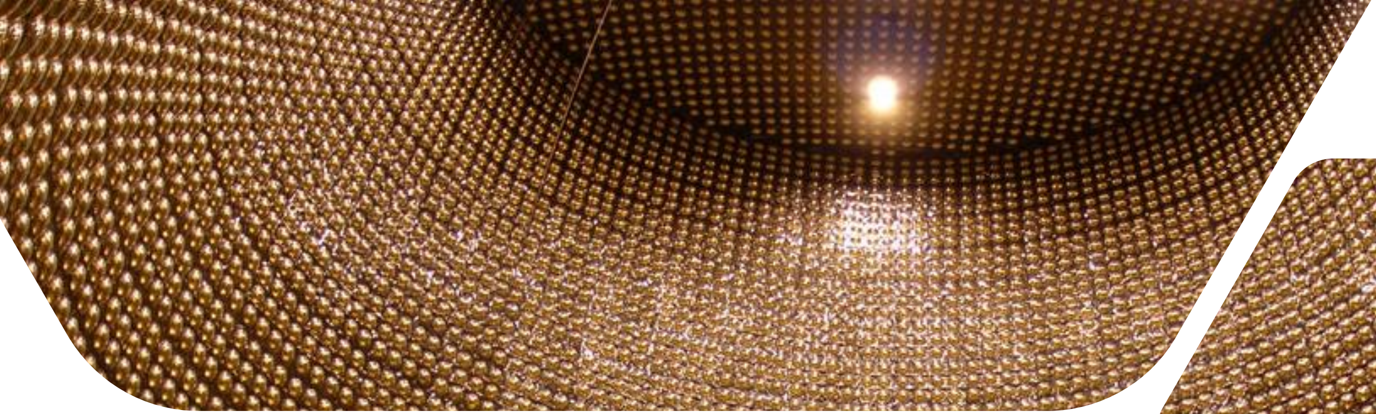
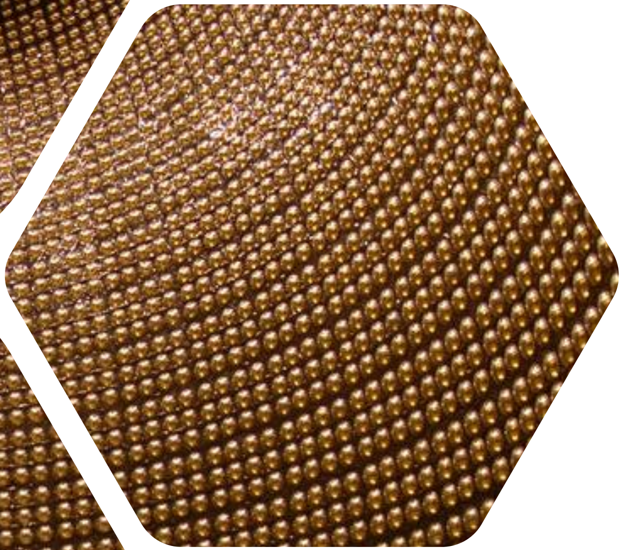
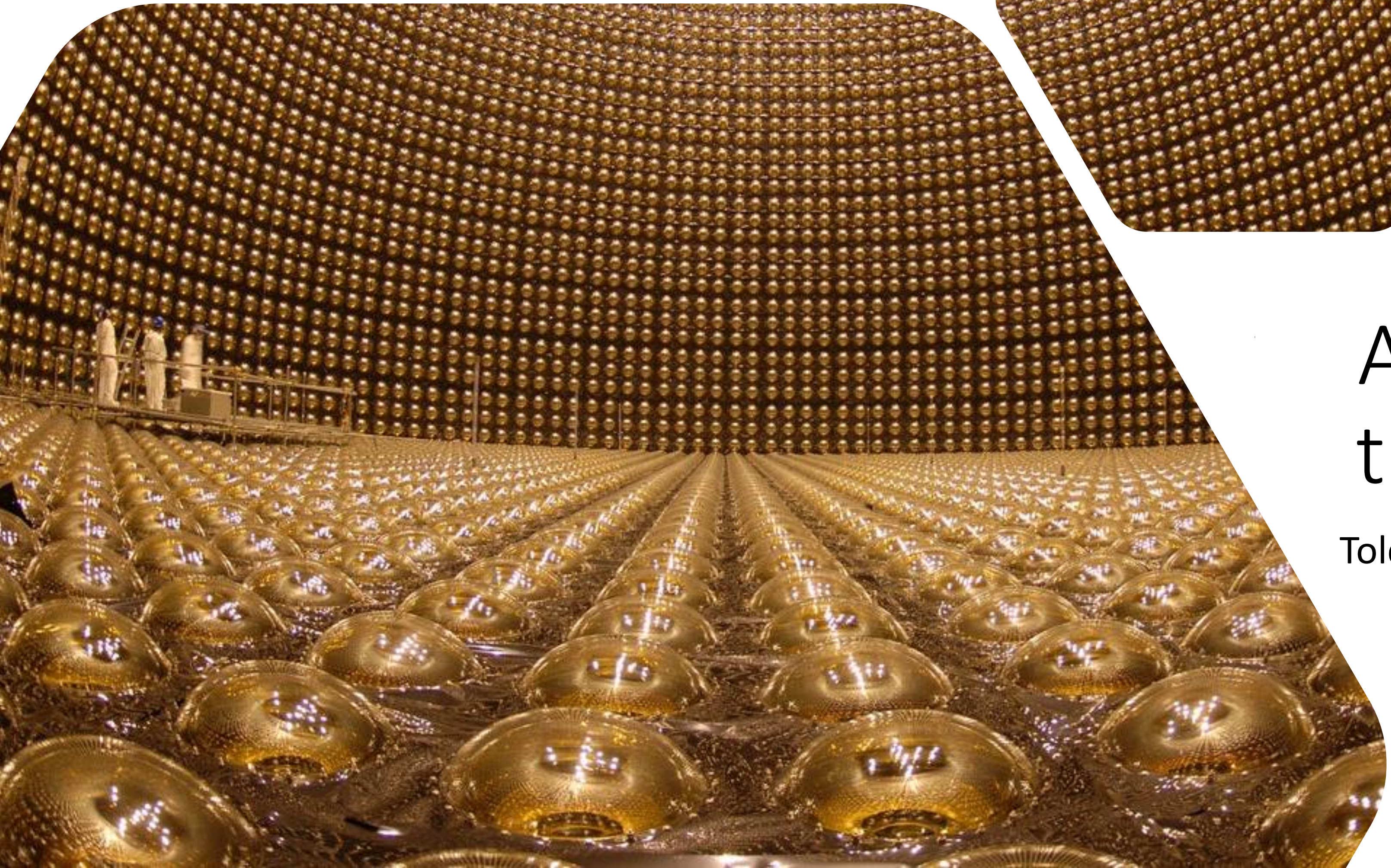




-Neutrinos- A brief overview of the ‘ghost particle’.

Told completely unbiased by Jaiden Parlone

Bananas emit around 1,000,000 neutrinos per day, mostly from Potassium-40 beta decay.

You emit around 300,000,000 per day as well.

Both are these are nothing compared to the sun’s output, which leads to around 100,000,000,000,000 (100 trillion) passing through your body every second.

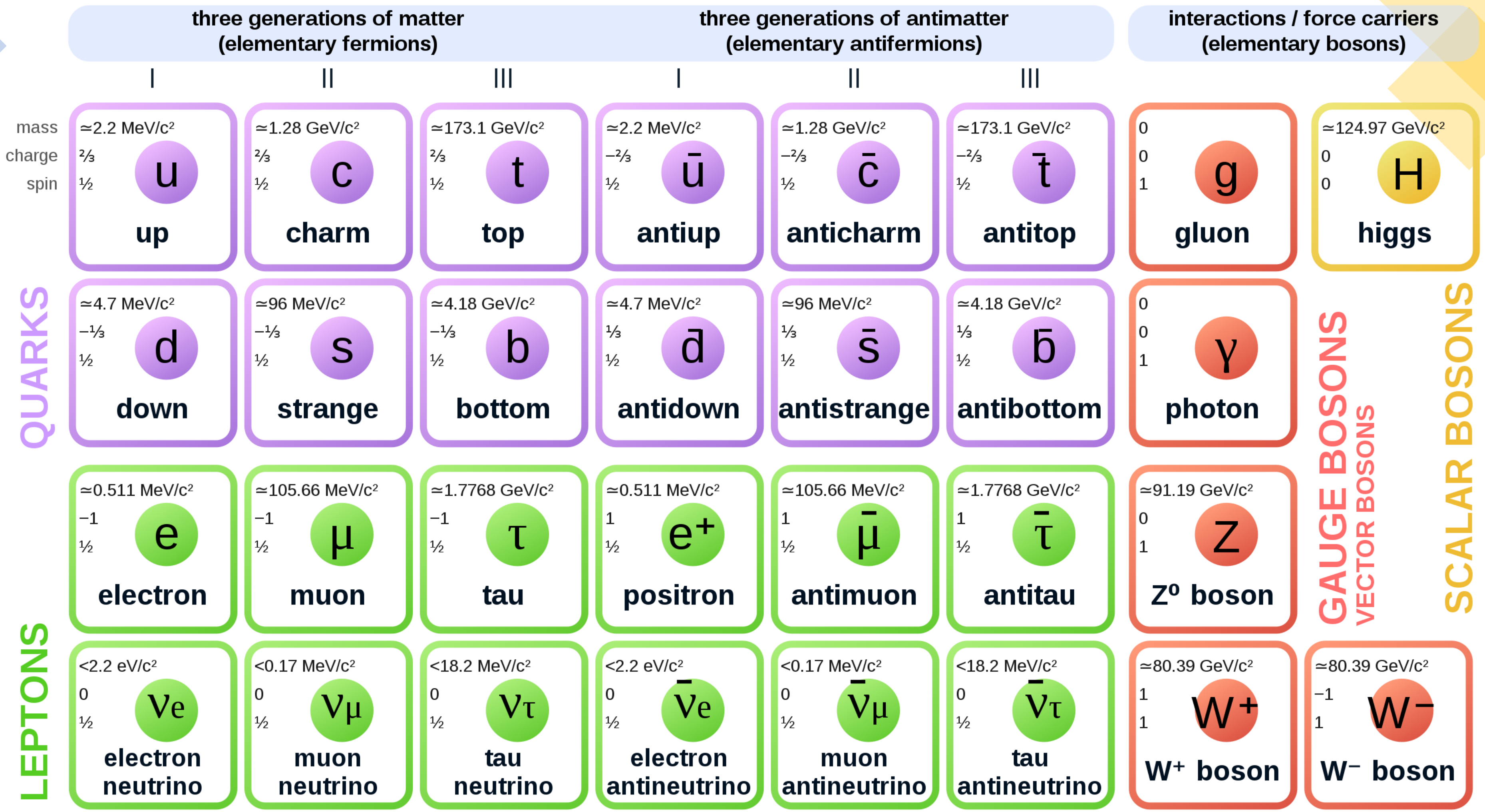
However…

There is only about a 1 in 4 chance of a neutrino interacting within your body within your lifetime.

And if you were holding a banana, it would take about 2 billion years before a neutrino from it interacted with you.

Plenty of time to learn about neutrinos.

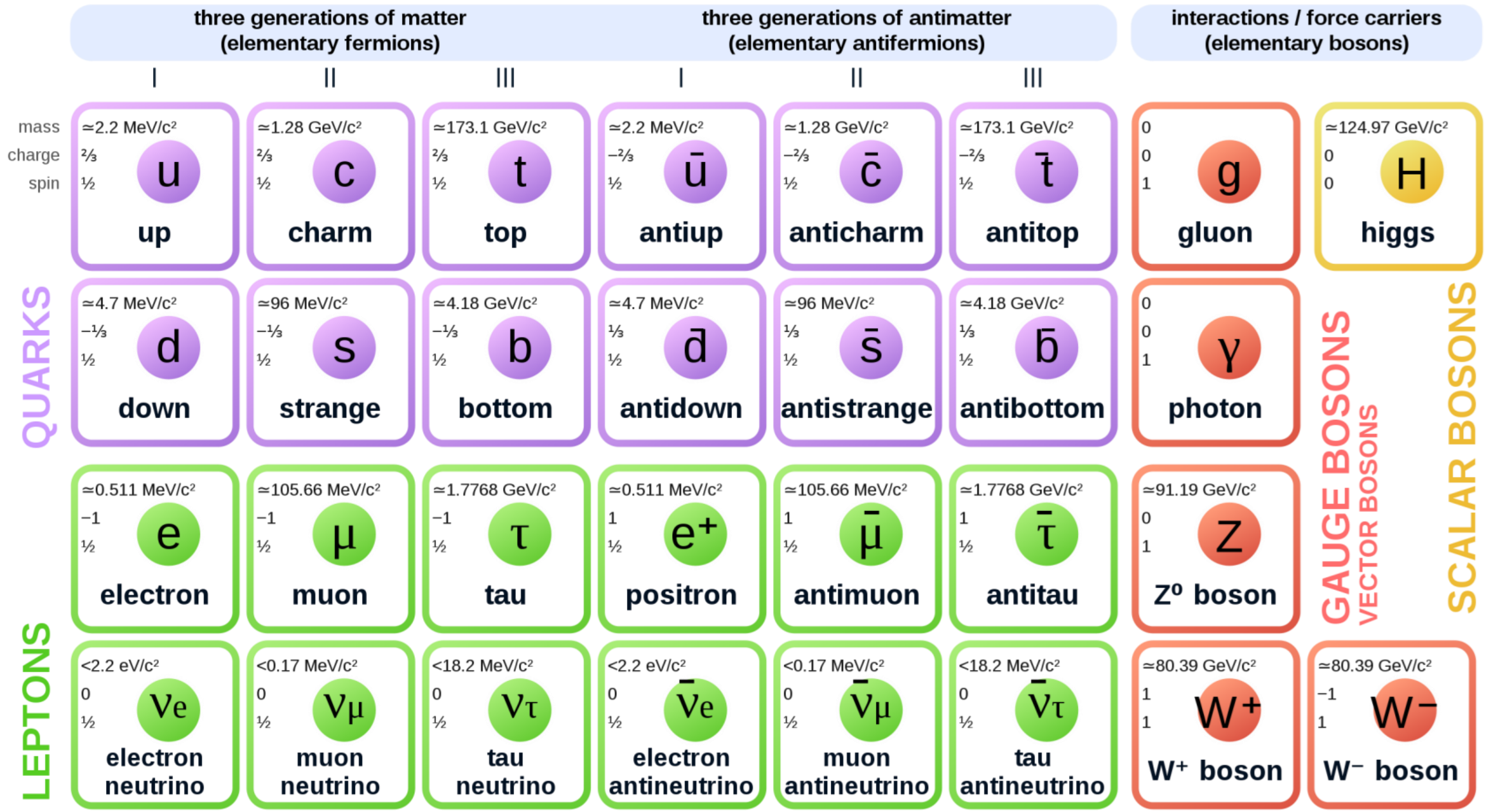
Standard Model of Elementary Particles



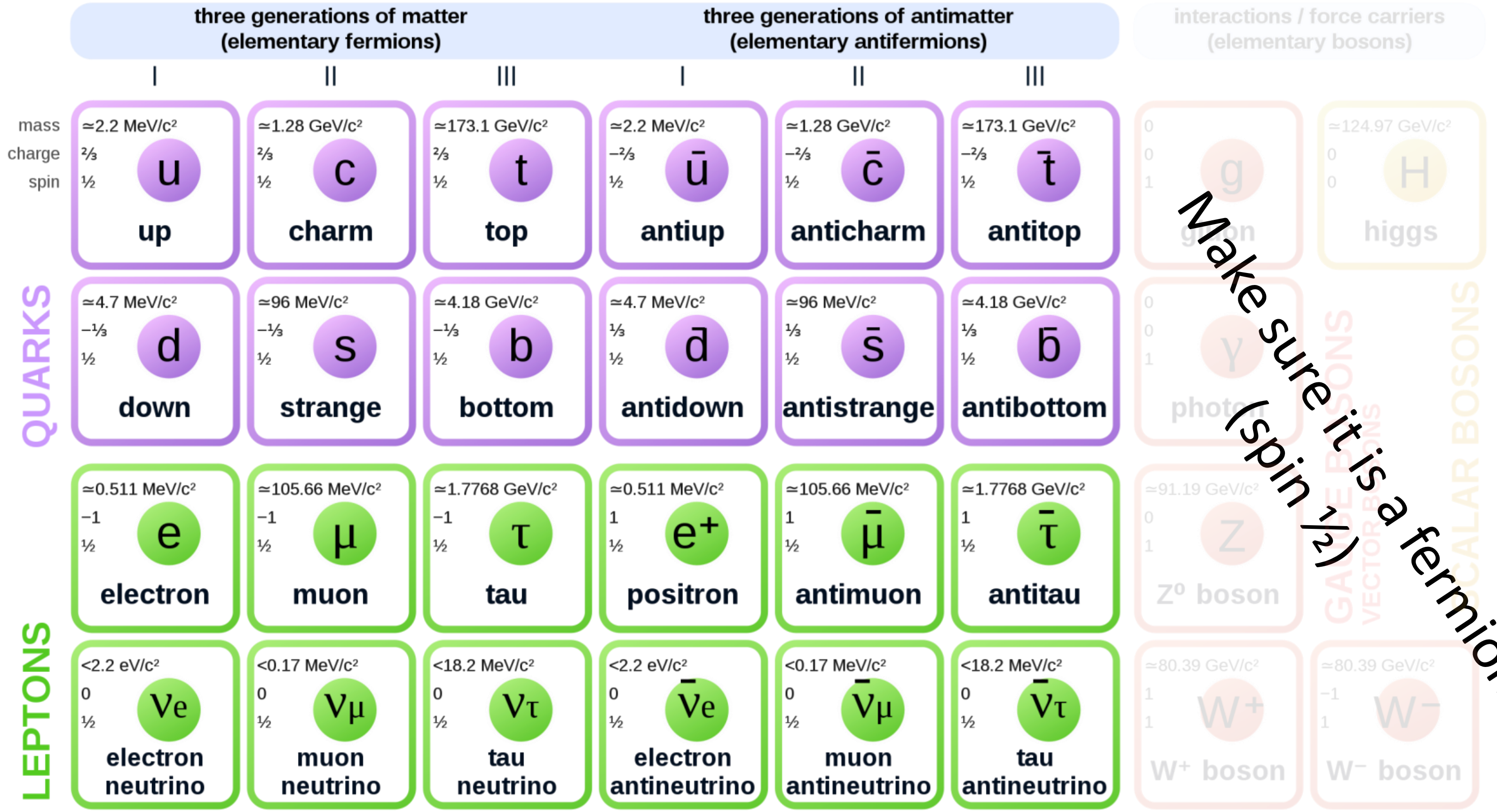
1 proton : 1 electron : 1 neutron : 1 billion neutrinos

Abundance 2nd only to photons!

