Status of the HVP contribution to the muon g-2 from lattice

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UNIVERSITY OF LIVERPOOL

III Workshop on Muon Precision Physics

Liverpool

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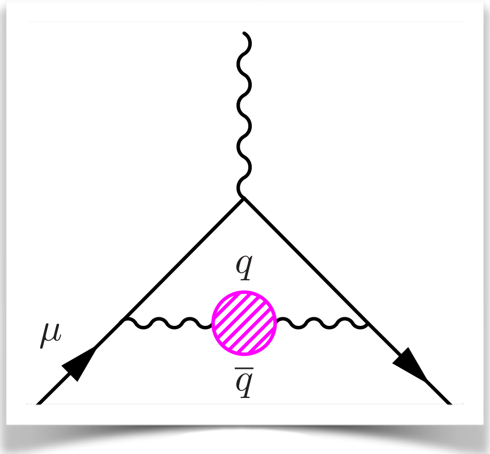
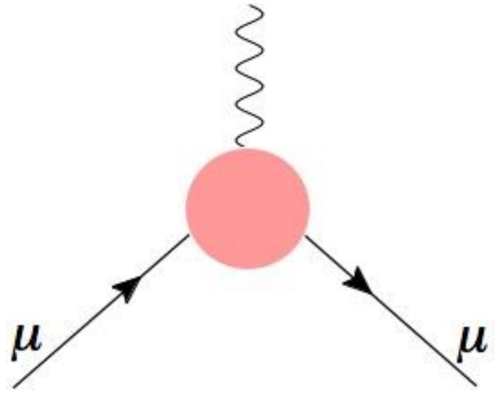
OUTLINE



- Introduction
- HVP from the lattice & window obs.
- Further connections

Introduction

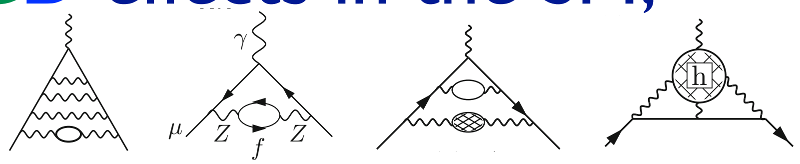
Muon magnetic anomaly



$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

muon anomalous magnetic moment: $a_\mu \equiv \dfrac{g_\mu - 2}{2} = F_2(0)$

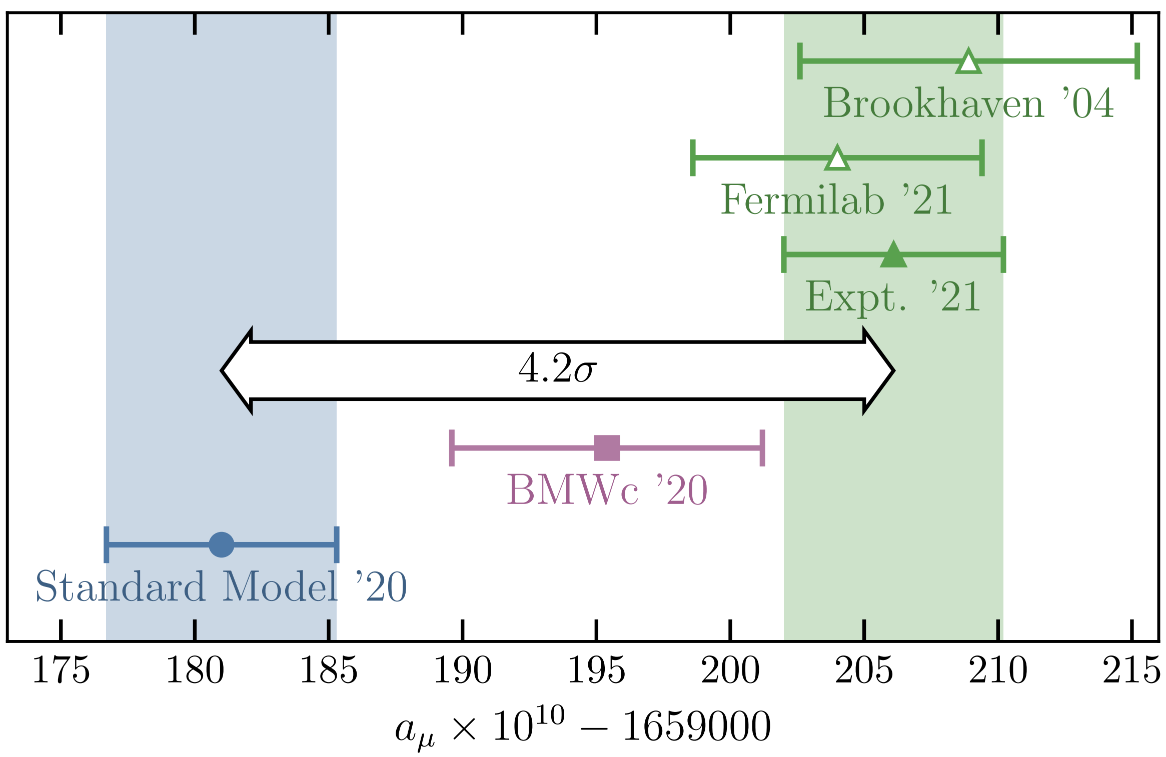
- is generated by quantum loops;
- receives contribution from QED, EW and QCD effects in the SM;
- is a sensitive probe of new physics



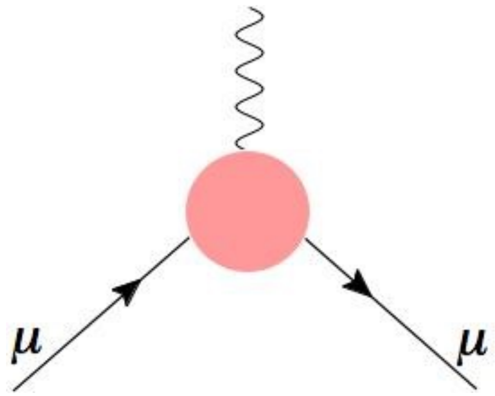
SM contributions to $a_\mu[\times 10^{10}]$

5-loop QED	11 658 471.8931(104)
2-loop EW	15.36(10)
HVP LO	693.1(4.0)
HVP NLO	-9.83(7)
HVP NNLO	1.24(1)
HLbL	9.2(1.8)

Theory error dominated by hadronic physics



Muon magnetic anomaly



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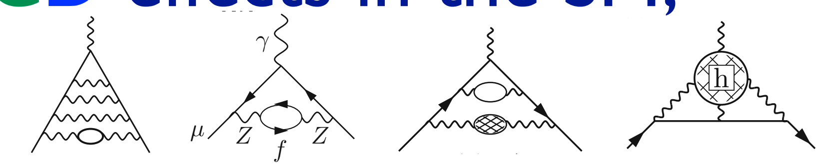

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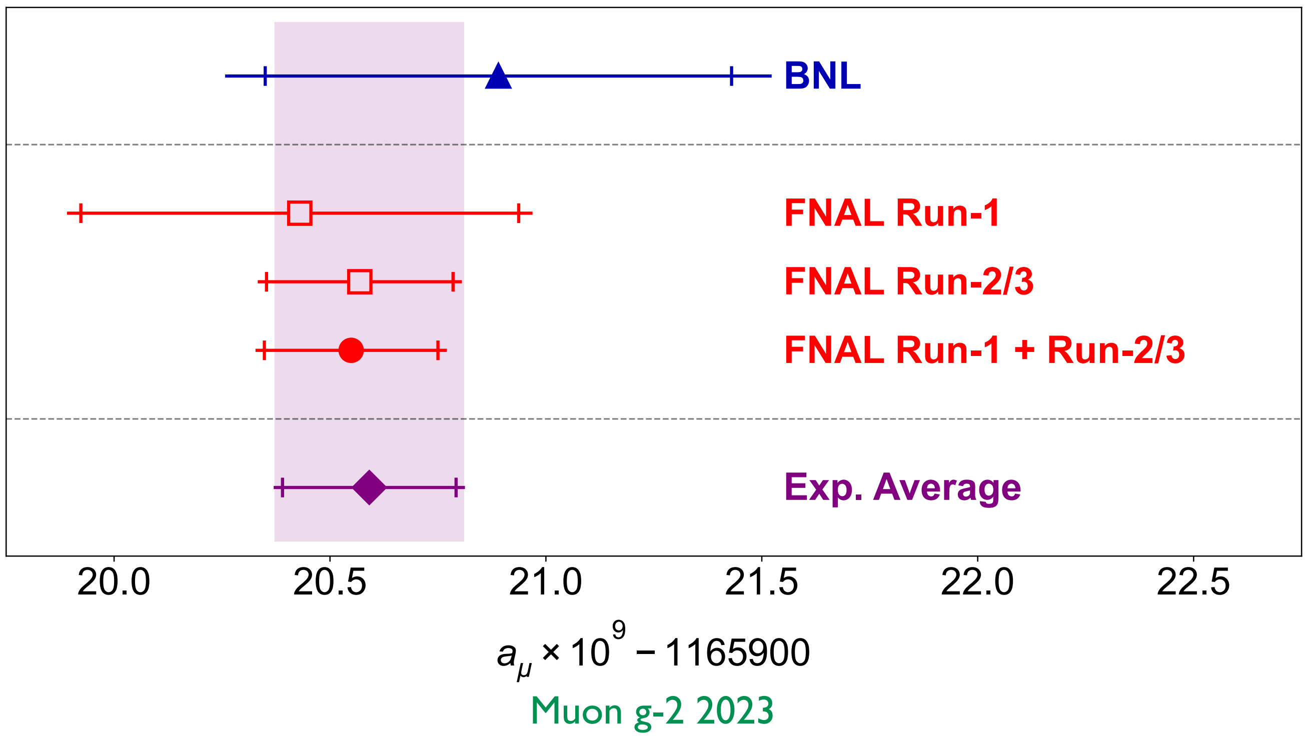
Theory error dominated by hadronic physics

Precision goal for Fermilab ×4 better implies knowing HVP at 0.2-0.3% accuracy



Hadronic contributions

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.9(4.1) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

Huge challenge: theory of strong interaction between quarks and gluons, QCD, hugely nonlinear at energies relevant for a_μ

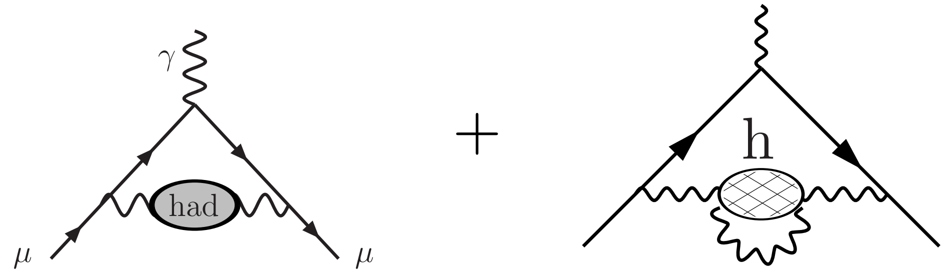
$\rightarrow$ perturbative methods used for electromagnetic and weak interactions do not work

