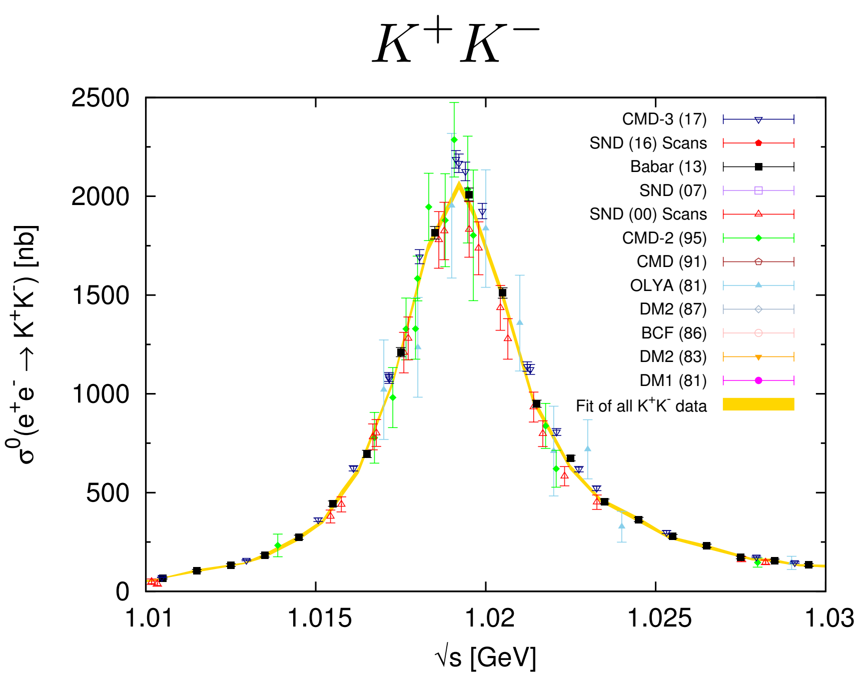
arXiv:1908.00921 Figure 5:



arXiv:1908.00921 Figure 6:



HVP: Kaon channels [KNT18, PRD97, 114025]


K^+K^-

New data:

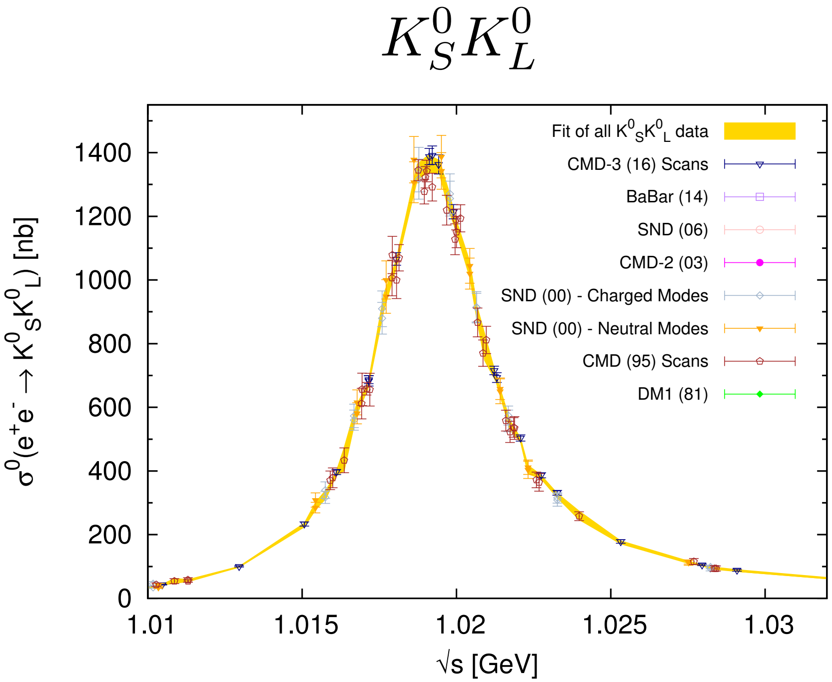
BaBar: [Phys. Rev. D 88 (2013), 032013.]
SND: [Phys. Rev. D 94 (2016), 112006.]
CMD-3: [arXiv:1710.02989.]

Note: CMD-2 data [Phys. Lett. B 669 (2008) 217.] omitted as waiting reanalysis.

$$a_\mu^{K^+K^-} = 23.03 \pm 0.22_{\mathrm{tot}}$$

HLMNT11: $22.15 \pm 0.46_{\mathrm{tot}}$

Large increase in mean value


$K_S^0 K_L^0$

New data:

BaBar: [Phys. Rev. D 89 (2014), 092002.]
CMD-3: [Phys. Lett. B 760 (2016) 314.]

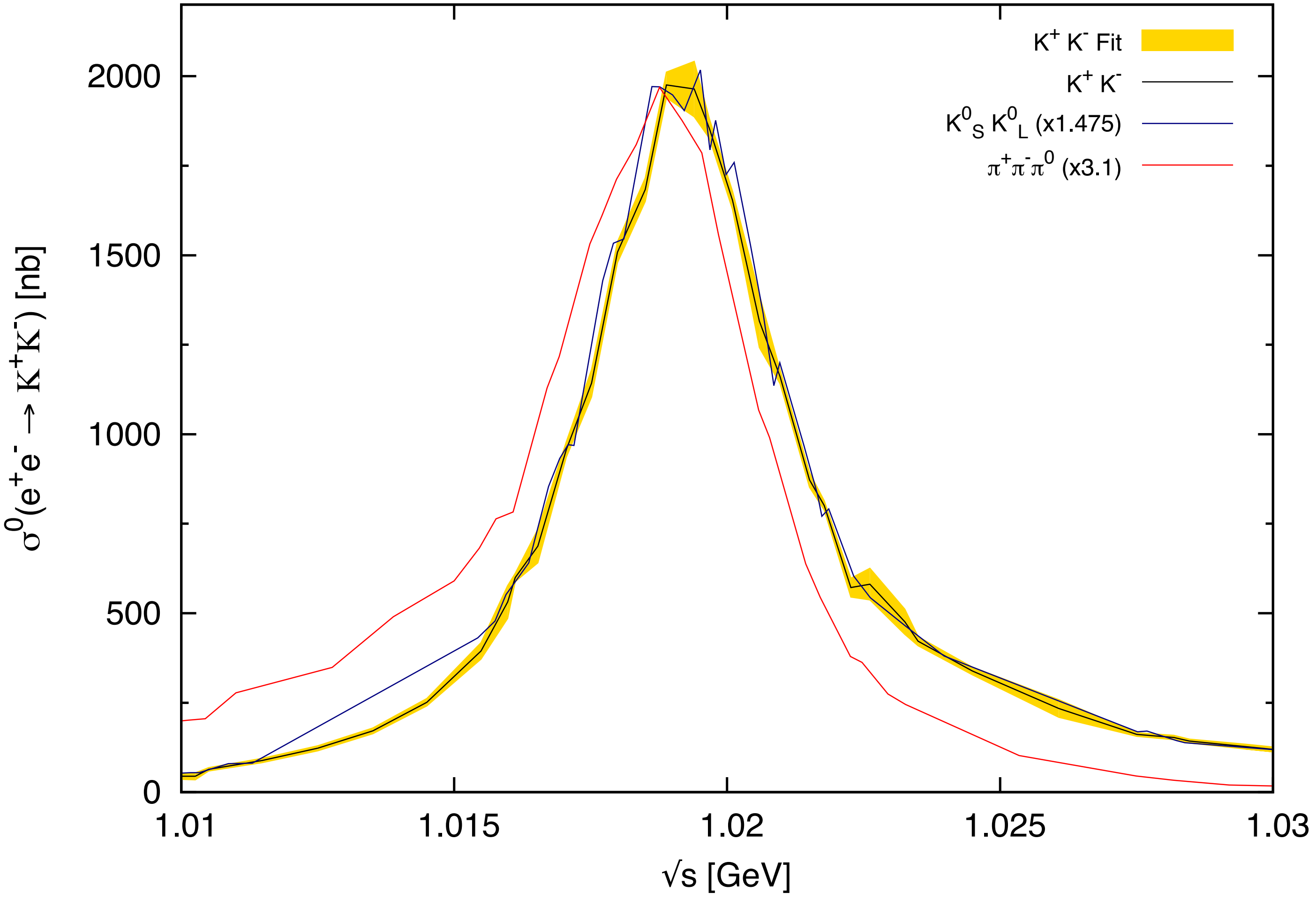
$$a_\mu^{K_S^0 K_L^0} = 13.04 \pm 0.19_{\mathrm{tot}}$$

HLMNT11: $13.33 \pm 0.16_{\mathrm{tot}}$

Large changes due to new precise measurements on ϕ

HVP: Φ in different final states K^+K^-, $K_S^0K_L^0$, $\pi^+\pi^-\pi^0$

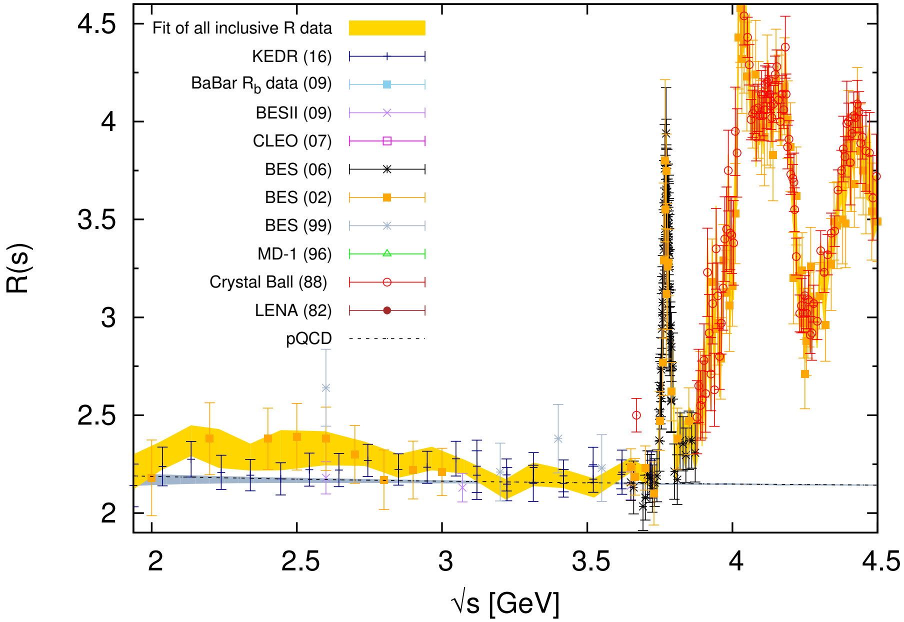
- Direct data integration automatically accounts for all hadronic dynamics, no resonance fits/parametrisations or estimates of mixing effects needed.



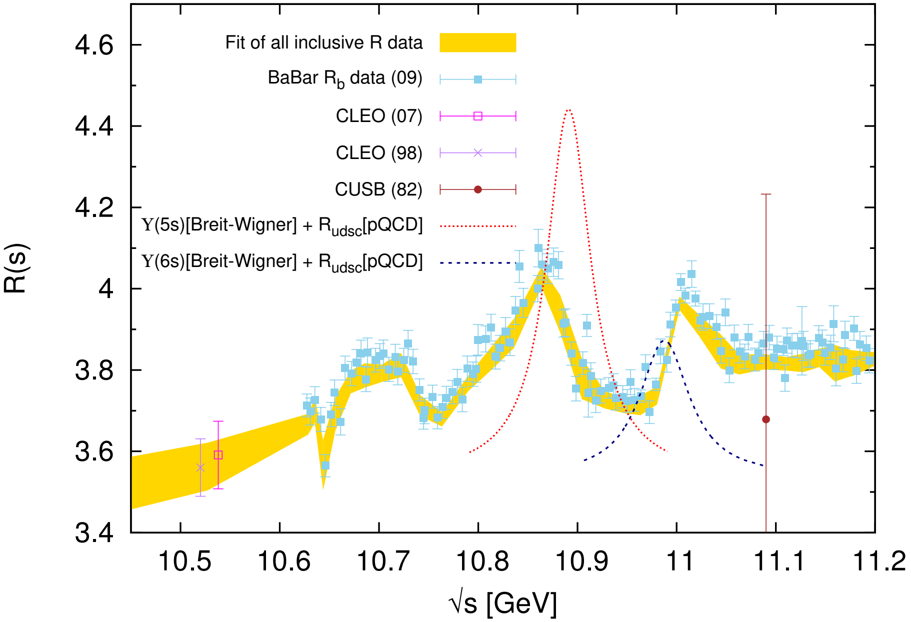
For demo. only, does not include latest data

HVP: σ_{had} inclusive region [KNT18]

$\Rightarrow$ New KEDR inclusive R data [Phys.Lett. B770 (2017) 174-181, Phys.Lett. B753 (2016) 533-541] and BaBar R_b data [Phys. Rev. Lett. 102 (2009) 012001.].