To get a neutrino:

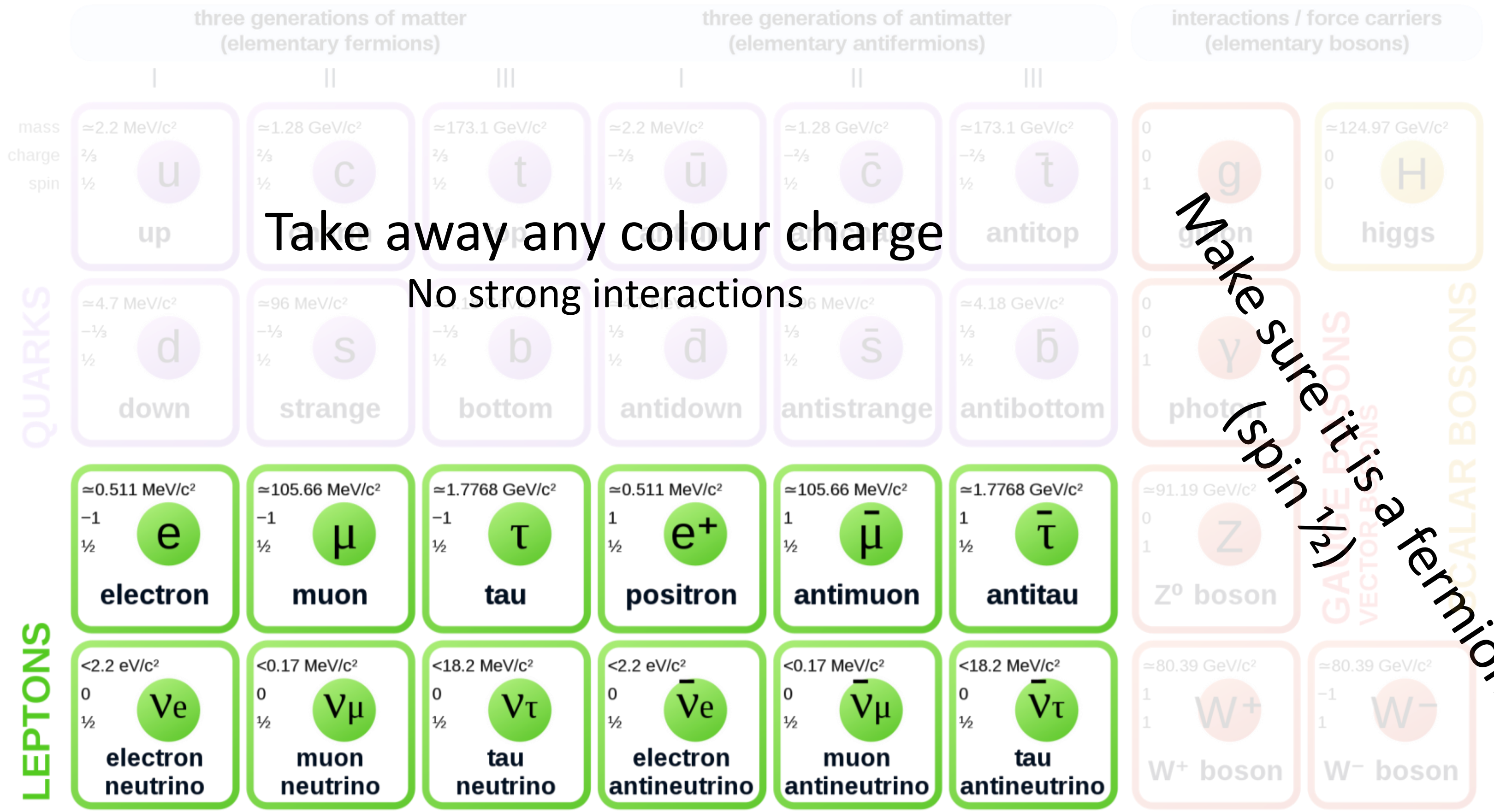


To get a neutrino:



Make sure it is a fermion
(spin ½)

To get a neutrino:



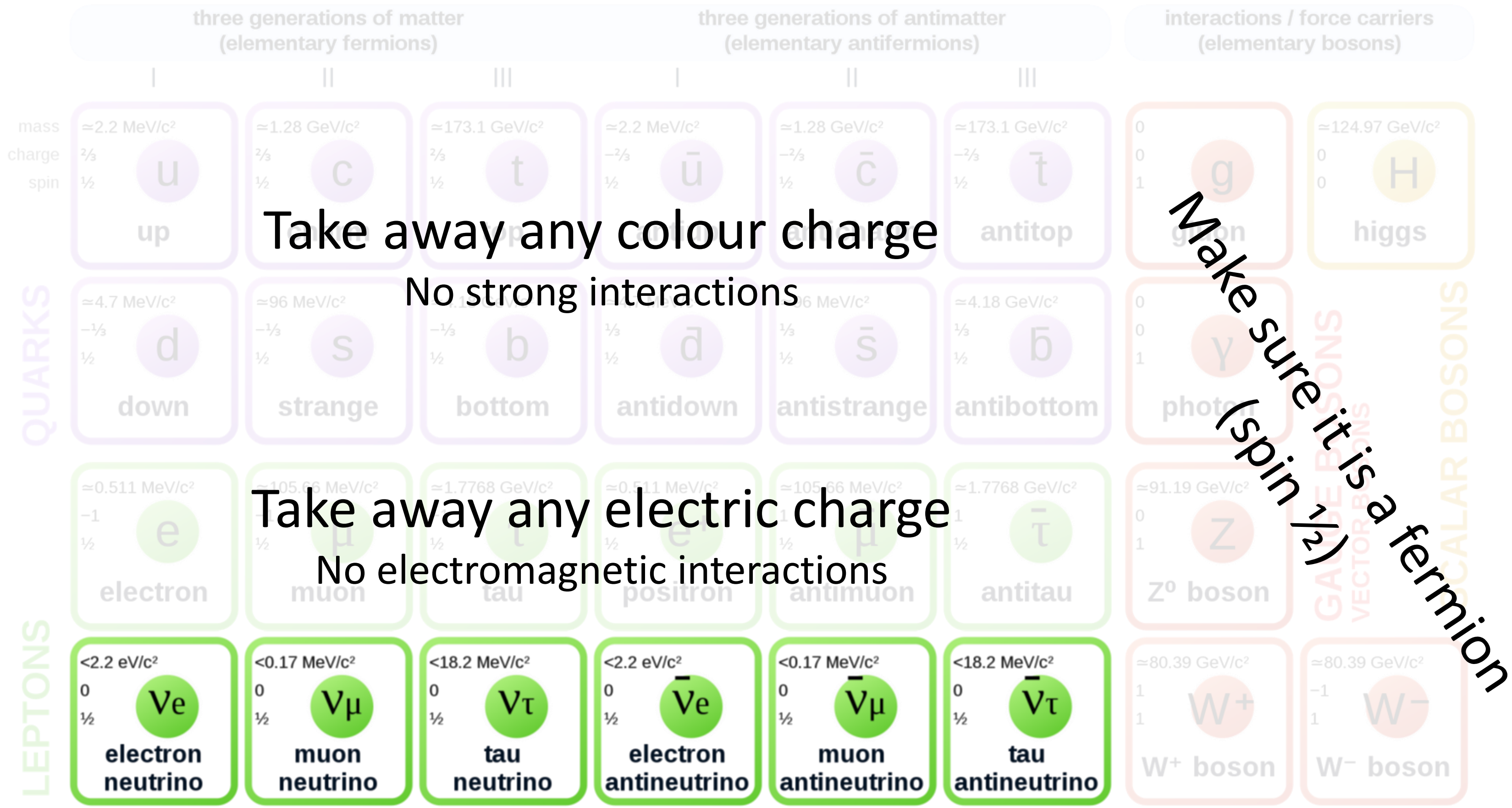
Take away any colour charge

No strong interactions

Make sure it is a fermion

(spin ½)

To get a neutrino:



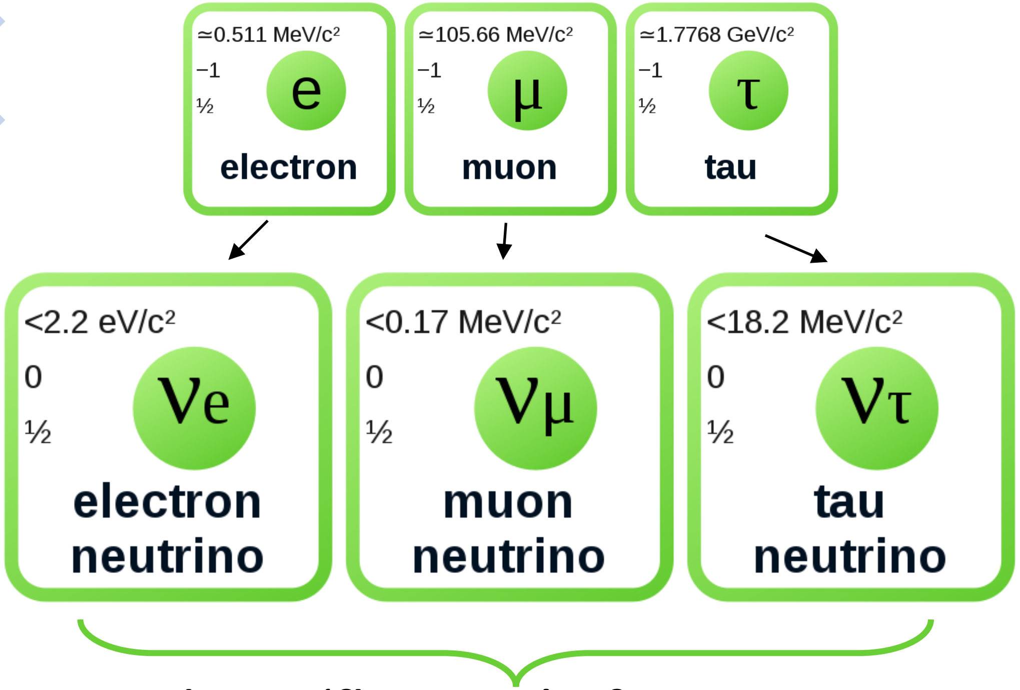
Take away any colour charge

No strong interactions

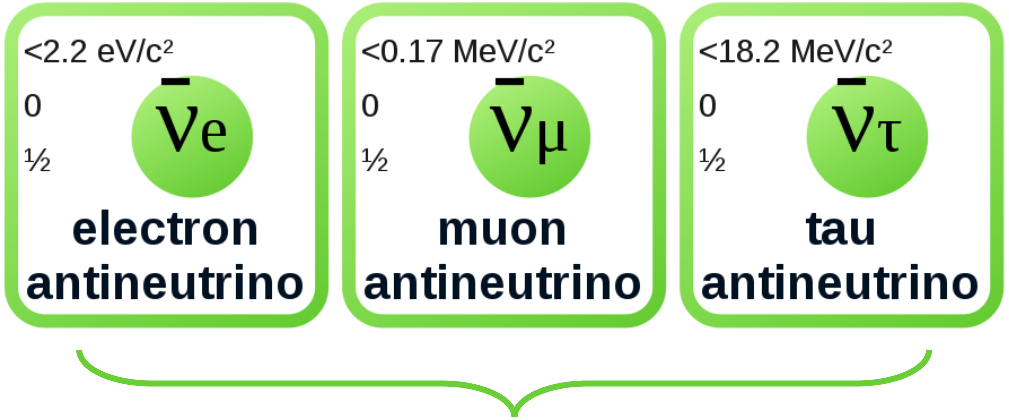
Take away any electric charge

No electromagnetic interactions

Make sure it is a fermion (spin ½)



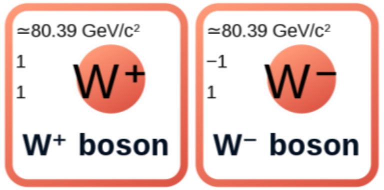
Three ‘flavours’ of neutrino

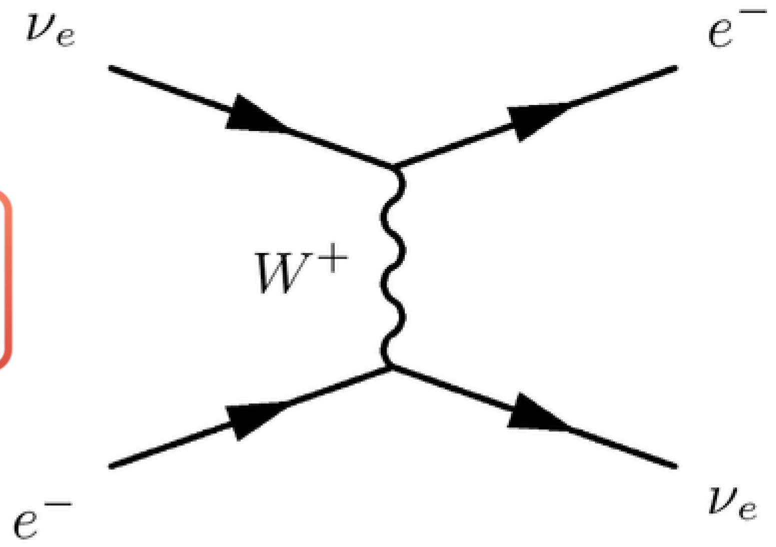


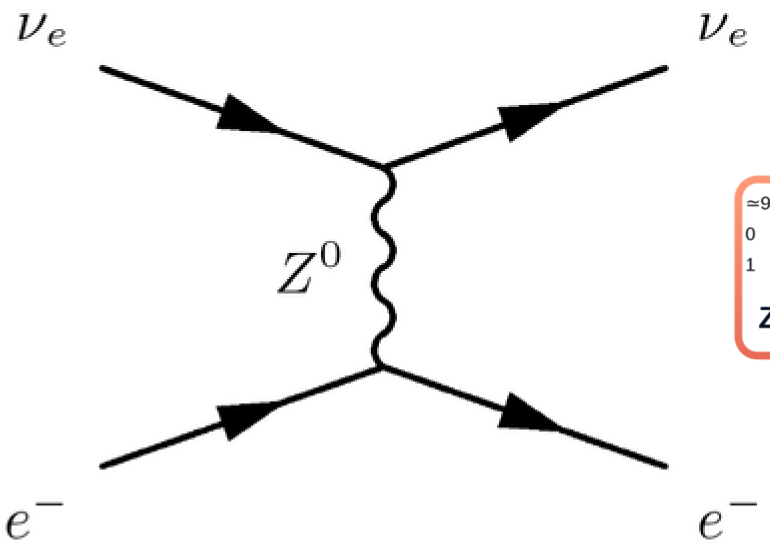
and antineutrino

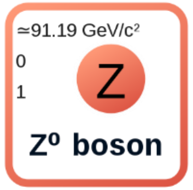
The standard model predicts that (anti)neutrinos should be massless!

Because they have no colour charge, or electric charge, neutrinos interact only through the Weak Force.







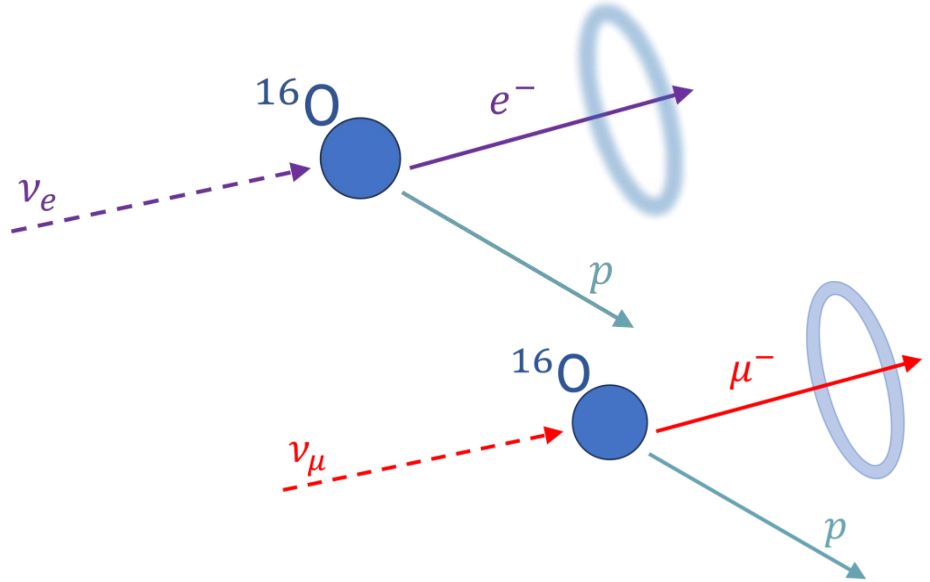


The charge related to this force is called 'Weak Isospin'.
(+Weak Hypercharge)

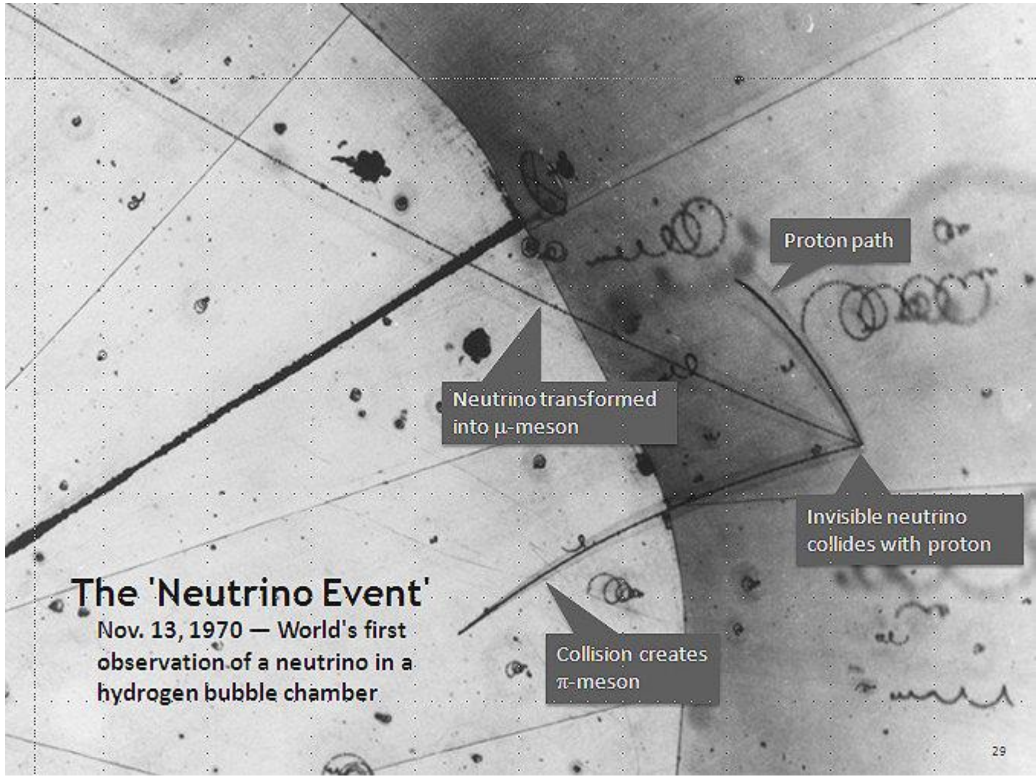
$W^{\pm}$ mediated interactions are the only known flavour changing interaction and are referred to as the 'Charged Current' in many sources.

The fact that the $W^{\pm}$ and Z^0 are massive is a result of symmetry breaking in the Higgs Field, the derivation of which tends to be covered in the 3rd or 4th year 'Gauge Theories' course.

- In most circumstances we measure neutrino interaction through neutrino-nucleon quasi-elastic scattering.
- Lepton flavour is preserved during neutrino interaction, with a charged lepton of the same generation produced.
- A neutrino has no charge so is invisible to electric and magnetic fields in a detector.
- Neutrinos interact rarely (low cross-section for interaction) meaning you very rarely see events.
- Electron, muon, tau leptons have different signatures in detectors.
- τ are suppressed due to the high energy threshold to produce (rest mass of $\sim$ 1.777 GeV), and decay very quickly.
- Signals get messy at higher energies as more pions start being produced during nucleon scatter.