$\rightarrow$ need nonperturbative approaches

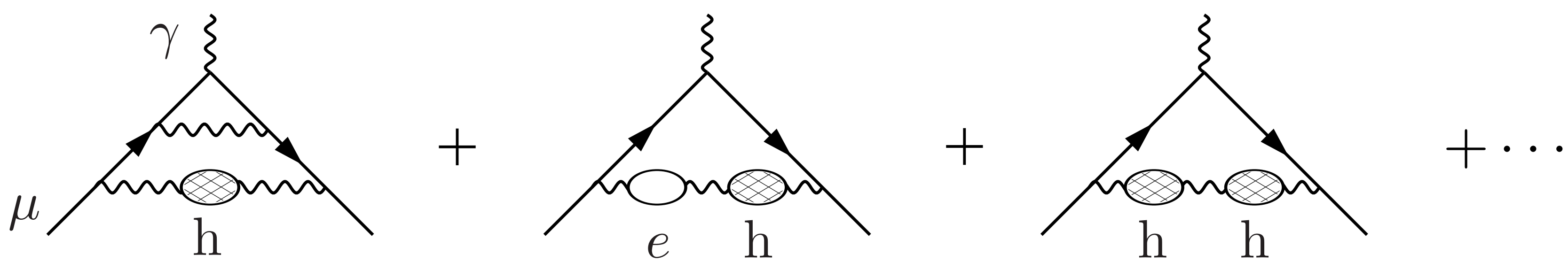
Write

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

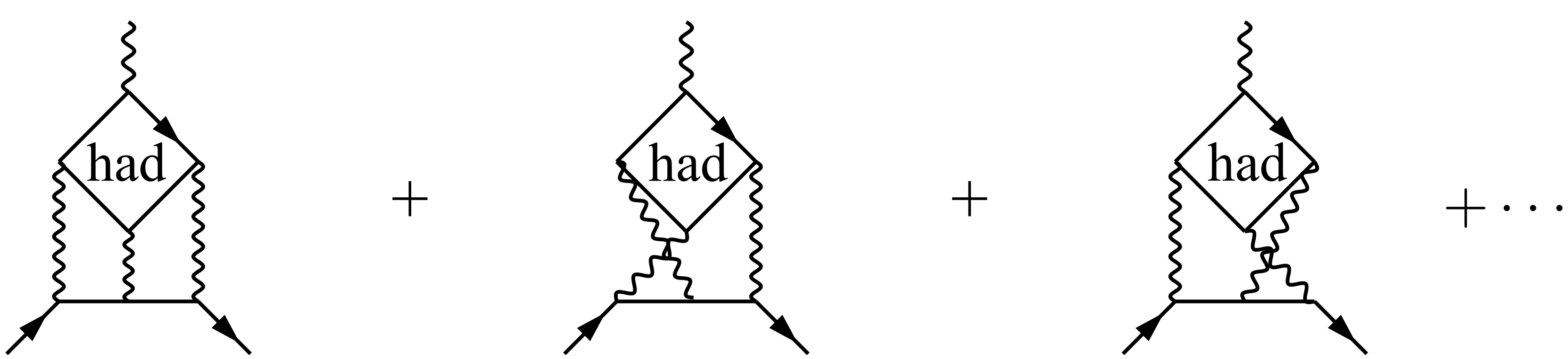
Hadronic contributions: diagrams



$\to a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$

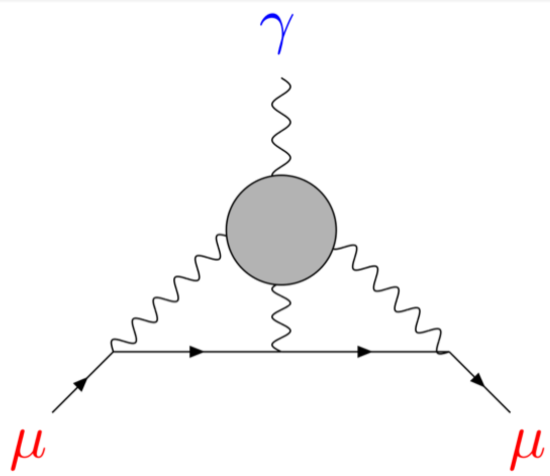


$\to a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

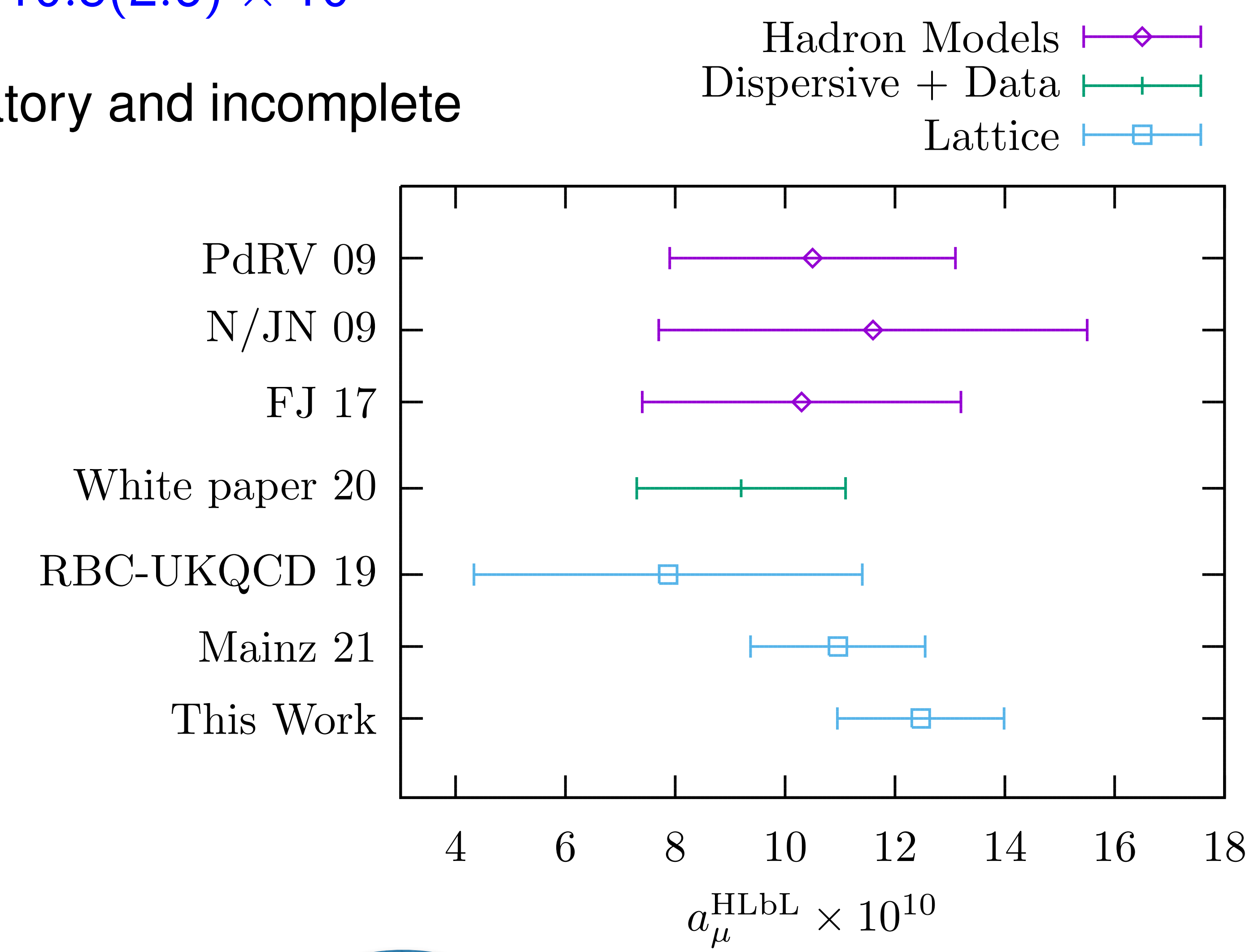


$\to a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Hadronic light-by-light



- HLbL much more complicated than HVP, but ultimate precision needed is $\simeq$ 10% instead of $\simeq$ 0.2%
- For many years, only accessible to models of QCD w/ difficult to estimate systematics (Prades et al '09): $a_\mu^{\text{HLbL}} = 10.5(2.6) \times 10^{-10}$
- Also, lattice QCD calculations were exploratory and incomplete
- Tremendous progress in past 5 years:
  - $\rightarrow$ Phenomenology: rigorous data driven approach [Colangelo, Hoferichter, Kubis, Procura, Stoffer,... '15-'20]
  - $\rightarrow$ Lattice: first two solid lattice calculations
- All agree w/ older model results but error estimate much more solid and will improve
- Agreed upon average w/ NLO HLbL and conservative error estimates [WP '20]
- $a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} - a_\mu^{\text{HLbL}} = 709.7(4.5) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{HVP}}$



[Blum et al '23]

NEW

BMWc '24 - preliminary

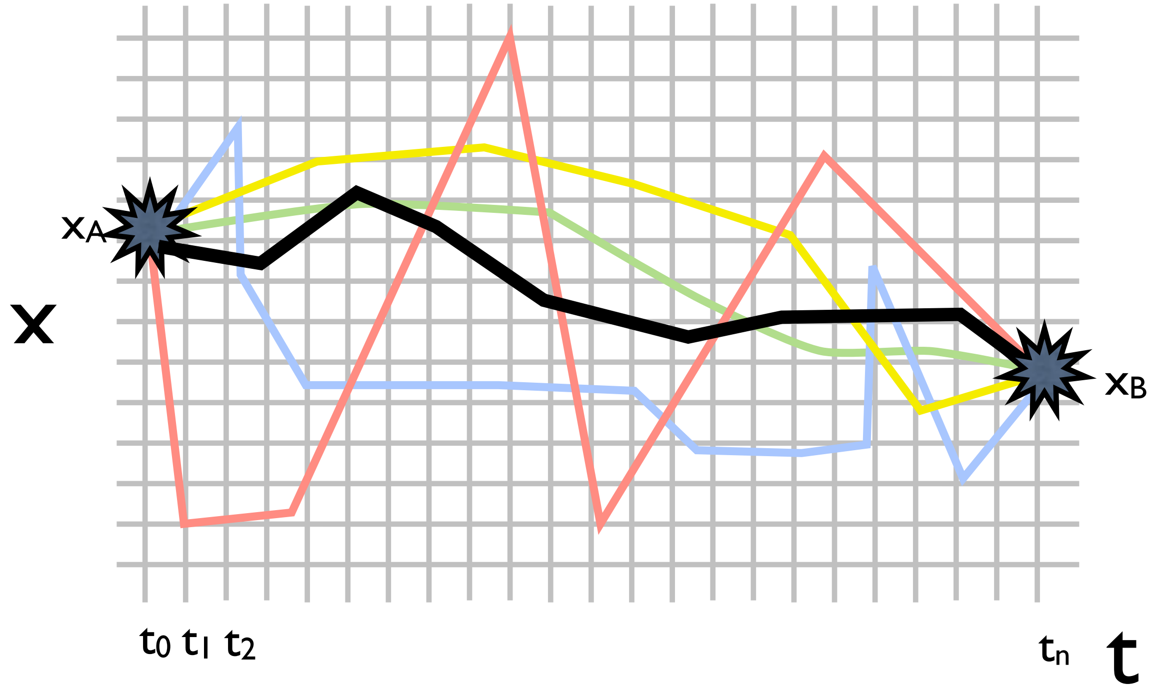
$a_\mu^{\text{HLbL}} = 12.6(1.2) \times 10^{-10}$