KEDR data improves the inclusive data combination below $c\bar{c}$ threshold



R_b resolves the resonances of the $\Upsilon(5S-6S)$ states.

$\Longrightarrow$ **Choose to adopt entirely data driven estimate from threshold to 11.2 GeV**

$$a_\mu^{\text{Inclusive}} = 43.67 \pm 0.17_{\text{stat}} \pm 0.48_{\text{sys}} \pm 0.01_{\text{vp}} \pm 0.44_{\text{fsr}} = 43.67 \pm 0.67_{\text{tot}}$$

HVP: White Paper comparison

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels [CHS 2018, HHKS 2019]

HVP: White Paper merging procedure

Conservative merging procedure developed during 2019 Seattle TI workshop:

- Accounts for the different results obtained by different groups based on the same or similar experimental input
- Includes correlations and their different treatment as much as possible
- Allows to give one recommended (merged) result, which is conservative w.r.t. the underlying (and possibly underestimated) systematic uncertainties
- Note: Merging leads to a bigger error estimate compared to individual evaluations; error `corridor' defined by embracing choices goes far beyond χ^2_{min} inflation

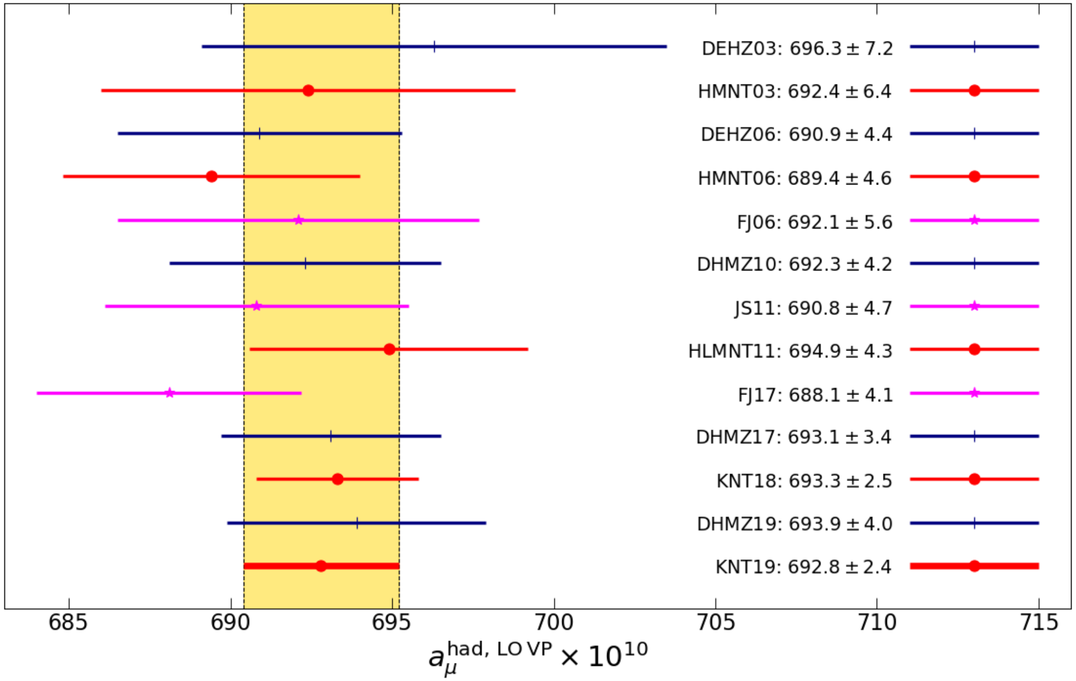
➠ $a_\mu^{HVP, LO} = 693.1\ (4.0) \times 10^{-10}$ is the result used in the WP `SM2020' value

- This result does not include lattice, but in 2020 was compatible with published full results, apart from the BMW prediction:

  $a_\mu^{HVP, LO}$ (BMW) = 707.5 (5.5) × 10^{-10} [Nature 2021] ⤳ **1.5/2.1 σ** tension w. exp/WP20

Many efforts are ongoing to understand this new puzzle!

a_μ^{HVP} : > 20 years of data based predictions, `pies’

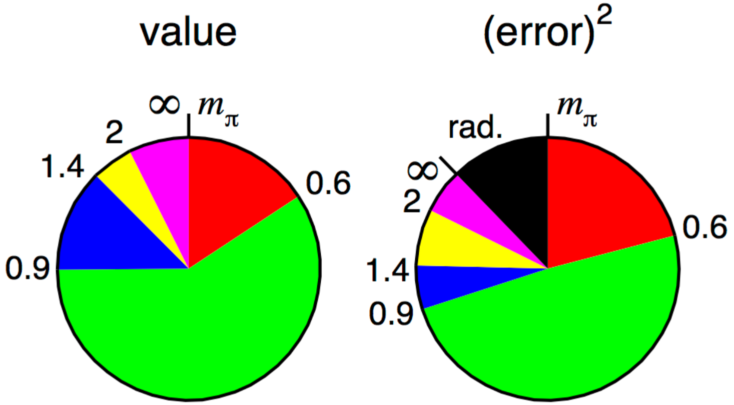


- **Stability** and **consolidation** over two decades thanks to more and better data input and improved compilation procedures

- Compare with **merged** DHMZ & KNT WP20 value:

$a_\mu^{\text{had, LO VP}}$(WP20) = 693.1(4.0)×$10^{-10}$

Pie diagrams for KNT compilation:



- error still dominated by the two pion channel
- significant contribution to error from additional **uncertainty from radiative corrections**
- **further puzzle from most recent CMD-3 data**

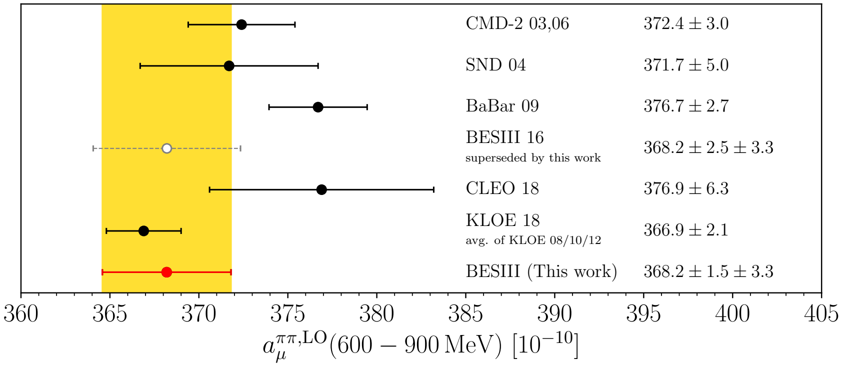
HVP: New/updated data sets since KNT19

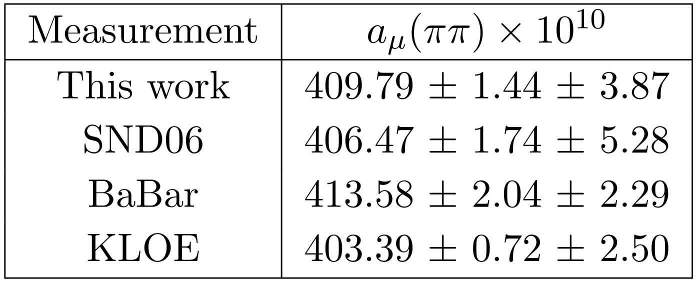
- **pi+pi-pi0**, BESIII (2019), arXiv:1912.11208
- **pi+pi-** [covariance matrix erratum], BESIII (2020), Phys.Lett.B 812 (2021) 135982 (erratum)
- **K+K-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 12, 1139
- **etapi0gamma** (res. only), SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-**, SND (2020), JHEP 01 (2021) 113
- **etaomega** → pi0gamma, SND (2020), Eur.Phys.J.C 80 (2020) 11, 1008
- **pi+pi-pi0**, SND (2020), Eur.Phys.J.C 80 (2020) 10, 993
- **pi+pi-pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112003
- **pi+pi-2pi0omega**, BaBar (2021), Phys. Rev. D 103, 092001
- **etaetagamma**, SND (2021), Eur.Phys.J.C 82 (2022) 2, 168
- **etaomega**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **omegaetapi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **pi+pi-4pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi0pi0eta**, BaBar (2021), Phys.Rev.D 103 (2021) 9, 092001
- **pi+pi-3pi0eta**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **2pi+2pi-3pi0**, BaBar (2021), Phys. Rev. D 103, 092001
- **omega3pi0**, BaBar (2021), Phys.Rev.D 104 (2021) 11, 112004
- **pi+pi-pi+pi-eta**, BaBar (2021), Phys. Rev. D 103, 092001
- **inclusive**, BESIII (2021), Phys.Rev.Lett. 128 (2022) 6, 062004
- …

HVP: New/updated data sets since KNT19

- No new full KNT update at this stage yet, *preliminary estimates* show no big surprises
- KNT analysis framework **blinded** in autumn 2022 (see Alex's talk at TI meeting in Edinburgh)

- **pi+pi-,** inclusion of BESIII (2020 erratum) & SND (2020):





Measurement	$a_\mu(\pi\pi)\times 10^{10}$
This work	$409.79 \pm 1.44 \pm 3.87$
SND06	$406.47 \pm 1.74 \pm 5.28$
BaBar	$413.58 \pm 2.04 \pm 2.29$
KLOE	$403.39 \pm 0.72 \pm 2.50$

(not yet full statistics, systematics?)

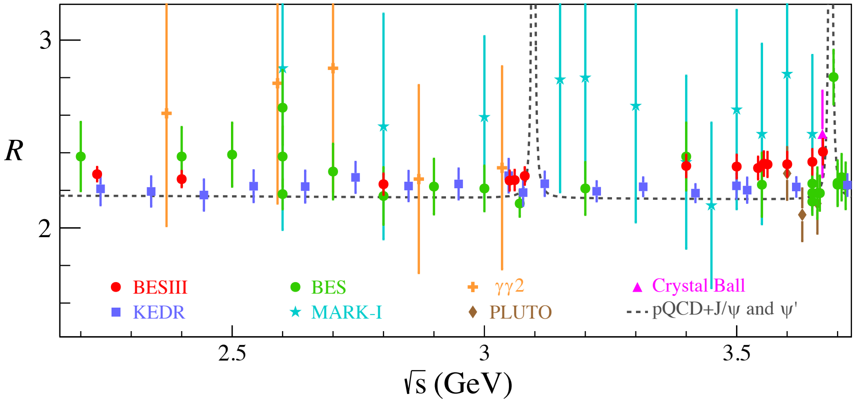
$a_\mu^{2\pi}$ [0.305 … 1.937 GeV] (KNT19) = (503.46 ± 1.91) × 10^{-10} ➡ (503.88 ± 1.79) × 10^{-10} (prel.)

- **inclusive**, inclusion of BESIII (2021):