Old language 'mu-meson'

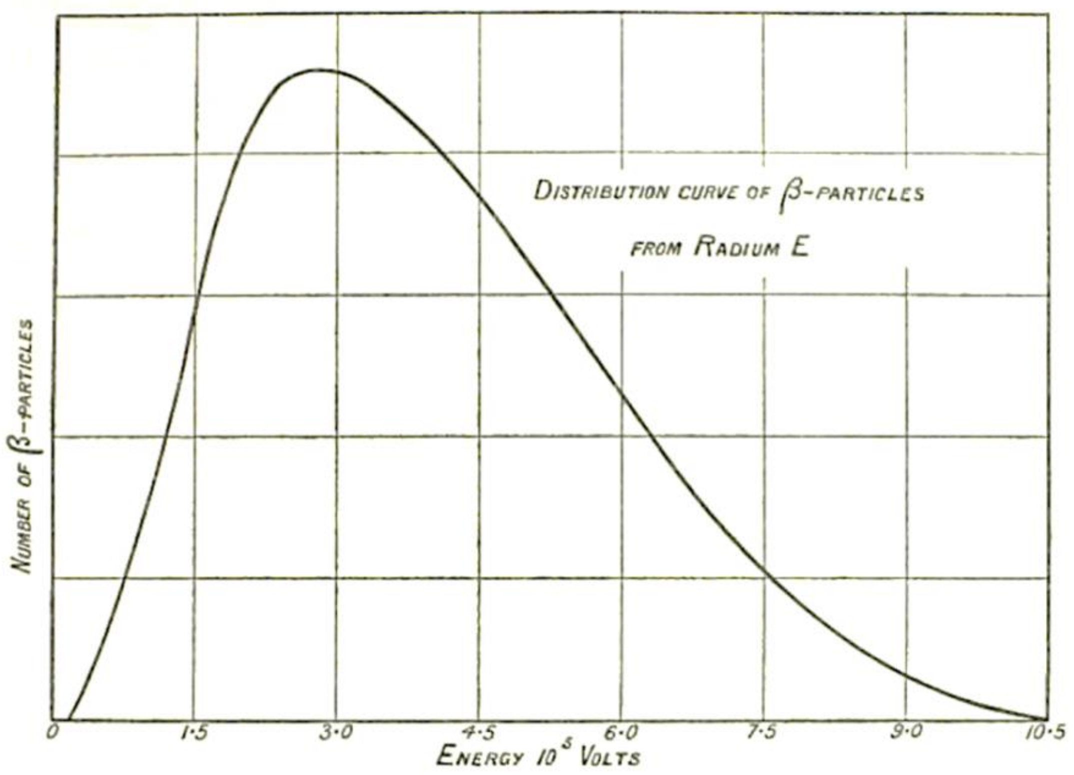


In the early 20th Century, beta decay was thought to be a two-body process, described by:

$$n \rightarrow p + e^{-}$$

In 1914 James Chadwick discovered that the energy spectrum of electrons emitted during beta decay of radium was continuous as opposed to discrete, apparently in violation of conservation of energy.

This type of decay would have also violated later laws of angular momentum and lepton number conservation.



Note that this diagram is actually from a confirmation paper from 1927; Doi: 10.1098/rspa.1927.0168.

In 1930 Pauli first postulated a neutral, spin-$\frac{1}{2}$ particle with negligible mass that was also produced in this process. This he named ‘neutron’ but it was later renamed to ‘neutrino’ (the italian equivalent of "little neutral one") after Chadwick’s discovery of the atomic neutron. This particle would later be fully realised as the electron antineutrino.

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

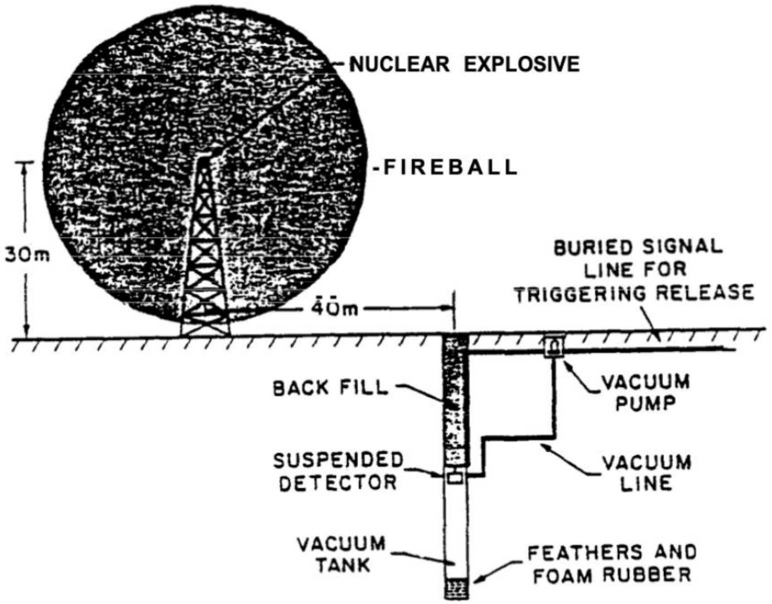
However, due to this particle’s properties Pauli famously stated:

In 1956 the team working on the Cowan-Reines neutrino experiment published results utilised antineutrinos created in a nuclear ~~blast~~ reactor to induce a process known as 'inverse beta decay':

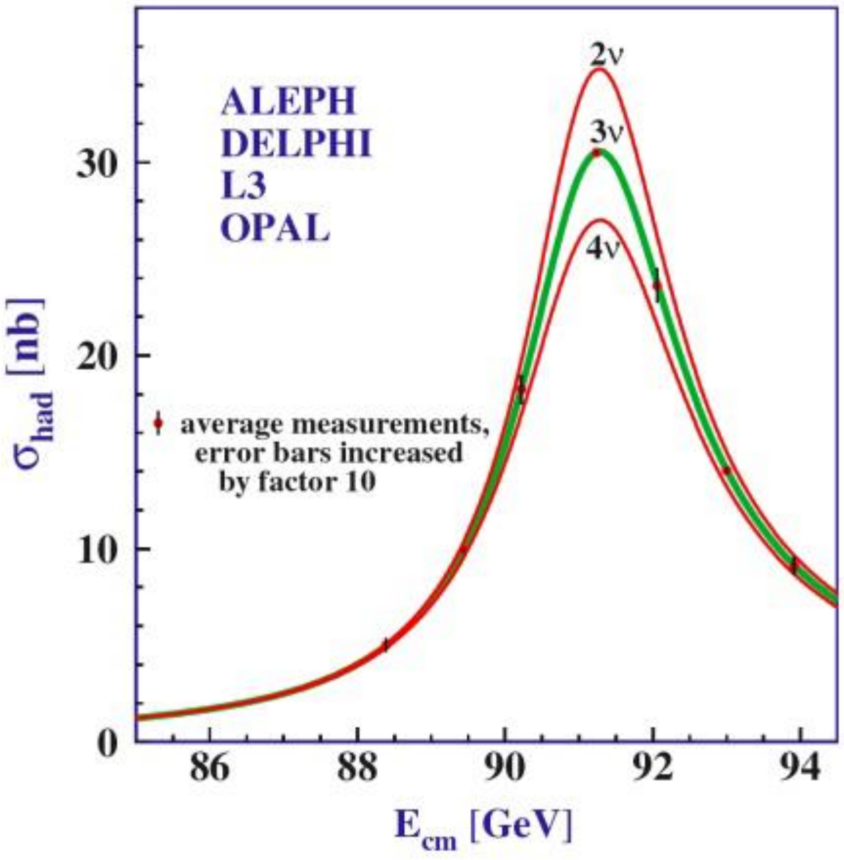
$$\bar{\nu}_e + p \rightarrow n + e^-$$

This effort was awarded a Nobel Prize in 1995, almost 45 years later.

However, open questions remained. These neutrinos had only been observed to produce electrons in interactions, whereas there was now another known lepton, the muon. This led to the discovery and distinction of the muon neutrino, ν_μ, by Lederman, Schwartz, and Steinberger in 1962.



Project "*Poltergeist*"



Studies of Z^0 boson decay allows for the number of light active neutrinos to be determined. This is done through the decay mode;

$$Z^0 \rightarrow \nu_l + \bar{\nu}_l$$

Where each neutrino flavour contributes to the decay width.

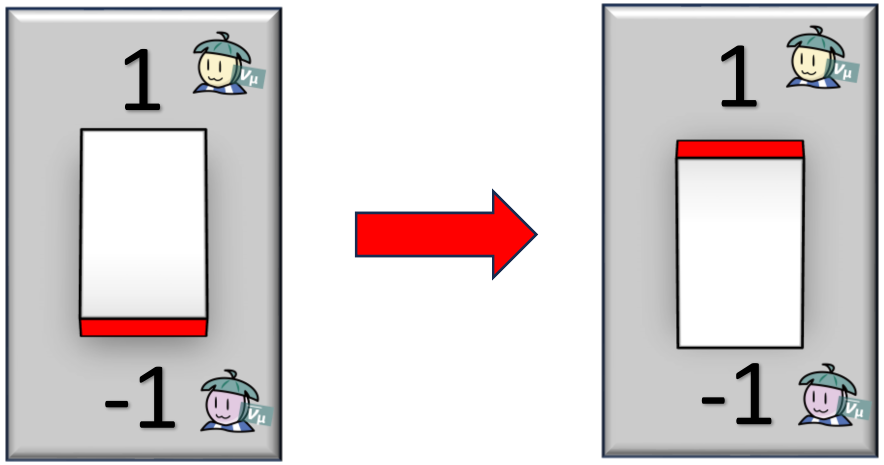
From this, the results from LEP determined to the number of light, active neutrino species to be: $N_\nu = 2.984 \pm 0.008$

This was achieved before the DONUT collaboration discovered the tau neutrino, ν_τ, in July 2000, completing the picture of the three flavours of leptons after finding τ in the 70s.

With this, our standard model is complete! (Apart from the Higgs boson in 2012 which obviously no-one cares about)

The standard model is constructed from discrete symmetries.

Charge (C) Conjugation



Invert all ‘charge’ quantum numbers:

- Colour
- Electric Charge
- Weak Isospin
- Isospin
- Baryon Number
- Lepton Number

Changes a particle for an antiparticle.

Invert all spatial properties. Flips signs of parity-odd variables:

- Momentum
- Helicity (momentum and spin alignment)

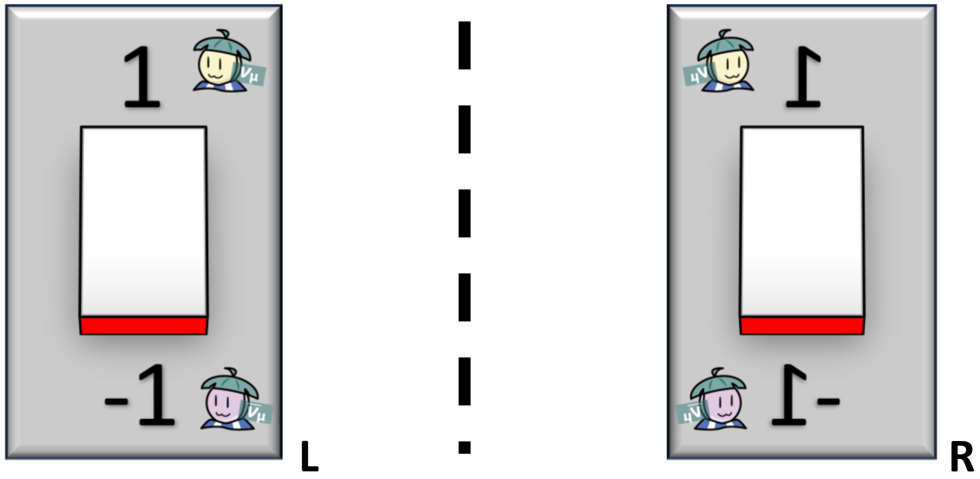
Does not change parity-even properties:

- Spin

Changes ‘handedness’ of particles!

Particles should act the same in a ‘mirror world’.

cough** if they are not chiral **cough

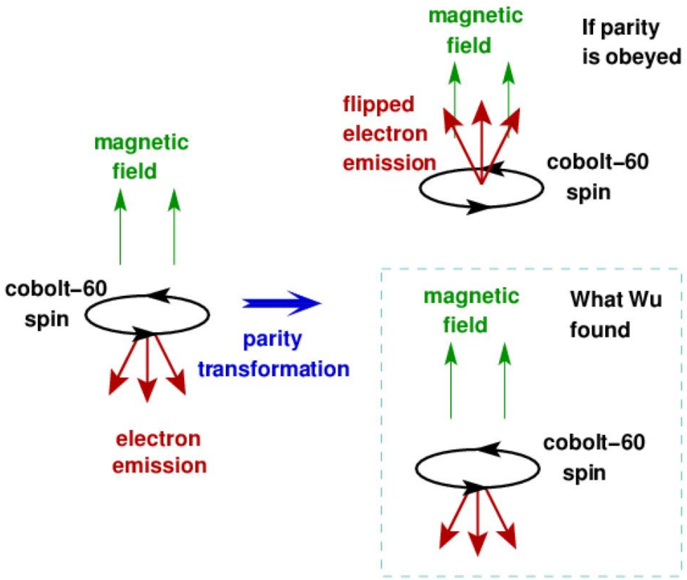


Parity (P) Transformation

C.S. Wu checked this was correct for Weak interactions and in 1957 found that it simply wasn't. Parity was violated, apparently maximally. This understandably traumatised many physicists. On the plus side, we now had a formal definition of left and right if we were to meet aliens.

$$^{60}_{27}\mathrm{Co} \rightarrow ^{60}_{28}\mathrm{Ni}^* + e^- + \bar{\nu}_e$$
$$^{60}_{28}\mathrm{Ni}^* \rightarrow ^{60}_{28}\mathrm{Ni} + 2\gamma$$

- Apply magnetic field
- Cool apparatus to 0.01 K
- High proportion of ^{60}Co spins are aligned with the magnetic field
- Examine distribution of emitted electrons and photons.

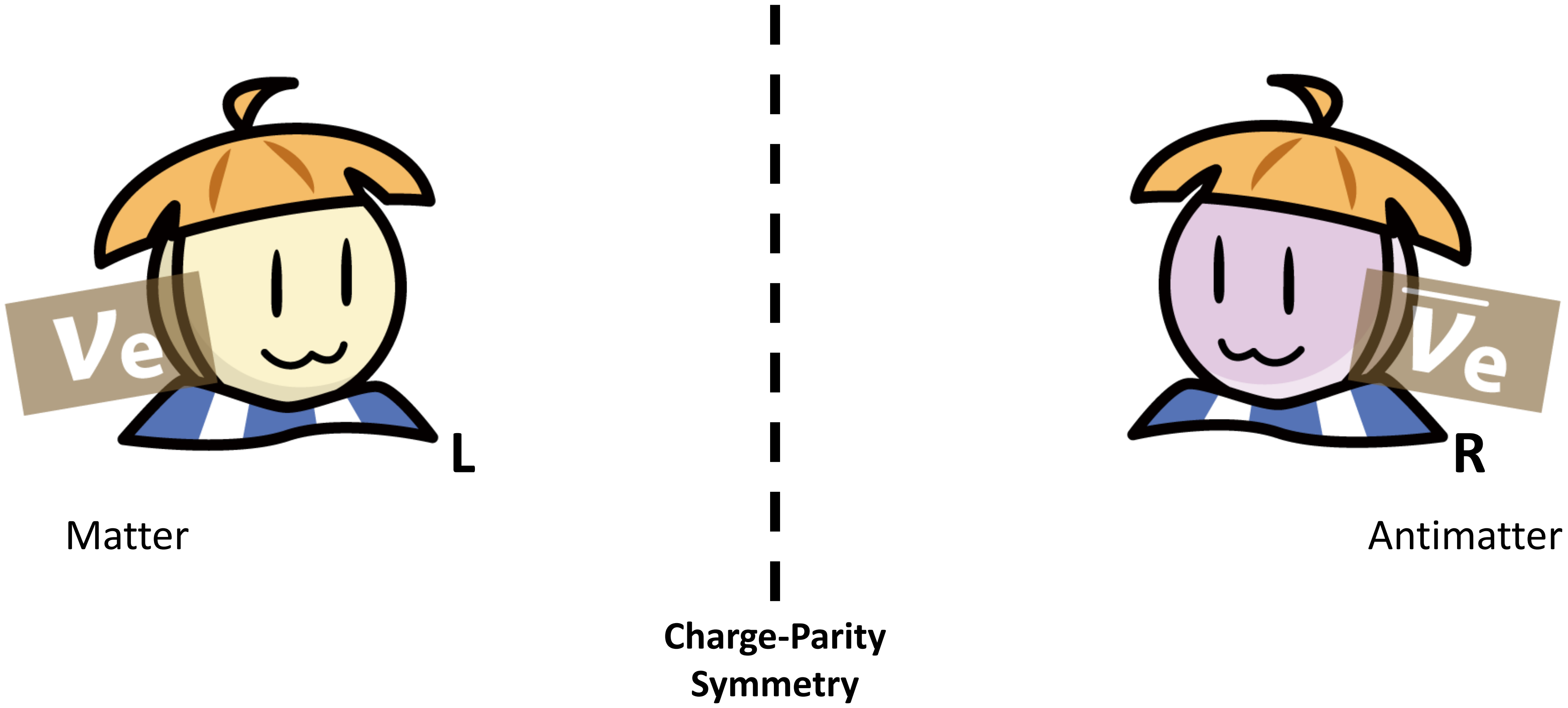


Lee and Yangs' contributions to the theory of Parity violation were rewarded with a Nobel prize in that same year. Though Wu's role was mentioned, she unfortunately wasn't officially honoured until 1978 with the Wolf Prize.

"Although I did not do research just for the prize, it still hurts me a lot that my work was overlooked for certain reasons."

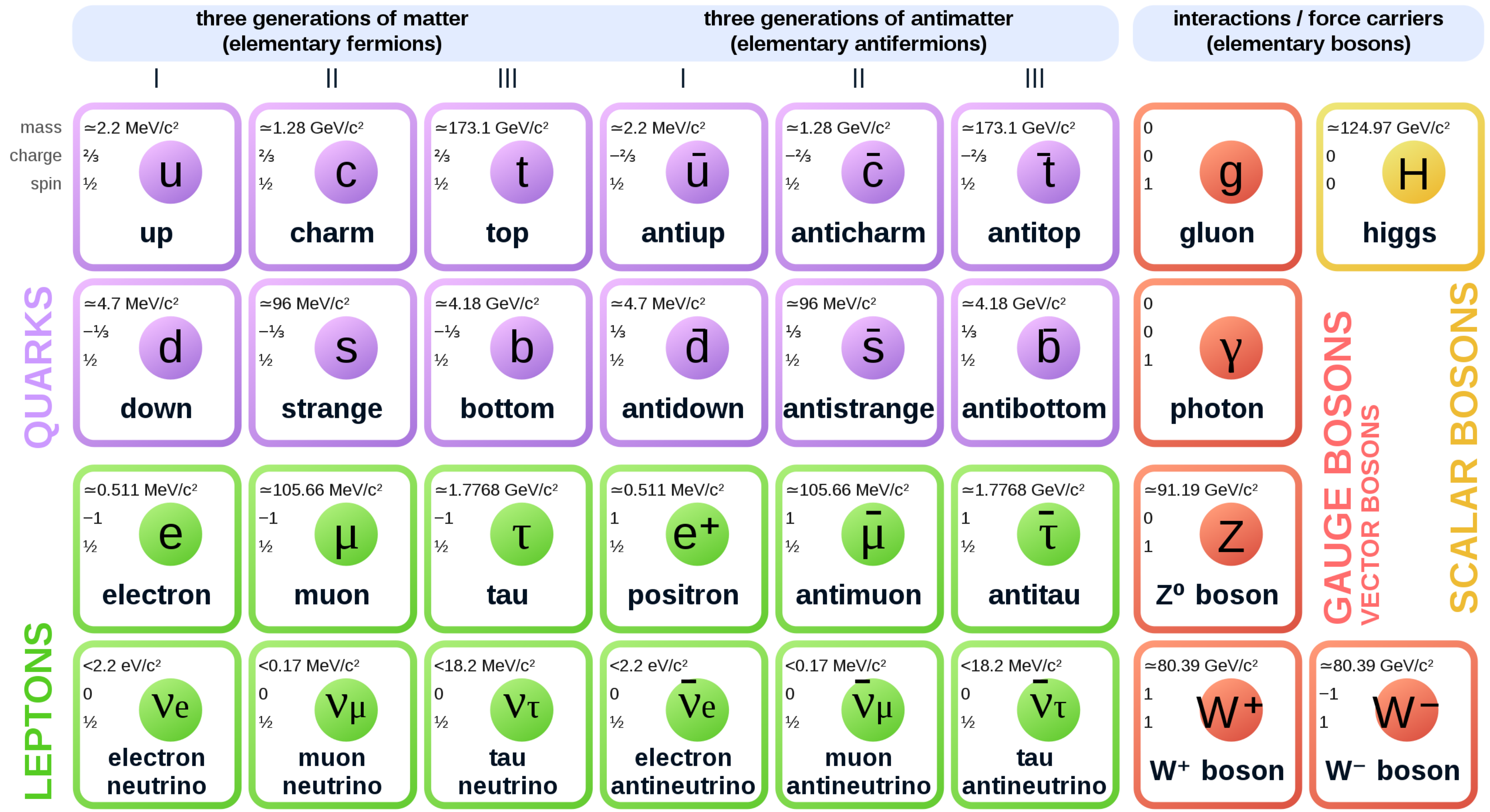
The charged weak bosons only couple to left-handed particles, and right-handed antiparticles. As neutrinos can only interact weakly, only left-handed neutrinos and right-handed antineutrinos exist, but their opposite chiral partners do not (in the standard model). This also means that Charge symmetry (C) is broken.