Small interlude:
Lattice QCD

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Discretise QCD onto 4D space-time lattice
- QCD equations ⟷ integrals over the values of quark and gluon fields on each site/link (QCD path integral)
- ~10^{12} variables (for state-of-the-art)

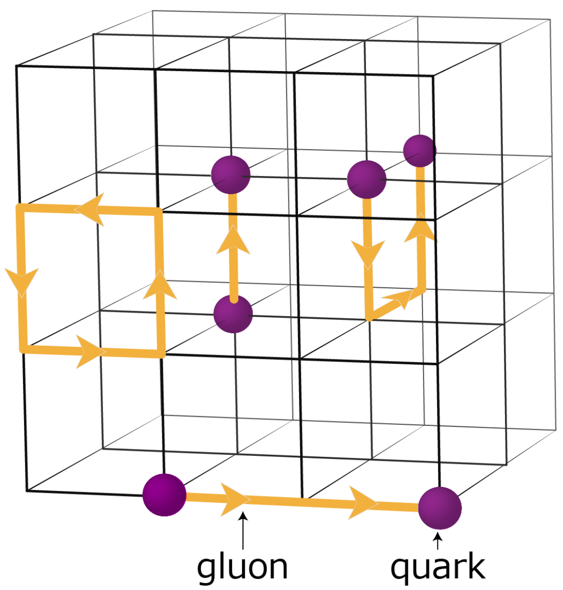


- Evaluate by importance sampling
- Paths near classical action dominate
- Calculate physics on a set (ensemble) of samples of the quark and gluon fields

Lattice QCD

Numerical first-principles approach to non-perturbative QCD

- Euclidean space-time $t \to i\tau$
- Finite lattice spacing a
- Volume $L^3 \times T = 64^3 \times 128$
- Boundary conditions



Approximate the QCD path integral by **Monte Carlo**

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\overline{\psi} \mathcal{D}\psi \mathcal{O}[A, \overline{\psi}\psi] e^{-S[A,\overline{\psi}\psi]} \quad \Rightarrow \quad \langle \mathcal{O} \rangle \simeq \frac{1}{N_{\mathrm{conf}}} \sum_{i}^{N_{\mathrm{conf}}} \mathcal{O}([U^i])$$

with field configurations U^i distributed according to $e^{-S[U]}$

Lattice QCD

Workflow of a lattice QCD calculation

1 Generate field configurations via Hybrid Monte Carlo

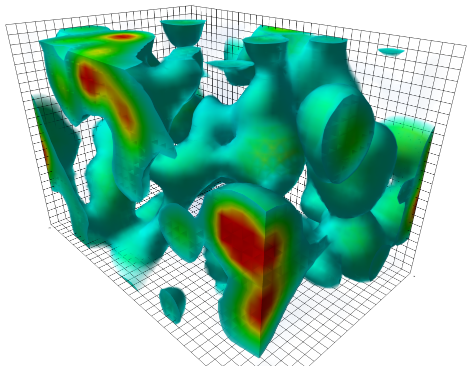
- Leadership-class computing
- ~100K cores or 1000GPUs, 10's of TF-years
- O(100-1000) configurations, each ~10-100GB

2 Compute propagators

- Large sparse matrix inversion
- ~few 100s GPUs
- 10x field config in size, many per config

3 Contract into correlation functions

- ~few GPUs
- O(100k-1M) copies



Lattice QCD action density, Leinweber, CSSM, Adelaide, 2003

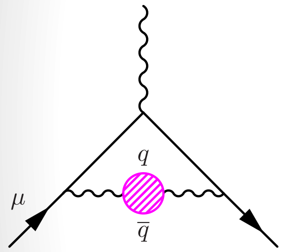
Hadrons are emergent phenomena of statistical average over background gluon configurations

- 1 year on supercomputer ~ 100k years on laptop

HVP from the lattice

&

Window observables

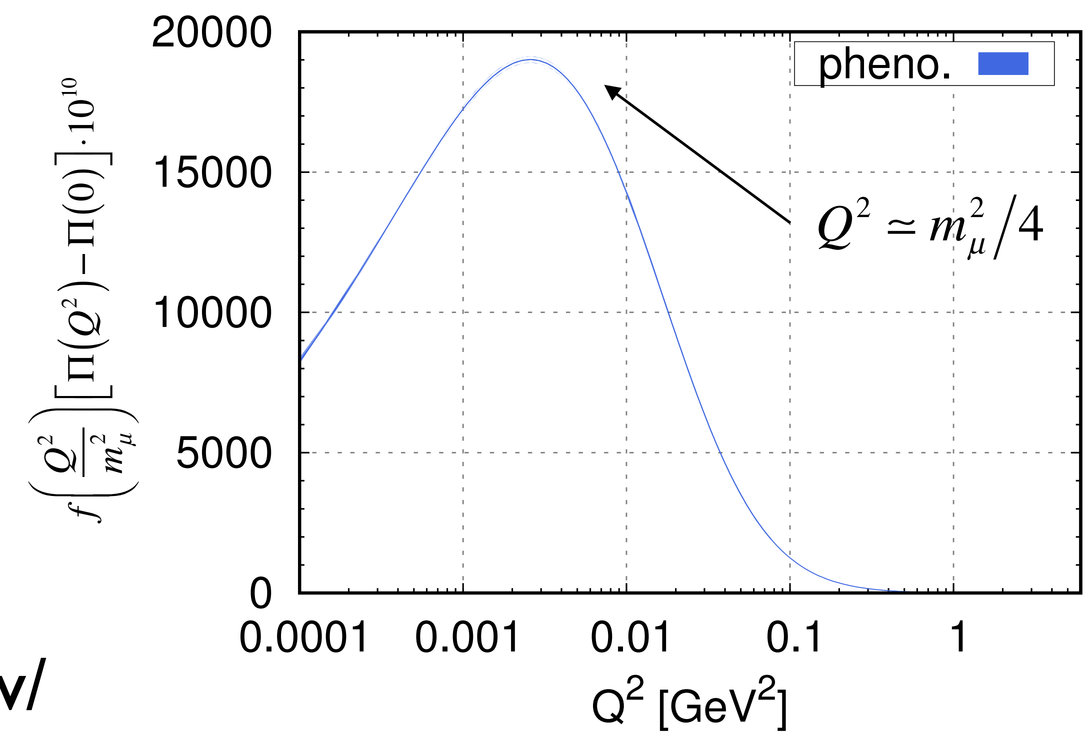


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \left\langle J_\mu(x) J_\nu(0) \right\rangle = \left[\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu\right] \Pi\left(Q^2\right)$$

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \, \frac{1}{m_\mu^2} f\left(\frac{Q^2}{m_\mu^2}\right) \left[\Pi\left(Q^2\right) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972



F. Jegerlehner, "alphaQEDc17"

FV & $a \neq 0$: A. discrete momenta ($Q_{\min} = 2\pi/T > m_\mu/2$); B. $\Pi_{\mu\nu}(0) \neq 0$ in FV contaminates $\Pi(Q^2) \sim \Pi_{\mu\nu}(Q)/Q^2$ for $Q^2 \to 0$ w/ very large FV effects; C. $\Pi(0) \sim \ln(a)$

Time-Momentum Representation

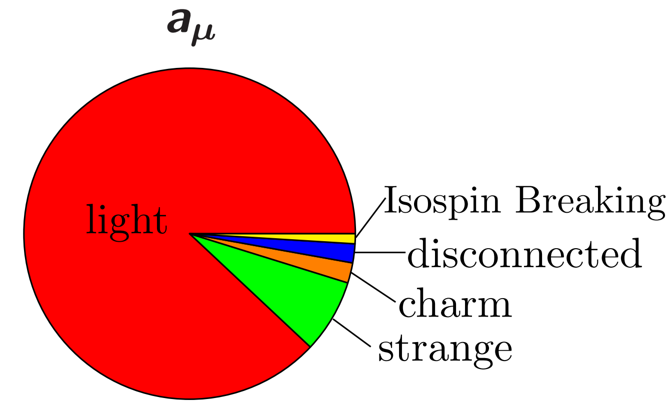
$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_0^\infty dt \; \tilde{f}(t)\, V(t)$$

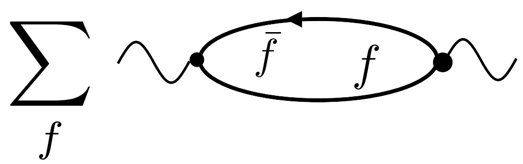
$$V(t) \equiv \frac{1}{3} \sum_{i=1,2,3} \int d\vec{x} \left\langle J_i(\vec{x},t) J_i(0) \right\rangle$$

D. Bernecker and H. B. Meyer, 2011

Time-Momentum Representation

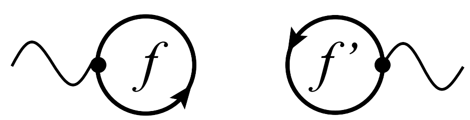
- No reliance on exp. data, except for hadronic quantities used to calibrate the simulation ($M_\pi, M_K, M_{nucl}, \ldots$)
- Can perform an explicit quark flavor separation of $a_\mu^{\mathrm{HVP,LO}}$

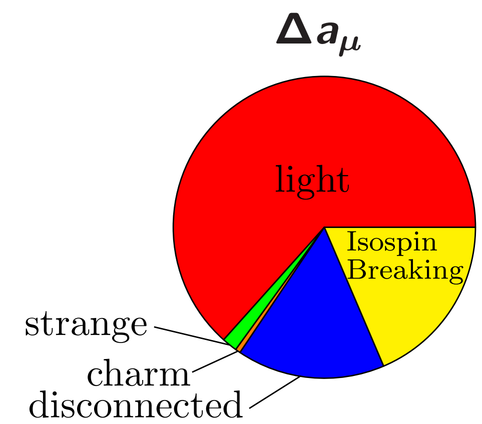




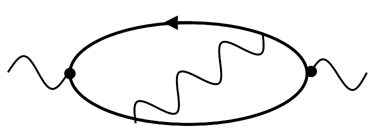
light-quark connected $a_\mu^{\mathrm{HVP,LO}}(\mathrm{ud}) \sim 90\,\%$ of total

s,c-quark connected $a_\mu^{\mathrm{HVP,LO}}(\mathrm{s,c}) \sim 8\,\%\,,\,2\,\%$ of total



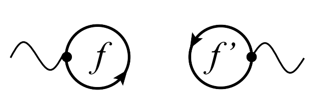
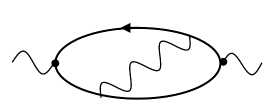