$a_\mu^{\mathrm{incl.}}$ [1.937 … 11.2 GeV] (KNT19) =

(43.55 ± 0.67) × 10^{-10} ➡

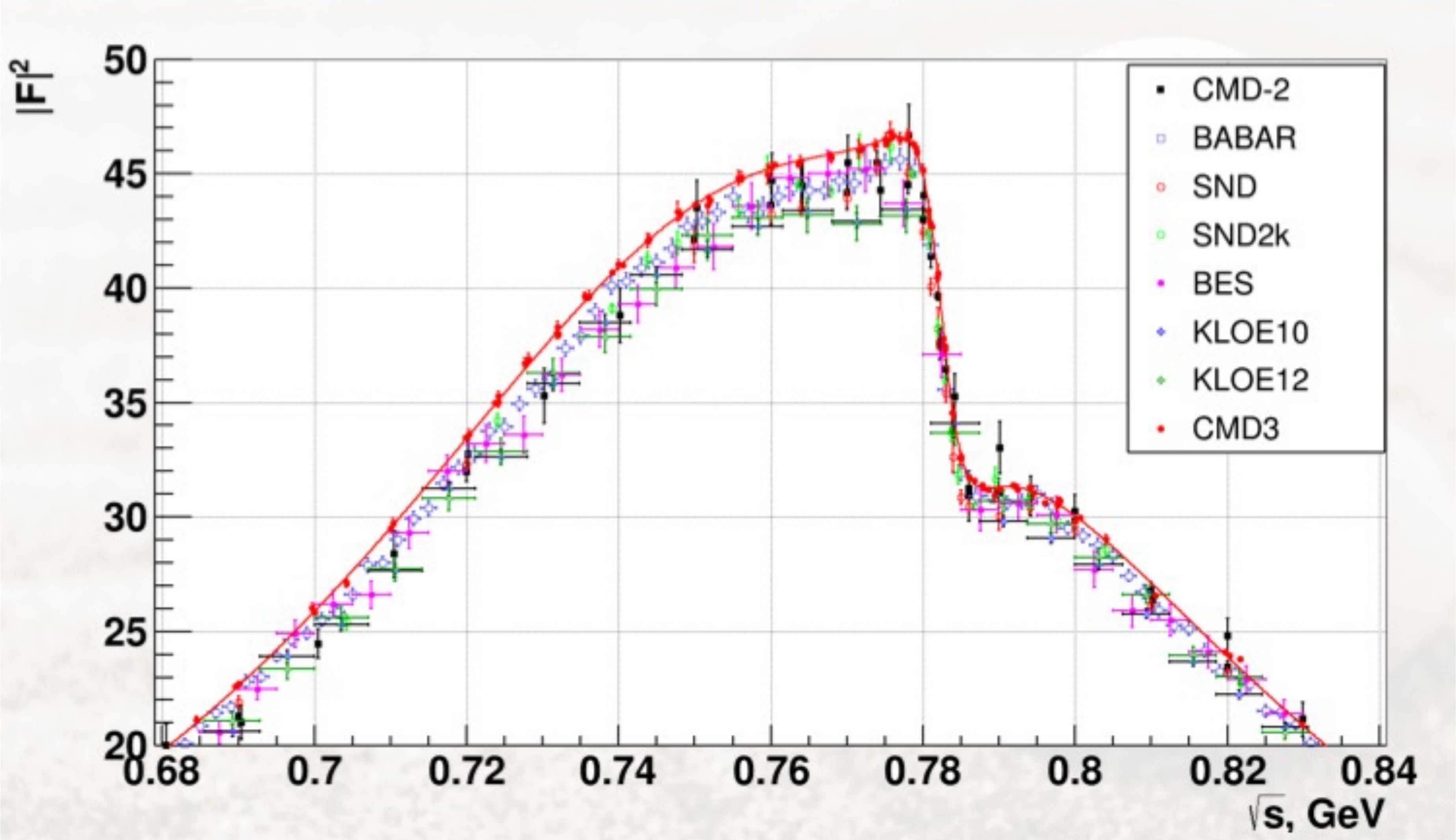
(43.16 ± 0.59) × 10^{-10} (prel.)



New CMD-3 $\pi^+\pi$- data vs. other experiments

Slides from Fedor Ignatov's TI talk 27.3.2023

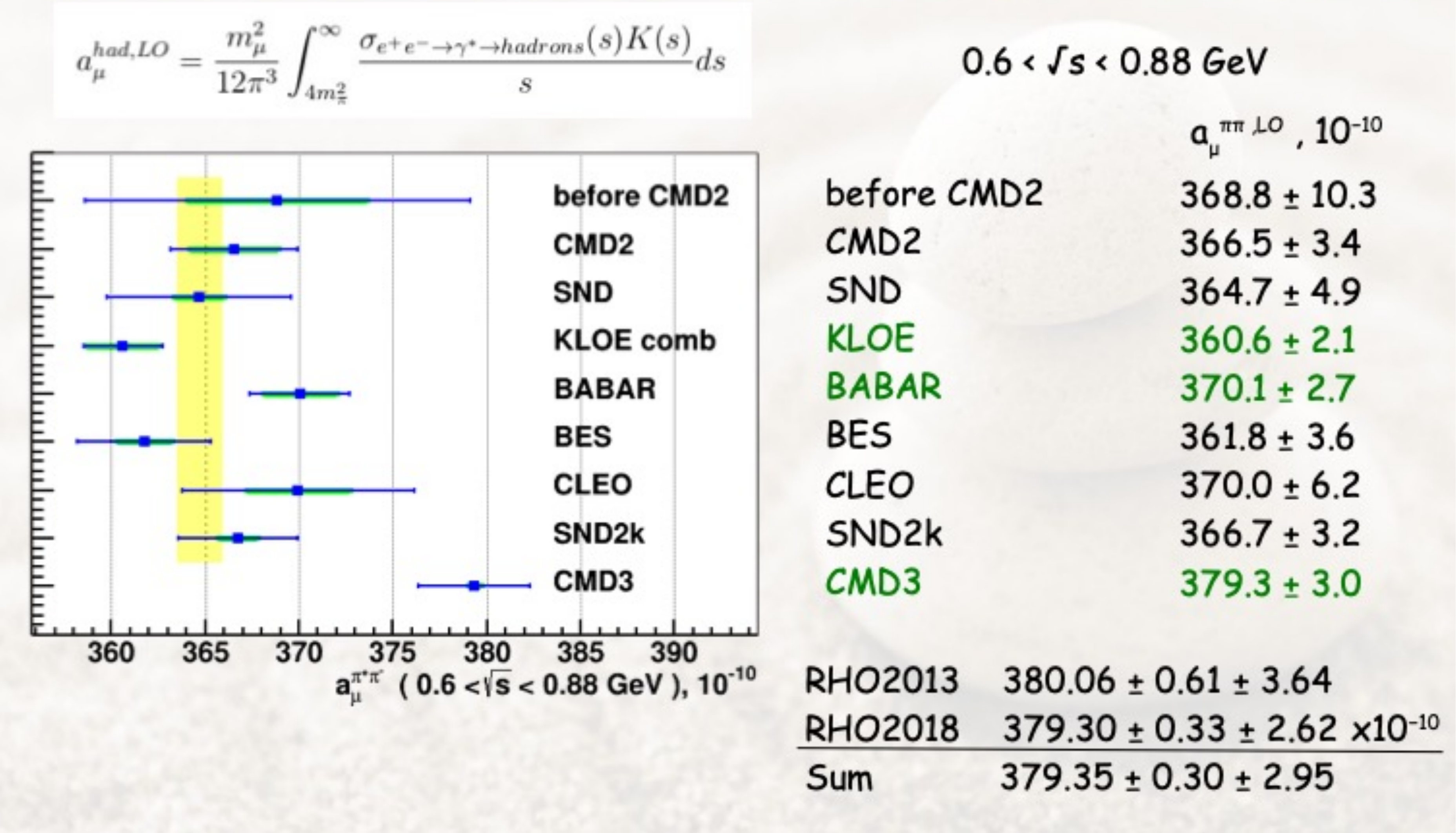
arXiv:2302.08834



New CMD-3 $\pi^+\pi^-$ puzzle for a_μ^{HVP}

Slides from Fedor Ignatov’s TI talk 27.3.2023

arXiv:2302.08834



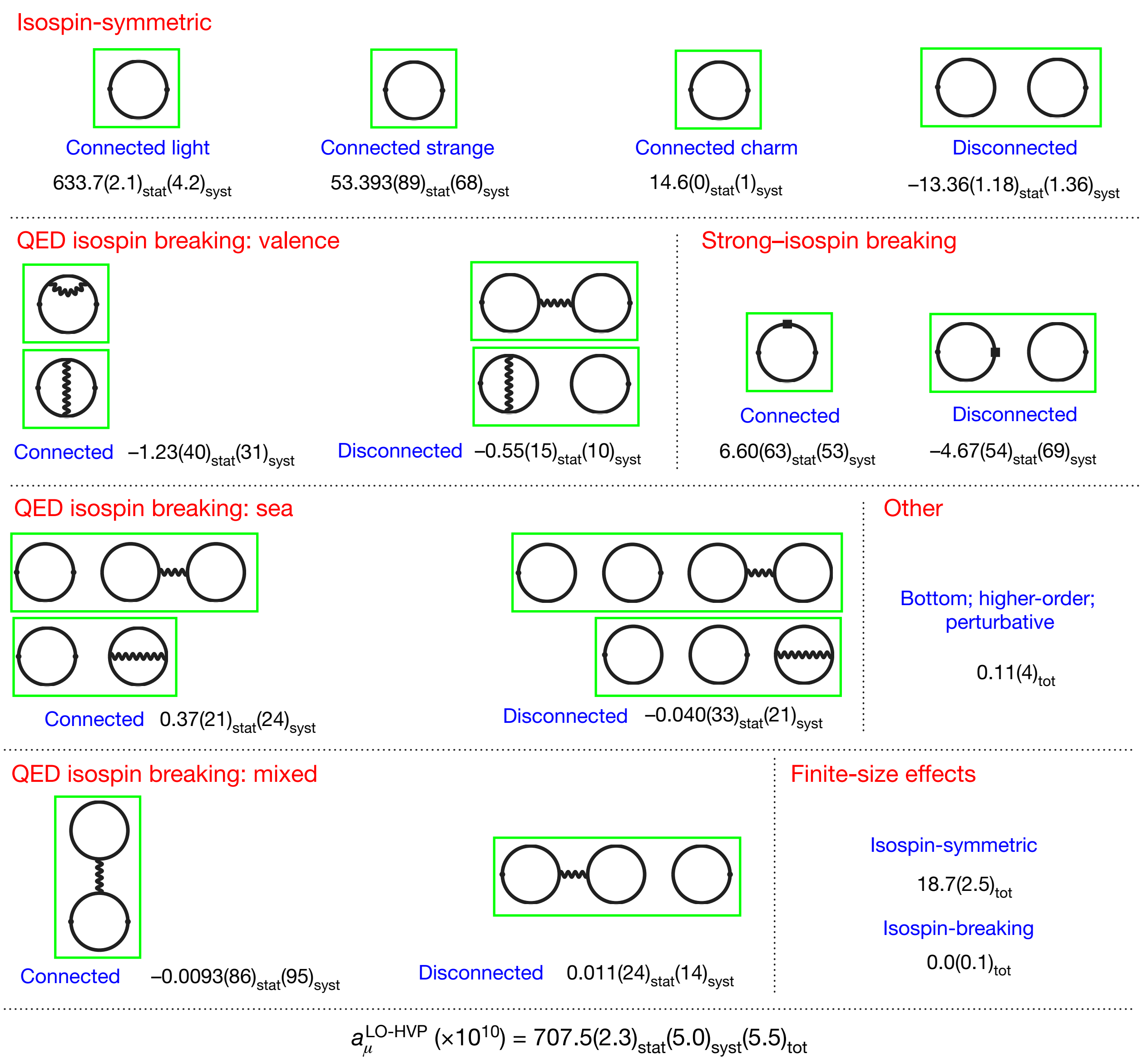
$$a_\mu^{had,LO} = \frac{m_\mu^2}{12\pi^3}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^+e^-\to\gamma^*\to hadrons}(s)K(s)}{s}ds$$

0.6 < √s < 0.88 GeV

	$a_\mu^{\pi\pi,LO}$, 10^{-10}
before CMD2	368.8 ± 10.3
CMD2	366.5 ± 3.4
SND	364.7 ± 4.9
KLOE	360.6 ± 2.1
BABAR	370.1 ± 2.7
BES	361.8 ± 3.6
CLEO	370.0 ± 6.2
SND2k	366.7 ± 3.2
CMD3	379.3 ± 3.0

RHO2013	380.06 ± 0.61 ± 3.64
RHO2018	379.30 ± 0.33 ± 2.62 x10^{-10}
Sum	379.35 ± 0.30 ± 2.95

a_μ^{HVP} : Lattice result from **BMW** [Borsanyi et al., Nature 2021]