Therefore, the combination of both, Charge-Parity (CP), was thought to be conserved:



[Elementary Particle Single Illustration – HiggsTan]

Standard Model of Elementary Particles



With this revelation of broken symmetries, the unified electroweak theory is fully within the standard model. So all is well! Neutrinos are massless, neutral, spin-$\frac{1}{2}$, chiral elementary fermions. Unless….

Solar neutrino problem

In the late 1960s Raymond Davis, a chemist by trade, knew that electron neutrinos interacting with chlorine would produce a radioactive isotope of argon:

$$\nu_e + Cl^{37} \rightarrow Ar^{37} + e^-$$

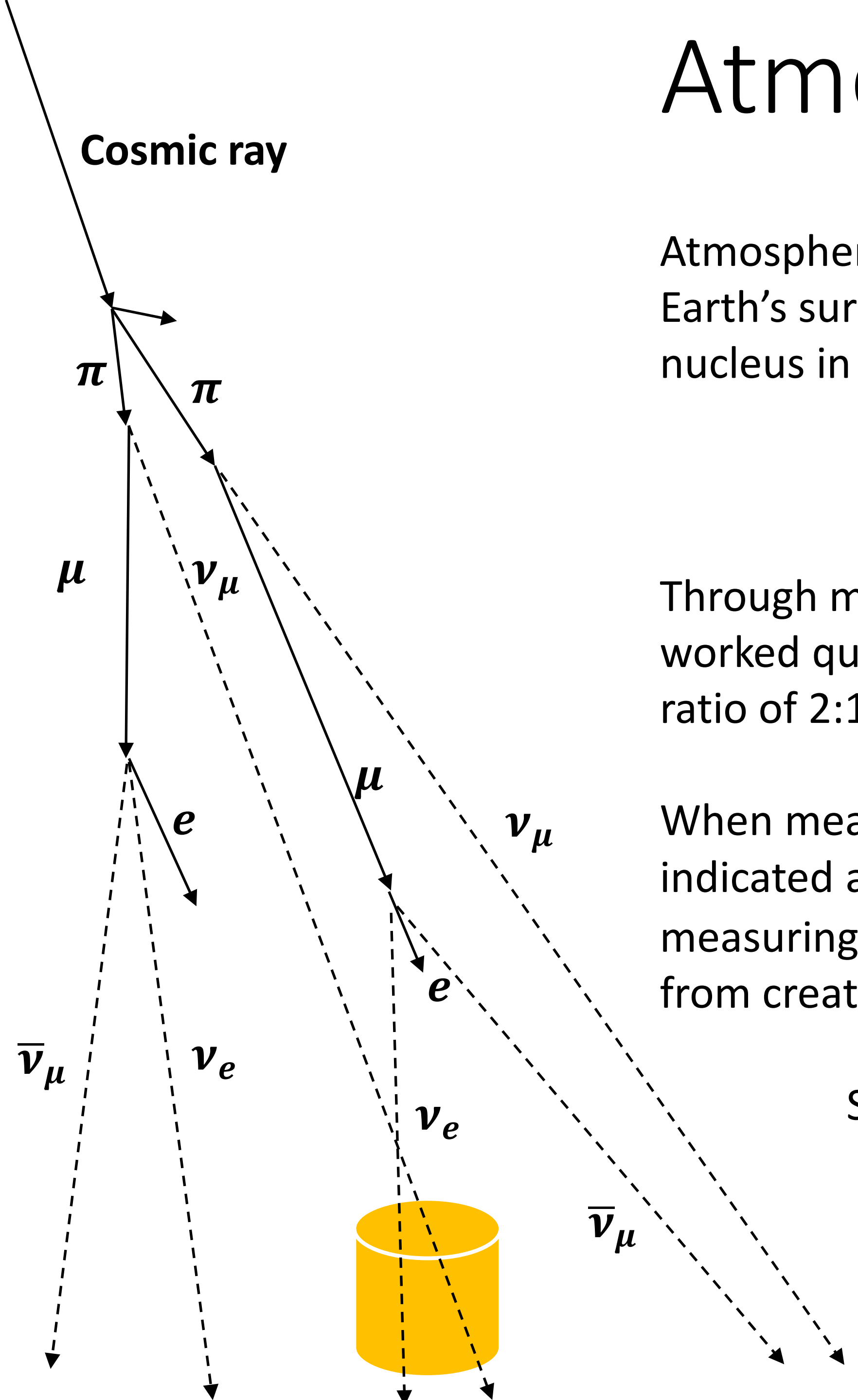
The idea was to fill a 380 cubic meter tank with perchloroethylene (dry-cleaning fluid), place it 1,478 meters underground to shield from cosmic rays, and count solar neutrinos (produced by fusion chains) by extracting and measuring the resulting argon.

With a week's operation, the experiment could expect to create 10 atoms of argon, compared to the 9×10^{30} chlorine atoms present in the tank.

However Davis did not find the expected 10 atoms, but instead 3. Many thought this to be experimental error, or an inaccurate prediction, but many later experiments confirmed this result.

This came to be known as the 'solar neutrino problem'.

Atmospheric neutrino problem



Atmospheric neutrinos are typically produced about 15 kilometres above the Earth's surface. When a cosmic ray (usually a proton) interact with an atomic nucleus in the atmosphere, it forms a cascade of particles:

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\mu^+ \to \bar{\nu}_\mu + e^+ + \nu_e$$

Through many decays of the unstable products (here be a great relativity worked question on the lifetime of muons), neutrinos are produced with a ratio of 2:1 of muon: electron flavour.

When measured, this ratio was different to the prediction. Many experiments indicated a disappearance of ν_μ and an appearance of ν_e, but only when measuring neutrinos coming up through the earth (longer distance travelled from creation)

Something was clearly going on.

A possible solution?

Inspired by Kaon oscillations proposed by Gell-Mann and Pais, Bruno Pontecorvo proposed a possibility that another neutral particle, neutrinos, might oscillate between two states.

The proposals he made were:

- $\nu_L \leftrightarrow \nu_R$
- $\nu_L \leftrightarrow \bar{\nu}_L$ and $\nu_R \leftrightarrow \bar{\nu}_R$

and more importantly:

- $\nu_e \leftrightarrow \nu_\mu$, allowing a neutrino to violate lepton flavour conservation, changing between the two lepton flavours known at the time.

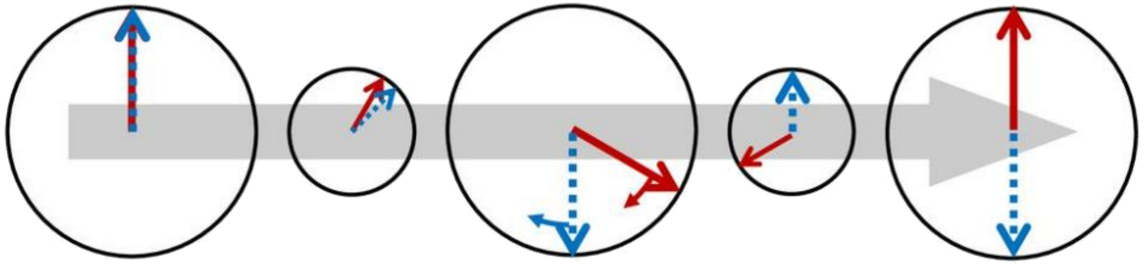
The change between these flavours is called oscillation, after the sinusoidal probability change over distance. This change can only occur if neutrinos had mass and arises because of difference between their definite mass and definite flavour.

Warning,
maths
incoming.

These proposals can all be found in:
Soviet Physics JETP. 26.984-988.

If we start off in a two flavour world, this rotation matrix becomes familiar;

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

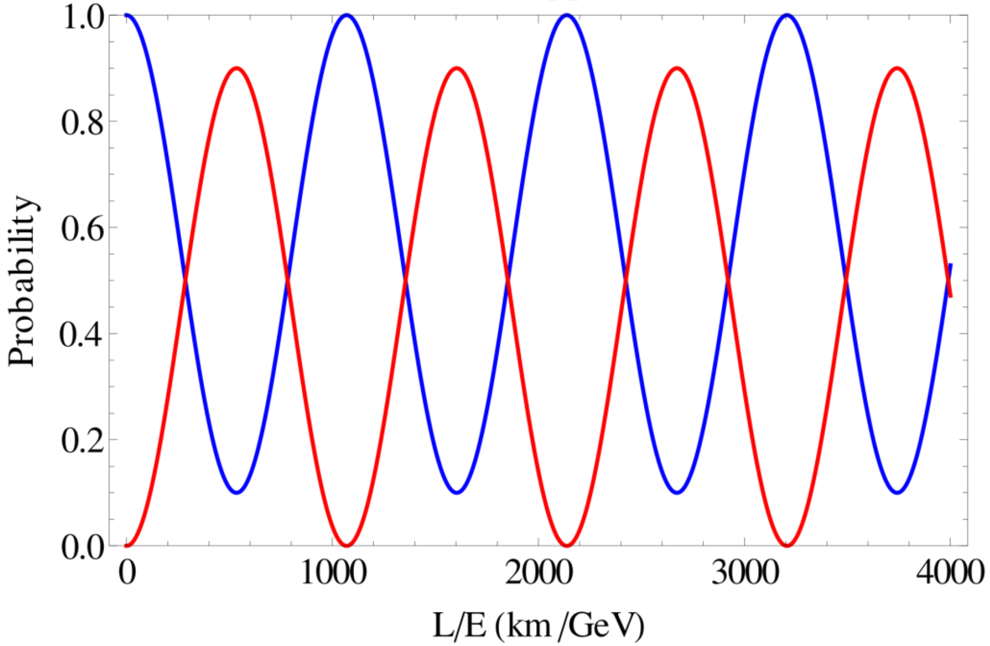


And the survival probability of a flavour can be derived to be;

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2 \left[\frac{(m_2^2 - m_1^2)x}{2E}\right]$$
$$= 1 - \sin^2 2\theta \sin^2 \left[\frac{1.27\Delta m^2 L}{E}\right]$$

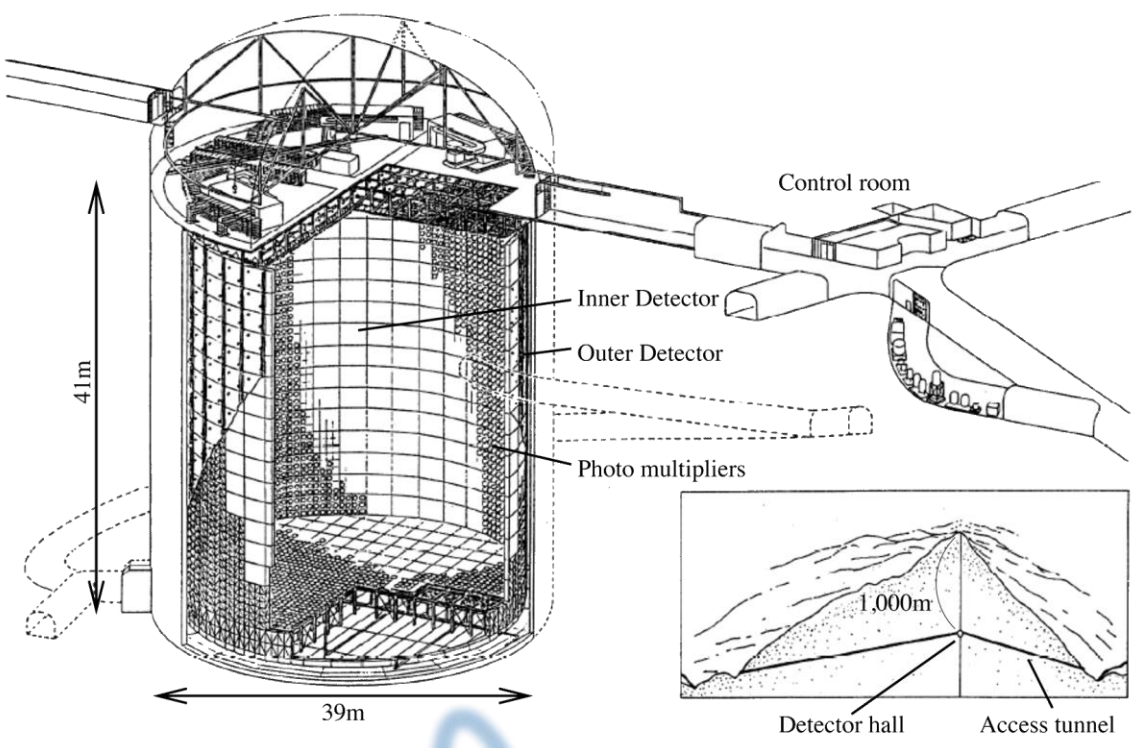
From here we can see a couple of things;

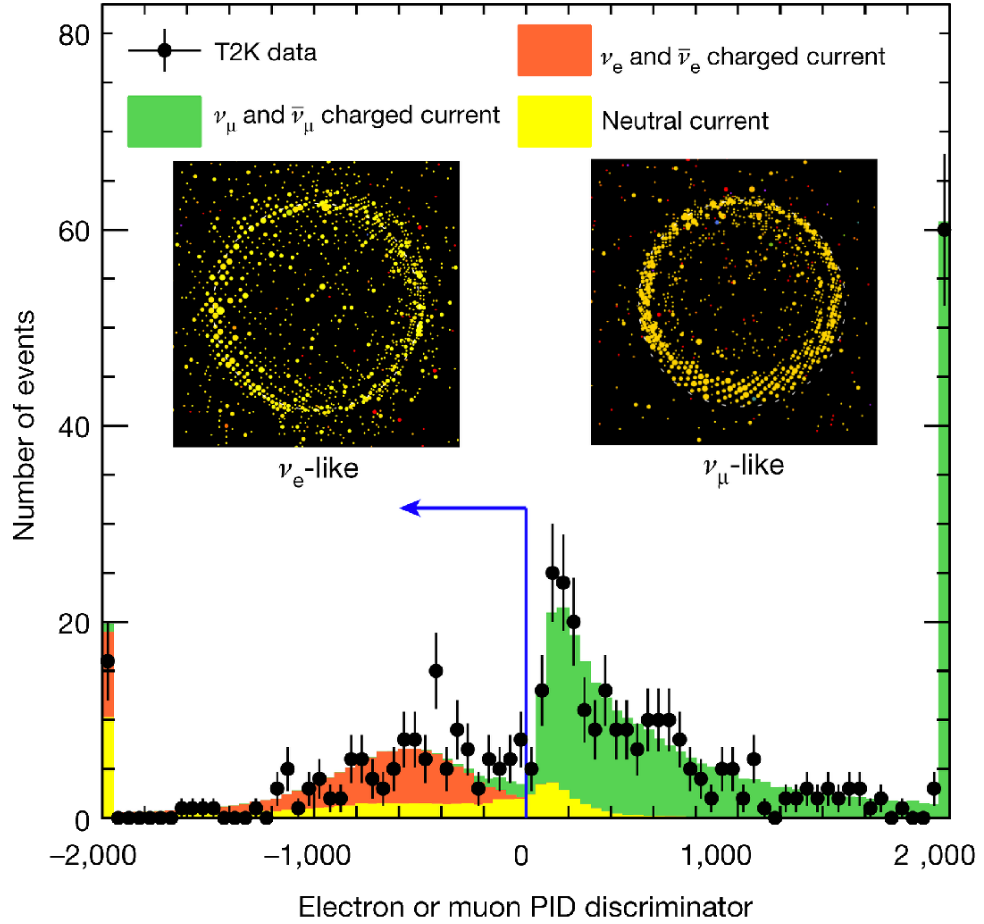
- If the mass difference is 0, no oscillations can occur.
- Theta, named the mixing angle, defines how different the flavour states are from mass states. This determines amplitude of oscillation and is maximal at $\frac{\pi}{4}$.
- There are two free parameters that are either set for us by nature, or we can freely control;
  - ❖ Length, L, is the distance a neutrino will propagate.
  - ❖ Energy, E, is the energy of that neutrino.

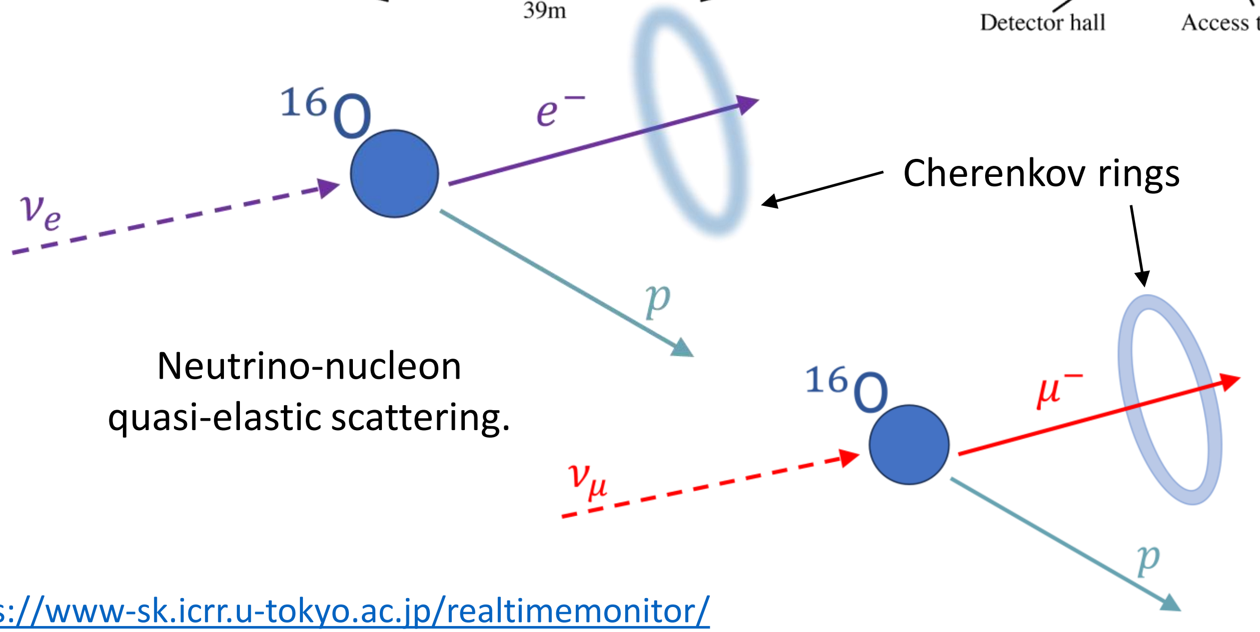


Super-size that Kamiokande

- 50 kton water Cherenkov detector.
- ~11,000 20” PMTs
- Super-K is located 1,000 m (3,300 ft) underground in the Mozumi Mine in Hida's Kamioka area, Japan.