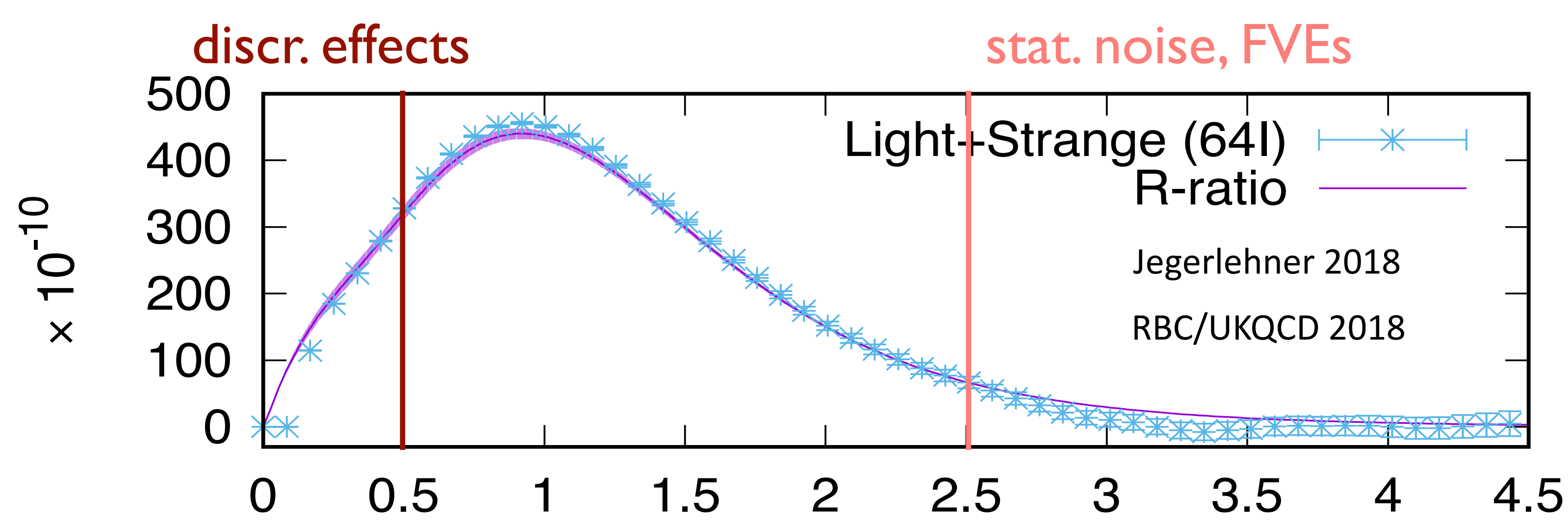
disconnected $a_{\mu,disc}^{\mathrm{HVP,LO}} \sim 2\,\%$ of total



IB ($m_u \neq m_d + \mathrm{QED}$) $\delta a_\mu^{\mathrm{HVP,LO}} \sim 1\,\%$ of total

Challenges:

- sub-percent stat. precision
  exp. growing StN ratio in $V(t)$ as $t \to \infty$
- correct for FVEs, control discr. effects (scale setting and continuum extrap.)
- quark-disconn. diagrams 
  control stat. & stochastic noise
- isospin-breaking: $m_u \neq m_d,\ \alpha_{em} \neq 0$ 



Windows "on the g-2 mystery"

Restrict integration over Euclidean time to sub-intervals

→ reduce/enhance sensitivity to systematic effects

$$a_\mu^{\rm HVP,LO} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f;t_0,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

$$a_\mu^{W}(f;t_0,t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f;t_1,\Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, \tilde{f}(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

"Standard" choice:

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm}$$

$$\Delta = 0.15 \text{ fm}$$

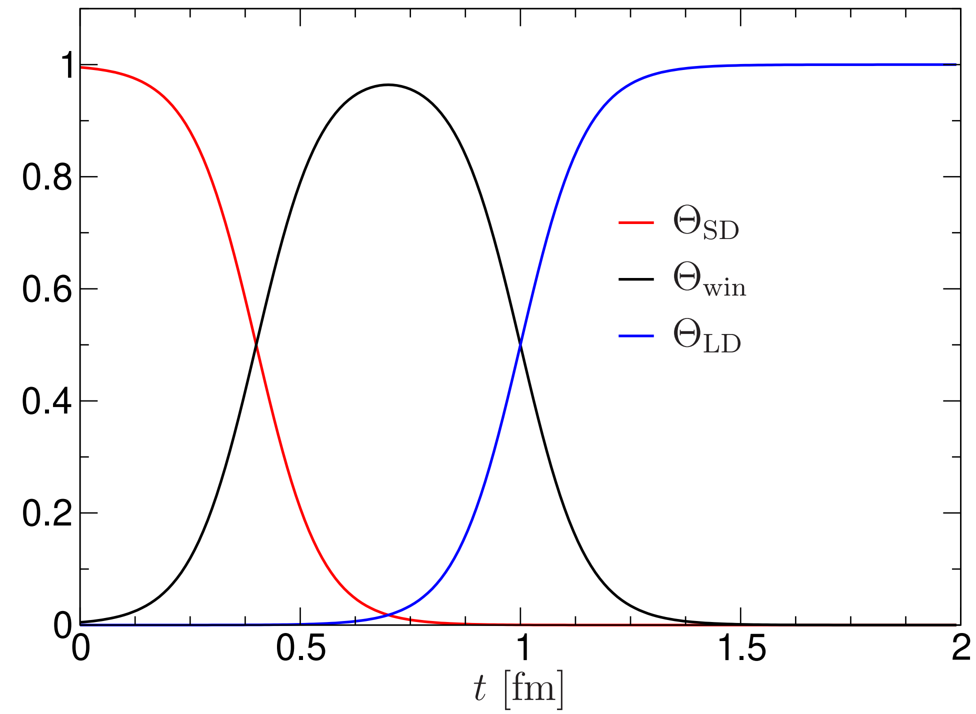
RBC/UKQCD 2018

Intermediate window

- Reduced FVEs
- Much better StN ratio

→ Precision test of different lattice calculations

→ Commensurate uncertainties compared to dispersive evaluations

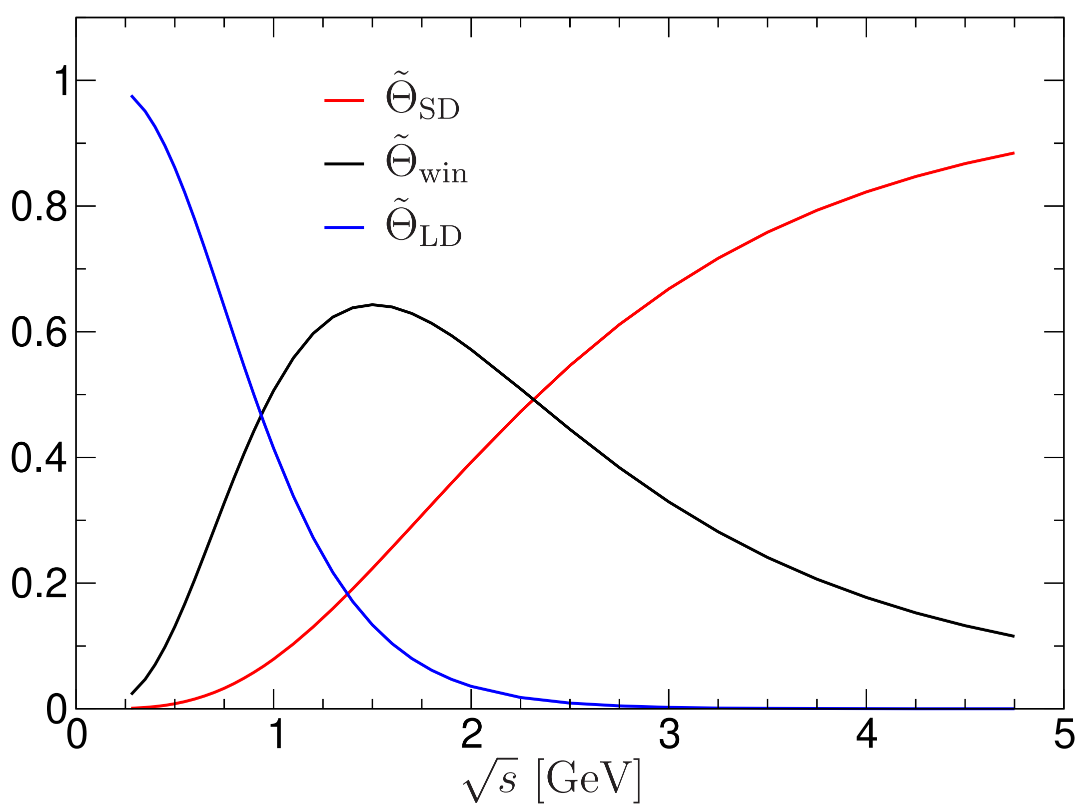


Comparison with R-ratio

$$V(t) = \frac{1}{12\pi^2}\int_{M_{\pi^0}}^{\infty} d(\sqrt{s})\, R(s)\, s\, e^{-\sqrt{s}t} \qquad R(s) = \frac{3s}{4\pi\alpha_{em}^2}\sigma(s, e^+e^- \to hadrons)$$

Insert $V(t)$ into the expression for TMR

$$a_{\mu,win}^{\mathrm{HVP,LO}} = 4\alpha_{em}^2 \int_{M_{\pi^0}}^{\infty} d(\sqrt{s})R(s)\, \frac{1}{12\pi^2}\, s \int_0^{\infty} dt\, \tilde{f}(t)\, \Theta_{win}(t)\, e^{-\sqrt{s}t}$$