Isospin-symmetric

Connected light $633.7(2.1)_{stat}(4.2)_{syst}$

Connected strange $53.393(89)_{stat}(68)_{syst}$

Connected charm $14.6(0)_{stat}(1)_{syst}$

Disconnected $-13.36(1.18)_{stat}(1.36)_{syst}$

QED isospin breaking: valence

Connected $-1.23(40)_{stat}(31)_{syst}$

Disconnected $-0.55(15)_{stat}(10)_{syst}$

Strong–isospin breaking

Connected $6.60(63)_{stat}(53)_{syst}$

Disconnected $-4.67(54)_{stat}(69)_{syst}$

QED isospin breaking: sea

Connected $0.37(21)_{stat}(24)_{syst}$

Disconnected $-0.040(33)_{stat}(21)_{syst}$

Other

Bottom; higher-order; perturbative

$0.11(4)_{tot}$

QED isospin breaking: mixed

Connected $-0.0093(86)_{stat}(95)_{syst}$

Disconnected $0.011(24)_{stat}(14)_{syst}$

Finite-size effects

Isospin-symmetric

$18.7(2.5)_{tot}$

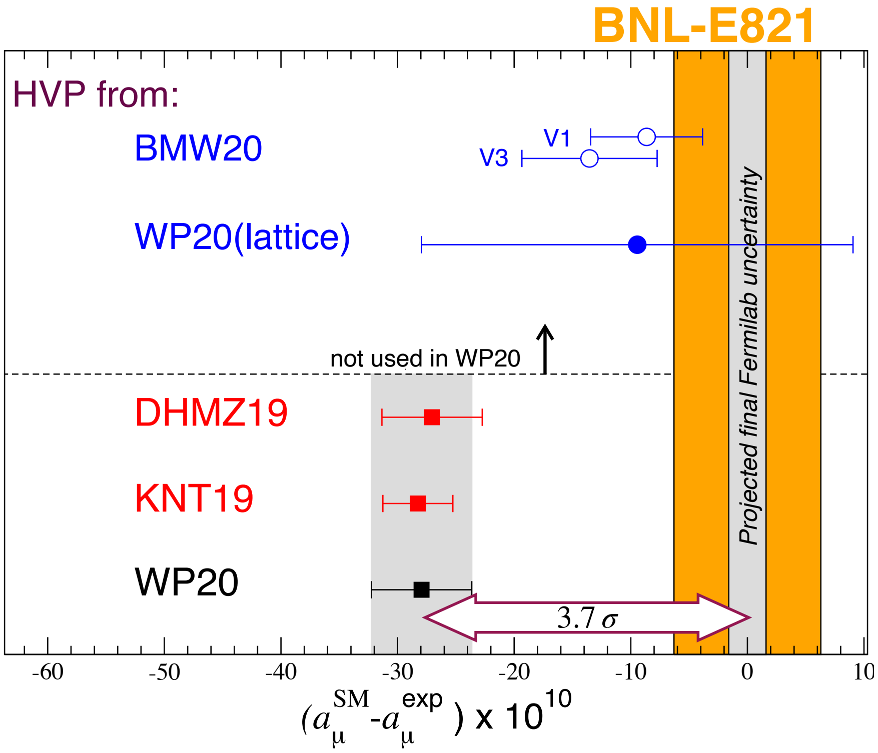
Isospin-breaking

$0.0(0.1)_{tot}$

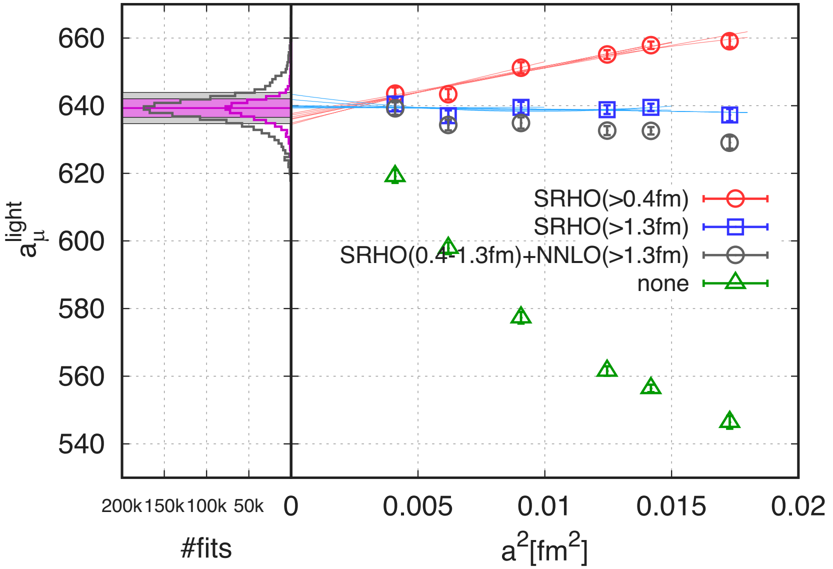
$a_\mu^{\text{LO-HVP}}\ (\times 10^{10}) = 707.5(2.3)_{stat}(5.0)_{syst}(5.5)_{tot}$

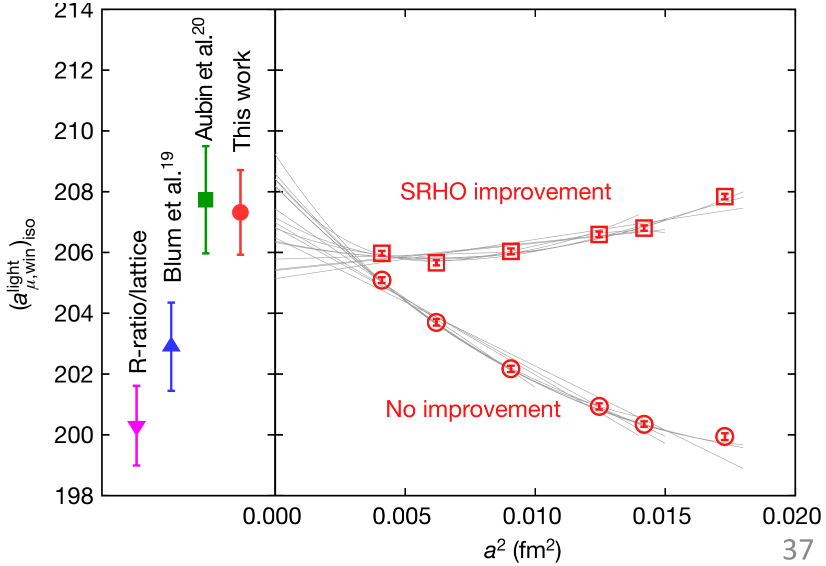
- First lattice prediction with errors matching the data-driven approach
- Current-current correlators, summed over all distances and integrated over time (TMR)
- Using a L~6fm lattice (11fm for finite size corrections)
- Physical quark masses
- Strong + QED isospin breaking corrections

a_μ^{HVP} : Tension between **data-driven & BMW**. Systematics



BMW20 [Borsanyi et al, arXiv:2002.12347, 2021 Nature]





BMW20: large systematics from **continuum limit,**
large taste-breaking corrections (`SRHO')

- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central `**window**' compared to other lattice and data-driven results (**3.7σ** tension)

a_μ^{HVP} : **Window method** for more detailed comparison

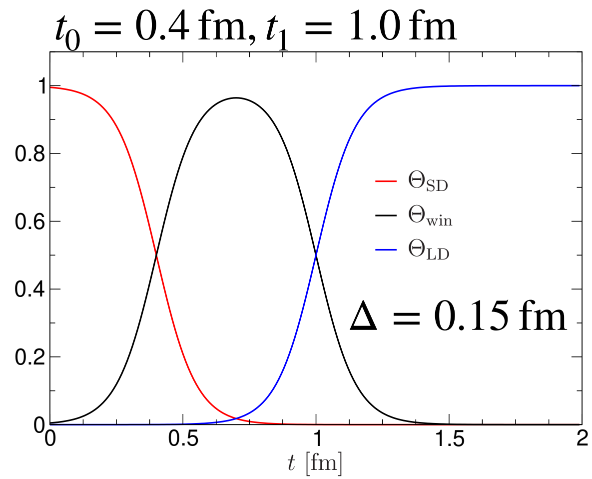
$$\boxed{a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)}$$

- Use windows in Euclidean time to consider the different time regions separately.

Short Distance (SD) $t : 0 \to t_0$

Intermediate (W) $t : t_0 \to t_1$

Long Distance (LD) $t : t_1 \to \infty$



- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

a_μ^{HVP} : **Window method** for more detailed comparison

Correspondence to kernels for comparison with (time-like) dispersive approach:

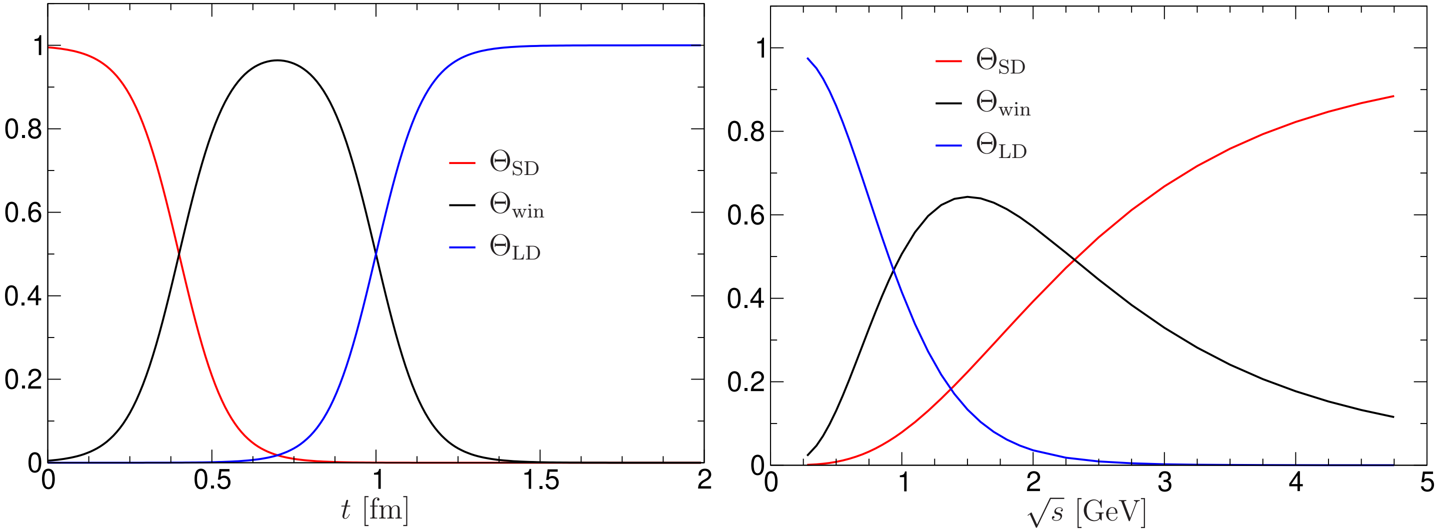
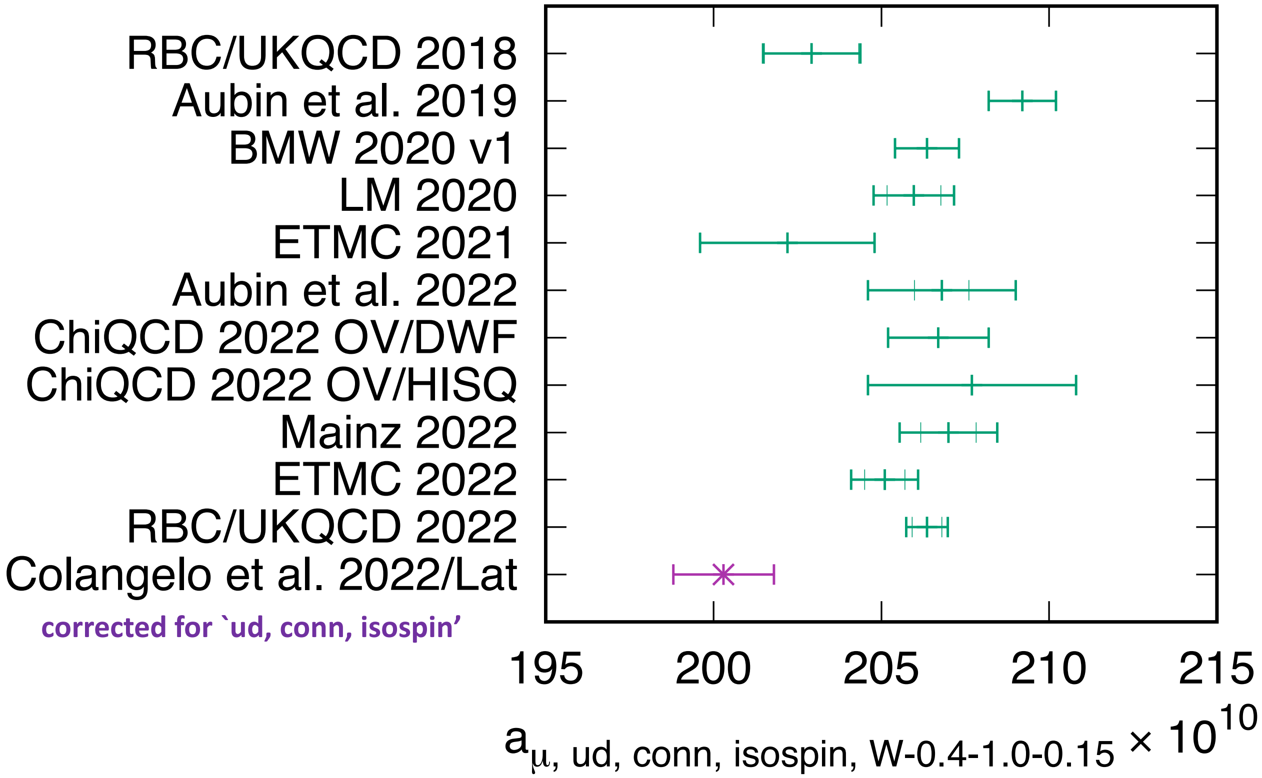


Fig.: G. Colangelo, PWA12/ATHOS7 2021

a_μ^{HVP} : `Window Fever’

Plot from C Lehner’s talk at the TI Edinburgh workshop 5-9.9.’22



Another ~4σ puzzle:

- Lattice QCD `easiest’ in the middle window
- Comparison not direct, but heavier quark and iso-spin breaking contributions unlikely to change much
- So why is there such a large disagreement w. the data?

