Status of the $(g-2)_\mu$ puzzle

Gilberto Colangelo

UNIVERSITÄT BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Muon Precision Physics Workshop 2024
Liverpool, November 12, 2024

Outline

Introduction: $(g-2)_\mu$ in the Standard Model

Hadronic light-by-light

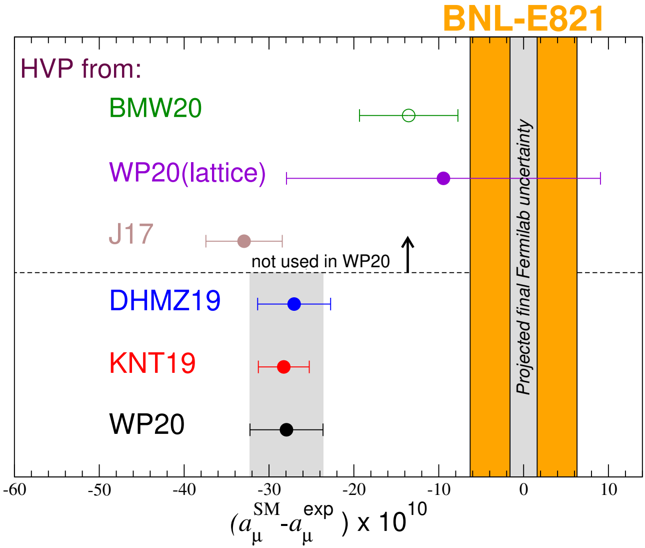
Hadronic Vacuum Polarization contribution
- Data-driven approach
- Data-driven vs. Lattice
- CMD3 measurement of $e^+e^- \to \pi^+\pi^-$
- Relevance of radiative corrections?

Conclusions and Outlook

Outline

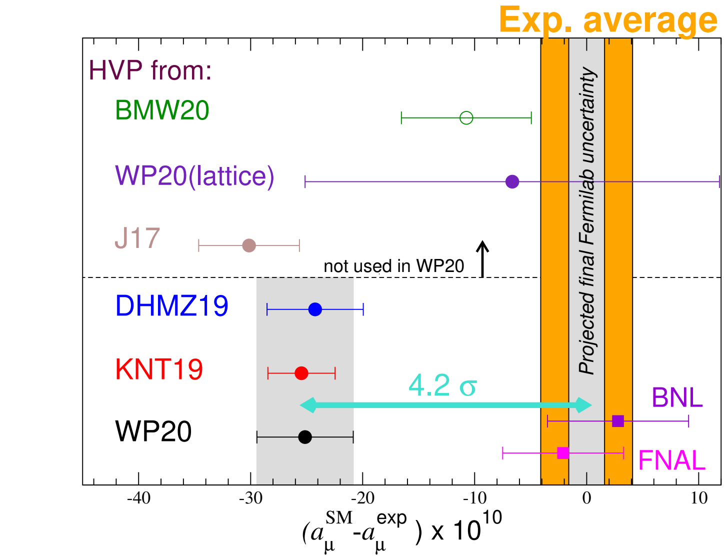
Present status of $(g-2)_\mu$: experiment vs SM

Before



Present status of $(g-2)_\mu$: experiment vs SM

After the 2021 Fermilab result



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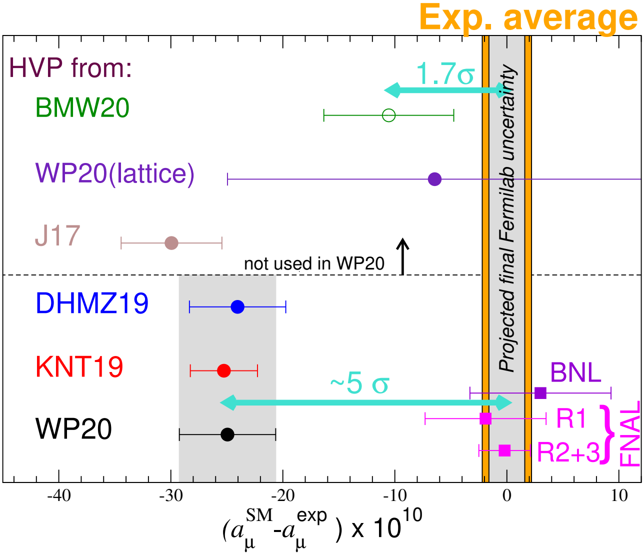
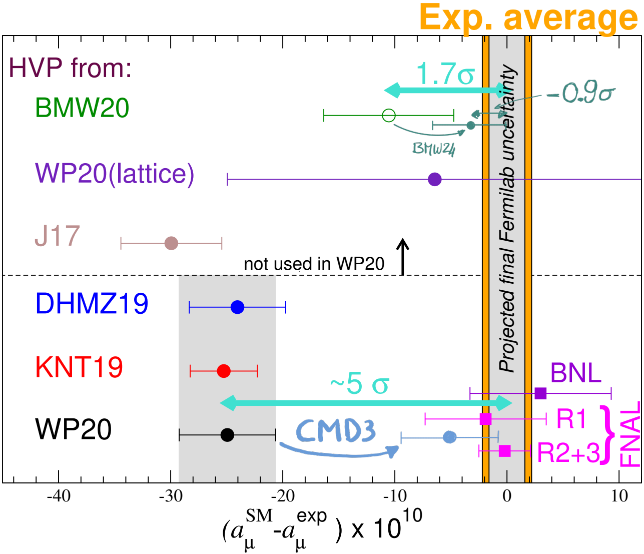


Present status of $(g-2)_\mu$: experiment vs SM

After the 2023 Fermilab result and CMD3 and BMW-24



White Paper (2020): $(g-2)_\mu$, experiment vs SM

Contribution	Value $\times 10^{11}$
HVP LO (e^+e^-)	6931(40)
HVP NLO (e^+e^-)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)
HVP LO (lattice, *udsc*)	7116(184)
HLbL (phenomenology)	92(19)
HLbL NLO (phenomenology)	2(1)
HLbL (lattice, *uds*)	79(35)
HLbL (phenomenology + lattice)	90(17)
QED	116 584 718.931(104)
Electroweak	153.6(1.0)
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)
HLbL (phenomenology + lattice + NLO)	92(18)
Total SM Value	116 591 810(43)
Experiment	116 592 059(22)
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	249(48)

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White Paper:
T. Aoyama et al. Phys. Rep. 887 (2020) = WP(20)

Muon $g-2$ Theory Initiative
Steering Committee:
GC
Michel Davier (vice-chair)
Aida El-Khadra (chair)
Martin Hoferichter
Laurent Lellouch
Christoph Lehner (vice-chair)
Tsutomu Mibe (J-PARC E34 experiment)
Lee Roberts (Fermilab E989 experiment)
Thomas Teubner
Hartmut Wittig

White Paper (2020): $(g-2)_\mu$, experiment vs SM

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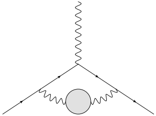
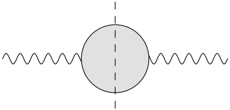
Muon $g-2$ Theory Initiative
Plenary Workshops:

- 1st, Q-Center (Fermilab), 3-6 June 2017
- 2nd, Mainz, 18-22 June 2018
- 3rd, Seattle, 9-13 September 2019
- 4th, KEK (virtual), 28 June-02 July 2021
- 5th, Higgs Center Edinburgh, 5-9 Sept. 2022
- 6th, Bern, 4-8 Sept. 2023
- 7th, KEK, 9-13 Sept. 2024

Theory uncertainty comes from hadronic physics

- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$





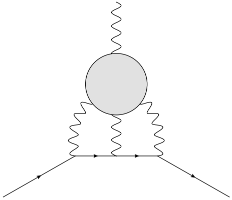
- unitarity and analyticity $\Rightarrow$ dispersive approach
- $\Rightarrow$ direct relation to experiment: $\sigma_{\text{tot}}(e^+e^- \to \text{hadrons})$
- e^+e^- Exps: BaBar, Belle, BESIII, CMD2/3, KLOE2, SND
- alternative approach: lattice, now competitive

(BMW, ETMC, Fermilab, HPQCD, Mainz, MILC, RBC/UKQCD)

$\rightarrow$ talk by D. Giusti

Theory uncertainty comes from hadronic physics

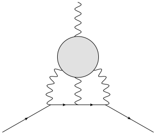
- Hadronic contributions responsible for most of the theory uncertainty
- Hadronic vacuum polarization (HVP) is $\mathcal{O}(\alpha^2)$, dominates the total uncertainty, despite being known to $< 1\%$
- Hadronic light-by-light (HLbL) is $\mathcal{O}(\alpha^3)$, known to $\sim 20\%$, second largest uncertainty (now subdominant)



- earlier: model-based—uncertainties difficult to quantify
- recently: dispersive approach $\Rightarrow$ data-driven, systematic treatment
- more recently: lattice QCD also competitive (Mainz, RBC/UKQCD)

Outline

HLbL contribution: Master Formula



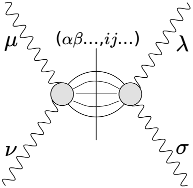
$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{48\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

Q_i^μ are the Wick-rotated four-momenta and τ the four-dimensional angle between Euclidean momenta: $Q_1 \cdot Q_2 = |Q_1||Q_2|\tau$
The integration variables $Q_1 := |Q_1|$, $Q_2 := |Q_2|$.