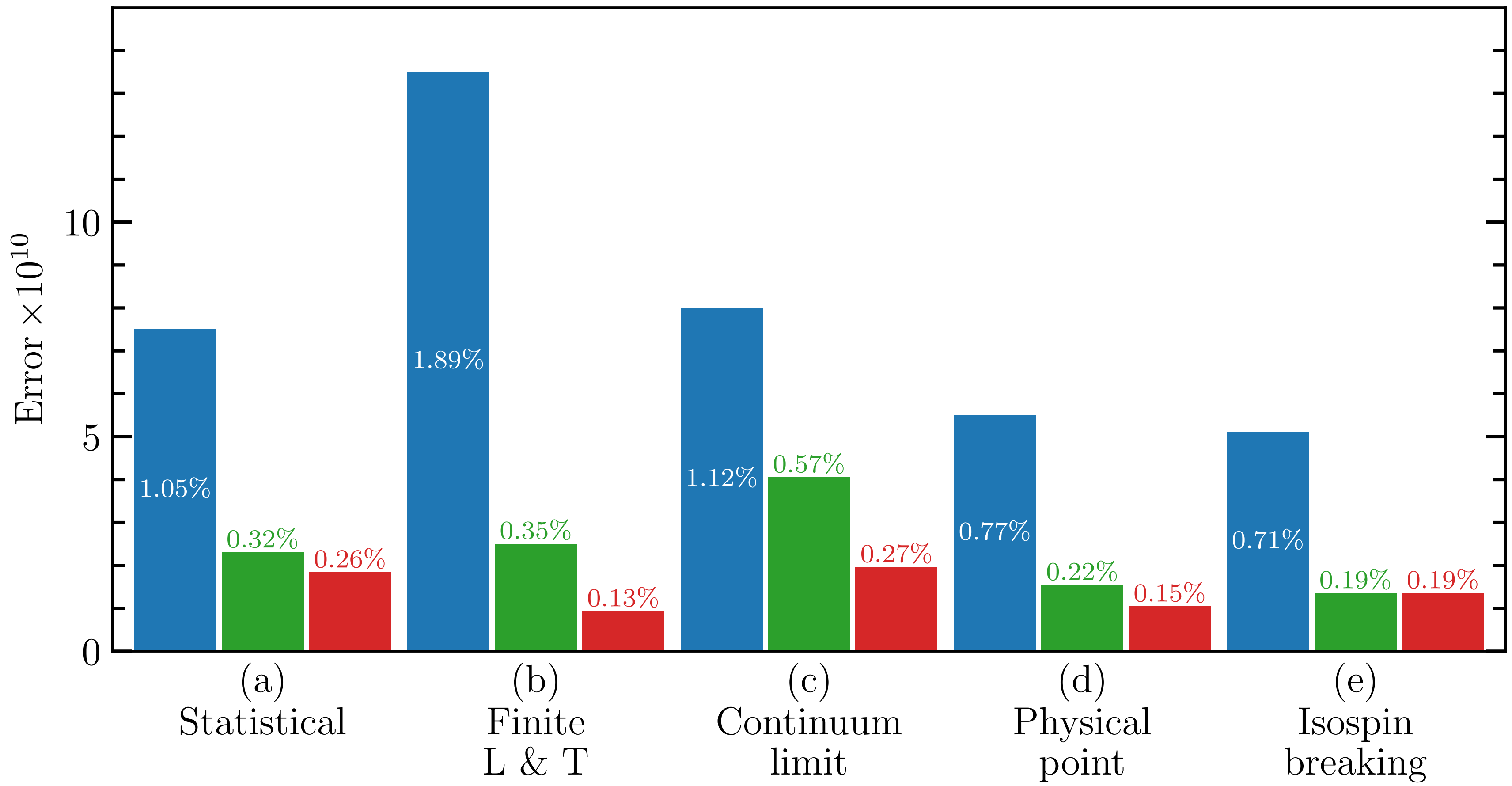
Colangelo *et al.* 2022

	$a_{\mathrm{SD}}^{\mathrm{HVP}}$	$a_{\mathrm{int}}^{\mathrm{HVP}}$	$a_{\mathrm{LD}}^{\mathrm{HVP}}$	$a_{\mathrm{total}}^{\mathrm{HVP}}$
All channels	68.4(5)	229.4(1.4)	395.1(2.4)	693.0(3.9)
	[9.9%]	[33.1%]	[57.0%]	[100%]
2π below 1.0 GeV	13.7(1)	138.3(1.2)	342.3(2.3)	494.3(3.6)
	[2.8%]	[28.0%]	[69.2%]	[100%]
3π below 1.8 GeV	2.5(1)	18.5(4)	25.3(6)	46.4(1.0)
	[5.5%]	[39.9%]	[54.6%]	[100%]
White Paper [1]	–	–	–	693.1(4.0)
RBC/UKQCD [24]	–	231.9(1.5)	–	715.4(18.7)
BMWc [36]	–	236.7(1.4)	–	707.5(5.5)
BMWc/KNT [7, 36]	–	229.7(1.3)	–	–
Mainz/CLS [99]	–	237.30(1.46)	–	–
ETMC [100]	69.33(29)	235.0(1.1)	–	–

BMW-DMZ '24 calculation

High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly

A. Boccaletti[1,2], Sz. Borsanyi[1], M. Davier[3], Z. Fodor[1,4,5,2,6,7,*], F. Frech[1], A. Gérardin[8], D. Giusti[2,9], A.Yu. Kotov[2], L. Lellouch[8], Th. Lippert[2], A. Lupo[8], B. Malaescu[10], S. Mutzel[8,11], A. Portelli[12,13], A. Risch[1], M. Sjö[8], F. Stokes[2,14], K.K. Szabo[1,2], B.C. Toth[1], G. Wang[8], Z. Zhang[3]



- New lattice spacing $a = 0.048$ fm (same cost as all of BMWc '20) $\longrightarrow$ divides a^2 effects by 2
- Over 30,000 gauge configurations, 10's of millions of measurements

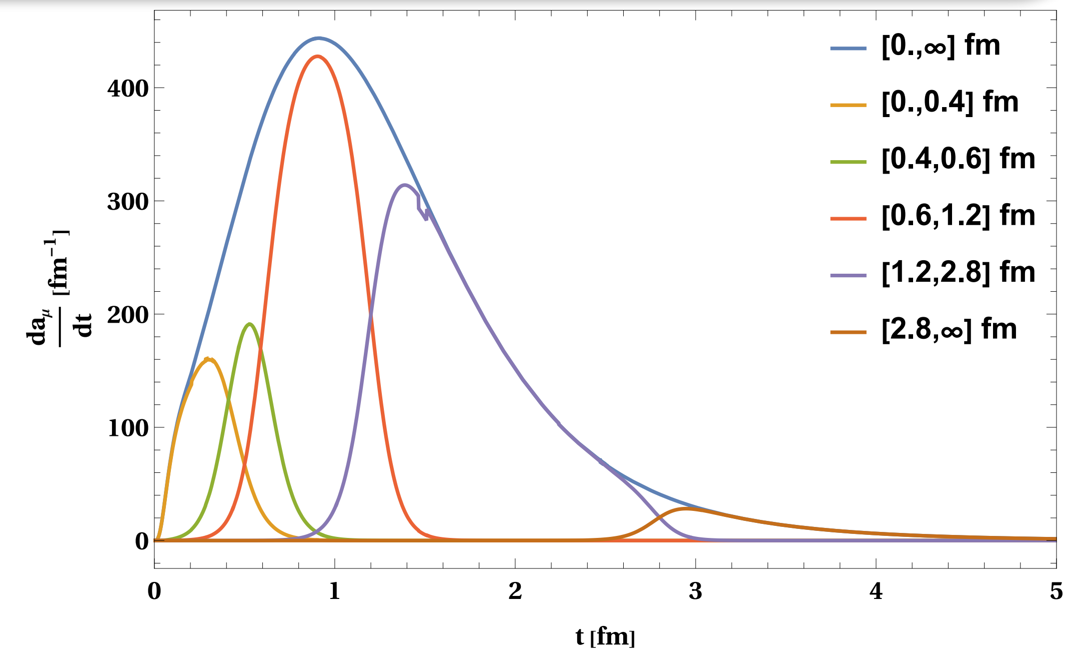
Strategy for improvement

- New simulations on finer lattice spacing: $128^3 \times 192$ w/ $a = 0.048\,\mathrm{fm}$
- Completely revamped analysis vs BMWc '20
- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$
- Combined fit to $a_{\mu,\text{win},04\text{-}06}^{\text{LO-HVP}}$, $a_{\mu,\text{win},06\text{-}12}^{\text{LO-HVP}}$, $a_{\mu,\text{win},12\text{-}28}^{\text{LO-HVP}}$
- Continuum extrapolate $I = 0$ instead of disconnected

$\to$ reduces statistical uncertainty
$\to$ reduces $a \to 0$ error

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ (proposed and used w/ $1\,\mathrm{fm} \to \infty$ [RBC/UKQCD '18])

$\to$ reduces FV effect $18.5(2.5) \to 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
$\to$ reduces LD noise
$\to$ reduces LD taste breaking and $a \to 0$ error



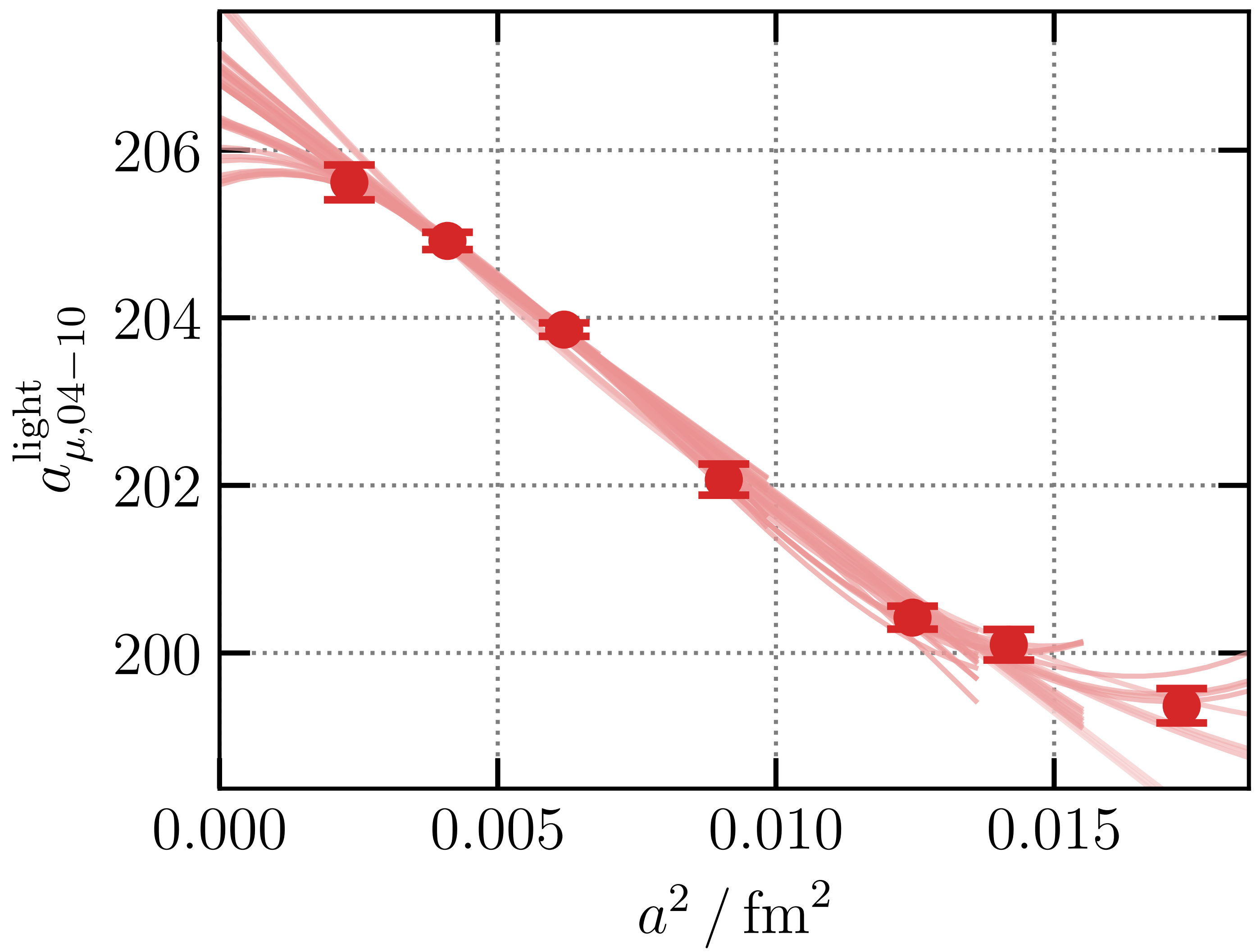
[plot made w/ KNT '18 data set]