- **3.9σ tension** betw. RBC/UKQCD 2022 and data-driven

  [Colangelo, El-Khadra, Hoferichter, Keshavarzi, Lehner, Stoffer, Teubner (22)]
- also new FNAL/HPQCD/MILC result: 206.1(1.0) [arXiv:2301.08274]
- **Agreement** of different lattice results, check of **universality** betw. lattice methods

Pathways to solving the (HVP) puzzles

- No easy way out! Signs for Beyond the Standard Model physics?
- BSM at high scales? Many explanations for `**4.2σ**' **puzzle**, few seem natural,
  NP smoking guns in the flavour sector weakened
- BSM `faking' low $\boldsymbol{\sigma}_{\textbf{had}}$? Possible but not probable
  [DiLuzio, Masiero, Paradisi, Passera *Phys.Lett.B* 829 (2022) 137037]
  .. a new Z' [Coyle, Wagner, 2305.02354]
  … or even new hadronic states (like sexa-quarks [Farrar, 2206.13460]) ?
- Situation now very complicated due to emerged **lattice & CMD-3 puzzles**
- **More & more precise data are needed (and coming) to solve puzzles**
- To avoid any possible bias, **blinded analyses** are now the standard, both for experiments (g-2 and $\boldsymbol{\sigma}_{\textbf{had}}$) and lattice
- The third way: **MUonE**

HVP from electron-muon scattering in the space-like

From Fulvio Piccinini @ HP2, September '22:

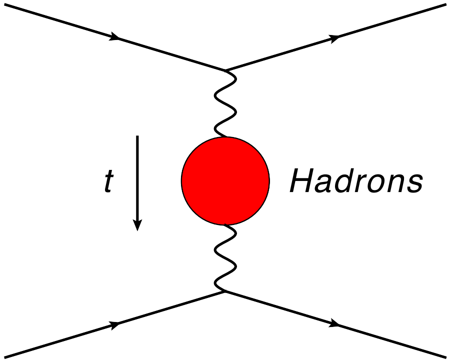
Master formula

- Alternatively (exchanging s and x integrations in a_μ^{HLO})

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\mathrm{had}}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



- ⇝ The hadronic VP correction to the running of α enters
- ⇝ Essentially the same formula used in lattice QCD calculation of a_μ^{HLO}

★ $\Delta\alpha_{\mathrm{had}}(t)$ (and a_μ^{HLO}) can be directly measured in a (single) experiment involving a space-like scattering process

Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

★ **Still a data-driven evaluation of a_μ^{HLO}, but with space-like data**

- By modifying the kernel function $\frac{\alpha}{\pi}(1-x)$, also a_μ^{HNLO} and a_μ^{HNNLO} can be provided

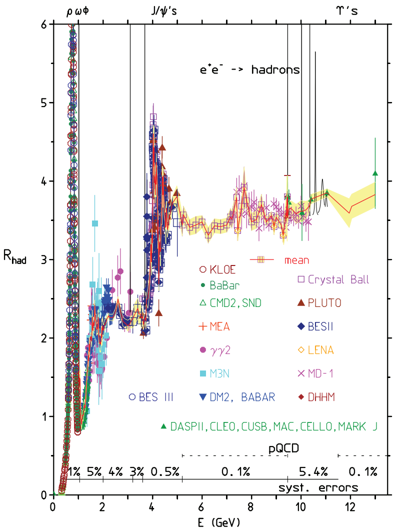
Balzani, Laporta, Passera, `arXiv:2112.05704 [hep-ph]`

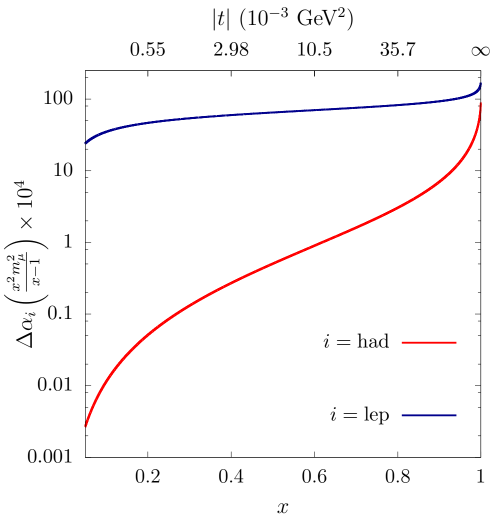
HVP from electron-muon scattering in the space-like

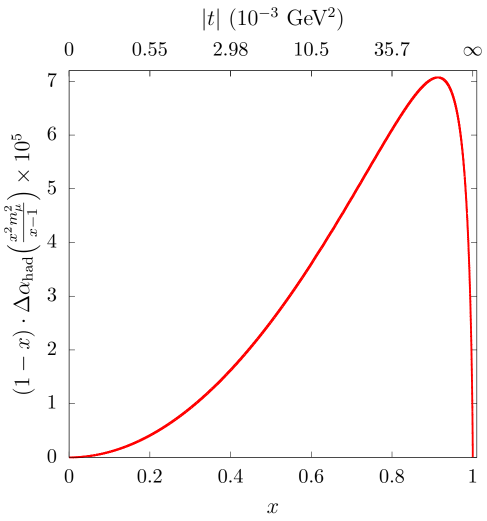
From Fulvio Piccinini @ HP2, September ‘22:

From time-like to space-like evaluation of a_μ^{HLO}

Time-like $\mapsto$ **Space-like**







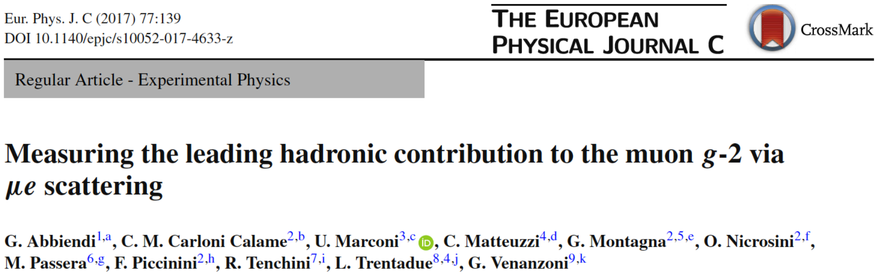
Smooth function

- $\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required
- $\mapsto$ **Space-like:** in principle, one single experiment, *it’s a one-loop effect, very high accuracy needed*

HVP from electron-muon scattering in the space-like

From Giovanni Abbiendi @ Strong2020, Zurich, June 7-9

MUonE experiment idea



Eur. Phys. J. C (2017) 77:139
DOI 10.1140/epjc/s10052-017-4633-z

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Experimental Physics

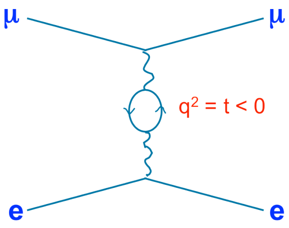
Measuring the leading hadronic contribution to the muon *g*-2 via *μe* scattering

G. Abbiendi[1,a], C. M. Carloni Calame[2,b], U. Marconi[3,c], C. Matteuzzi[4,d], G. Montagna[2,5,e], O. Nicrosini[2,f], M. Passera[6,g], F. Piccinini[2,h], R. Tenchini[7,i], L. Trentadue[8,4,j], G. Venanzoni[9,k]

Eur.Phys.J.C77(2017)139

Very precise measurement of the running of α_{QED} from the shape of the differential cross section of elastic scattering of μ(150-160GeV) on atomic electrons of a fixed target with low Z (Be or C)

→ **CERN SPS**



$$\frac{d\sigma}{dt} \approx \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha(0)}\right|^2 \approx \frac{d\sigma_0}{dt}\underbrace{\left|\frac{1}{1-\Delta\alpha(t)}\right|^2}_{\text{running of }\alpha} \Rightarrow \Delta\alpha(t) = \underbrace{\Delta\alpha_{lep}(t)}_{\text{known from QED}} + \underbrace{\Delta\alpha_{had}(t)}_{\text{to be measured}}$$

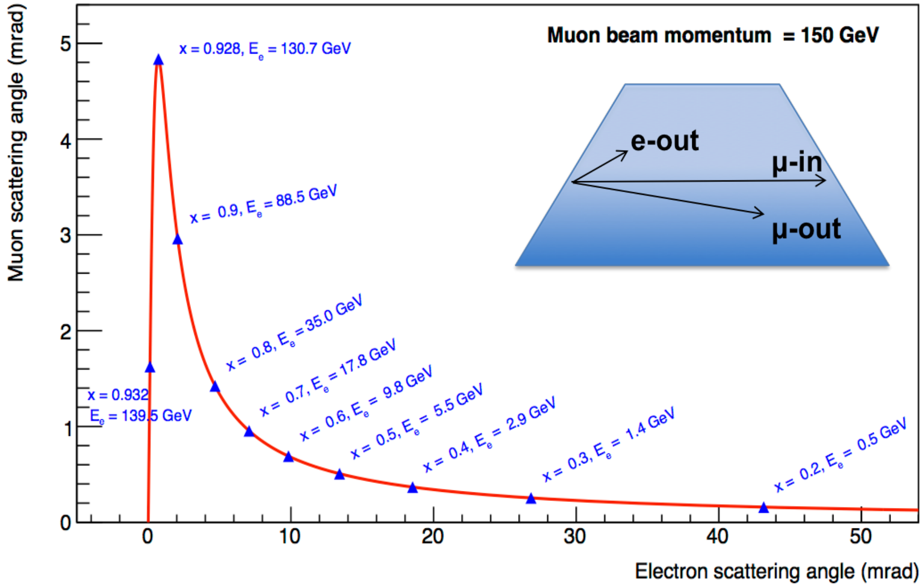
From $\Delta\alpha_{had}(t)$ determine a_μ^{HLO} by the space-like approach: Phys.Lett.B746(2015)325

$$\boxed{a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]}$$

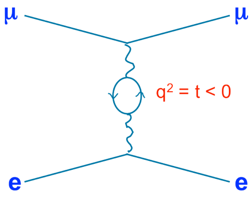
HVP from electron-muon scattering in the space-like

From Giovanni Abbiendi @ Strong2020, Zurich, June 7-9

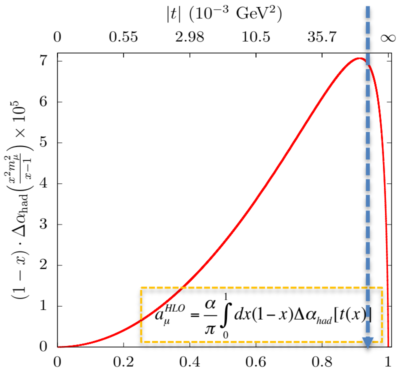
μ-e Elastic scattering: pros



- **Simple kinematics:** $t \cong -2\, m_e E_e$
  E_e can be determined from the scattering angle θ_e and the beam energy
- Scattering angles θ_e and θ_μ are correlated
- Events are planar



Kin.Limit: x~0.93



- For E(beam)=160 GeV the phase space covers 88% of the a_μ^{HLO} integral
  - Smooth extrapolation to the full integral with a proper fit model

New idea to get optimal reach of kinematically accessible region; work in progress in Liverpool.

HVP from electron-muon scattering in the space-like

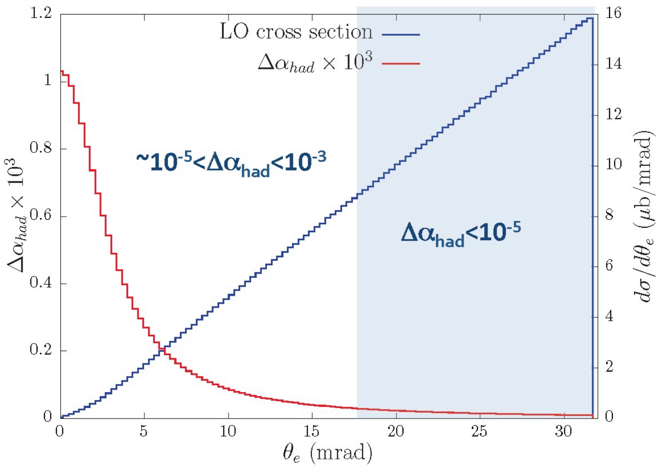
From Giovanni Abbiendi @ Strong2020, Zurich, June 7-9

μ-e elastic scattering: challenges

Observable effect **~ 10^{-3}**

wanted accuracy **~10^{-2}**

Required precision **~10^{-5}**
on the shape of dσ/dt



- Large statistics to reach the necessary sensitivity
- Minimal distortions of the outgoing e/μ trajectories within the target material and small rate of radiative events

Requirements for very precise Radiative Corrections and MCs:

- High order real + virtual QED (massive NNLO, resummation)
- Higher order kernels to disentangle LO from HO VP effects
- Two dedicated **MC** groups: **McMule** and **Mesmer**

a_μ^{HVP} : Hadronic tau decay data

- Historically, hadronic tau decay data, e.g. $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$, were used to improve precision of e^+e^- based evaluations

- However, with the increased precision of the e^+e^- data there is now limited merit in this (there are some conflicting evaluations, DHMZ have dropped it)