GC, Hoferichter, Procura, Stoffer (15)

- T_i: known kernel functions
- $\bar{\Pi}_i$ are amenable to a dispersive treatment:
  imaginary parts are related to measurable subprocesses

“Amenable to a dispersive treatment”



+ crossed diagrams

$$\mathrm{Im}\,\Pi^{\mu\nu\lambda\sigma} = \sum_{\alpha\beta\ldots,ij\ldots} \Gamma^{\mu\nu\alpha\beta\ldots}_{ij\ldots} \Gamma^{\lambda\sigma\alpha\beta\ldots\,\star}_{ij\ldots}$$

- projection on the BTT basis for $\Pi^{\mu\nu\lambda\sigma}$ $\Rightarrow$ DR for Π_i
- result for $\Pi^{\mu\nu\lambda\sigma}$ (and a_μ) depends on the basis choice unless a set of sum rules is satisfied CHPS 17
- even for single-particle intermediate states this is in general not the case, other than for pseudoscalars

Improvements obtained with the dispersive approach

Contribution	PdRV(09) *Glasgow consensus*	N/JN(09)	J(17)	WP(20)
π^0, η, η'-poles	114(13)	99(16)	95.45(12.40)	93.8(4.0)
π, K-loops/boxes	−19(19)	−19(13)	−20(5)	−16.4(2)
S-wave $\pi\pi$ rescattering	−7(7)	−7(2)	−5.98(1.20)	−8(1)
subtotal	88(24)	73(21)	69.5(13.4)	69.4(4.1)
scalars	–	–	–	} − 1(3)
tensors	–	–	1.1(1)	
axial vectors	15(10)	22(5)	7.55(2.71)	6(6)
u, d, s-loops / short-distance	–	21(3)	20(4)	15(10)
c-loop	2.3	–	2.3(2)	3(1)
total	105(26)	116(39)	100.4(28.2)	92(19)

- significant reduction of uncertainties in the first three rows

  CHPS (17), Masjuan, Sánchez-Puertas (17) Hoferichter, Hoid et al. (18), Gerardin, Meyer, Nyffeler (19)

- resonances and short-distance constraints need to be improved

  Danilkin, Hoferichter, Stoffer (21), Lüdtke, Procura, Stoffer (23), Melnikov, Vainshtein (04), Nyffeler (09), Bijnens et al. (20,21), Cappiello et al. (20), Leutgeb, Rebhan (19,21)

Recent progress on HLbL

- Pseudoscalars:
  dispersive analysis for $\eta^{(\prime)}$ just completed

  Hoferichter, Hoid, Holz, Kubis, to appear
- Axials:
  - TFF analyzed in terms of VMD, including phenom. constraints Hoferichter, Kubis, Zanke '23
  - Optimized basis (no singularities, ok for pion box)

    Hoferichter, Stoffer, Zillinger '24 and t.a.
- Tensors: no proper basis for general kinematics
  $\Rightarrow$ dispersion relation for $g-2$ kinematics ($q_4 = 0$)

  Lüdtke, Procura, Stoffer '23
- SDC:
  - complete analysis in QCD at NLO in all regimes (Melnikov-Vainshtein and beyond)

    Bijnens, Hermansson-Truedsson, Rodríguez-Sánchez, '23 and t.a.
  - hQCD models have been further refined (axial-vector contrib. $\gtrsim$ than in WP) Leutgeb, Mager, Rebhan '23, and t.a.

Updates on HLbL from $(g-2)_7$ @KEK 2024

Pole Contributions

Results

$$a_\mu^{\eta-\text{pole}} \times 10^{11} = 14.64\,(32)_{\text{disp.}}\,(56)_{\text{norm}}\,(23)_{\text{BL}}\,(35)_{\text{asym.}}\,[77]_{\text{tot.}}$$

$$a_\mu^{\eta'-\text{pole}} \times 10^{11} = 13.44\,(15)_{\text{disp.}}\,(48)_{\text{norm}}\,(13)_{\text{BL}}\,(48)_{\text{asym.}}\,[70]_{\text{tot.}}$$

rel. err of $a_\mu^{\eta^{(\prime)}-\text{pole}}$	disp.	norm	BL	asym.	total
η / %	2.2	3.8	1.6	2.4	5.3
η' / %	1.1	3.6	1.0	3.6	5.2

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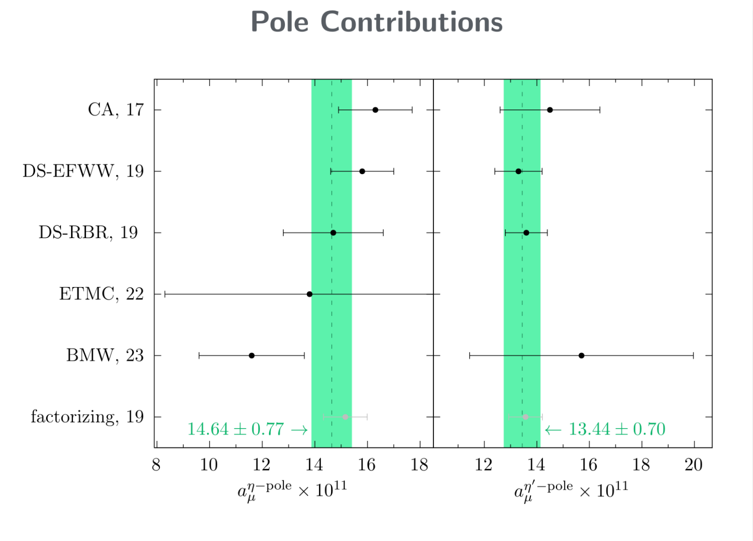
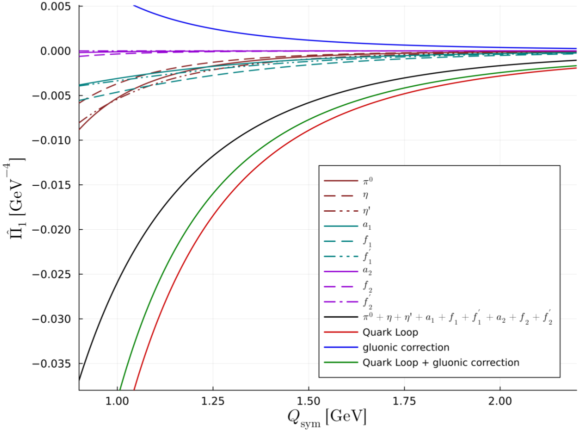


Updates on HLbL from $(g-2)_7$ @KEK 2024

Comparison of OPE expressions and hadronic states for $\hat{\Pi}_1$

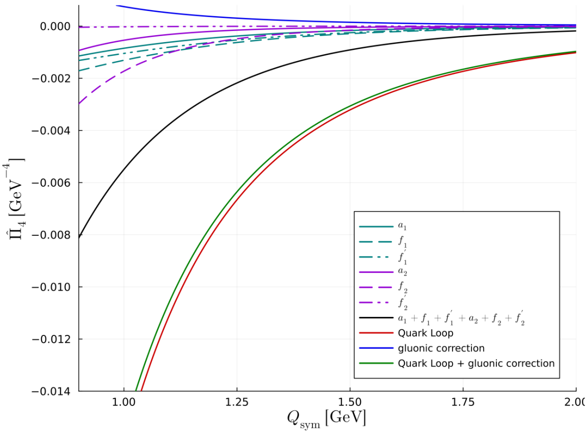
- $\hat{\Pi}_1(Q_{\rm sym}) := \hat{\Pi}_1(Q_{\rm sym}, Q_{\rm sym}, Q_{\rm sym})$, $\alpha_{\rm QCD}(\mu)$ with $\mu = 1.0\,\rm GeV$



Updates on HLbL from $(g-2)_7$ @KEK 2024

Comparison of OPE expressions and hadronic states for $\hat{\Pi}_4$

- $\hat{\Pi}_4(Q_{\rm sym}) := \hat{\Pi}_4(Q_{\rm sym}, Q_{\rm sym}, Q_{\rm sym})$, $\alpha_{\rm QCD}(\mu)$ with $\mu = 1.0\,\mathrm{GeV}$



Updates on HLbL from $(g-2)_7$ @KEK 2024

Attempts for further improvements

Issues with LMR2022 model:

- equivalent photon decay rate of f_1, f_1' higher than L3 data indicate
- f_1-f_1' mixing angles unrealistic, too far from ideal mixing

To appear soon: LMR2024 with scalar-extended CS term (Quillen's superconnection)
(adaption of open-string-tachyon condensation model of Casero, Kiritsis & Paredes 2007)

preliminary results:

- f_1-f_1' mixing angle closer to ideal, lower equivalent photon rate:
- $\rightarrow$ lower contribution from ground-state a_1, f_1's, but more from excited AV
- but less perfect fit of η and η', excessive π^0 TFF!
- total sum almost unchanged:

$a_\mu^{\cdot\cdot\cdot} \times 10^{11}$	LMR(OPE fit)	LMR(F_ρ-fit)	WP2020
PS poles	104$\rightarrow$ 113	97.5$\rightarrow$ 104	93.8(4.0)
AV+LSDC	34$\rightarrow$ 25	30.5$\rightarrow$ 24.7	19(12)
total	138$\rightarrow$ 138	128$\rightarrow$ 128	113(16)

Range of quantitatively successful $N_f = 2+1$ hQCD models:

$a_\mu^{\text{AV+LSDC}} = (34 \ldots \underline{30.5} \ldots 24.7) \times 10^{-11}$ $a_\mu^{PS*} = (1.4 \ldots \underline{1.6} \ldots 5.5) \times 10^{-11}$

- New feature: scalar nonet naturally couples to photons, unlike minimal model, with one of the terms (ζ_+) considered by Cappiello, Cata, D'Ambrosio 2110.05962

Updates on HLbL from $(g-2)_7$ @KEK 2024

Conclusions

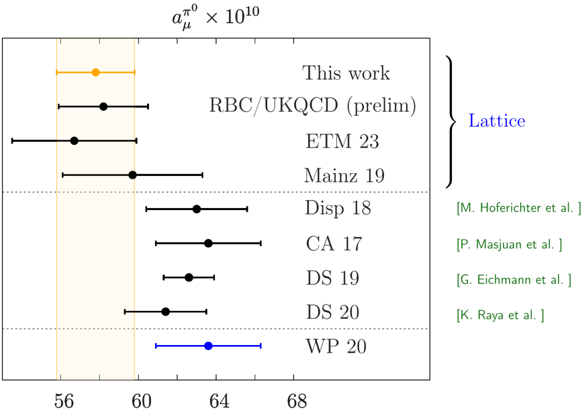
- Simple HW holographic QCD models as well as SHW improvements reproduce remarkably well the π^0 HLBL contribution from dispersive and lattice approaches, in particular with reduced g_5^2 to fit F_ρ (90% of OPE limit ↔ typical gluonic corrections)
- Extension with strange quark and WV η_0 mass (LMR2022): nice fit of η, η' data
- Melnikov-Vainshtein constraint naturally satisfied by tower of axial vector mesons
- Axial vector and LSDC contribution estimated together (≈58% of AV is longitudinal) with good agreement among various (flavor-symmetric) models
  - U(3)-symmetric models with OPE fit: $a_\mu^{\text{AV+LSDC}} = 40(3) \times 10^{-11}$
  - Best guess (LMR2022): $\boxed{a_\mu^{\text{AV+LSDC}} = 30.5^{+3.2(\text{OPE})}_{-6(\text{Quillen})} \times 10^{-11}}$

    around upper end of WP20 estimate $a_\mu^{\text{AV+LSDC}} = 19(12) \times 10^{-11}$
- Excited pseudoscalars (in WP20 contained in LSDC estimate)
  - U(3)-symmetric HW models with OPE fit: $a_\mu^{P*} = 4a_\mu^{\pi^*} = 5(2) \times 10^{-11}$
  - Best guess (LMR2022): $\boxed{a_\mu^{P*} = 1.6^{+4(\text{Quillen})}_{-0.2(\text{OPE})} \times 10^{-11}}$
- Scalar and tensor contributions very model dependent

  BL short-distance behavior of scalar and tensor TFFs not reproduced

Updates on HLbL from $(g-2)_7$ @KEK 2024

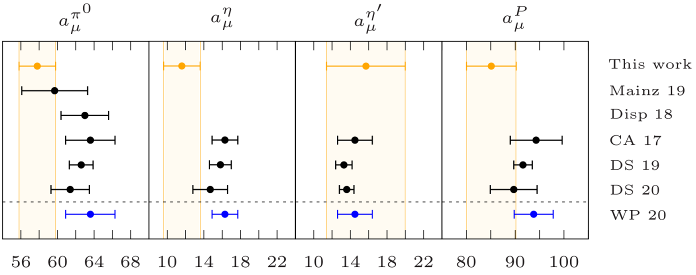
Pion-pole contribution : status



- Excellent agreement among lattice calculations
- 1.5σ compared with the previous WP estimate

Updates on HLbL from $(g-2)_7$ @KEK 2024

Comparison



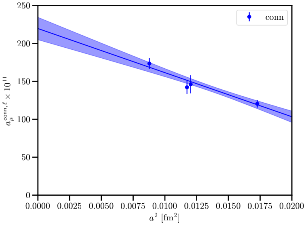
- Our final estimate

$$a_\mu^{\mathrm{HLbL;ps-poles}} = (85.1 \pm 4.7_{\mathrm{stat}} \pm 2.3) \times 10^{-11}\,.$$

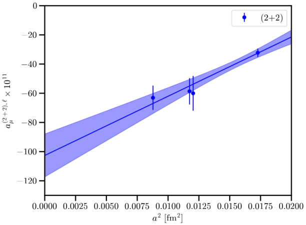
Updates on HLbL from $(g-2)_7$ @KEK 2024

Continuum extrapolation

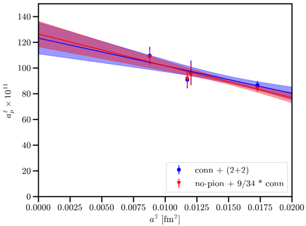
Connected



Disconnected



Total



Preliminary result : $a_\mu^{\mathrm{light}} = 122.6(11.6) \times 10^{-11}$

Mainz '22 : $107.4(11.3)(9.2) \times 10^{-11}$

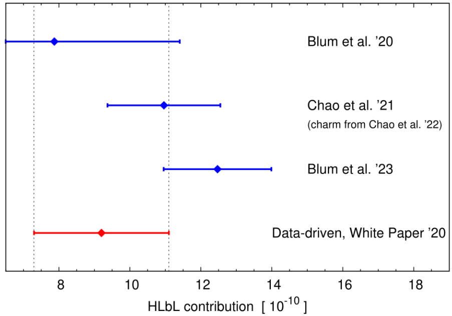
RBC/UKQCD '23 : $122.0(10.1)(9.5) \times 10^{-11}$

$\rightarrow$ taste-breaking effects well described by π^0-pole contribution

$\rightarrow$ new simulation at finer lattice spacing : on-going

Updates on HLbL from $(g-2)_7$ @KEK 2024

Compilation of a_μ^{HLbL} determinations



Good consistency of different determinations.
Lattice'24: $a_\mu^{\mathrm{HLbL}} = 12.6(1.2)(3) \cdot 10^{-10}$ (Ch. Zimmermann, BMW).

Results from the Bern dispersive framework and from three independent lattice QCD calculations since 2021 are in agreement with comparable uncertainties.

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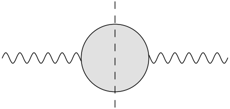
- Relevance of radiative corrections?

Conclusions and Outlook

HVP contribution: Master Formula

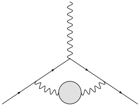
Unitarity relation: simple, same for all intermediate states



$$\mathrm{Im}\bar{\Pi}(q^2) \propto \sigma(e^+e^- \to \mathrm{hadrons}) = \sigma(e^+e^- \to \mu^+\mu^-)R(q^2)$$

Analyticity $\left[\bar{\Pi}(q^2) = \frac{q^2}{\pi}\int ds \frac{\mathrm{Im}\bar{\Pi}(s)}{s(s-q^2)}\right] \Rightarrow$ Master formula for HVP

Bouchiat, Michel (61)