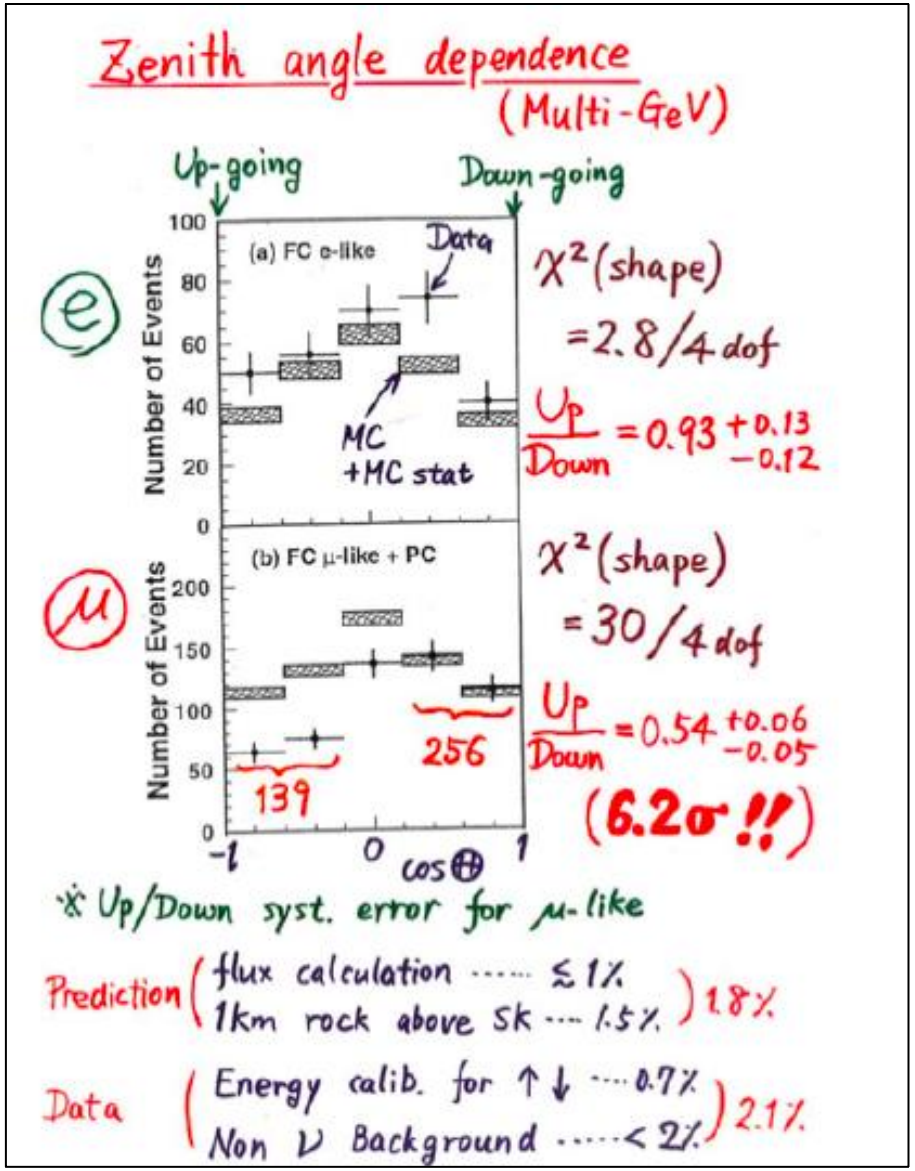
Neutrino-nucleon quasi-elastic scattering.

https://www-sk.icrr.u-tokyo.ac.jp/realtimemonitor/

Disclaimer: Almost all events are cosmic ray muon events.

A complete confirmation

In 1998, the Super-Kamiokande water Cherenkov detector achieved the first measurement of atmospheric neutrino flavour disappearance, with the energy dependant pattern expected of oscillation phenomena.

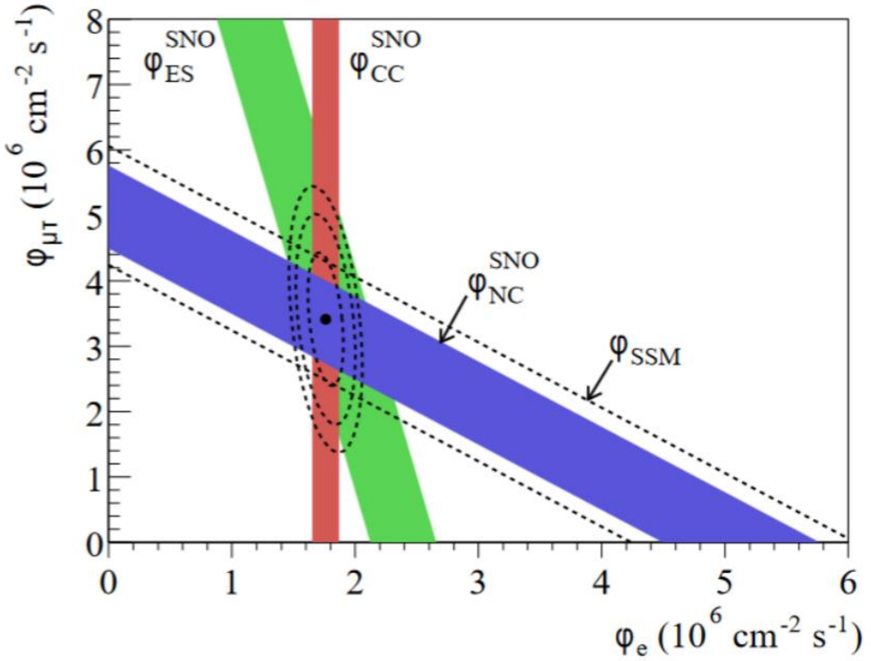


On the right is shown the dependence on zenith angle (i.e. the angle the neutrino entered the detector from with from below being $\cos\Theta = -1$ and above $\cos\Theta = 1$) and therefore the distance travelled by the neutrino (from atmosphere above or through Earth as well).

SNO place like home.

In 2002 the Sudbury Neutrino Observatory, a kiloton heavy water Cherenkov detector announced that it had measured solar neutrinos using three different interaction channels.

- The first, a charged current interaction, was sensitive to only ν_e. This saw the deficit Davis had at the Homestake experiment.
- The second, a neutral current interaction, was sensitive to all ν. This saw neutrino flux matching that predicted by the solar model.
- The third, an Elastic Scattering interaction, primarily sensitive to ν_e but had sensitivity to other flavours. This mostly gave directional info.

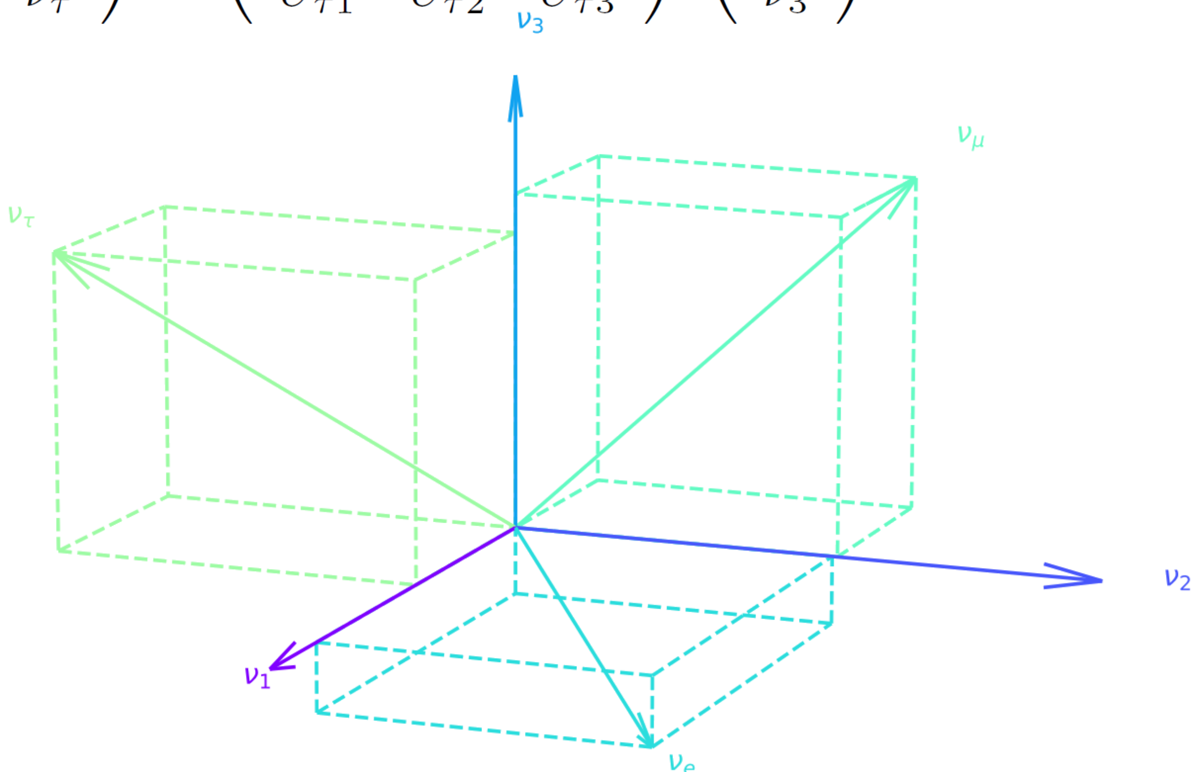


With these detections, the community was sure that neutrino flavour oscillations were occurring! However this realisation came with a troubling conclusion. Neutrinos had mass.

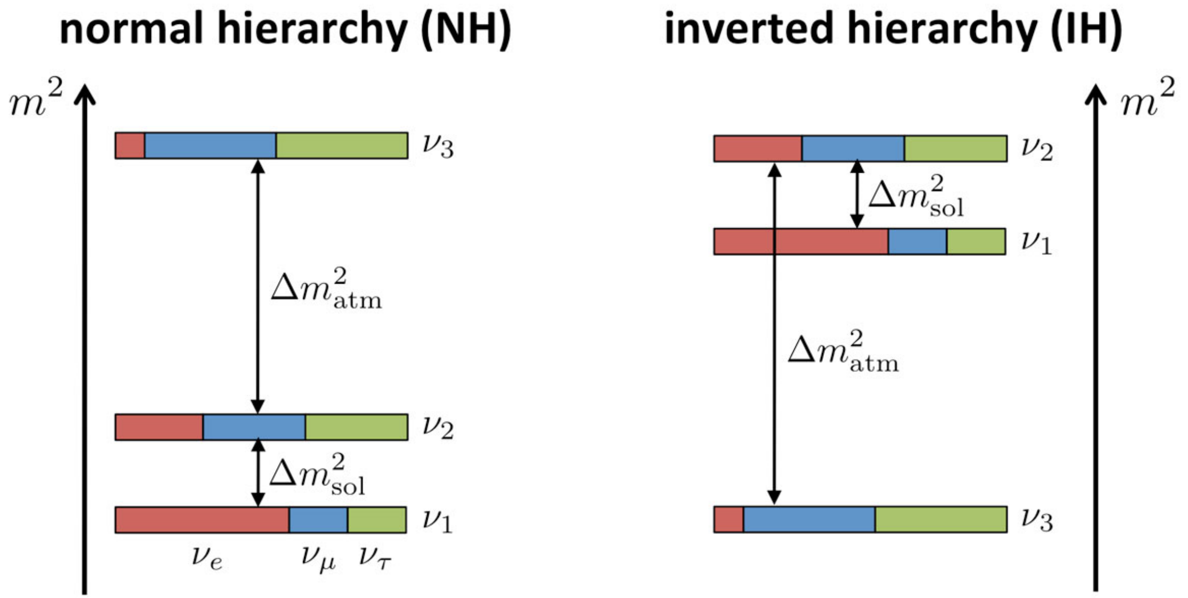
With 3 flavour oscillations, which we now need to consider, things get more complex (no pun intended) but remain familiar.

Rotate from flavour to mass states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



Now we have three states, we aren't sure what order they are in.



Warning, maths incoming.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{bmatrix}.$$

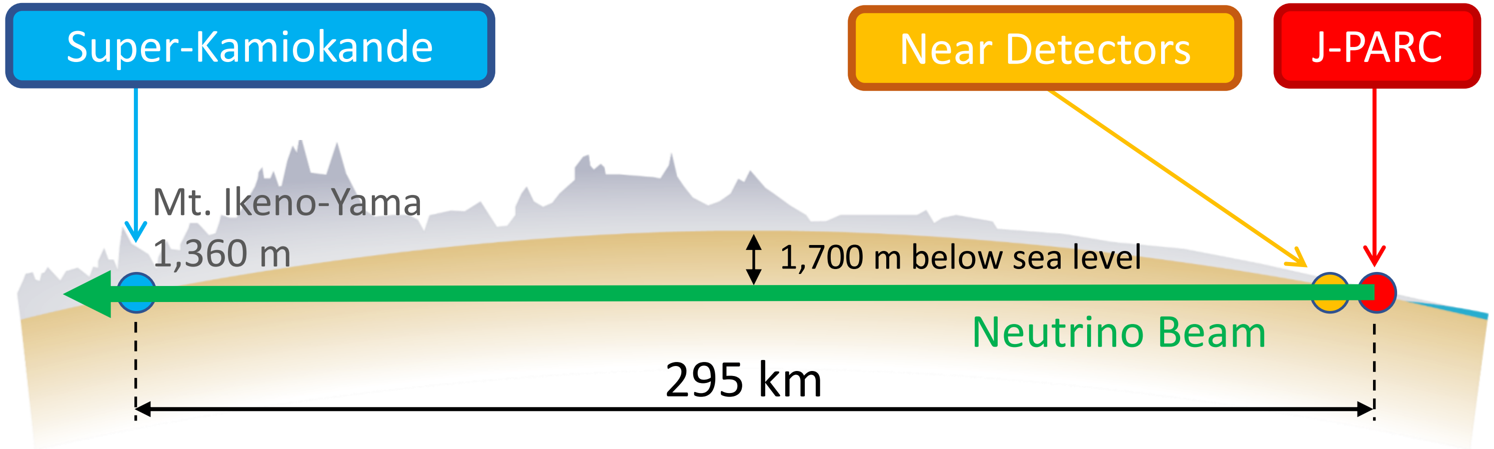
$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right) + 2\sum_{i>j}\mathrm{Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right),$$

- There are now 2 mass differences, the large and the small; Δm^2_{32} & Δm^2_{21}. Note that because of the squared mass terms, the signs are not detectable without extra effects.
  - ❖ $\Delta m^2_{21} = 7.53 \times 10^{-5} eV^2$
  - ❖ $\Delta m^2_{32} \approx \Delta m^2_{13} = 2.45 \times 10^{-3} eV^2$
- There are 3 mixing angles; θ_{12}, θ_{23}, and θ_{13}. These values (shown below in terms of $\sin^2(\theta)$) are large, especially θ_{23} which is nearly maximal.
  - ❖ $\sin^2(\theta_{12}) = 0.307$
  - ❖ $\sin^2(\theta_{23}) = 0.547$
  - ❖ $\sin^2(\theta_{13}) = 0.0220$

The value for θ_{13} was found to be surprisingly large (though still smaller than the others) and opened up a whole new possibility; measuring the CP-violating phase factor, δ_{CP}.

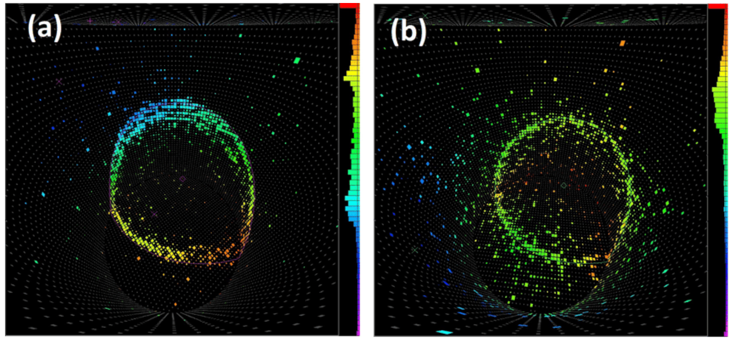
Parameter values taken from PDG interactive listings, shown without error
https://pdglive.lbl.gov/Particle.action?node=S067&init=0

T2K (Tokai to Kamioka) is a long-baseline neutrino experiment that utilises multiple detectors in the goal of measuring the properties of neutrinos and their oscillations. This is a ‘precision era’ of neutrino physics.



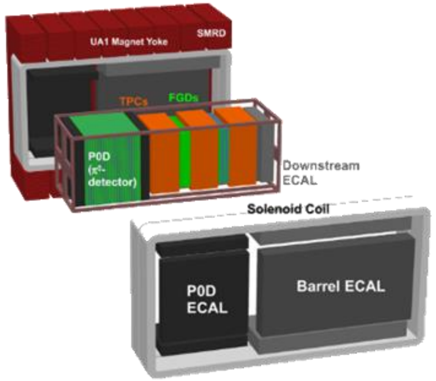
Super-K

- Off-axis water-based Cerenkov far detector.
- Topology based PID.
- CCQE dominant interactions.



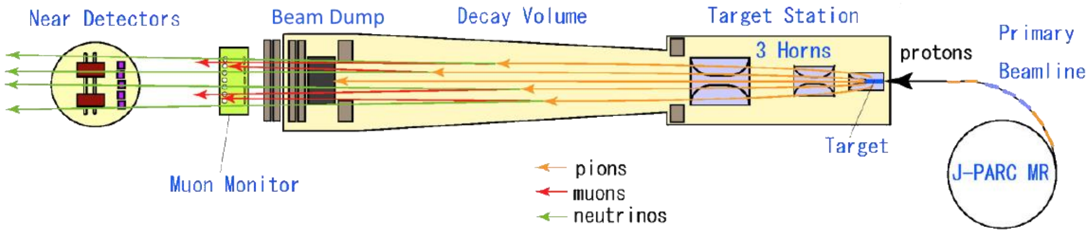
Cerenkov rings detected at SK. (a) is a muon event, (b) is an electron event.