- The required iso-spin breaking corrections re-introduce a model-dependence and connected systematic uncertainty (there is, e.g., no ρ–ω mixing in τ decays)

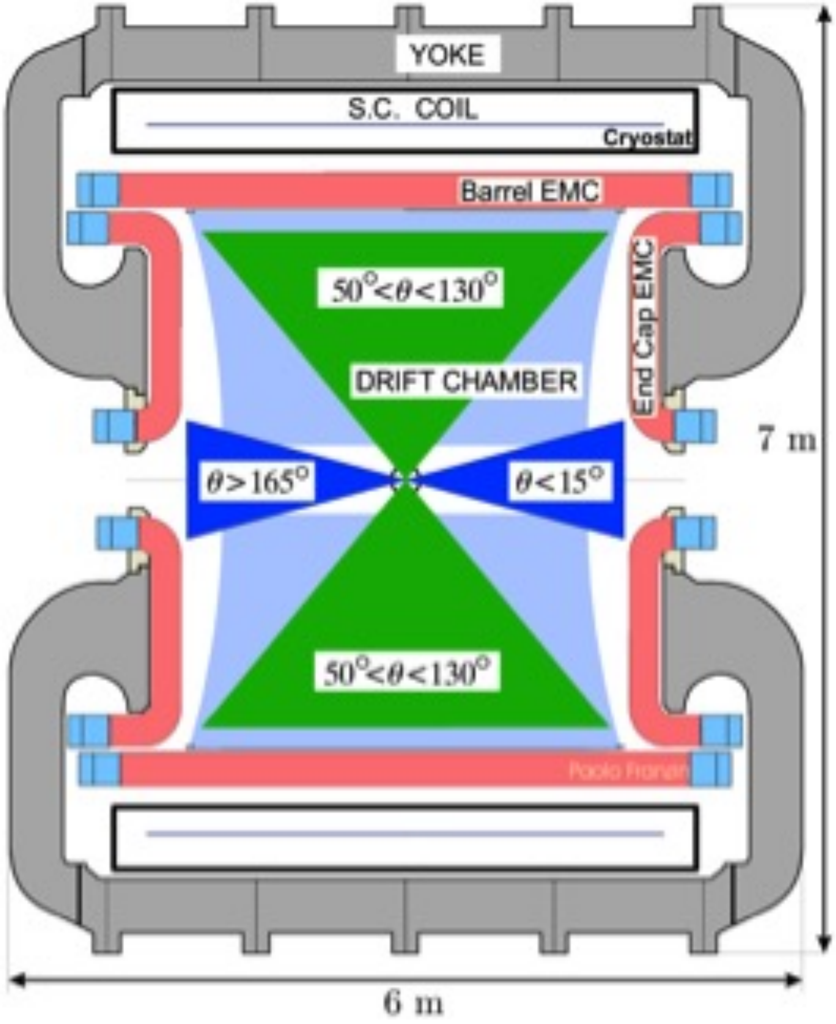
- Quote from the WP, where this approach is discussed in detail:

"Concluding this part, it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Sec. 3.4.2, may provide a solution to this problem."

- New contribution to the discussion by Masjuan, Miranda, Roig: arXiv:2305.20005
  `τ data-driven evaluation of Euclidean windows for the hadronic vacuum polarization'

- Opportunities for Belle-2

KLOE 2π analyses



Large Angle:

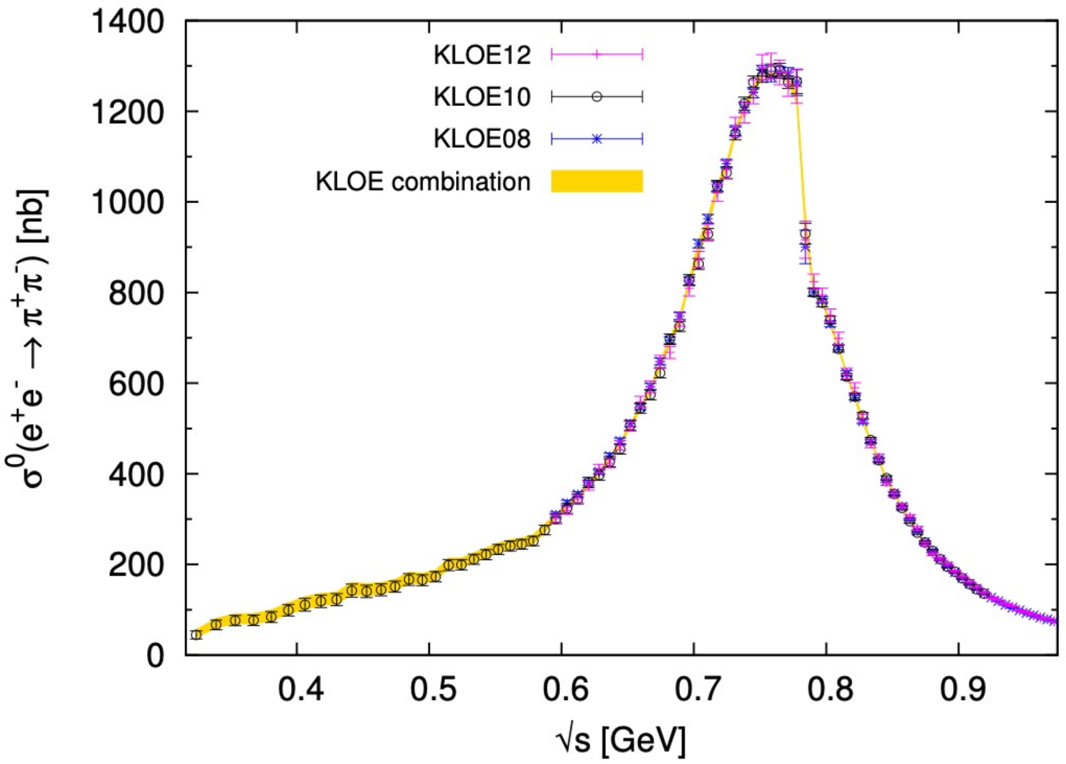
2 pion (muon) tracks at 50° < $\vartheta_{\pi,\mu}$ < 130°

Small angle photon selection:

$\vartheta_{miss} < 15°$; $\vartheta_{miss} > 165°$

- high statistics for ISR events
- low FSR contribution
- easy to suppress $\varphi \longrightarrow \pi^+\pi^-\pi^0$ background
- photon momentum from kinematics: $\vec{p}_\gamma = \vec{p}_{miss} = -(\vec{p}^+ + \vec{p}^-)$
- threshold region not accessible

KLOE 2π results



KLOE05

Small Angle analysis of 140 pb^{-1} @ m_φ

KLOE Coll. Phys. Lett. B 606 (2005)

KLOE08

Small Angle analysis of 240 pb^{-1} @ m_φ

KLOE Coll. Phys. Lett. B 670 (2009)

KLOE10

Large angle analysis of 250 pb^{-1} @ 1 GeV

KLOE Coll. Phys. Lett. B 700 (2011)

KLOE12

KLOE08 with normalisation to $e^+e^- \to \mu^+\mu^-$

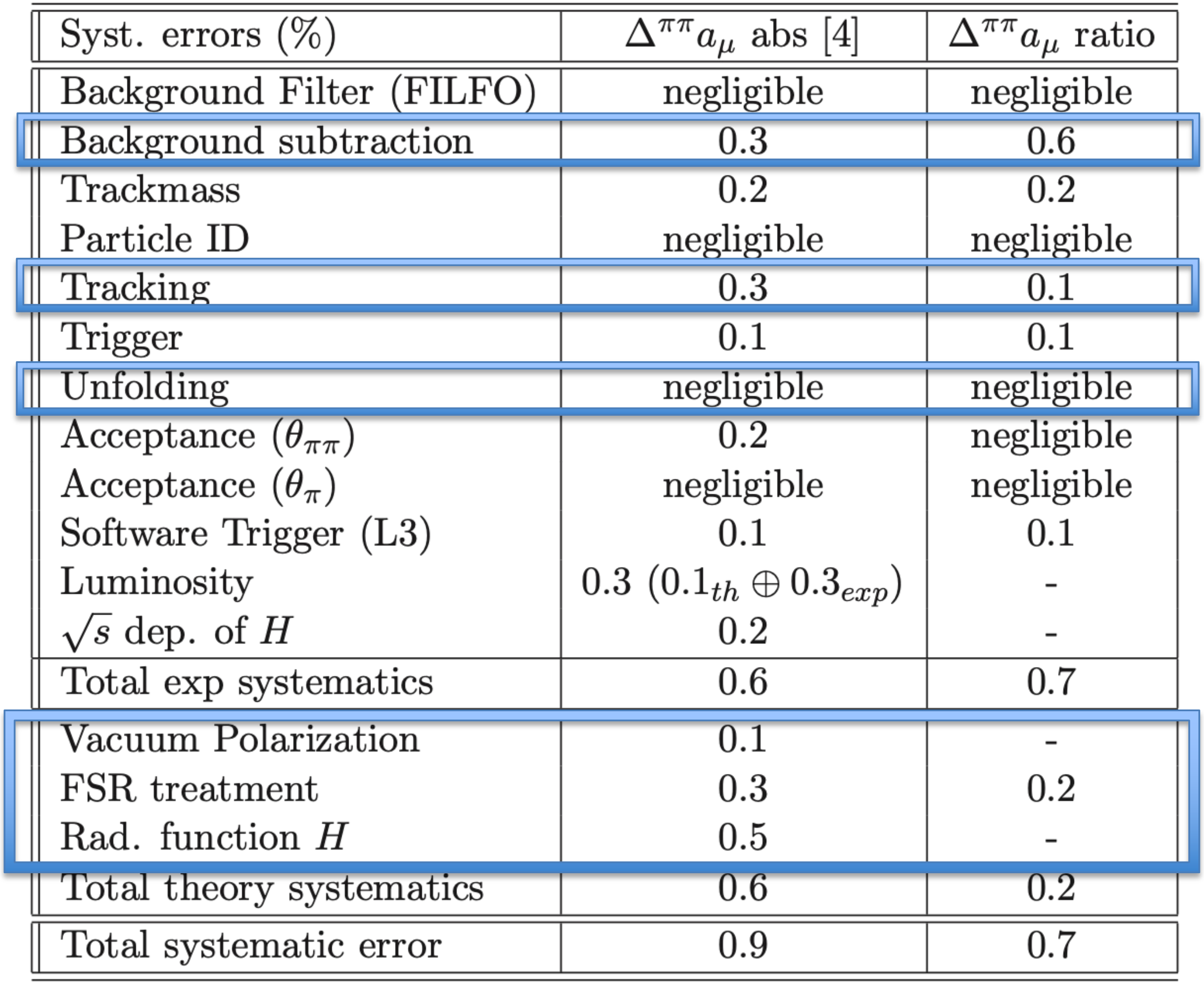
KLOE Coll. Phys. Lett. B 720 (2013)

Combination of three sets *JHEP 1803 (2018) 173:*

$$a_\mu^{\pi\pi}\,[0.1 < s < 0.95\ \mathrm{GeV}^2] = (489.8 \pm 1.7_{\mathrm{stat}} \pm 4.8_{\mathrm{sys}}) \times 10^{-10}$$

KLOE 2π uncertainties

We aim to improve:



Syst. errors (%)	$\Delta^{\pi\pi}a_\mu$ abs [4]	$\Delta^{\pi\pi}a_\mu$ ratio
Background Filter (FILFO)	negligible	negligible
Background subtraction	0.3	0.6
Trackmass	0.2	0.2
Particle ID	negligible	negligible
Tracking	0.3	0.1
Trigger	0.1	0.1
Unfolding	negligible	negligible
Acceptance ($\theta_{\pi\pi}$)	0.2	negligible
Acceptance (θ_{π})	negligible	negligible
Software Trigger (L3)	0.1	0.1
Luminosity	0.3 ($0.1_{th} \oplus 0.3_{exp}$)	-
$\sqrt{s}$ dep. of H	0.2	-
Total exp systematics	0.6	0.7
Vacuum Polarization	0.1	-
FSR treatment	0.3	0.2
Rad. function H	0.5	-
Total theory systematics	0.6	0.2
Total systematic error	0.9	0.7

↖

possible corrs. to naïve ISR-FSR factorization for radiator function

↙

KLOE 2π activities

- New effort to analyse the full statistics KLOE **2π** data (**integrated *L* ~ 1.7 fb^{-1}**)
- New **blind analysis**, unbiased from previous results of KLOE & other experiments
- Significant involvement from theoretical groups
  => improvement of MC(s) to describe ISR and FSR events (`PHOKHARA`, …)
- Goal: sub-percent accuracy:
  improvement of a factor of ~2 on the total uncertainty => **$\Delta a_\mu^{HLO} \lesssim 0.4\%$**
- Challenges and opportunities to get a clearer understanding of the puzzles
- The Liverpool + externals team:
  - Leverhulme International Professorship: **G. Venanzoni**
    F. Ignatov, P. Beltrame, E. Zaid; A. Kumari, N. Vestergaard, C. Devanne
  - Theory efforts: T. Teubner; W. Torres Bobadilla, J. Paltrinieri; T. Dave, P. Petit Rosas