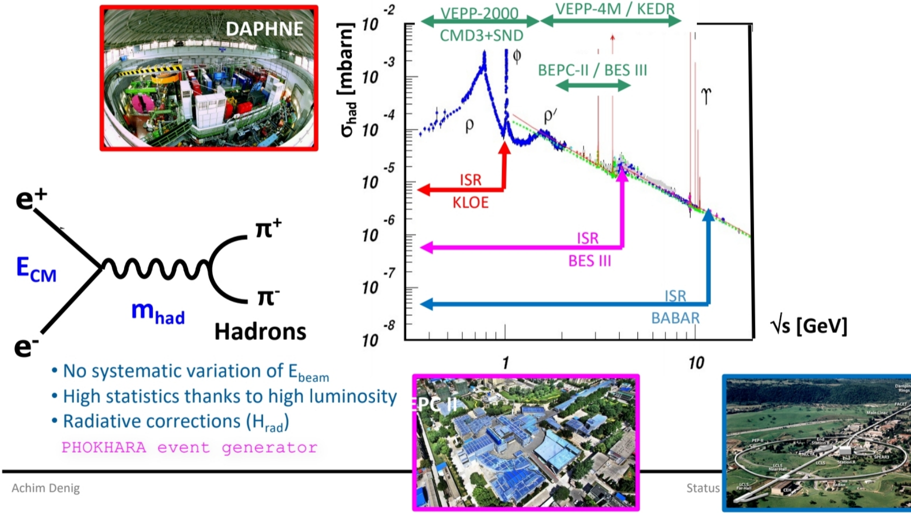


$\Leftrightarrow$

$$a_\mu^{\mathrm{hvp}} = \frac{\alpha^2}{3\pi^2}\int_{s_{th}}^{\infty}\frac{ds}{s}K(s)R(s)$$

$K(s)$ known, depends on m_μ and $K(s) \sim \frac{1}{s}$ for large s

HVP contribution: Master Formula



Slide by Achim Denig, MUonE Workshop, June 2024

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(3.38)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(1.45)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.30)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.55)	18.15(74)	−0.12
K^+K^-	23.08(0.44)	23.00(22)	0.08
K_SK_L	12.82(0.24)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.10)	4.58(10)	−0.17
Sum of the above	626.08(3.90)	623.62(2.27)	2.46
$[1.8, 3.7]\,\mathrm{GeV}$ (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)\,\mathrm{GeV}$	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

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Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

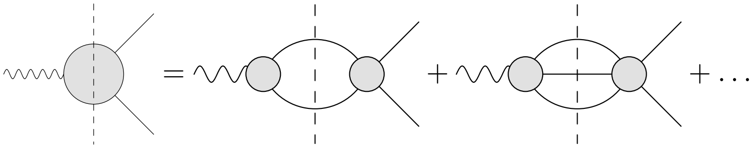
For the dominant $\pi\pi$ channel more theory input can be used

Comparison between DHMZ19 and KNT19

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(3.38)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(1.45)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.30)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.55)	18.15(74)	−0.12
K^+K^-	23.08(0.44)	23.00(22)	0.08
K_SK_L	12.82(0.24)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.10)	4.58(10)	−0.17
Sum of the above	626.08(3.90)	623.62(2.27)	2.46
$[1.8, 3.7]\,\mathrm{GeV}$ (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)\,\mathrm{GeV}$	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(4.0)	692.8(2.4)	1.2

For the 3π and KK channels also Hoferichter, Hoid, Kubis, Stamen, Hariharan, Stoffer

Omnès representation including isospin breaking



Omnès representation including isospin breaking

- Omnès representation

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

- Split elastic ($\leftrightarrow \pi\pi$ phase shift, δ_1^1) from inelastic phase

$$\delta = \delta_1^1 + \delta_{\rm in} \quad \Rightarrow \quad F_V^\pi(s) = \Omega_1^1(s)\Omega_{\rm in}(s)$$

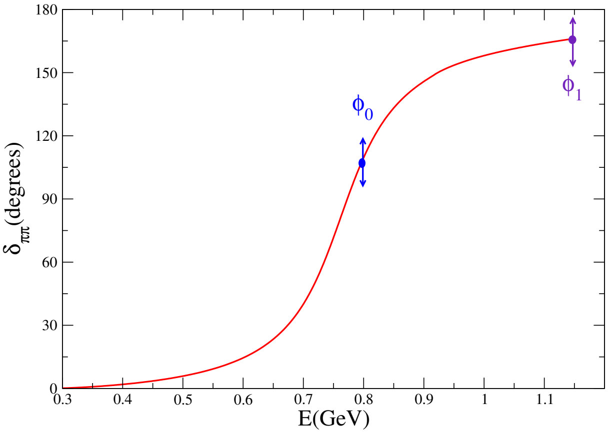
Eidelman-Lukaszuk: unitarity bound on $\delta_{\rm in}$

$$\sin^2\delta_{\rm in} \le \frac{1}{2}\left(1-\sqrt{1-r^2}\right), \; r = \frac{\sigma^{I=1}_{e^+e^-\to\neq 2\pi}}{\sigma_{e^+e^-\to 2\pi}} \Rightarrow s_{\rm in} = (M_\pi + M_\omega)^2$$

- $\rho-\omega$–mixing $\qquad F_V(s) = \Omega_{\pi\pi}(s)\cdot\Omega_{\rm in}(s)\cdot G_\omega(s)$

$$G_\omega(s) = 1 + \epsilon\frac{s}{s_\omega - s} \qquad \text{where} \qquad s_\omega = (M_\omega - i\Gamma_\omega/2)^2$$

Essential free parameters

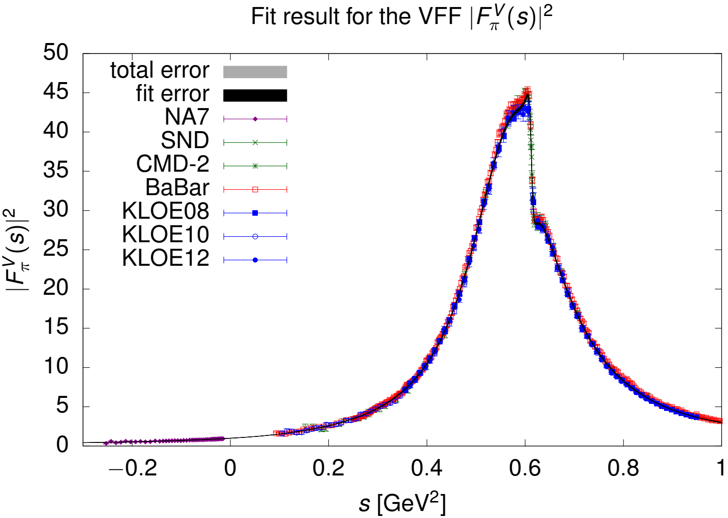


Estimated range ($\pi N \to \pi\pi N$): Caprini, GC, Leutwyler (12)

$$\phi_0 = 108.9(2.0)^\circ \qquad \phi_1 = 166.5(2.0)^\circ$$

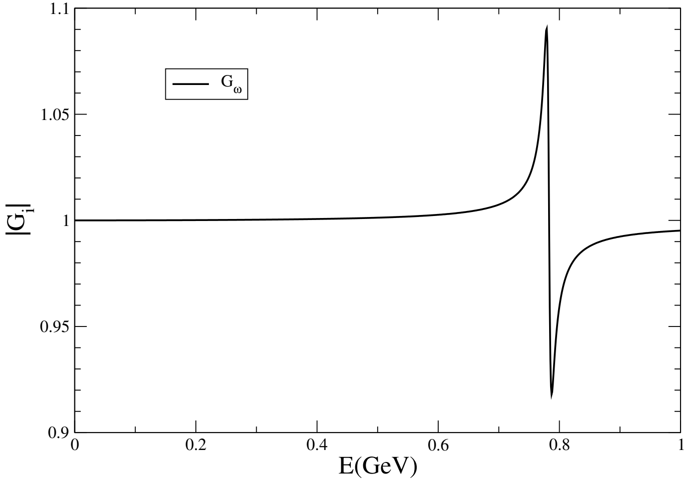
Fit results

GC, Hoferichter, Stoffer (18)



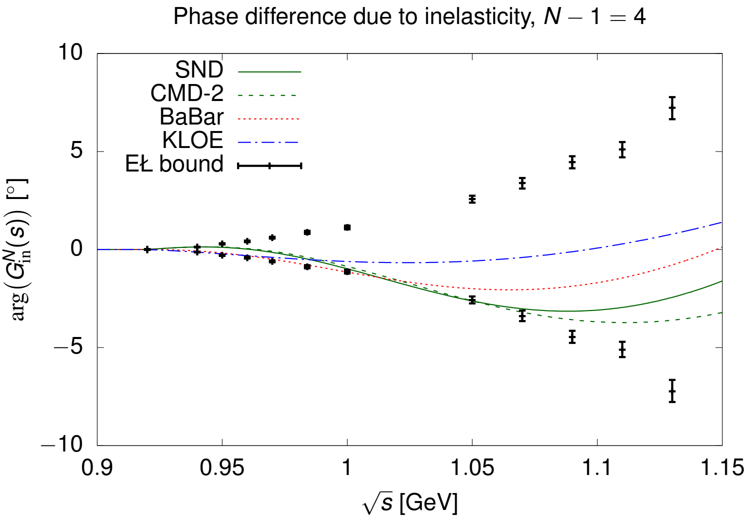
Fit results

GC, Hoferichter, Stoffer (18)