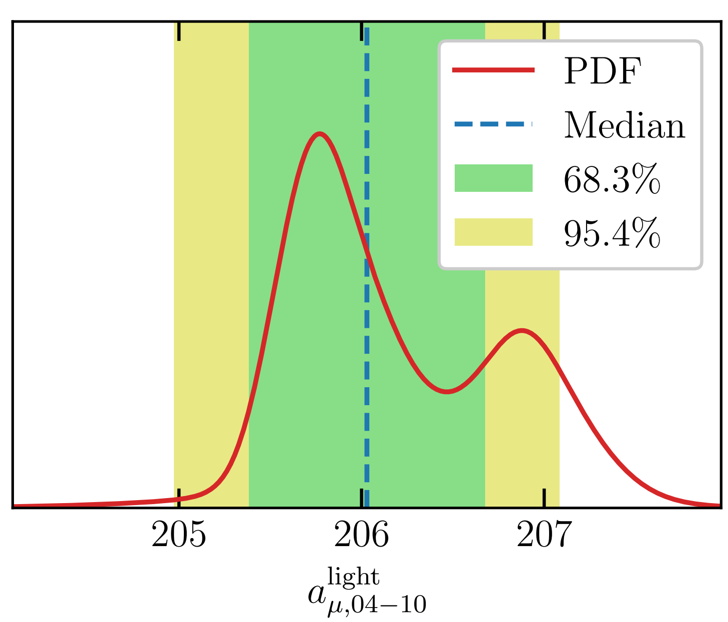
Fully blinded analysis:

- Independent blinding by factor $\pm 3\%$ on correlator for each window and component, including data-driven tail
- $\gtrsim 2$ independent analyses of all blinded $a_\mu^{\text{LO-HVP}}$ contributions (and of other aspects)
- Once agreement reached, partial unblinding to allow sum of contributions
- Full unblinding on July 12, 2024, w/ automatic script that made appropriate changes in all figures and text
- Paper submitted to arXiv on July 15, 2024

July 12, 2024: unblinding

ID window

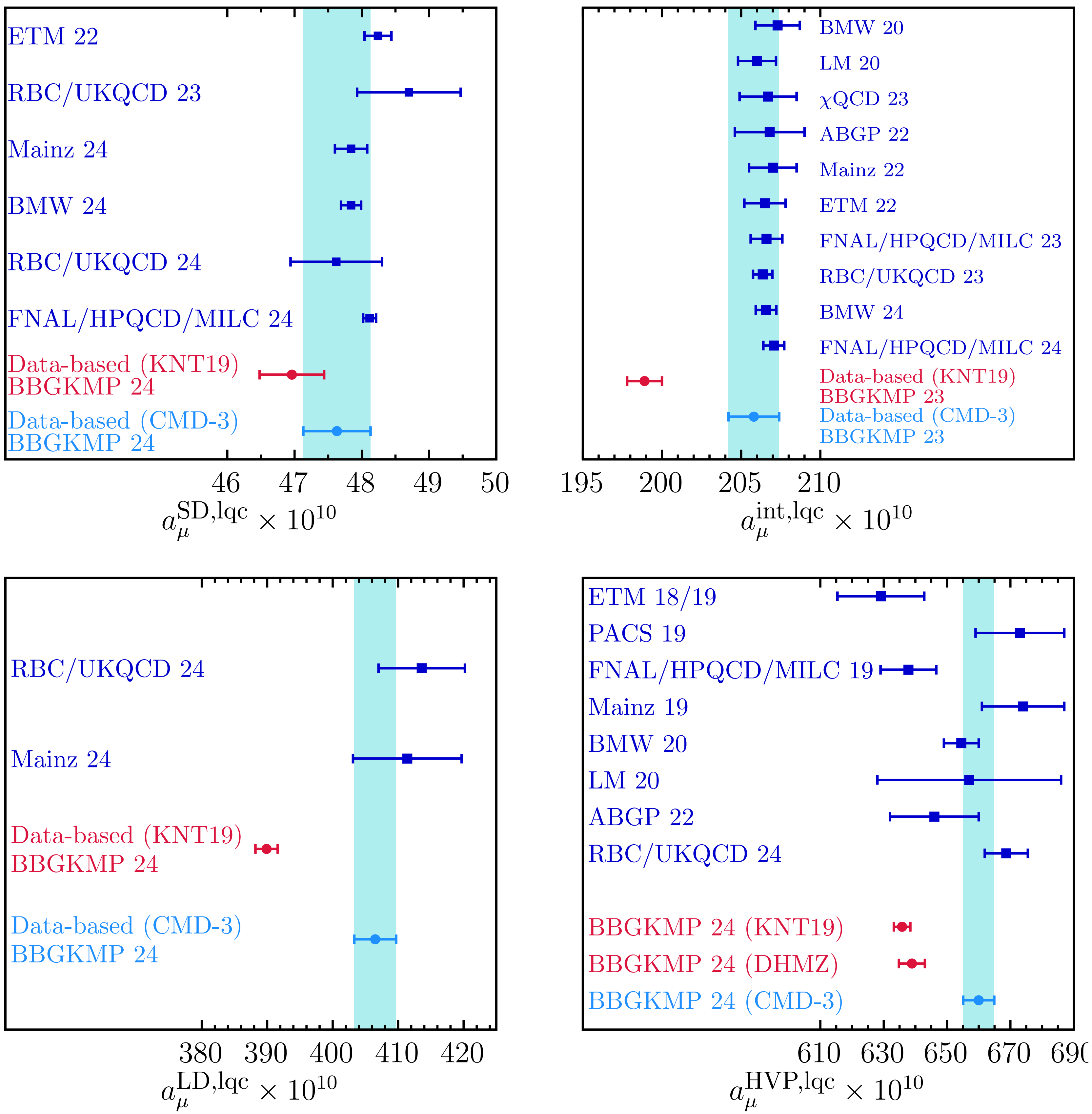




Median	206.03	
Total error	0.65	0.31 %
Statistical error	0.25	0.12 %
Systematic error	0.60	0.29 %
Pseudoscalar fit range	0.01	$<$ 0.01 %
Physical value of M_{ss}	0.01	$<$ 0.01 %
w_0 scale setting	0.21	0.10 %
Lattice spacing cuts	0.14	0.07 %
Order of fit polynomials	0.20	0.10 %
Continuum parameter (Δ_{KS} or a^2)	0.40	0.20 %