ND280

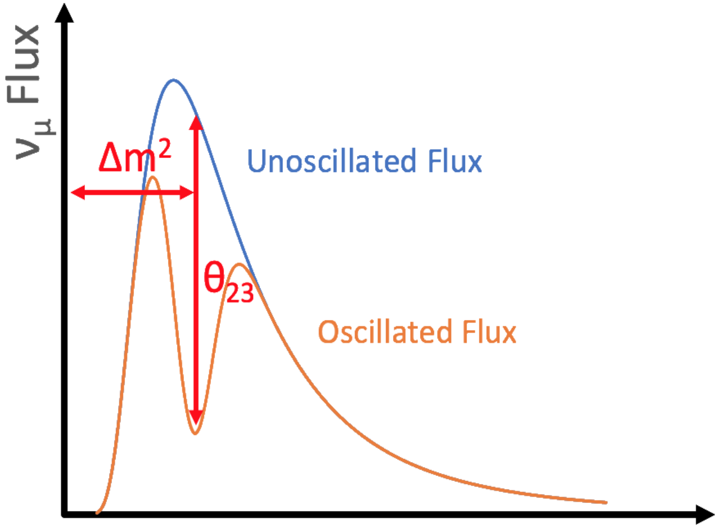


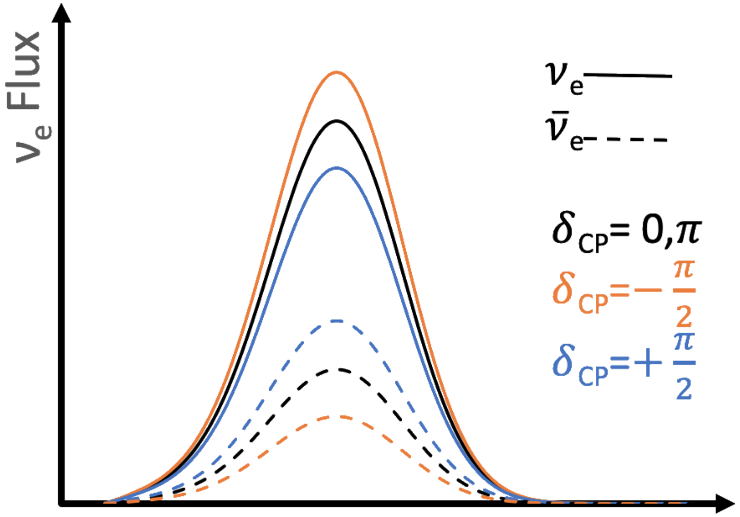
- Magnetised composite near detector.
- Off-axis (replicates SK energy spectra).
- Constrains flux and cross-section uncertainties.

J-PARC



- ‘Off axis’ beam tuned to 0.6 GeV for oscillation max at SK.
- Produces pure $\nu_\mu/\bar{\nu}_\mu$ flux.
- Able to be run in ν or $\bar{\nu}$ mode.





CP violation would mean: $P(\nu_\alpha \rightarrow \nu_\beta) \neq P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$

The magnitude of CP effect is given by the Jarlskog Invariant:

$$J_{CP}^{PMNS} = \frac{1}{8}\, sin2\theta_{12}\; sin2\theta_{13}\; sin2\theta_{23}\; cos\theta_{13}\; sin\delta_{CP}$$

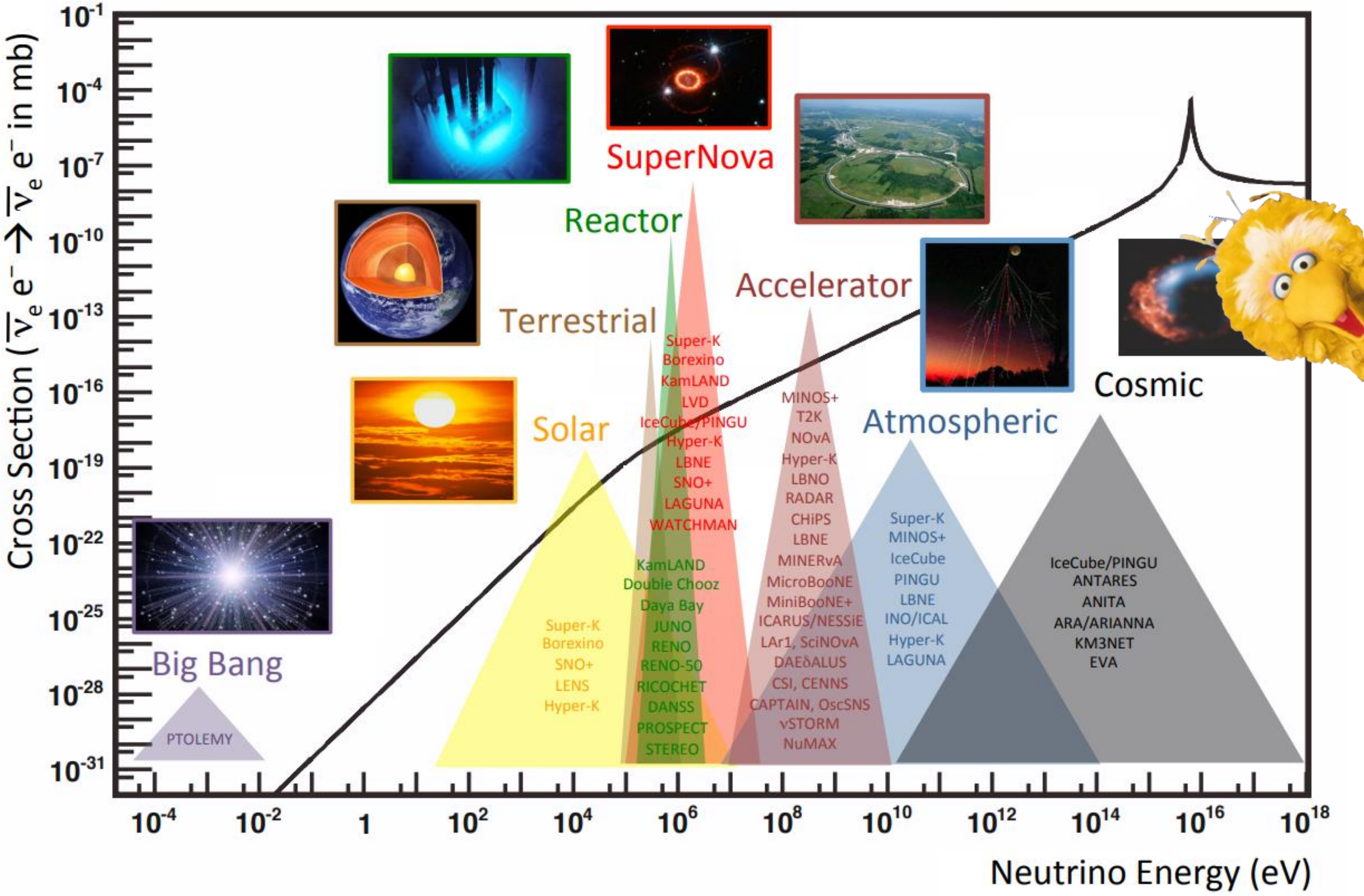
$$J_{CP}^{PMNS} = 0.035\; sin\delta_{CP}$$

$$J_{CP}^{CKM} \approx (3 \pm 1) \times 10^{-5}$$

Indications that the value of δ_{CP} is near maximal, and thus leptonic CP violation is large. This could explain the matter-antimatter asymmetry!

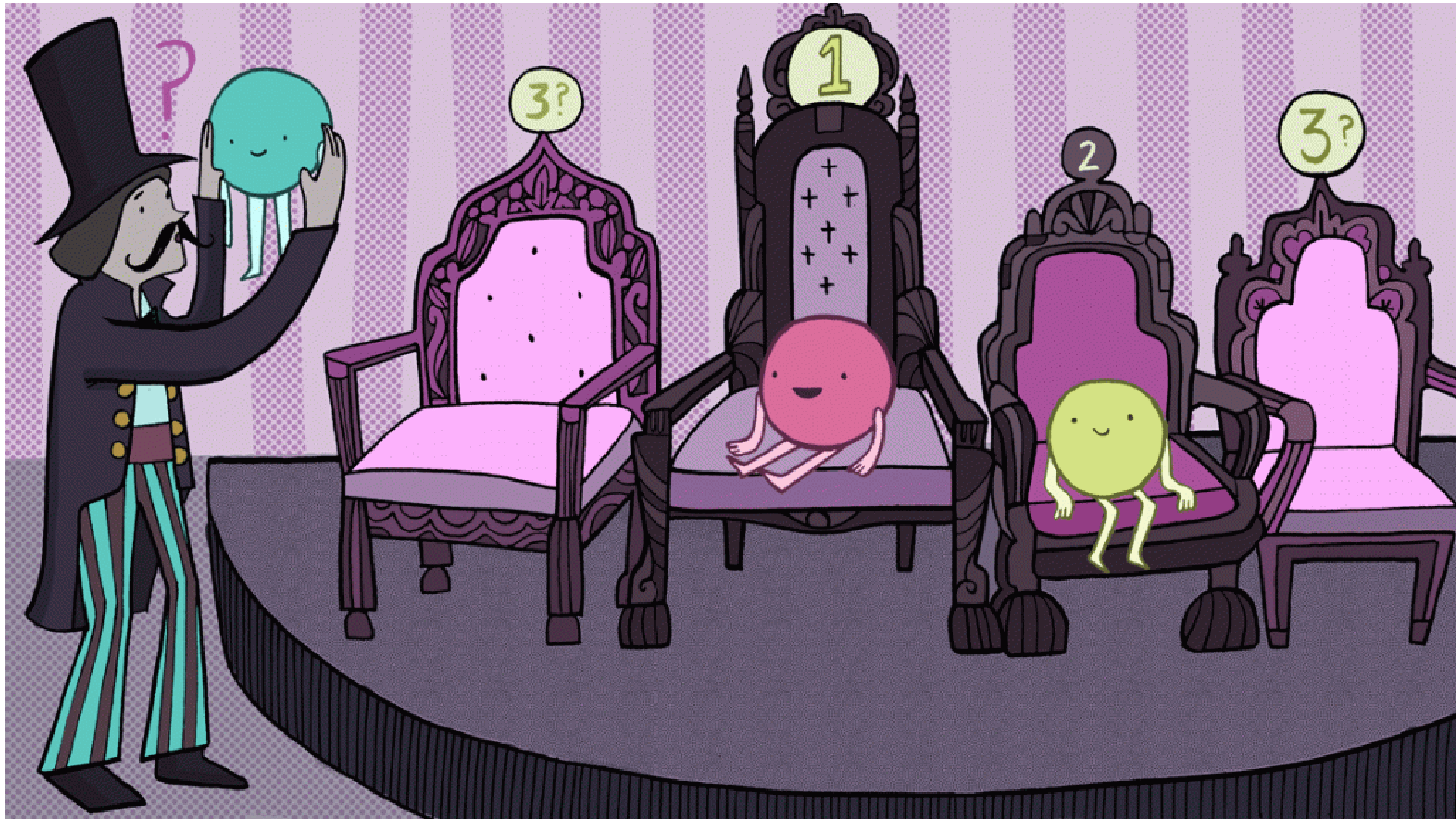
The highest energy neutrino confirmed was dubbed 'Big Bird' at about $\sim 10^{15}$ PeV. Before this was Bert and Ernie.

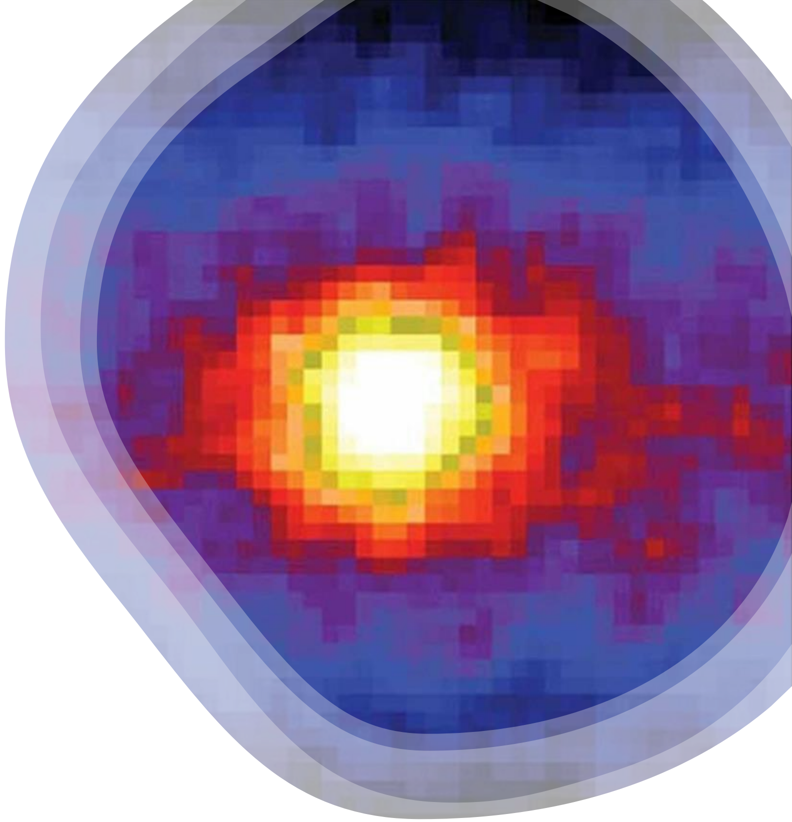
Our neutrino detection capabilities span an incredible range of energies, with many neutrino experiments trying to achieve many different measurements.



Other open questions

- Neutrinos have mass, where does this come from? (Neutrinoless double beta decay)
- Which order are these masses in? This is called neutrino mass ‘hierarchy’. (Matter effects)
- Is there another neutrino which does not interact like the rest? (Short baseline experiments)
    - Anomalies detected, anomalies ruled out.
- Is θ_{23} maximal or which side of $\frac{\pi}{4}$ does it lie.
- What is a neutrino’s speed? Looking at you OPERA.





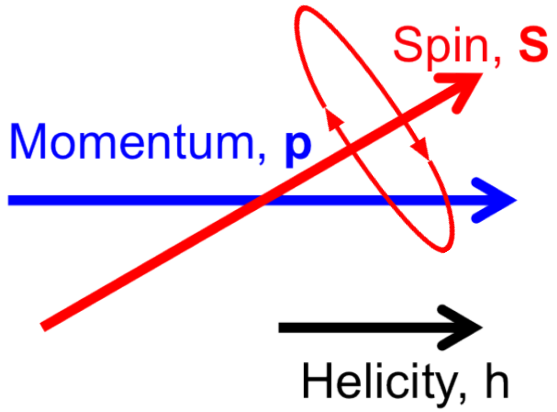
Neutrino Astronomy

Questions?

Is it not a strange fate that we should suffer so much fear and doubt for so small a thing?- Boromir

BACKUP

Helicity is defined as the projection of the spin on to the direction of the particle's momentum.



$$h = \frac{\sigma \cdot \mathbf{p}}{|p|}$$

Right helicity = spin aligned with direction of momentum. $\vec{\sigma} \uparrow\uparrow \vec{p}$

Left helicity = spin anti-aligned with direction of momentum. $\vec{\sigma} \uparrow\downarrow \vec{p}$

Chirality is a bit more abstract, and is best thought about in terms of transformations (i.e. a reflection of a chiral object is not the same), but in the relativistic limit, helicity and chirality become equivalent.

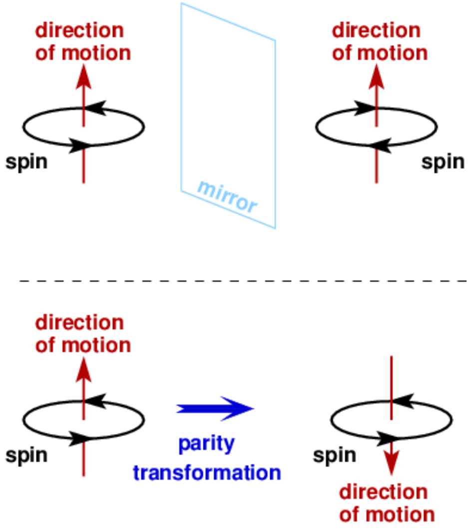
When a parity operation (flip coordinates) is performed on helicity:

Momentum: Polar vector, changes sign: **p** → -**p**
Spin: Axial vector, $\sigma \rightarrow \sigma$

Therefore helicity should always change under parity transformation… right?

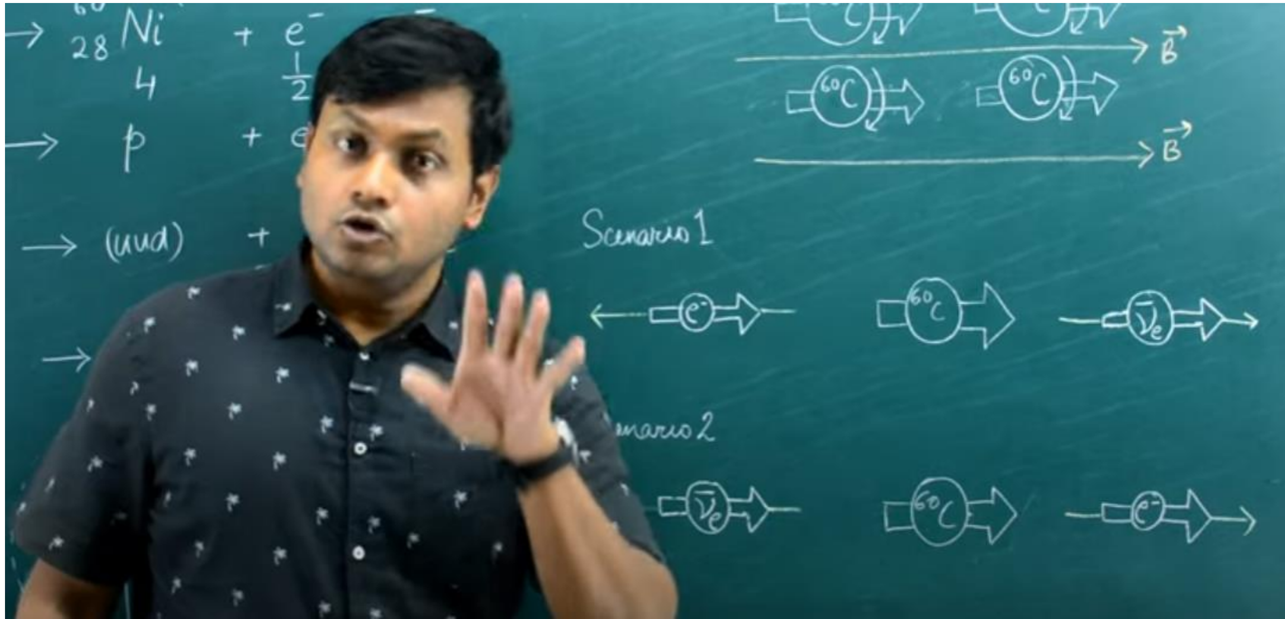
Time to get interactive

- Curl your hands!
- The direction of axial vector/ pseudo-vector follows a curling right hand.
  - The curl is the rotation (or similar in other examples), and in classical examples is formed of the combination (cross product) of two perpendicular polar (regular) vectors.
  - The thumb is the direction of the vector.