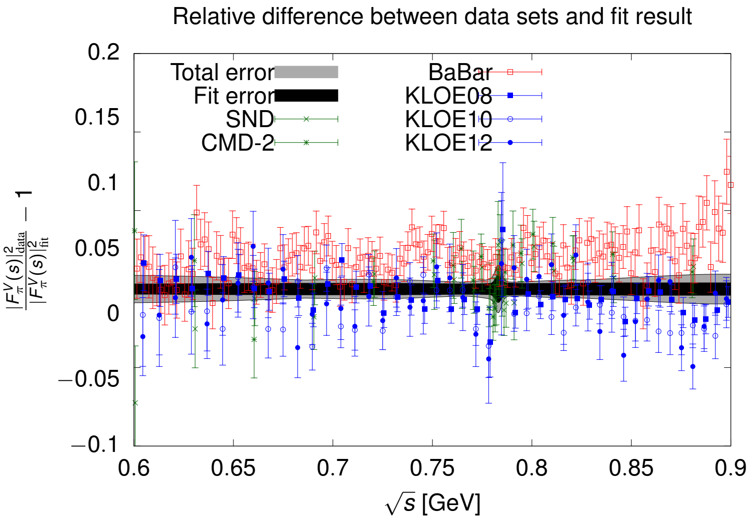
Fit results

GC, Hoferichter, Stoffer (18)



Fit results

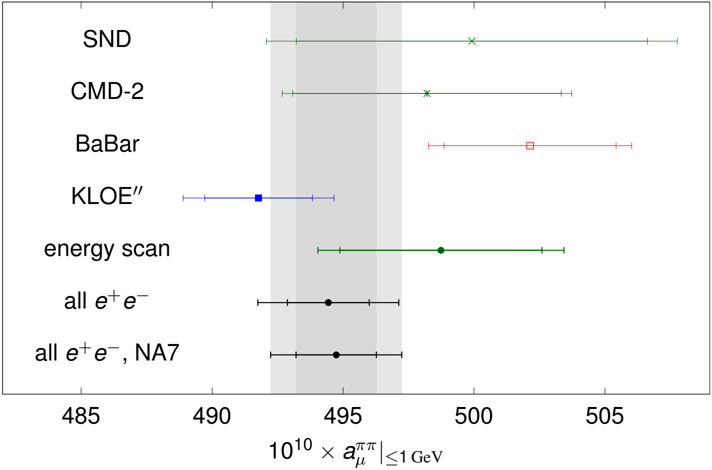
GC, Hoferichter, Stoffer (18)



Fit results

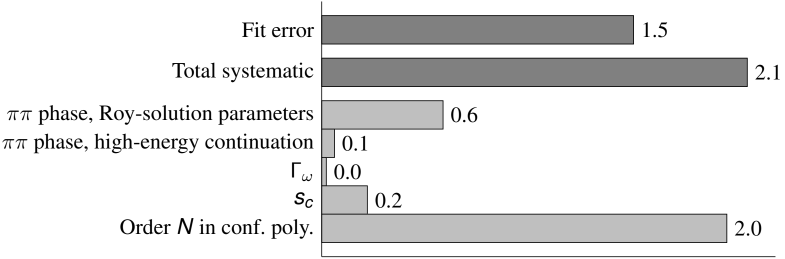
GC, Hoferichter, Stoffer (18)

Result for $a_\mu^{\pi\pi}|_{\leq 1\,\mathrm{GeV}}$ from the VFF fits to single experiments and combinations



Fit results

GC, Hoferichter, Stoffer (18)



Uncertainties on $a_\mu^{\pi\pi}|_{\leq 1\,\text{GeV}}$ in combined fit to all experiments

Order $N = 5$ conformal polynomial has zeros. Reasonable?

2π: comparison with the dispersive approach

2π channel described dispersively $\Rightarrow$ more theory constraints

Ananthanarayan, Caprini, Das (19), GC, Hoferichter, Stoffer (18) WP(20)

Energy range	CHS18	DHMZ19	KNT19
$\leq 0.6\,\mathrm{GeV}$	110.1(9)	110.4(4)(5)	108.7(9)
$\leq 0.7\,\mathrm{GeV}$	214.8(1.7)	214.7(0.8)(1.1)	213.1(1.2)
$\leq 0.8\,\mathrm{GeV}$	413.2(2.3)	414.4(1.5)(2.3)	412.0(1.7)
$\leq 0.9\,\mathrm{GeV}$	479.8(2.6)	481.9(1.8)(2.9)	478.5(1.8)
$\leq 1.0\,\mathrm{GeV}$	495.0(2.6)	497.4(1.8)(3.1)	493.8(1.9)
$[0.6, 0.7]\,\mathrm{GeV}$	104.7(7)	104.2(5)(5)	104.4(5)
$[0.7, 0.8]\,\mathrm{GeV}$	198.3(9)	199.8(0.9)(1.2)	198.9(7)
$[0.8, 0.9]\,\mathrm{GeV}$	66.6(4)	67.5(4)(6)	66.6(3)
$[0.9, 1.0]\,\mathrm{GeV}$	15.3(1)	15.5(1)(2)	15.3(1)
$\leq 0.63\,\mathrm{GeV}$	132.8(1.1)	132.9(5)(6)	131.2(1.0)
$[0.6, 0.9]\,\mathrm{GeV}$	369.6(1.7)	371.5(1.5)(2.3)	369.8(1.3)
$\left[\sqrt{0.1}, \sqrt{0.95}\right]\,\mathrm{GeV}$	490.7(2.6)	493.1(1.8)(3.1)	489.5(1.9)

Combination method and final result

Complete analyses DHMZ19 and KNT19, as well as CHS19 (2π) and HHK19 (3π), have been so combined:

HHK=Hoferichter, Hoid, Kubis

- central values are obtained by simple averages (for each channel and mass range)
- the largest experimental and systematic uncertainty of DHMZ and KNT is taken
- $1/2$ difference DHMZ$-$KNT (or BABAR$-$KLOE in the 2π channel, if larger) is added to the uncertainty

Final result:

$$a_\mu^{\text{HVP, LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{DV+QCD}} \times 10^{-10}$$
$$= 693.1(4.0) \times 10^{-10}$$

WP(20)

The BMW result

Borsanyi et al. Nature 2021

State-of-the-art lattice calculation of $a_\mu^{\text{HVP, LO}}$ based on

- current-current correlator, summed over all distances, integrated in time with appropriate kernel function (TMR)
- using staggered fermions on an $L \sim 6$ fm lattice ($L \sim 11$fm used for finite volume corrections)
- at (and around) physical quark masses
- including isospin-breaking effects

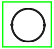
- update (24) confirms result and reduces uncertainty intermediate and SD windows confirmed by several lattice collab. — LD window just confirmed by RBC/UKQCD and Mainz

$\rightarrow$ talk by D. Giusti

The BMW result

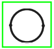
Borsanyi et al. Nature 2021

Isospin-symmetric



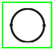
Connected light

$633.7(2.1)_{stat}(4.2)_{syst}$



Connected strange

$53.393(89)_{stat}(68)_{syst}$



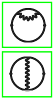
Connected charm

$14.6(0)_{stat}(1)_{syst}$

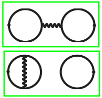
Disconnected

$-13.36(1.18)_{stat}(1.36)_{syst}$

QED isospin breaking: valence

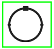


Connected $-1.23(40)_{stat}(31)_{syst}$



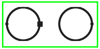
Disconnected $-0.55(15)_{stat}(10)_{syst}$

Strong–isospin breaking



Connected

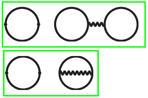
$6.60(63)_{stat}(53)_{syst}$



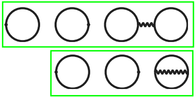
Disconnected

$-4.67(54)_{stat}(69)_{syst}$

QED isospin breaking: sea



Connected $0.37(21)_{stat}(24)_{syst}$



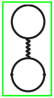
Disconnected $-0.040(33)_{stat}(21)_{syst}$

Other

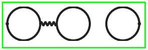
Bottom; higher-order; perturbative

$0.11(4)_{tot}$

QED isospin breaking: mixed



Connected $-0.0093(86)_{stat}(95)_{syst}$



Disconnected $0.011(24)_{stat}(14)_{syst}$

Finite-size effects

Isospin-symmetric

$18.7(2.5)_{tot}$

Isospin-breaking

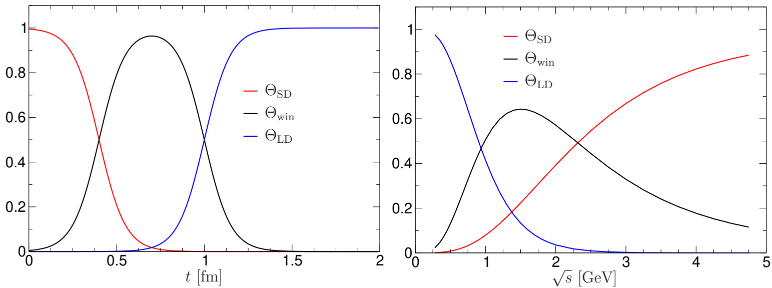
$0.0(0.1)_{tot}$

$$a_\mu^{\text{LO-HVP}} (\times 10^{10}) = 707.5(2.3)_{stat}(5.0)_{syst}(5.5)_{tot}$$