    + contributors from the wider Liverpool Theoretical Physics group
  - External collaborators: A. Kupsc, S. Müller, L. Punzi, O. Shekhovstova,
    A. Keshavarzi, W. Wislicki, A. Lusiani, J. Wiechnik

Outlook / Conclusions

- The still **unresolved muon g-2 discrepancy** has triggered a lot of experimental & theory activities, including experiments, the Muon g-2 Theory Initiative & **lattice**

- **Much progress** has been made for **HLbL** (disp. & lattice), previously the bottleneck

- For **HVP dispersive**, the **TI published a conservative consensus (WP20)**

  -- no significant changes since WP20 yet, but

  ➤ the resolution of the **puzzles** in the crucial **2π** channel requires further new data

  -- expected/**puzzling new σ_{had} data for 2π** and other channels from

  BaBar, **CMD-3**, **SND**, **BES III**, **Belle II**, and **KLOE** (Liverpool analysis has started)

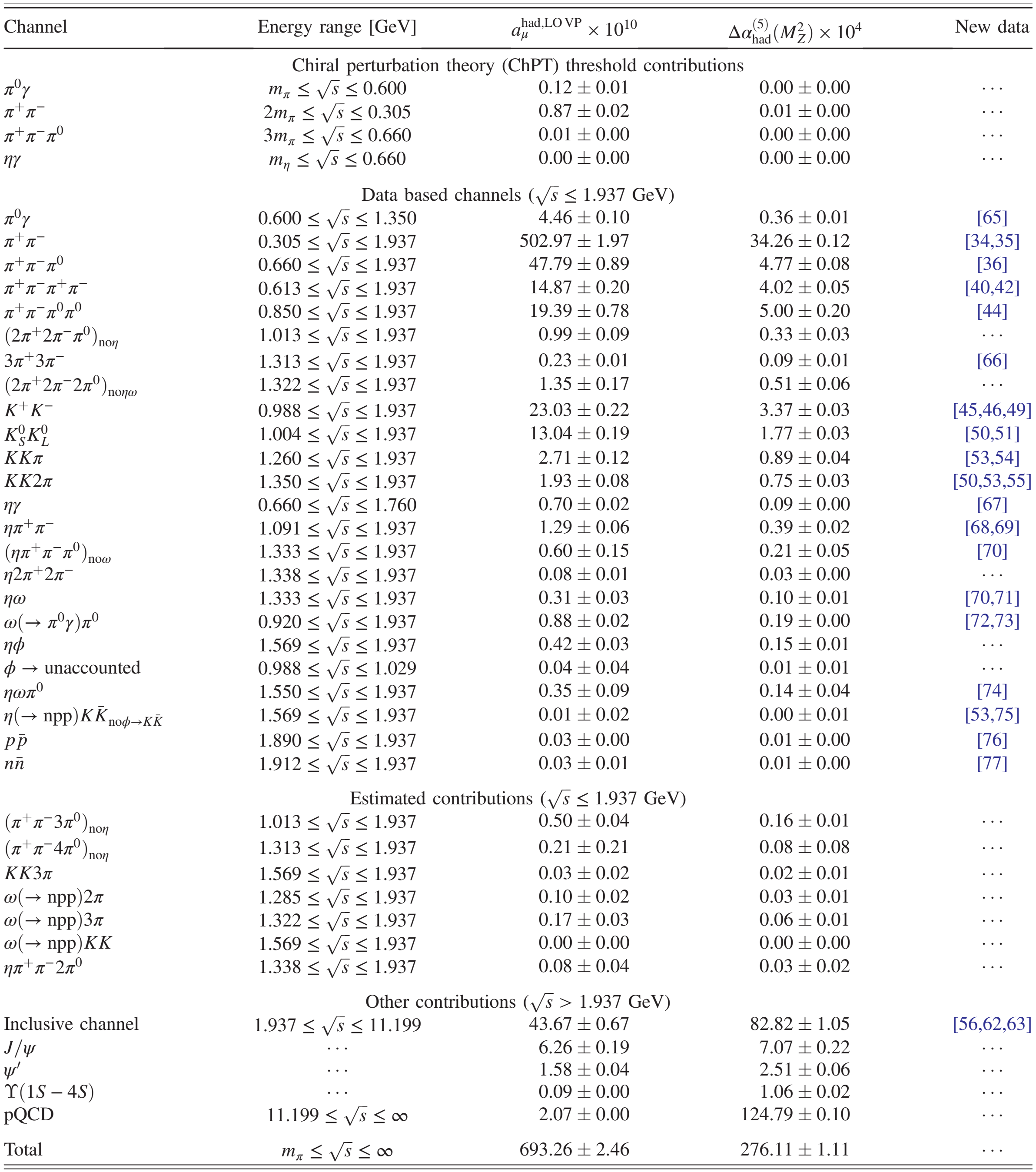
  ➤ **if** precise data **agree**, the **$a_\mu^{HVP, LO (dispersive)}$ puzzle** will go away and the error down

  -- but **further theory input** (**NNLO$^+$ rad. corrs. & MCs**) **will be crucial**

  ➤ may solve the **puzzle w. lattice HVP predictions.** **Longer term, 3rd way: MUonE**

✤ ***There is a lot to do in the field of RCs and MCs beyond/before the HL LHC …***

Extras



Channel	Energy range [GeV]	$a_\mu^{\text{had,LO VP}} \times 10^{10}$	$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) \times 10^4$	New data
Chiral perturbation theory (ChPT) threshold contributions				
$\pi^0\gamma$	$m_\pi \le \sqrt{s} \le 0.600$	0.12 ± 0.01	0.00 ± 0.00	⋯
$\pi^+\pi^-$	$2m_\pi \le \sqrt{s} \le 0.305$	0.87 ± 0.02	0.01 ± 0.00	⋯
$\pi^+\pi^-\pi^0$	$3m_\pi \le \sqrt{s} \le 0.660$	0.01 ± 0.00	0.00 ± 0.00	⋯
$\eta\gamma$	$m_\eta \le \sqrt{s} \le 0.660$	0.00 ± 0.00	0.00 ± 0.00	⋯
Data based channels ($\sqrt{s} \le 1.937$ GeV)				
$\pi^0\gamma$	$0.600 \le \sqrt{s} \le 1.350$	4.46 ± 0.10	0.36 ± 0.01	[65]
$\pi^+\pi^-$	$0.305 \le \sqrt{s} \le 1.937$	502.97 ± 1.97	34.26 ± 0.12	[34,35]
$\pi^+\pi^-\pi^0$	$0.660 \le \sqrt{s} \le 1.937$	47.79 ± 0.89	4.77 ± 0.08	[36]
$\pi^+\pi^-\pi^+\pi^-$	$0.613 \le \sqrt{s} \le 1.937$	14.87 ± 0.20	4.02 ± 0.05	[40,42]
$\pi^+\pi^-\pi^0\pi^0$	$0.850 \le \sqrt{s} \le 1.937$	19.39 ± 0.78	5.00 ± 0.20	[44]
$(2\pi^+2\pi^-\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.99 ± 0.09	0.33 ± 0.03	⋯
$3\pi^+3\pi^-$	$1.313 \le \sqrt{s} \le 1.937$	0.23 ± 0.01	0.09 ± 0.01	[66]
$(2\pi^+2\pi^-2\pi^0)_{\text{no}\eta\omega}$	$1.322 \le \sqrt{s} \le 1.937$	1.35 ± 0.17	0.51 ± 0.06	⋯
K^+K^-	$0.988 \le \sqrt{s} \le 1.937$	23.03 ± 0.22	3.37 ± 0.03	[45,46,49]
$K_S^0K_L^0$	$1.004 \le \sqrt{s} \le 1.937$	13.04 ± 0.19	1.77 ± 0.03	[50,51]
$KK\pi$	$1.260 \le \sqrt{s} \le 1.937$	2.71 ± 0.12	0.89 ± 0.04	[53,54]
$KK2\pi$	$1.350 \le \sqrt{s} \le 1.937$	1.93 ± 0.08	0.75 ± 0.03	[50,53,55]
$\eta\gamma$	$0.660 \le \sqrt{s} \le 1.760$	0.70 ± 0.02	0.09 ± 0.00	[67]
$\eta\pi^+\pi^-$	$1.091 \le \sqrt{s} \le 1.937$	1.29 ± 0.06	0.39 ± 0.02	[68,69]
$(\eta\pi^+\pi^-\pi^0)_{\text{no}\omega}$	$1.333 \le \sqrt{s} \le 1.937$	0.60 ± 0.15	0.21 ± 0.05	[70]
$\eta 2\pi^+2\pi^-$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.01	0.03 ± 0.00	⋯
$\eta\omega$	$1.333 \le \sqrt{s} \le 1.937$	0.31 ± 0.03	0.10 ± 0.01	[70,71]
$\omega(\to \pi^0\gamma)\pi^0$	$0.920 \le \sqrt{s} \le 1.937$	0.88 ± 0.02	0.19 ± 0.00	[72,73]
$\eta\phi$	$1.569 \le \sqrt{s} \le 1.937$	0.42 ± 0.03	0.15 ± 0.01	⋯
$\phi \to$ unaccounted	$0.988 \le \sqrt{s} \le 1.029$	0.04 ± 0.04	0.01 ± 0.01	⋯
$\eta\omega\pi^0$	$1.550 \le \sqrt{s} \le 1.937$	0.35 ± 0.09	0.14 ± 0.04	[74]
$\eta(\to \text{npp})K\bar{K}_{\text{no}\phi\to K\bar{K}}$	$1.569 \le \sqrt{s} \le 1.937$	0.01 ± 0.02	0.00 ± 0.01	[53,75]
$p\bar{p}$	$1.890 \le \sqrt{s} \le 1.937$	0.03 ± 0.00	0.01 ± 0.00	[76]
$n\bar{n}$	$1.912 \le \sqrt{s} \le 1.937$	0.03 ± 0.01	0.01 ± 0.00	[77]
Estimated contributions ($\sqrt{s} \le 1.937$ GeV)				
$(\pi^+\pi^-3\pi^0)_{\text{no}\eta}$	$1.013 \le \sqrt{s} \le 1.937$	0.50 ± 0.04	0.16 ± 0.01	⋯
$(\pi^+\pi^-4\pi^0)_{\text{no}\eta}$	$1.313 \le \sqrt{s} \le 1.937$	0.21 ± 0.21	0.08 ± 0.08	⋯
$KK3\pi$	$1.569 \le \sqrt{s} \le 1.937$	0.03 ± 0.02	0.02 ± 0.01	⋯
$\omega(\to \text{npp})2\pi$	$1.285 \le \sqrt{s} \le 1.937$	0.10 ± 0.02	0.03 ± 0.01	⋯
$\omega(\to \text{npp})3\pi$	$1.322 \le \sqrt{s} \le 1.937$	0.17 ± 0.03	0.06 ± 0.01	⋯
$\omega(\to \text{npp})KK$	$1.569 \le \sqrt{s} \le 1.937$	0.00 ± 0.00	0.00 ± 0.00	⋯
$\eta\pi^+\pi^-2\pi^0$	$1.338 \le \sqrt{s} \le 1.937$	0.08 ± 0.04	0.03 ± 0.02	⋯
Other contributions ($\sqrt{s} > 1.937$ GeV)				
Inclusive channel	$1.937 \le \sqrt{s} \le 11.199$	43.67 ± 0.67	82.82 ± 1.05	[56,62,63]
J/ψ	⋯	6.26 ± 0.19	7.07 ± 0.22	⋯
ψ'	⋯	1.58 ± 0.04	2.51 ± 0.06	⋯
$\Upsilon(1S-4S)$	⋯	0.09 ± 0.00	1.06 ± 0.02	⋯
pQCD	$11.199 \le \sqrt{s} \le \infty$	2.07 ± 0.00	124.79 ± 0.10	⋯
Total	$m_\pi \le \sqrt{s} \le \infty$	693.26 ± 2.46	276.11 ± 1.11	⋯