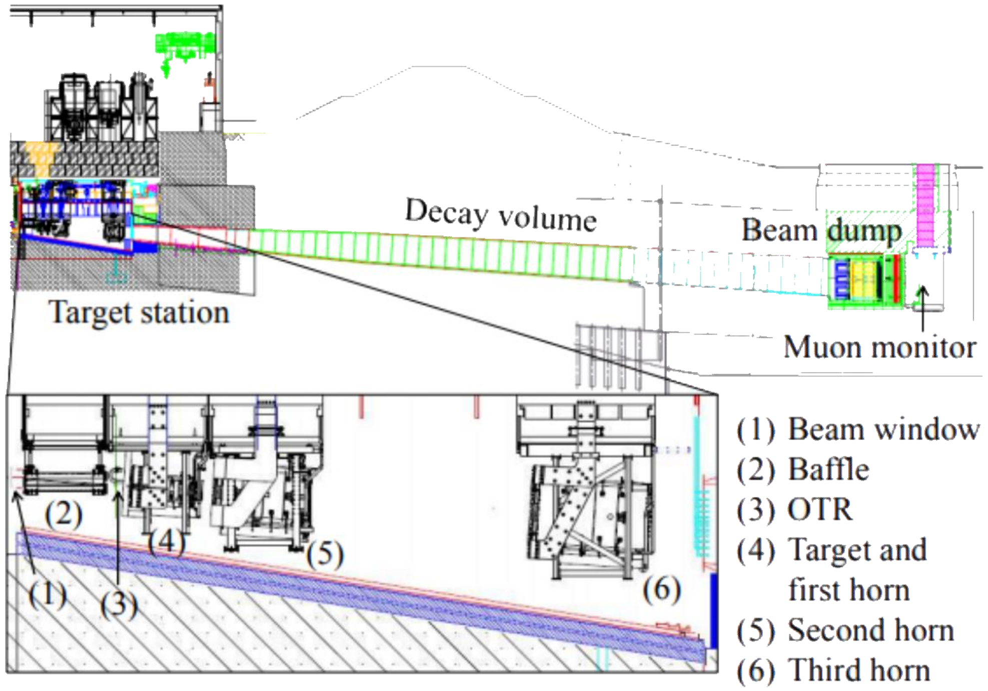


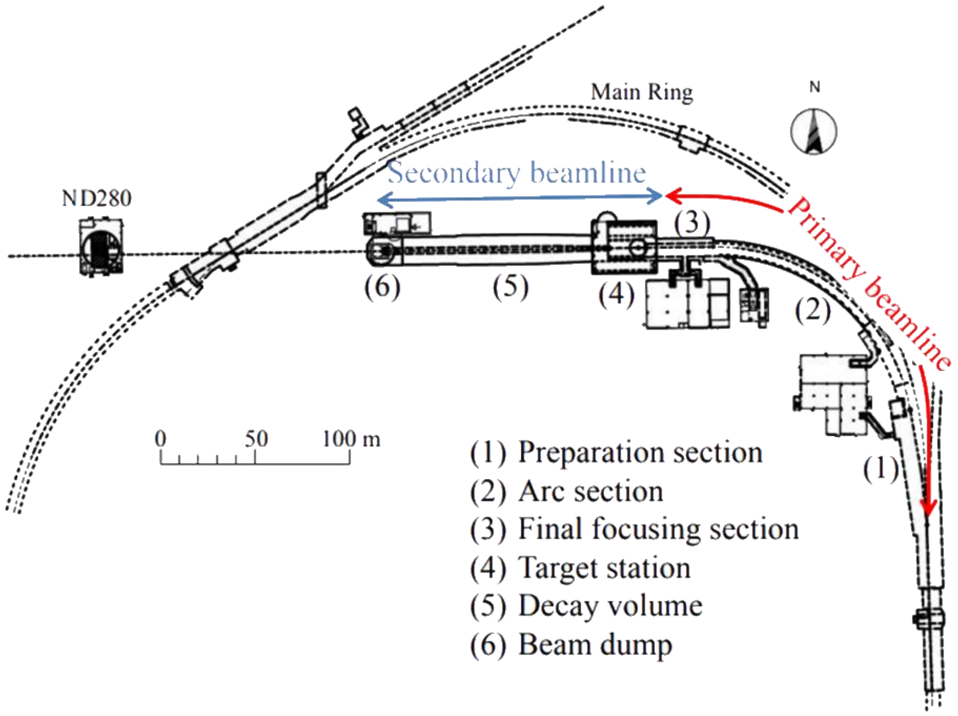
- We can also use this curling hand trick to visualise the ‘handedness’ of a particle, aligning or unaligning momentum with spin.

https://www.youtube.com/watch?v=v44jEXN4sSY



J-PARC beam production facility



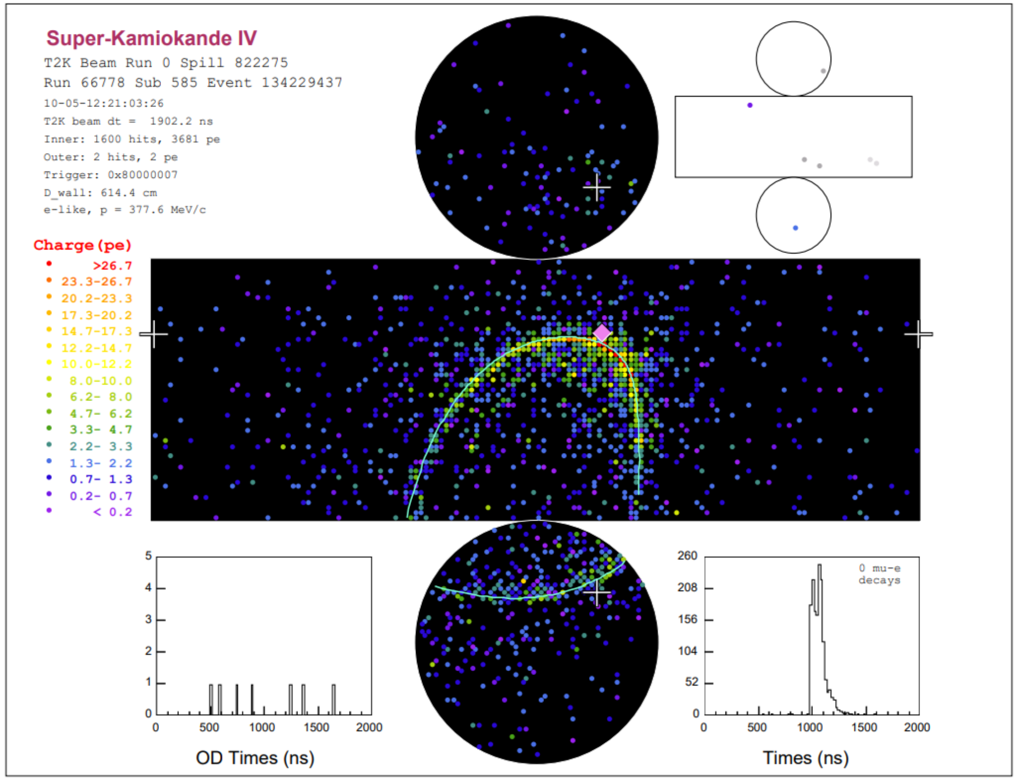


Multi-purpose beam production facility (meaning that not all the time is neutrino time ☹)
Utilises pion decay to produce almost pure flavour beam.
Able to run in ν or $\bar{\nu}$ mode by selection of pion charge. This is known as Forward or Reverse Horn Current.

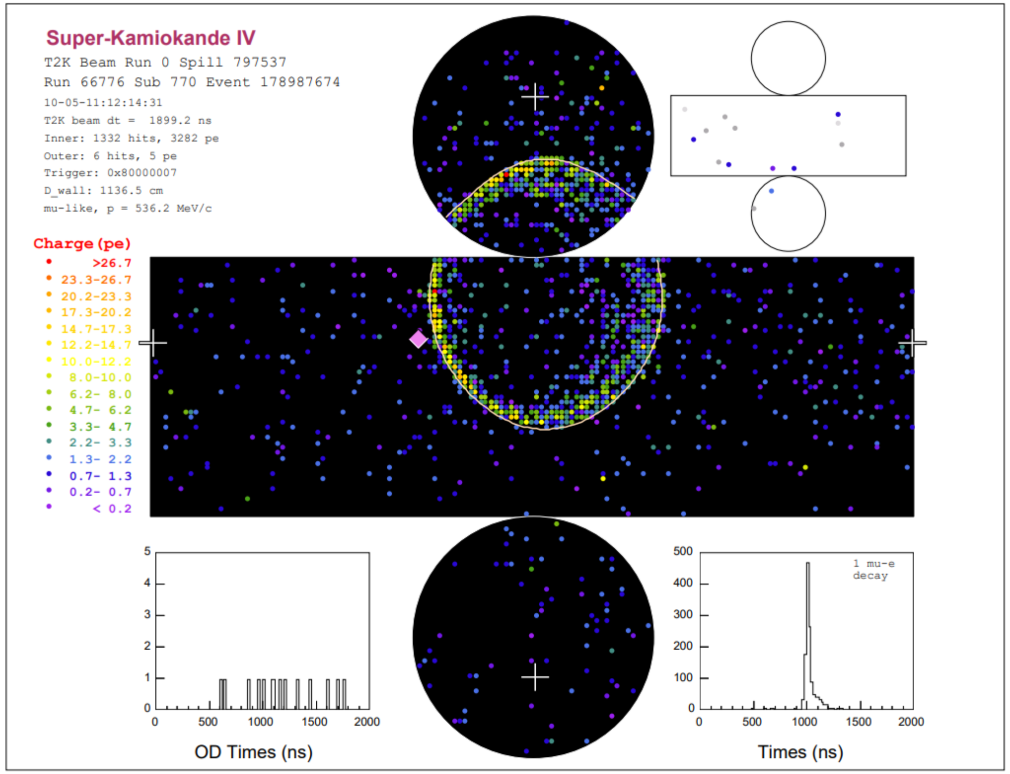
$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

More clear SK displays



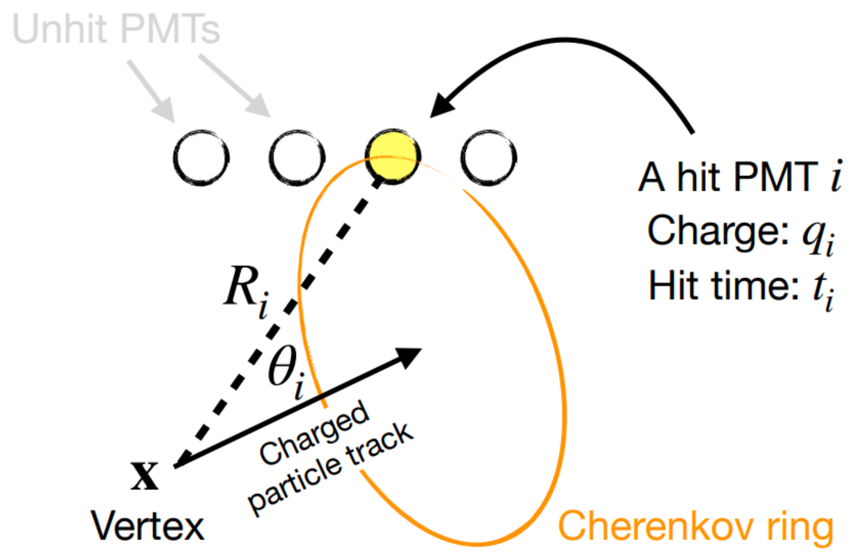
An electron neutrino candidate

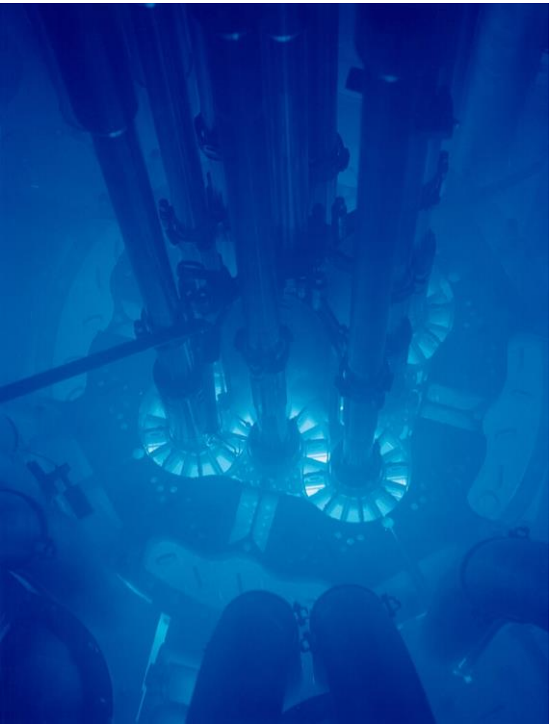


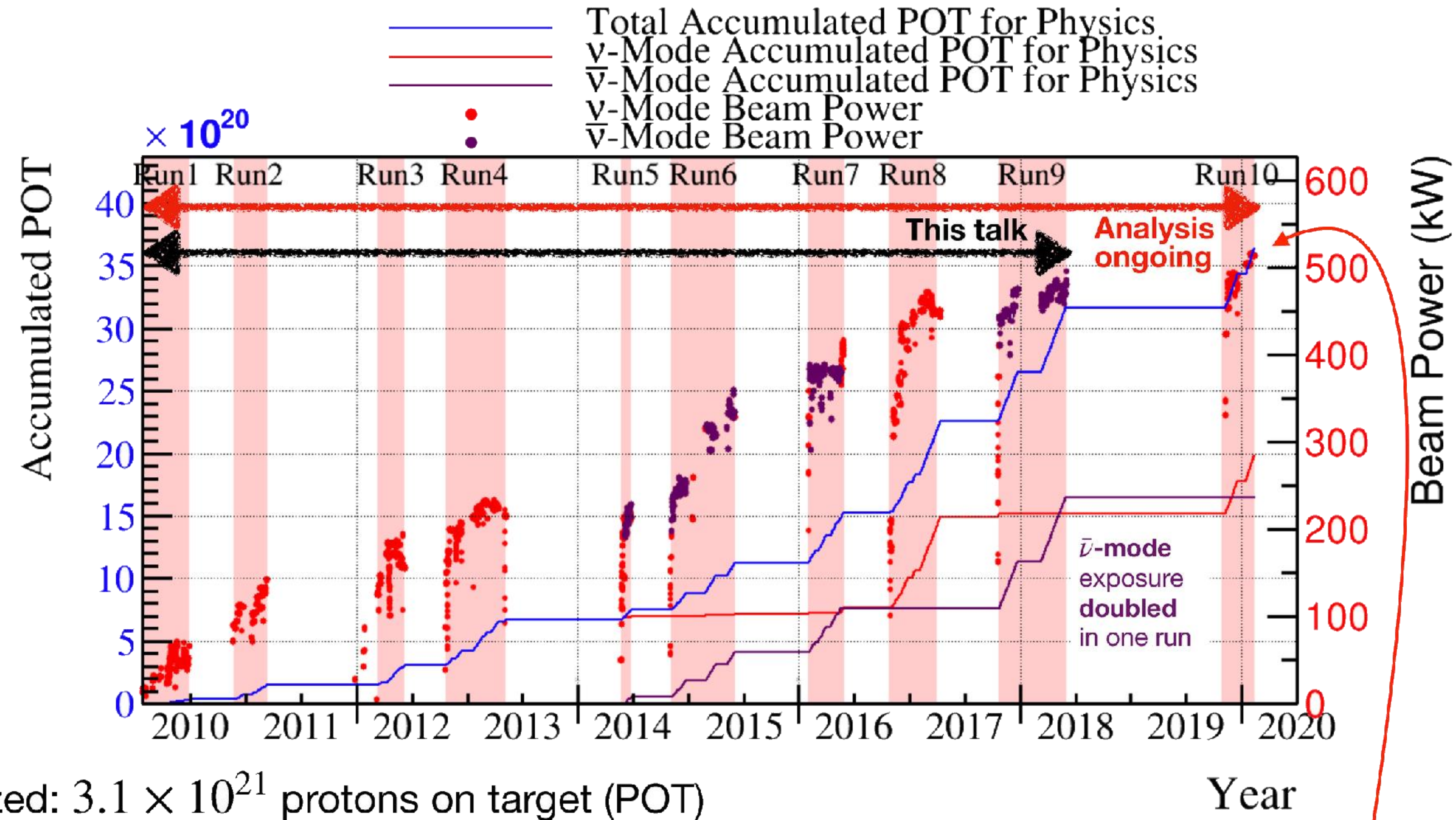
A muon neutrino candidate

Cherenkov Radiation

Blue glow resultant when a charged particle passes through a polarisable medium at ultra-relativistic speeds.







Analyzed: 3.1×10^{21} protons on target (POT)

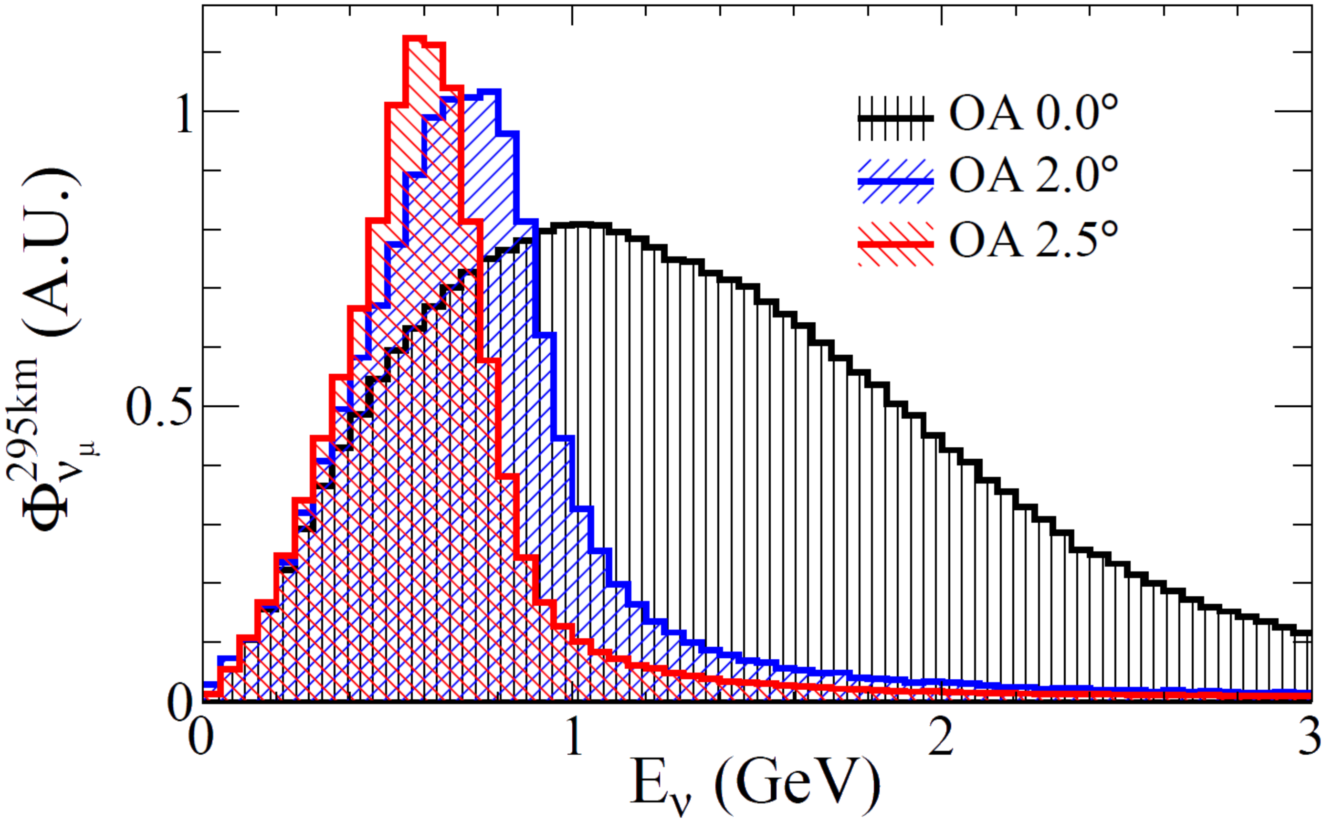
ν-mode : $\bar{\nu}$-mode ~ 50 : 50

515 kW operation achieved recently!

33% increase of ν-mode data in upcoming analysis.

$$E = \frac{E^*}{\gamma(1 - \beta\cos\theta)}$$

E* is neutrino energy in centre of mass frame of the decaying meson.
γ is the neutrino's Lorentz factor.
β is the neutrino's Lorentz velocity.
θ is the neutrino's angle in the lab frame.



T2K is the first experiment in which the off-axis concept was implemented. This decreases the amount of neutrinos at high energies (decreasing more complicated interaction types and also tightening flux around osc max).