Present status of the window quantities

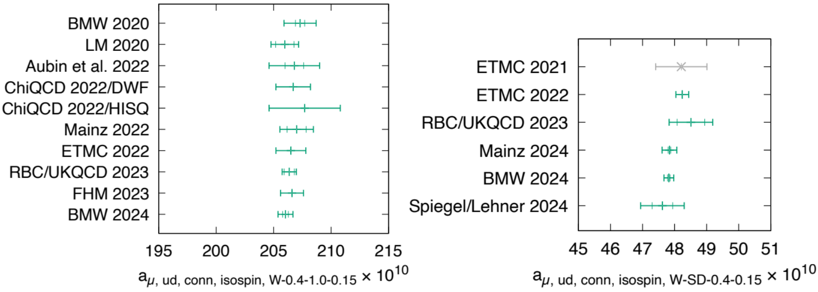
Weight functions for window quantities

RBC/UKQCD (18)



C. Lehner, talk at Lattice 2024

Present status of the window quantities



C. Lehner, talk at Lattice 2024

Individual-channel contributions to $a_\mu^{\rm win}$

Channel	total	window
$\pi^+\pi^-$	504.23(1.90)	144.08(49)
$\pi^+\pi^-\pi^0$	46.63(94)	18.63(35)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19)	8.88(12)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74)	11.20(46)
K^+K^-	23.00(22)	12.29(12)
K_SK_L	13.04(19)	6.81(10)
$\pi^0\gamma$	4.58(10)	1.58(4)
Sum of the above	623.62(2.27)	203.47(78)
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	34.45(56)	15.93(26)
J/ψ, $\psi(2S)$	7.84(19)	2.27(6)
$[3.7, \infty)$ GeV	16.95(19)	1.56(2)
WP(20) / GC, El-Khadra *et al.* (22)	693.1(4.0)	229.4(1.4)
BMWc	707.5(5.5)	236.7(1.4)
Mainz/CLS		237.3(1.5)
ETMc		235.0(1.1)
RBC/UKQCD		235.6(0.8)

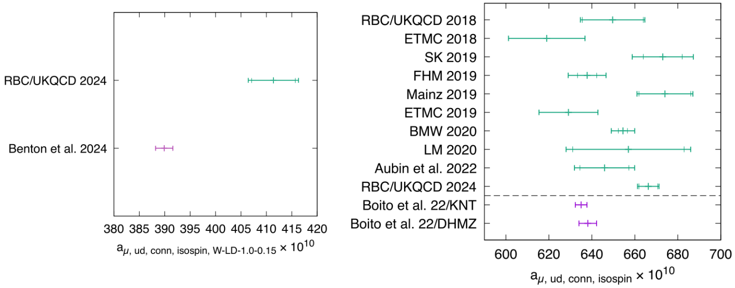
Numbers for the channels refer to KNT19 — thanks to Alex Keshavarzi for providing them

$$\Delta a_\mu^{\text{HVP, LO}} = 14.4(6.8)\,(2.1\sigma), \qquad \Delta a_\mu^{\rm win} \sim 6.5(1.5)\,(\sim 4.3\sigma)$$

Updates on lattice HVP from $(g-2)_7$ @KEK 2024

Slide by C. Lehner

Unblinded results in BMW20 isospin-symmetric world



Result for $a_\mu^{\rm iso\ lqc}$ with 7.5/1000 precision.

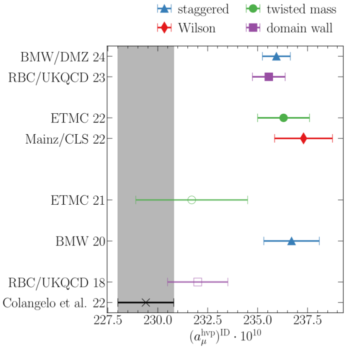
$$a_\mu^{\rm LD\ iso\ lqc} = 411.4(4.3)_{\rm stat.}(2.3)_{\rm syst.} \times 10^{-10}\,,$$
$$a_\mu^{\rm iso\ lqc} = 666.2(4.3)_{\rm stat.}(2.5)_{\rm syst.} \times 10^{-10}\,.$$

More high-precision lattice results needed for consolidation of full $a_\mu^{\rm iso\ lqc}$!

Updates on lattice HVP from $(g-2)_7$ @KEK 2024

Slide by S. Kuberski

$a_\mu^{\rm hvp}$ FROM LATTICE QCD



- Use windows in the time-momentum representation to compute [Blum et al., 1801.07224]

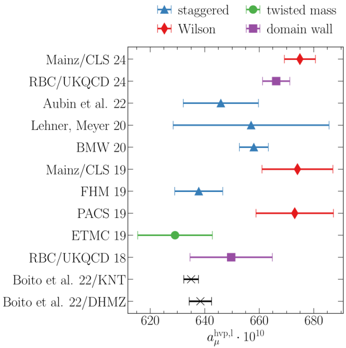
$$a_\mu^{\rm hvp} = (a_\mu^{\rm hvp})^{\rm SD} + (a_\mu^{\rm hvp})^{\rm ID} + (a_\mu^{\rm hvp})^{\rm LD}$$

  - Intermediate distance (✓): [Cè et al., 2206.06582]

Updates on lattice HVP from $(g-2)_7$ @KEK 2024

Slide by S. Kuberski

CONTRIBUTIONS TO a_μ^{hvp} IN ISOQCD

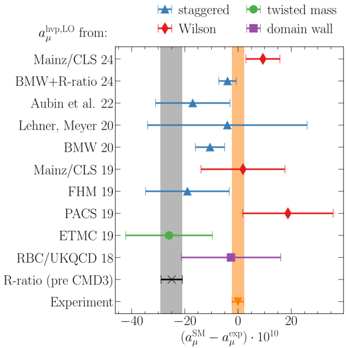


- Compute contributions to a_μ^{hvp} in isoQCD (Mainz world) by combinations with $(a_\mu^{\text{hvp}})^{\text{SD}}$ and $(a_\mu^{\text{hvp}})^{\text{ID}}$.
- We (will) publish the derivatives w.r.t. the input that defines our scheme. See [Portelli] for a comparison of schemes.
- $a_\mu^{\text{hvp,l}}$ determined to 0.8% precision
- Excellent compatibility of Mainz/CLS 19 with Mainz/CLS 24.

Updates on lattice HVP from $(g-2)_7$ @KEK 2024

Slide by S. Kuberski

THE LEADING-ORDER HADRONIC VACUUM POLARIZATION CONTRIBUTION

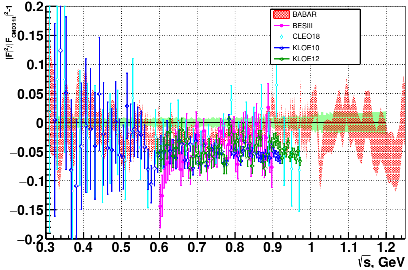


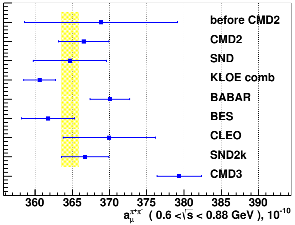
[BNL $g-2$, hep-ex/0602035]
[FNAL $g-2$, 2104.03281, 2308.06230]

- The estimate of IB corrections allows to compute a **preliminary** a_μ^{hvp}.
- Our result supports the no new physics scenario.
- Ongoing work to compute IB corrections. So far
  - no IB in scale setting
  - electroquenched approximation
  - **preliminary estimate**