Other windows and comparison

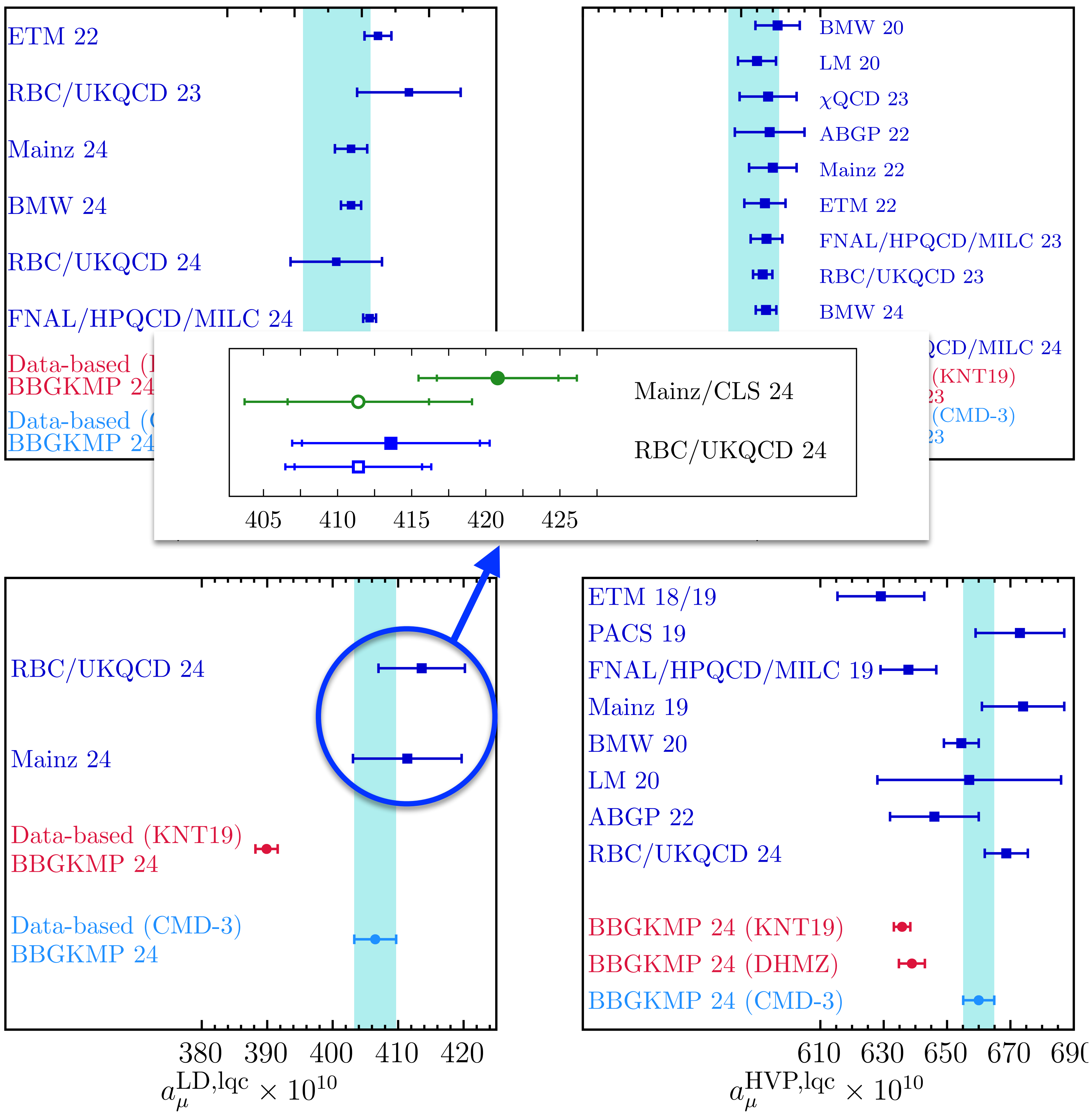
Benton *et al.* 2024



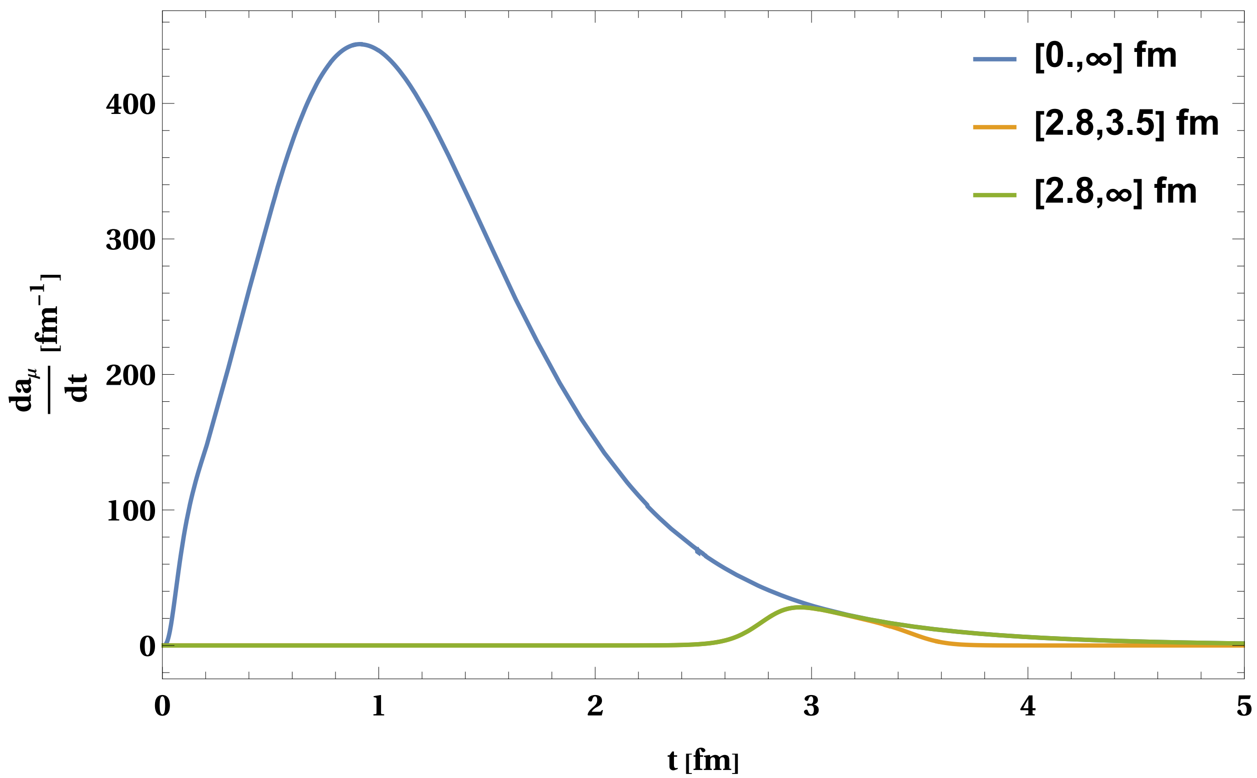
Other windows and comparison

Benton *et al.* 2024

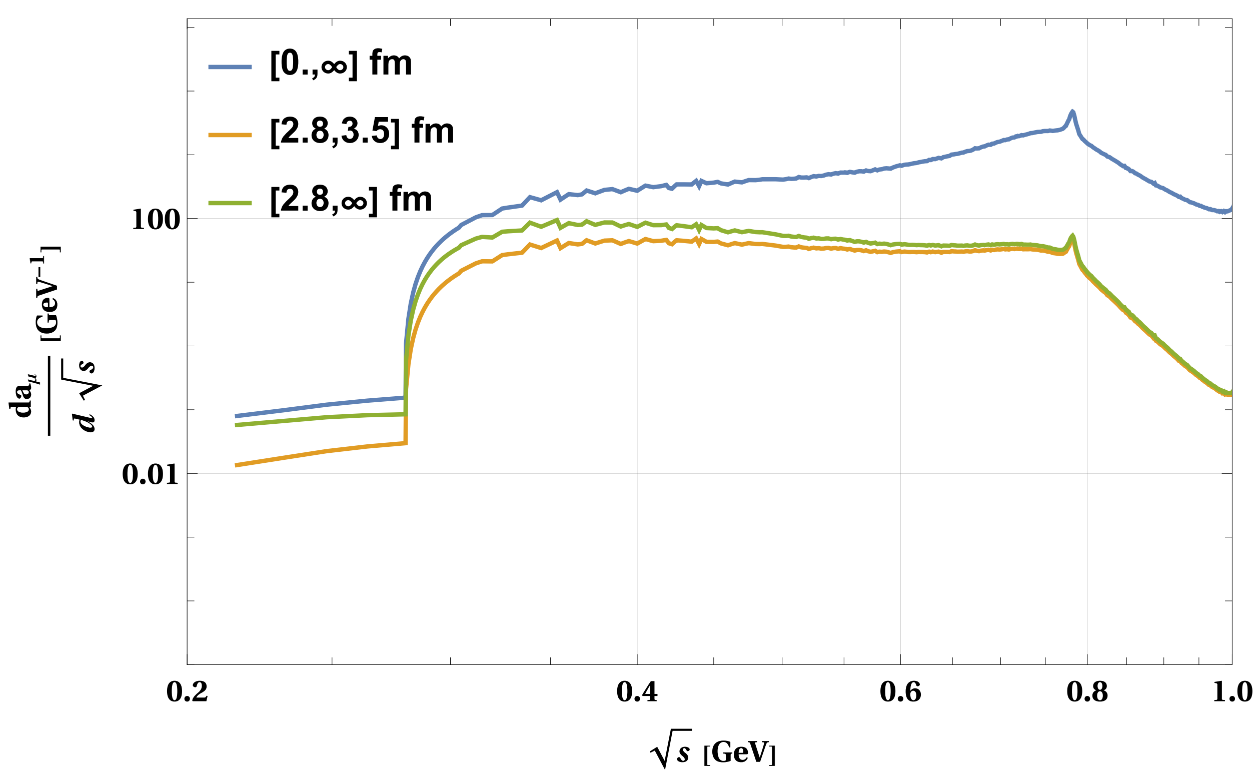
Mainz '24



Tail contribution



$\updownarrow$

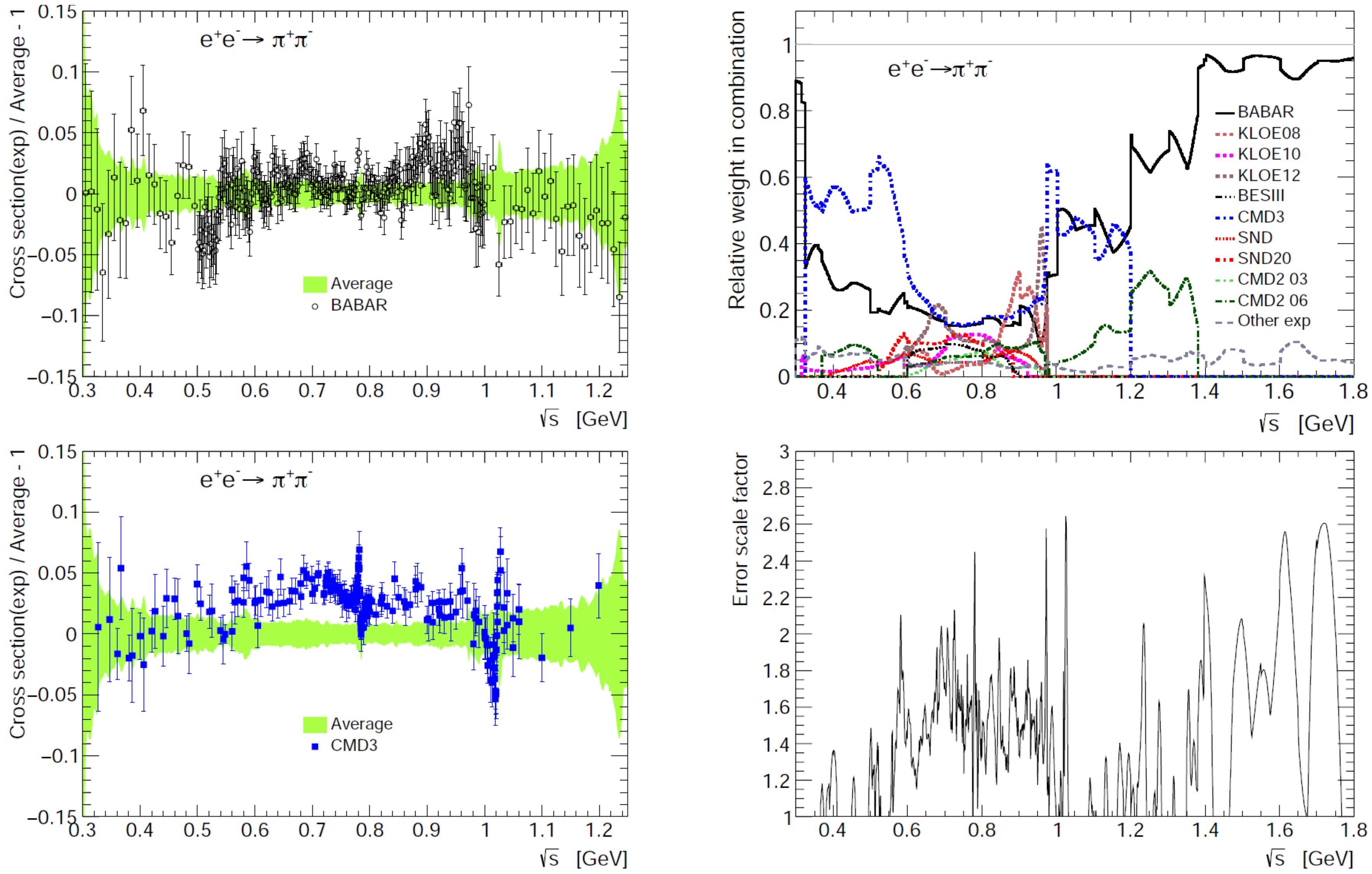


- Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ contributes $\lesssim 5\%$ to final result for a_μ
- Tail dominated by cross section below ρ peak: $\sim 75\%$ for $\sqrt{s} \leq 0.63\,\mathrm{GeV}$
- Partial tail $a_{\mu,28\text{-}35}^{\text{LO-HVP}}$ for comparison with lattice dominated by cross section below ρ peak: $\sim 70\%$ for $\sqrt{s} \leq 0.63\,\mathrm{GeV}$
- For small $\sqrt{s}$ possible radiative-correction issues are less pronounced [DHLMZ '23]
- Region well controlled by theory (χPT, analyticity, unitarity, ...) and other experimental constraints (e.g. $\langle r_\pi^2 \rangle$)

[plots made w/ KNT '18 data set]