ND280 detector suite

- Same off-axis angle as Super-K (2.5 degrees).
- Measures ν_μ and ν_e spectrum before the oscillation → TPCs + FGDs
- Measure background processes to oscillation (NCπ0, NC1π, CC1π…)
- Compare Carbon and Oxygen interactions (FGD2 and P0D)

ND280 installed in ex-UA1 magnet (0.2 T) 3.5x3.6x7.3 m

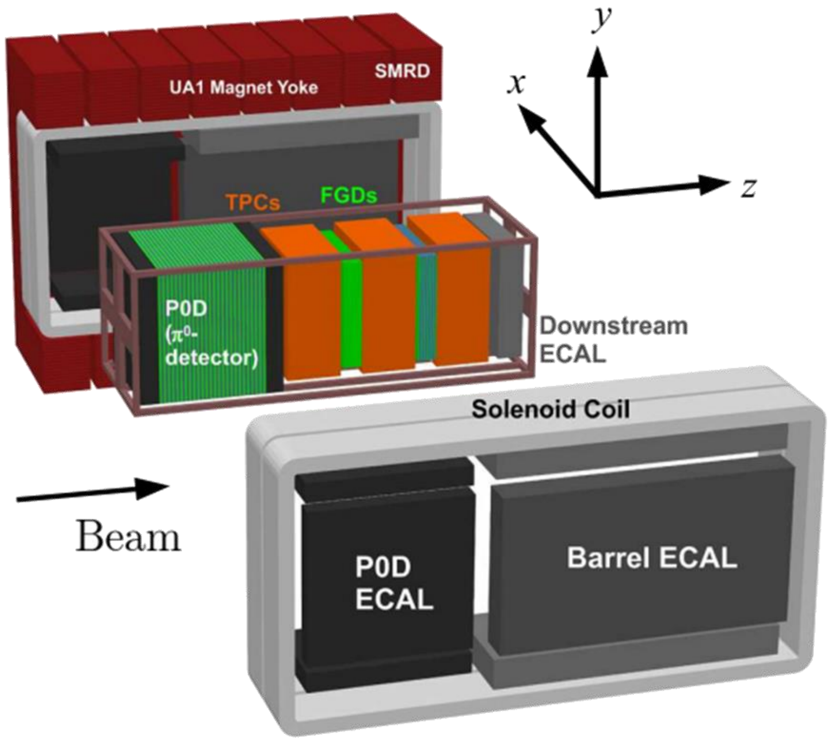
SMRD (Side Muon Range Detector): scintillator planes in magnet yokes. Measure high angle muons

2 FGDs (Fine Grained Detector): active target mass for the tracker, optimized for p/π separation Carbon+Water target in FGD2

P0D (π0 detector): scintillator bars interleaved with fillable water target bags and lead and brass sheets. Optimised for γ detection

3 TPCs (Time Projection Chambers): measure momentum and charge of particles from FGD and P0D, PID capabilities through dE/dx

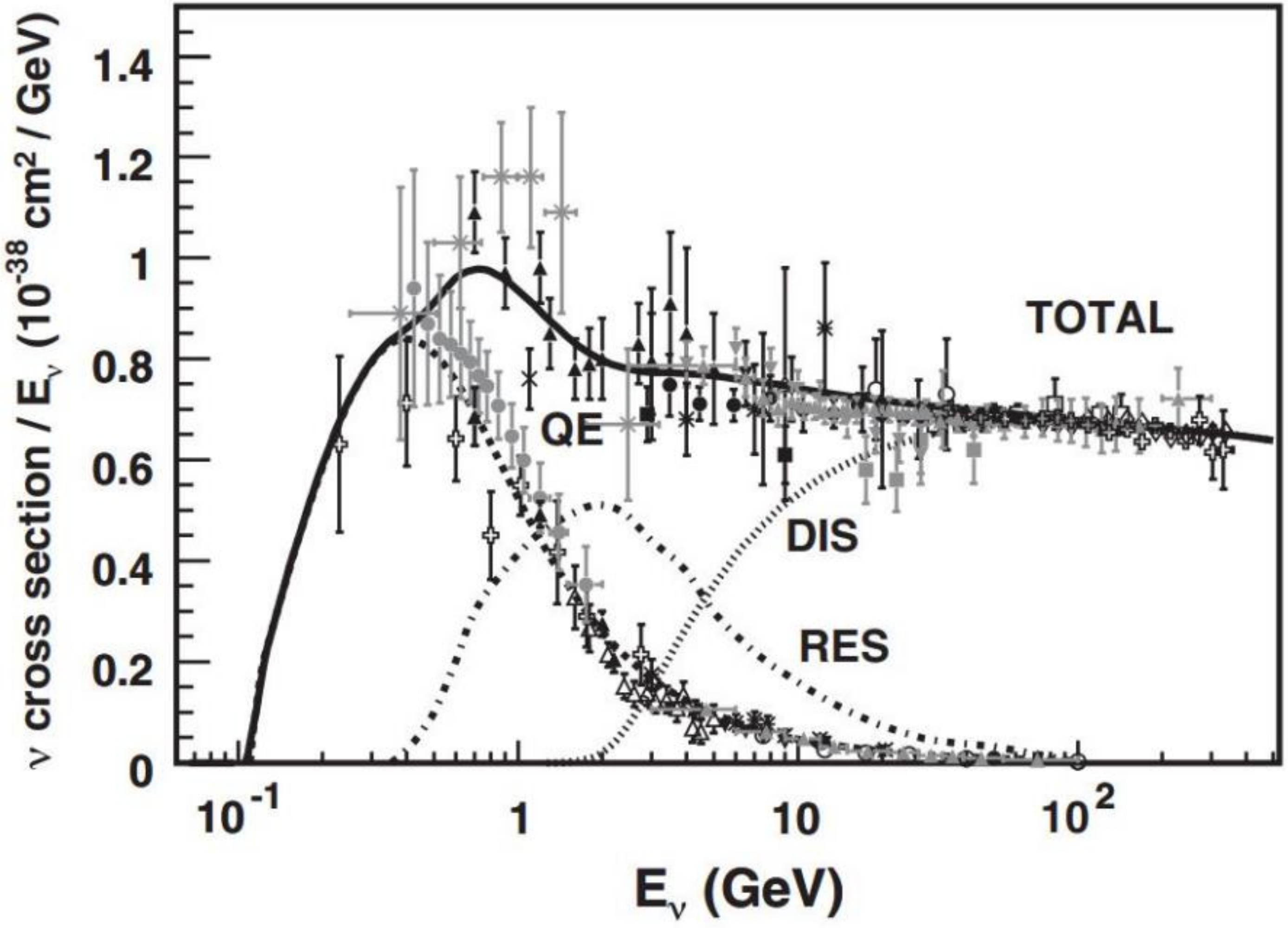
P0D, Barrel and Downstream ECAL: scintillator planes with radiator to measure EM showers



Neutrinos in cosmology.
Neutrinos decoupled during the big bang at ~ 2seconds.
Neutrinos carry away 100 times the energy of photons from core collapse supernovae.

Cross section between any two particles is the area transverse to their relative motion within which they must meet in order to interact (effective size).

There are no hard spheres and cross section depends on many things, we mostly deal with cross section in terms of energy.



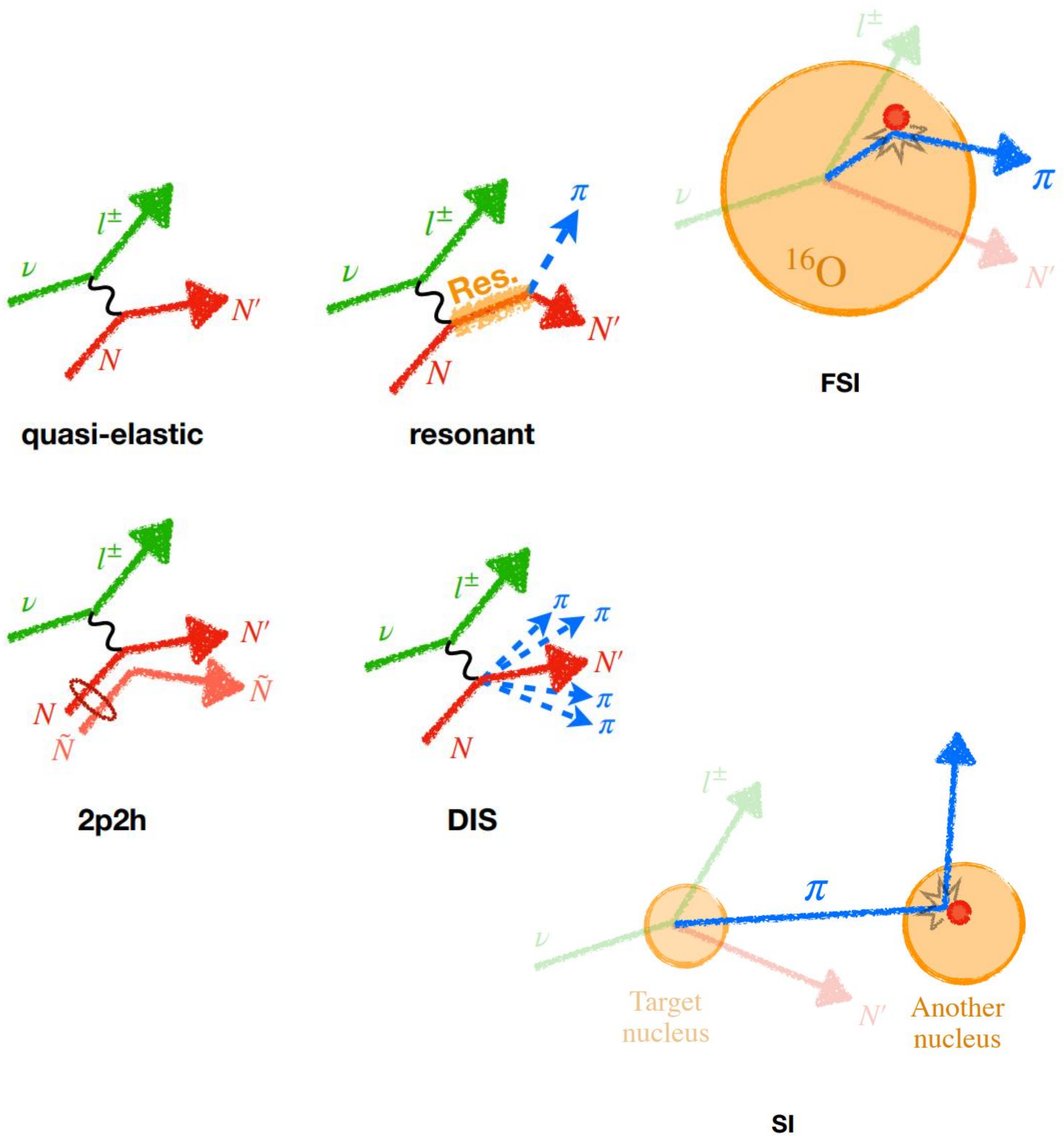
Phase Space

As neutrino energy increases, there is more ‘room’ available to create more particles, and have increasingly inelastic collisions.

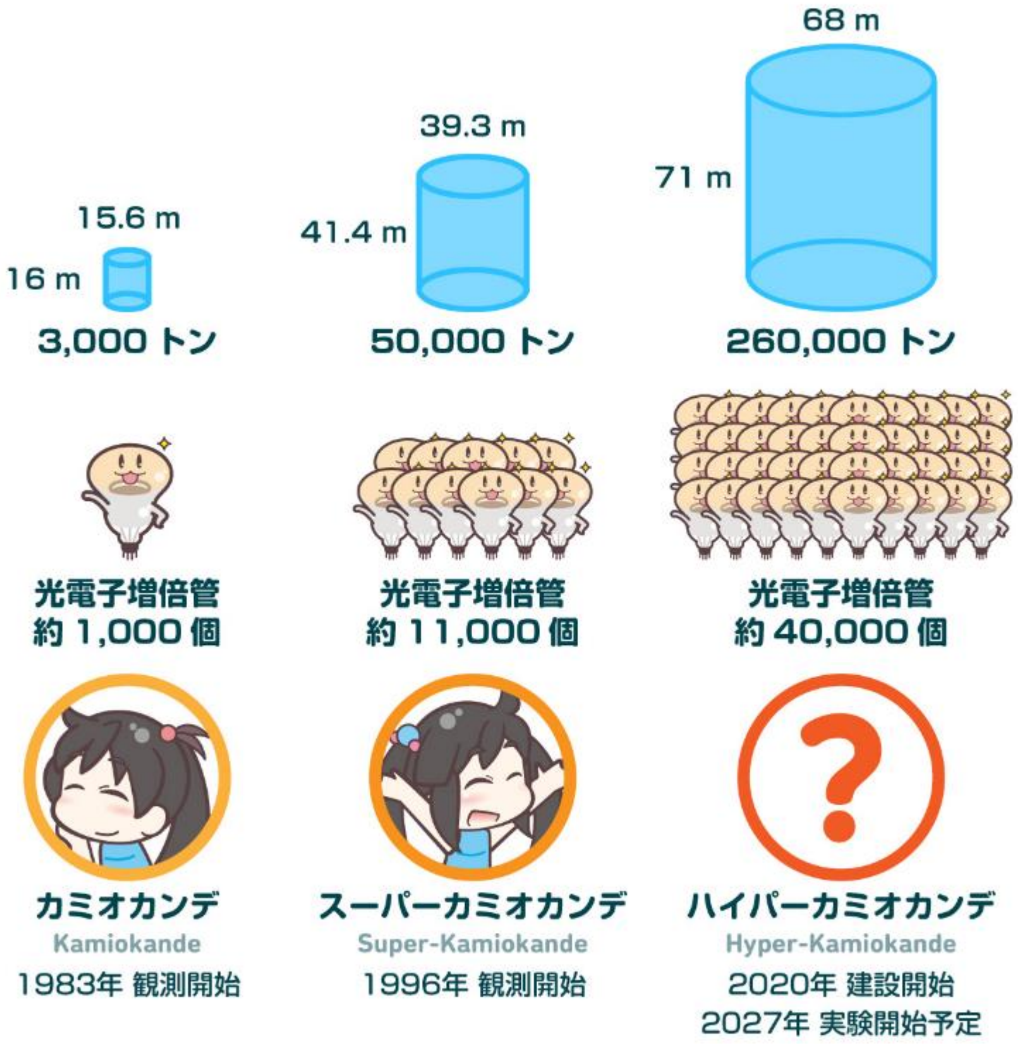
Things get messy, as things always do with hadrons, and new signals get introduced, with charged pions often looking very similar to muons in our detector.

Deep Inelastic Scattering processes occur when neutrinos resolve the quarks themselves instead of the nucleon as a whole and create large showers.

Finally extra nuclear effects come into play, where pions can re-interact within a nucleus or with another nearby nucleus, and that signal gets lost or changed.



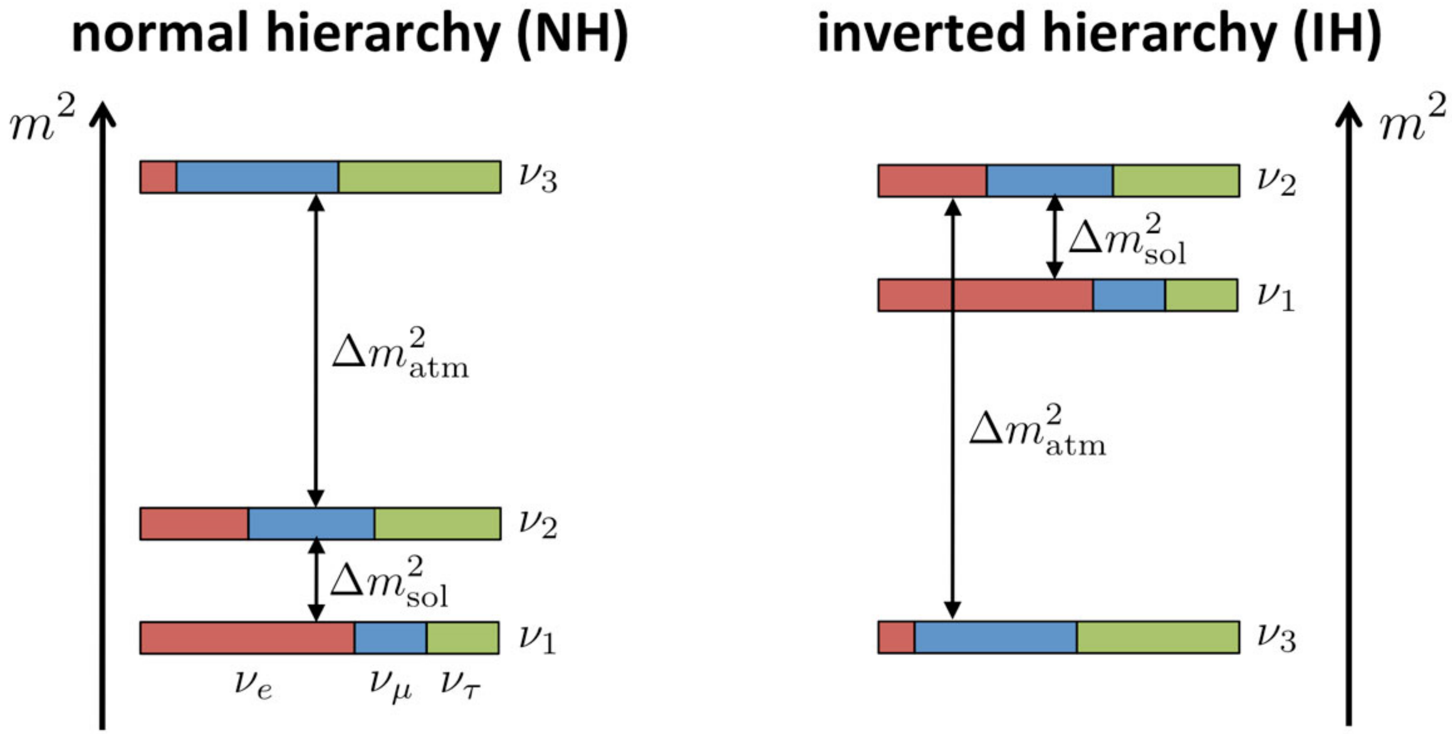
Hyper-Kamiokande



With these detections, the community was sure that neutrino flavour oscillations were occurring! However this realisation came with a troubling conclusion. Neutrinos had mass.

Well, kind of. What they actually required in theory was a mass difference between three, set, well defined states. And then each set neutrino flavour must be a transformation from these states, each flavour ending up as a mix of the three masses. This transformation is given by a rotation (contained in a matrix) from the mass states to the flavour states (or visa versa).

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$



Warning, maths incoming.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\rm CP}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\rm CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\rm CP}} & c_{23}c_{13} \end{bmatrix}.$$

$$P_{\alpha\to\beta} = \delta_{\alpha\beta} - 4\sum_{i>j} {\rm Re}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin^2\left(\frac{\Delta m^2_{ij}L}{4E}\right) + 2\sum_{i>j} {\rm Im}\left(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\right)\sin\left(\frac{\Delta m^2_{ij}L}{2E}\right),$$

With 3 flavour oscillations, things get more complex (no pun intended), but remain familiar;

- There are now 2 mass differences, the large and the small; Δm^2_{32} & Δm^2_{21}. Note that because of the squared mass terms, the signs are not detectable without extra effects.
  - ❖ $\Delta m^2_{21} = 7.53 \times 10^{-5} eV^2$
  - ❖ $\Delta m^2_{32} \approx \Delta m^2_{13} = 2.45 \times 10^{-3} eV^2$
- There are 3 mixing angles; θ_{12}, θ_{23}, and θ_{13}. These values (shown below in terms of $\sin^2(\theta)$) are large, especially θ_{23} which is nearly maximal.
  - ❖ $\sin^2(\theta_{12}) = 0.307$
  - ❖ $\sin^2(\theta_{23}) = 0.547$
  - ❖ $\sin^2(\theta_{13}) = 0.0220$

The value for θ_{13} was found to be surprisingly large (though still smaller than the others) and opened up a whole new possibility; measuring the CP-violating phase factor, δ_{CP}.

Parameter values taken from PDG interactive listings, shown without error
https://pdglive.lbl.gov/Particle.action?node=S067&init=0