CMD-3 measurement of $e^+e^- \to \pi^+\pi^-$

F. Ignatov et al., CMD-3, arXiv:2302.08834

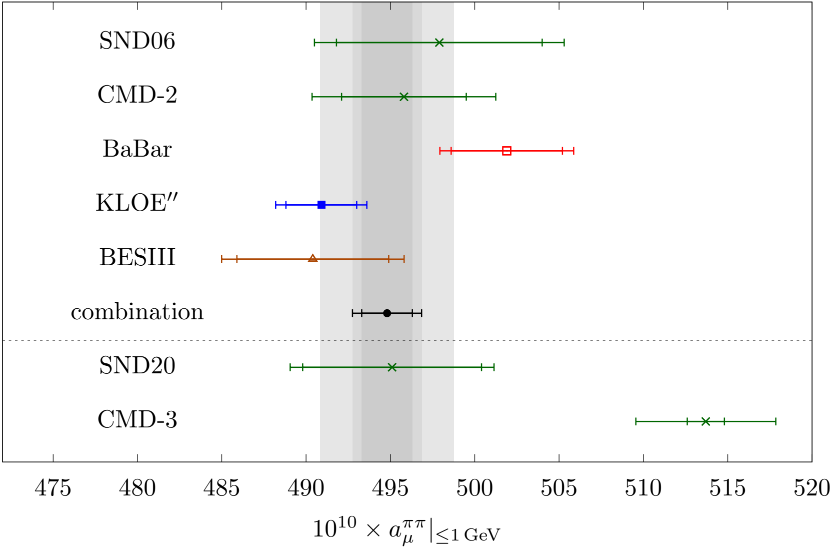




The comparison of pion form factor measured in this work with the most recent ISR experiments (BABAR [21], KLOE [18, 19], BES [22]) is shown in Fig. 34. The comparison with the most precise previous energy scan experiments (CMD-2 [12, 13, 14, 15], SND [16] at the VEPP-2M and SND [23] at the VEPP-2000) is shown in Fig. 35. The new result generally shows larger pion form factor in the whole energy range under discussion. The most significant difference to other energy scan measurements, including previous CMD-2 measurement, is observed at the left side of ρ-meson ($\sqrt{s} = 0.6 - 0.75$ GeV), where it reach up to 5%, well beyond the combined systematic and statistical errors of the new and previous results. The source of this difference is unknown at the moment.

Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217



Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217

$10^{10}\times$	$a_\mu^{\pi\pi}\|_{\leq 1\mathrm{GeV}}$	$a_\mu^{\pi\pi,\mathrm{win}}\|_{\leq 1\mathrm{GeV}}$	χ^2/dof
SND06	497.9(6.1)(4.2)	139.6(1.8)(1.0)	1.09
CMD-2	495.8(3.7)(4.0)	139.4(1.0)(0.8)	1.01
BaBar	501.9(3.3)(2.2)	140.6(1.0)(0.7)	1.17
KLOE"	490.9(2.1)(1.7)	137.1(0.6)(0.4)	1.13
BESIII	490.4(4.5)(3.0)	137.8(1.3)(0.4)	1.01
SND20	495.1(5.3)(2.9)	139.2(1.5)(0.4)	1.88
CMD-3	513.7(1.1)(4.0)	144.0(0.3)(1.1)	1.09
Combination	494.8(1.5)(1.4)(3.4)	138.3(0.4)(0.3)(1.1)	1.21

Combination: NA7 + all data sets other than SND20 and CMD-3

$$\Delta a_\mu^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1)\,, \qquad \Delta a_\mu^{\mathrm{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_\mu^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8), \qquad \Delta a_\mu^{\mathrm{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

Preliminary analysis of the CMD-3 measurement

GC, Hoferichter and Stoffer, arXiv:2308.04217

Discrepancy	$a_\mu^{\pi\pi}\big\vert_{[0.60,0.88]\,\text{GeV}}$	$a_\mu^{\pi\pi}\big\vert_{\leq 1\,\text{GeV}}$	int window
SND06	1.8σ	1.7σ	1.7σ
CMD-2	2.3σ	2.0σ	2.1σ
BaBar	3.3σ	2.9σ	3.1σ
KLOE′′	5.6σ	4.8σ	5.4σ
BESIII	3.0σ	2.8σ	3.1σ
SND20	2.2σ	2.1σ	2.2σ
Combination	4.2σ [6.1σ]	3.7σ [5.0σ]	3.8σ [5.7σ]

Uncertainties in brackets exclude KLOE-BaBar systematic eff.

Combination: NA7 + all data sets other than SND20 and CMD-3

$$\Delta a_\mu^{\text{HVP, LO}}(\text{CMD-3-Comb.}) = 18.9(5.1)\,, \qquad \Delta a_\mu^{\text{win}}(\text{CMD-3-Comb.}) = 5.7(1.5)$$

$$\Delta a_\mu^{\text{HVP, LO}}(\text{BMW-WP20}) = 14.4(6.8), \qquad \Delta a_\mu^{\text{win}}(\text{Lattice-WP20}) \sim 6.5(1.5)$$

Origin of the differences

GC, Hoferichter and Stoffer, arXiv:2308.04217

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \mathrm{Re}\epsilon_\omega$	δ_ϵ [°]
SND06	1.09	33%	782.12(33)(2)	2.03(5)(2)	8.6(2.3)(0.3)
CMD-2	1.01	46%	782.65(33)(4)	1.90(6)(3)	11.5(3.1)(1.0)
BaBar	1.17	3.0%	781.89(18)(4)	2.06(4)(2)	0.4(1.9)(0.6)
KLOE″	1.13	11%	782.45(24)(5)	1.96(4)(2)	6.1(1.7)(0.6)
BESIII	1.01	45%	783.07(61)(2)	2.03(19)(7)	17.8(6.9)(1.2)
SND20	1.88	3.8×10^{-3}	782.34(28)(6)	2.07(5)(2)	9.9(2.4)(1.3)
CMD-3	1.09	20%	782.33(6)(3)	2.08(1)(2)	7.4(4)(3)
Combin.	1.21	1.4×10^{-4}	782.07(12)(5)(8)	1.99(2)(2)(0)	3.8(0.9)(0.8)(1.6)

$\pi\pi$ phase shift consistent among all experiments:

Combination (2018) $\phi_0 = 110.4(1)(7)^\circ$ $\quad \phi_1 = 165.9(0.1)(2.4)^\circ$

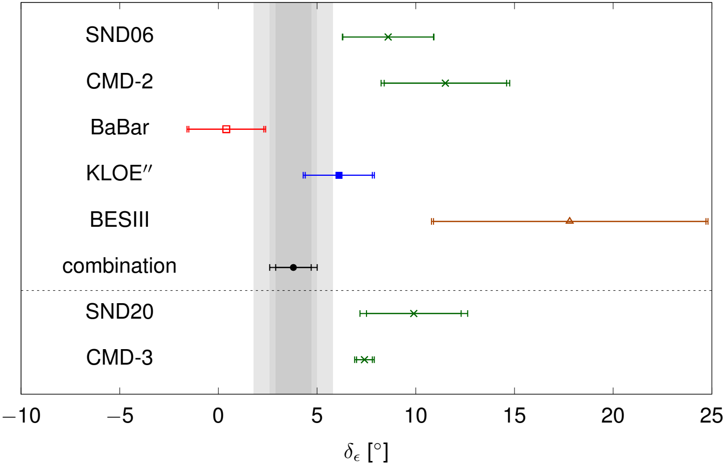
CMD-3 $\phi_0 = 110.7(1)(..)^\circ$ $\quad \phi_1 = 166.2(1)(..)^\circ$

and with earlier estimate ($\pi N \to \pi\pi N$): Caprini, GC, Leutwyler (12)

$$\phi_0 = 108.9(2.0)^\circ \qquad \phi_1 = 166.5(2.0)^\circ$$

Origin of the differences

GC, Hoferichter and Stoffer, arXiv:2308.04217 GC, Hoferichter, Kubis, Stoffer (22)



Origin of the differences

GC, Hoferichter and Stoffer, arXiv:2308.04217

	χ^2/dof	p-value	M_ω [MeV]	$10^3 \times \mathrm{Re}\epsilon_\omega$	δ_ϵ [°]
SND06	1.09	33%	782.12(33)(2)	2.03(5)(2)	8.6(2.3)(0.3)
CMD-2	1.01	46%	782.65(33)(4)	1.90(6)(3)	11.5(3.1)(1.0)
BaBar	1.17	3.0%	781.89(18)(4)	2.06(4)(2)	0.4(1.9)(0.6)
KLOE″	1.13	11%	782.45(24)(5)	1.96(4)(2)	6.1(1.7)(0.6)
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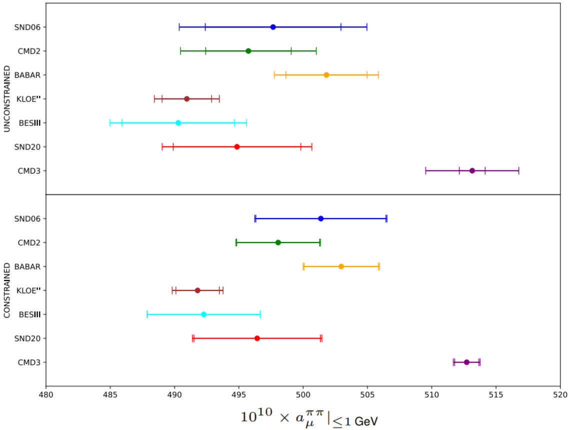
$\Rightarrow$ Main difference due to inelastic contributions

Theory uncertainties in $F_\pi(s)^V$ overestimated?