Table from KNT18,
PRD 97(2018)114025

Update: KNT19
LO+NLO HVP for
$a_{e,\mu,\tau}$ & hyperfine splitting of muonium

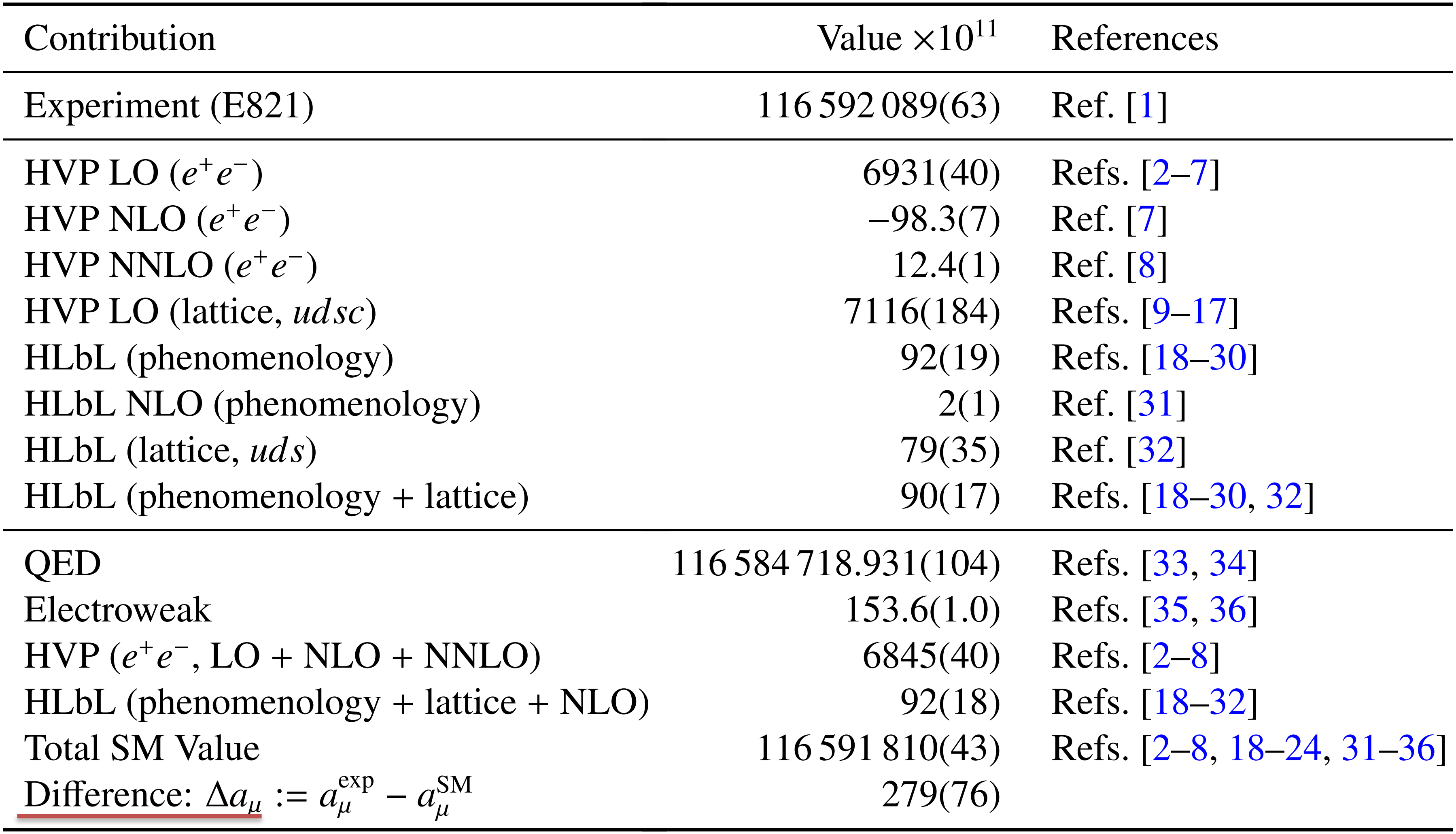
PRD101(2020)014029

Breakdown of HVP contributions in ∼35 hadronic channels

From 2-11 GeV, use of inclusive data, pQCD only beyond 11 GeV

a_μ (SM): White Paper https://doi.org/10.1016/j.physrep.2020.07.006

White Paper [T. Aoyama et al, arXiv:2006.04822], 132 authors, 82 institutions, 21 countries



Contribution	Value $\times 10^{11}$	References
Experiment (E821)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, uds)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}}$	279(76)	

w.r.t. BNL only

Window method (introduced in RBC/UKQCD 2018)

We also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

with

$$\begin{aligned} a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\ a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\ a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\ \Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]\,/2\,. \end{aligned}$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+e^- \to \mathrm{had})$.

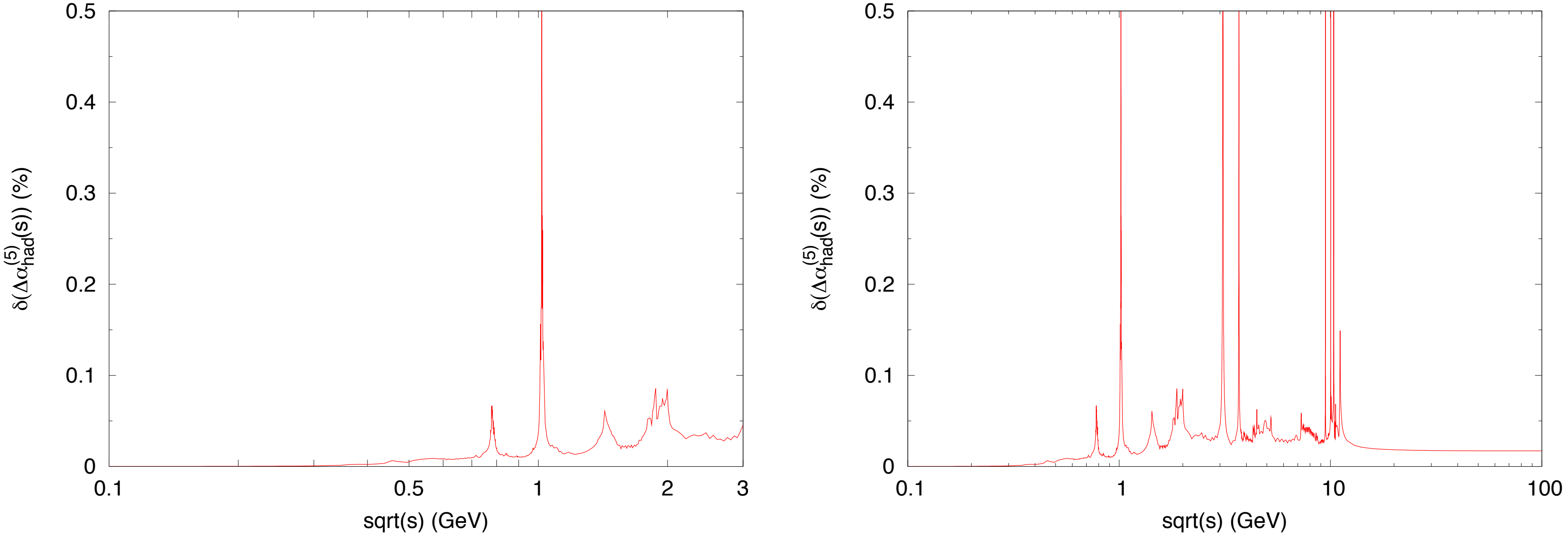
a_μ^{W} has small statistical and systematic errors on lattice!

(slide: C. Lehner, muon g-2 TI 5th Plenary Workshop, Edinburgh 2022)

Rad Corrs: HVP for running α(q²). Accuracy

- Typical accuracy $\delta\left(\Delta\alpha^{(5)}_{\rm had}(s)\right)$

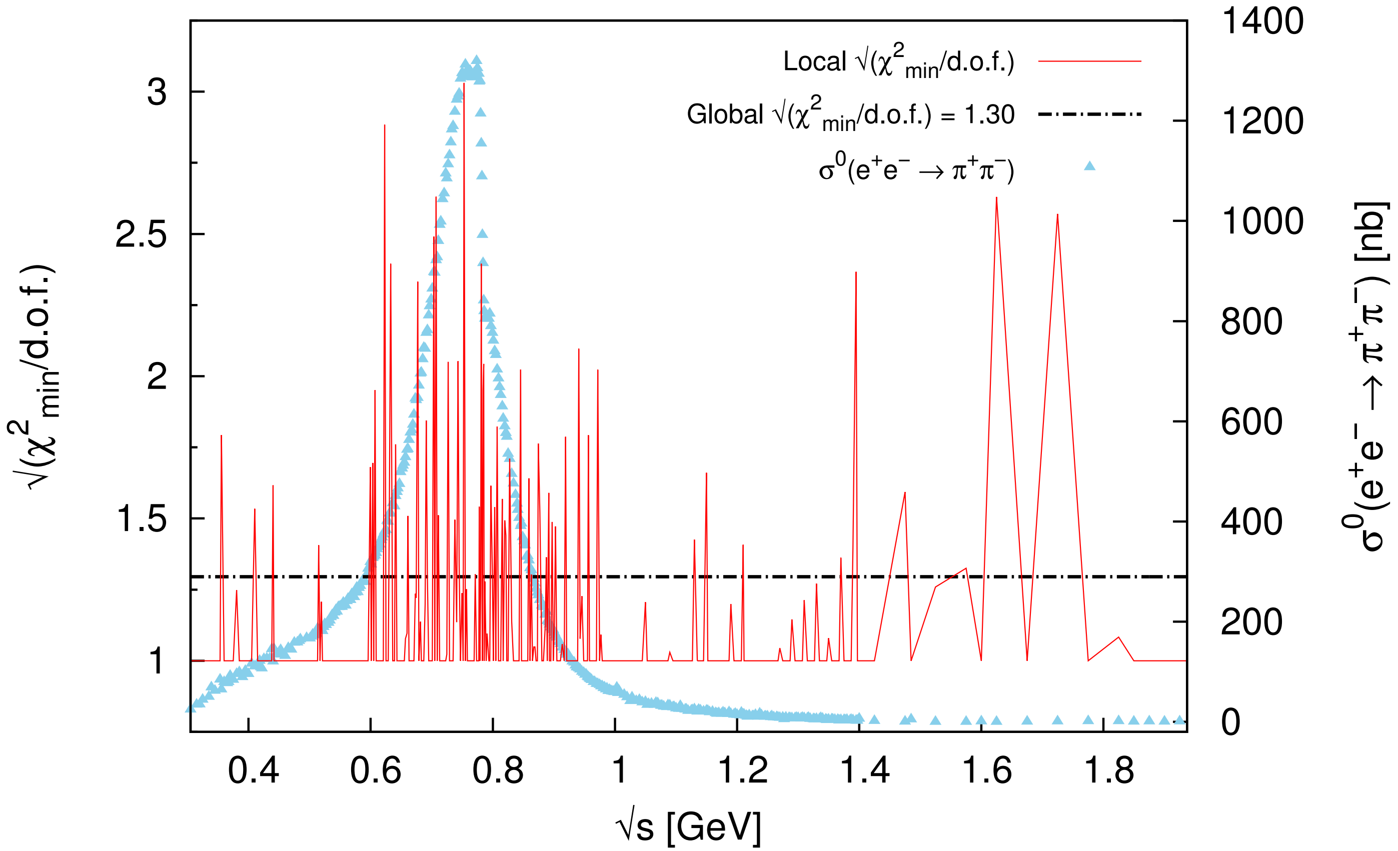
Error of VP in the timelike regime at low and higher energies (HLMNT compilation):



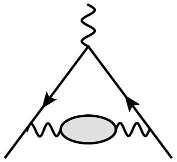
$\longrightarrow$ Below one per-mille (and typically $\sim 5\cdot 10^{-4}$), apart from Narrow Resonances where the bubble summation is not well justified.

HVP: $\pi^+\pi^-$ channel. Error inflation in KNT

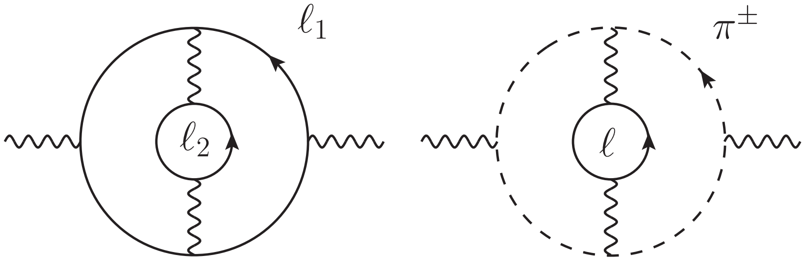
- Inflation of error with **local χ^2_{min}** accounts for tensions, leading to a ~14% error inflation, with overlay of 2**π** cross section fit (blue markers) and global χ^2_{min} (dash-dotted line)



a_μ^{HVP} : short detour into double-bubbles

- What if the blob in  is a `double-bubble’ ?

- Purely leptonic graphs (left diagram below) are part of four-loop QED corrections



- But possibly enhanced contributions from mixed hadronic-leptonic double bubble graphs (right diagram above) are not included in the hadronic **NNLO** HVP corrections quoted above

- Our recent work has estimated these remaining NNLO contributions to $\mathbf{a_\mu}$ to be **below 1 × $\mathbf{10^{-11}}$** and hence not critical at the level of the experimental accuracy

M Hoferichter + TT, *Phys. Rev. Lett.* 128 (2022) 11, 112002