Constrained fits without zeros: $a_\mu^{\pi\pi}$

$\rightarrow$ work in progress with **Thomas Leplumey** (ETH master student)



Slide by Peter Stoffer, MUonE Workshop, Mainz June 3-7, 2024

Dispersive treatment of FSR in $e^+e^- \to \pi^+\pi^-$

Work in collab. with M. Cottini, J. Monnard and J. Ruiz de Elvira

Goal: pion vector FF in QCD+QED at $\mathcal{O}(\alpha)$: $F_\pi^{V,\alpha}$:

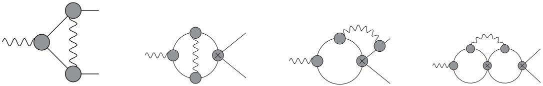
$$\begin{aligned}\frac{\mathrm{Disc}F_\pi^{V,\alpha}(s)}{2i} &= \frac{(2\pi)^4}{2}\int d\Phi_2 F_\pi^V(s)\times T_{\pi\pi}^{\alpha *}(s,t)\\ &+ \frac{(2\pi)^4}{2}\int d\Phi_2 F_\pi^{V,\alpha}(s)\times T_{\pi\pi}^{*}(s,t)\\ &+ \frac{(2\pi)^4}{2}\int d\Phi_3 F_\pi^{V,\gamma}(s,t)T_{\pi\pi}^{\gamma *}(s,\{t_i\})\end{aligned}$$

Approximation: only 2π intermediate states for $F_\pi^{V,\gamma}$ and $T_{\pi\pi}^\gamma$:

All subamplitudes known $\Rightarrow$ $T_{\pi\pi}^\alpha$, $F_\pi^{V,\gamma}$ and $T_{\pi\pi}^\gamma$ ✓

Evaluation of $F_\pi^{V,\alpha}$

Having evaluated all the following diagrams J. Monnard, PhD thesis 2021



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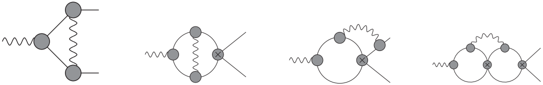
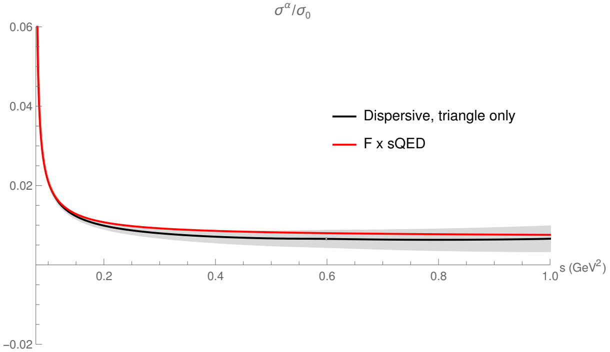


the results for $\sigma(e^+e^- \to \pi^+\pi^-(\gamma))$ look as follows:

Preliminary!



Red curve corresponds to Hoefer, Gluza, Jegerlehner (02) and Campanario et al. (19) (?)

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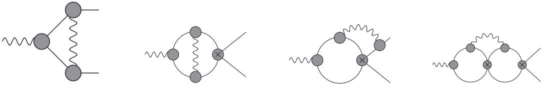

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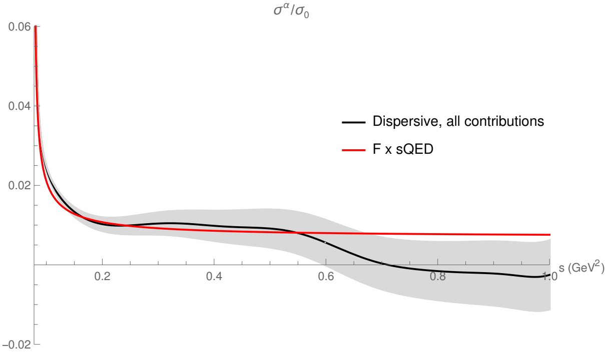

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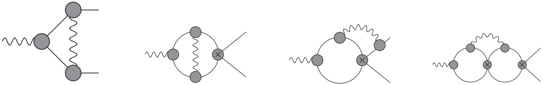

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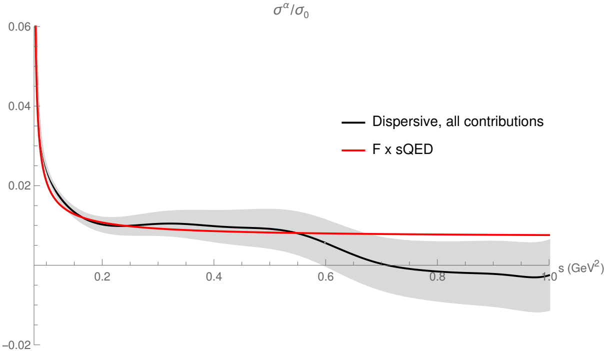


$\Rightarrow$ No large effects

Updates on IB corrections from $(g-2)_7$ @KEK 2024

- KLOE and BESIII have rebutted claims that higher-order radiative corrections might have impacted their analyses

  talks by A. Denig and G. Venanzoni @KEK24

- claim that initial/final radiation interference on the box diagram might impact significantly radiative-return experiments is under scrutiny

  F. Ignatov @STRONG2020 Zürich (23)

- reconsideration of τ decays as input for HVP has been advocated by DHMZ

  TI Virtual workshops on Nov. 8 and Dec. 9

- analysis of IB for τ decays on the lattice is ongoing

  talk by M. Bruno @KEK24

- dispersive analysis of IB for τ decays is ongoing

  talk by M. Cottini @KEK24

Outline

Introduction: $(g-2)_\mu$ in the Standard Model

Hadronic light-by-light

Hadronic Vacuum Polarization contribution

- Data-driven approach
- Data-driven vs. Lattice
- CMD3 measurement of $e^+e^- \to \pi^+\pi^-$
- Relevance of radiative corrections?

Conclusions and Outlook

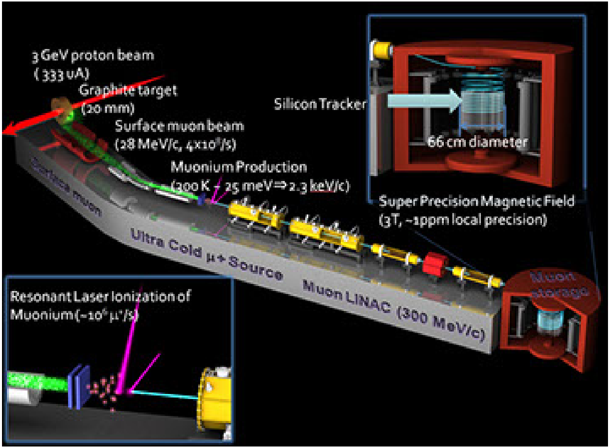
Conclusions

- Data-driven evaluation of the HVP contribution (WP20): 0.6% error ⇒ dominates the theory uncertainty
- Dominant contribution to HVP: $\pi\pi$ (<1 GeV). WP20 based on: CMD-2, SND06, BaBar, KLOE, BES-III
  New puzzle: measurement by CMD-3 significantly higher!
- The BMW lattice calculation [BMW(20,24)] has reached a similar precision but differs from the dispersive one (=from e^+e^- data).
  Discrepancy with experiment ↘ below 1σ
- Intermediate and SD windows of BMW have been confirmed by several other lattice collaborations (Aubin et al., Mainz, ETMc, RBC/UKQCD, Fermilab-HPQCD-MILC) and disagrees with data-driven [other than CMD-3]
  LD window of BMW recently also confirmed (RBC/UKQCD, Mainz)
- Evaluation of the HLbL contribution based on the dispersive approach: 20% accuracy. Two recent lattice calculations [RBC/UKQCD(23), Mainz(21)] agree with it

Outlook

- The Fermilab experiment aims to reduce the BNL uncertainty by a factor four $\Rightarrow$ final result in early 2025
- Improvements on the SM theory/data side:
  - Situation for HVP data-driven urgently needs to be clarified:
    - New CMD-3 result—after thorough scrutiny—is a puzzle
    - Forthcoming measur./analyses: BaBar, Belle II, BESIII, KLOE,SND
    - Model-independent evaluation of RadCorr underway
    - Monte Carlo codes used by experiments: what is their role?
    - MuonE will provide an alternative way to measure HVP
  - HVP lattice:
    calculations w/ precision $\sim$ BMW for $a_\mu^{\text{HVP, LO}}$ expected soon
  - HLbL: goal of $\sim$ 10% uncertainty within reach
    (both data-driven and lattice)

Future: Muon $g-2$/EDM experiment @ J-PARC



Credit: J-PARC