Cross section and the tail

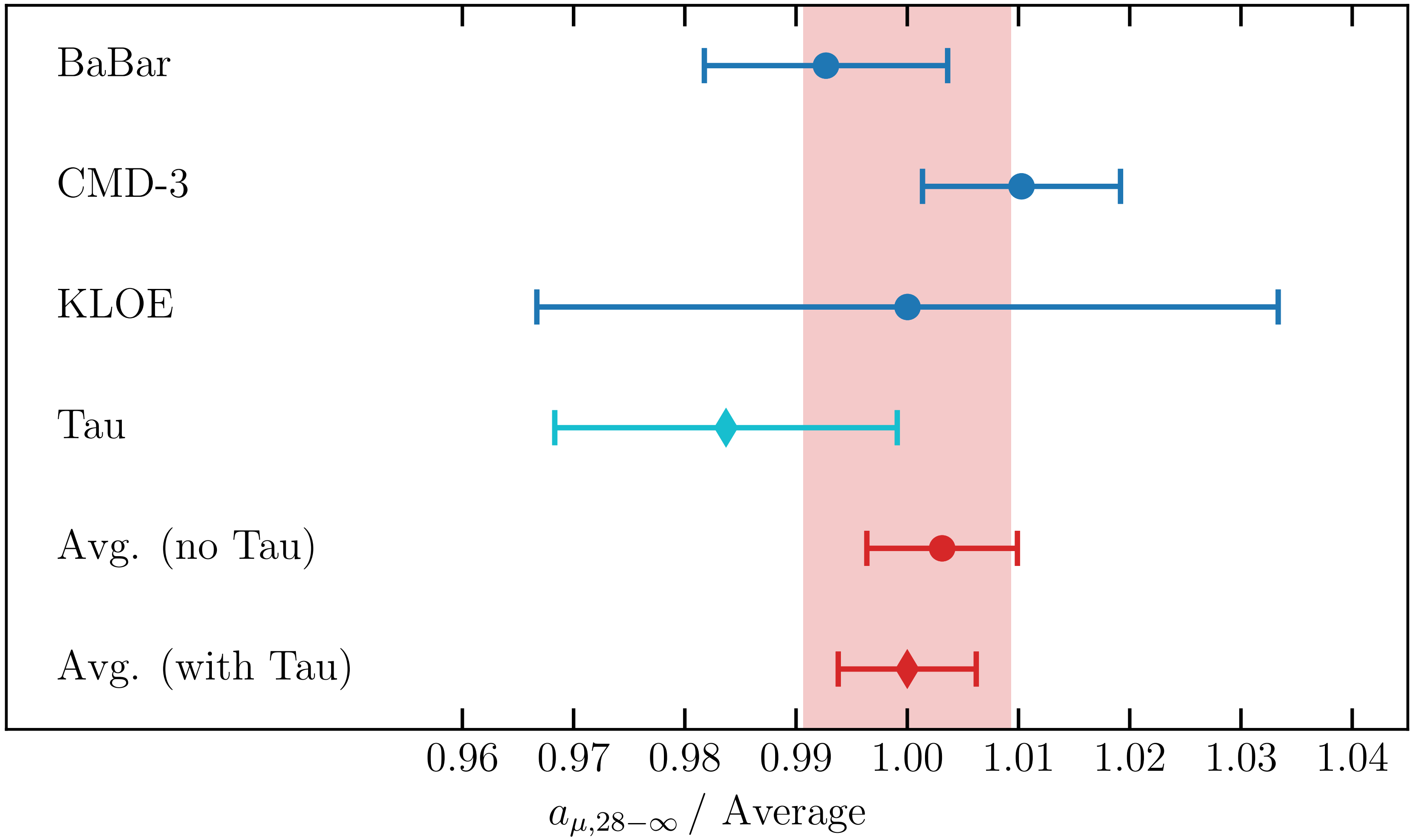
Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ dominated cross section below ρ peak: $\sim 75\%$ for $\sqrt{s} \leq 0.63\,\mathrm{GeV}$



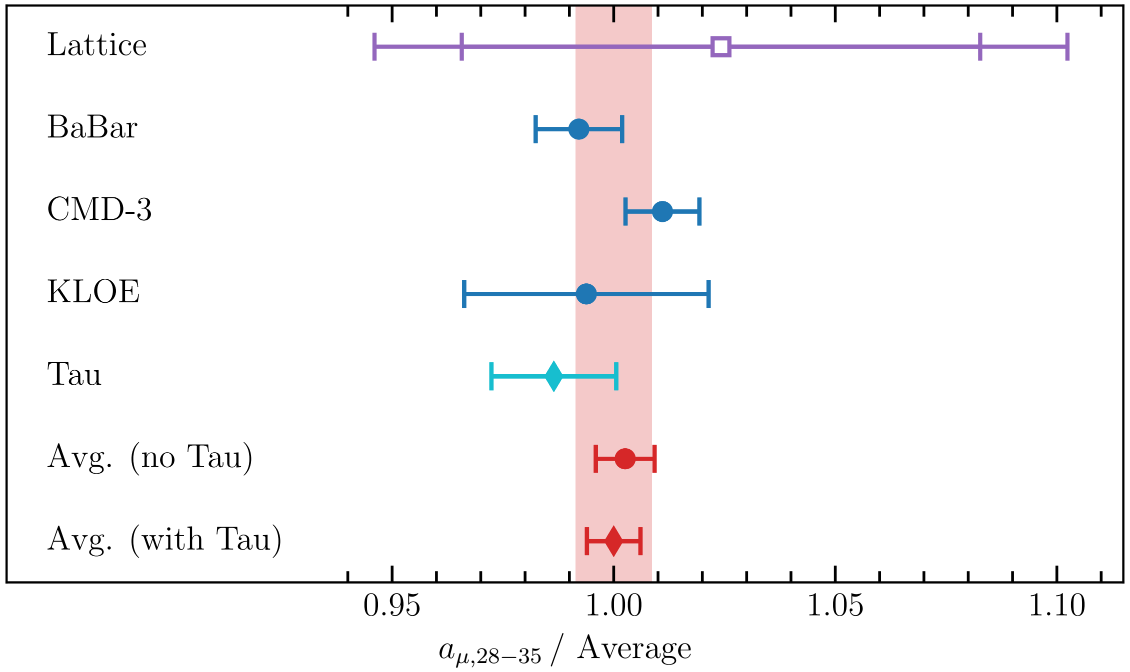
All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55\,\mathrm{GeV}$

$\Rightarrow$ tensions that plague $a_\mu^{\text{LO-HVP}}$ & $a_{\mu,\text{win}}^{\text{LO-HVP}}$ not present here

Data-driven tail



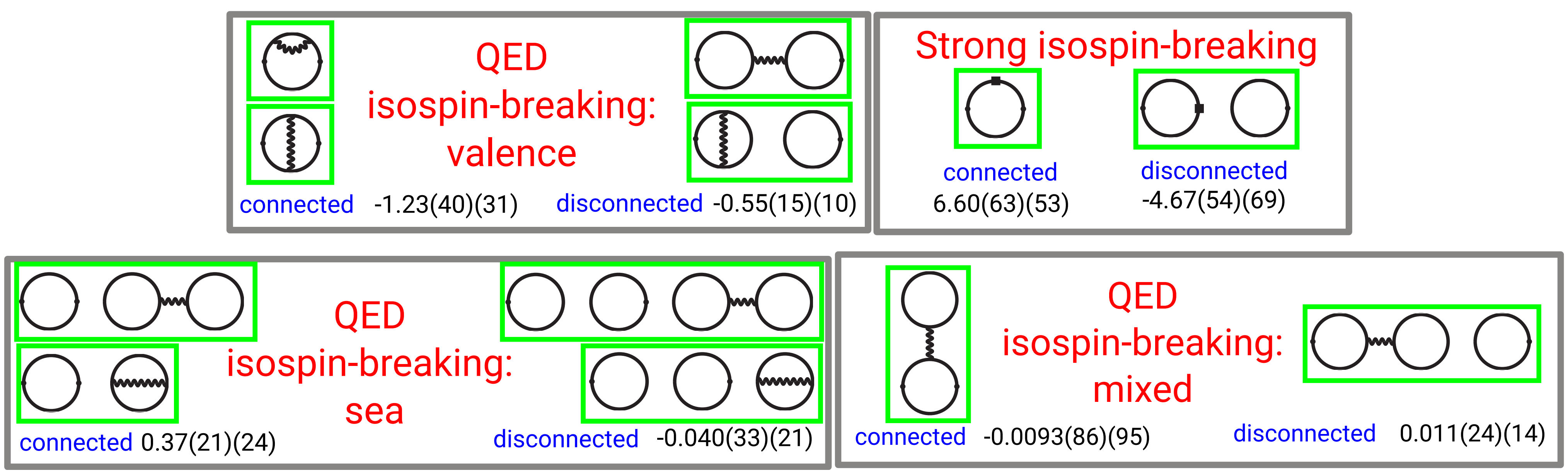
- Only $\lesssim$ 5% of final result for a_μ
- Contributes $\sim$ 65% to total squared uncertainty improvement: 5.5 $\rightarrow$ 3.3



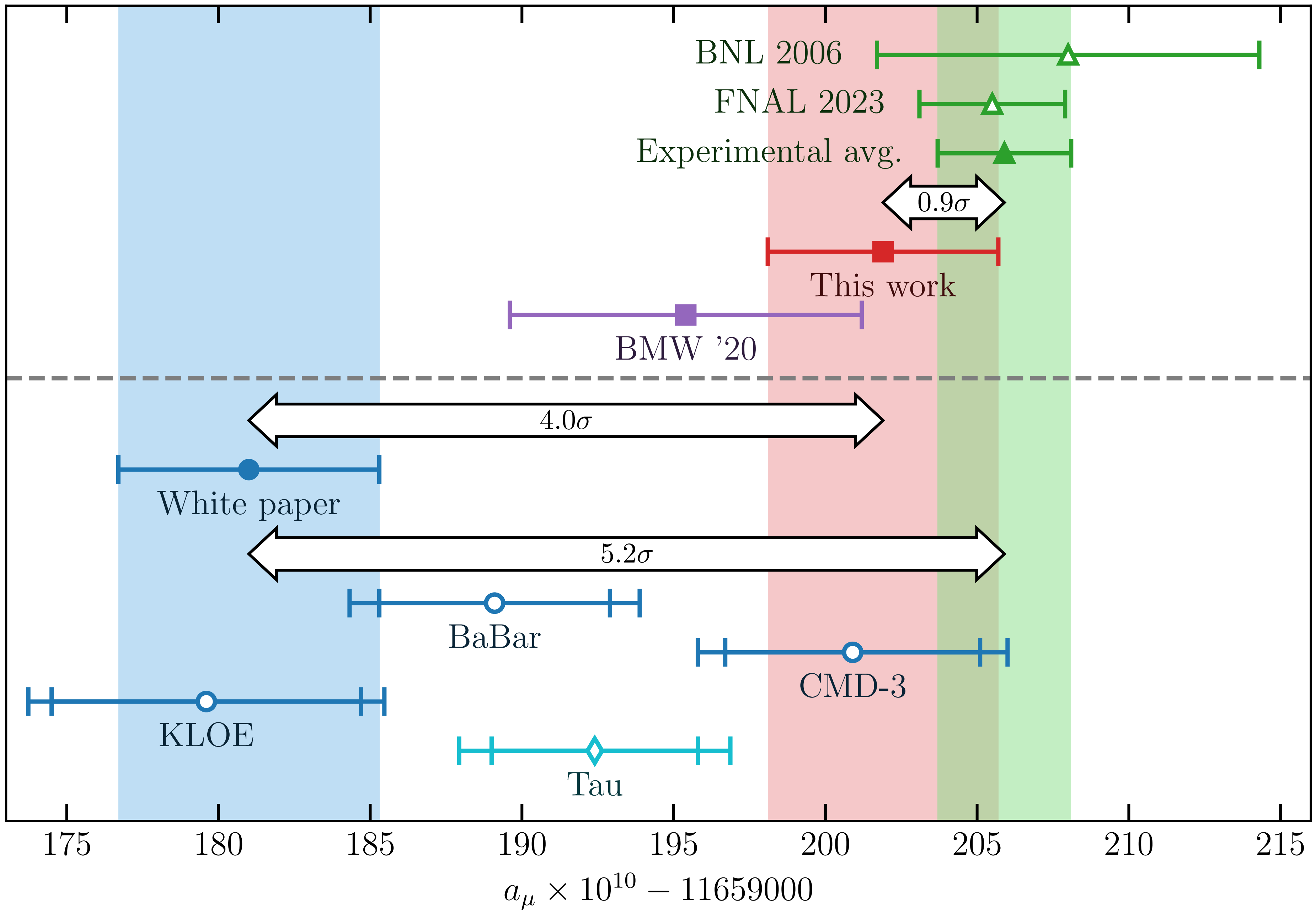
Summary of all contributions [BMW-DMZ ’24]

light and disconnected $00-28$	$618.6(1.9)(2.3)[3.0]$	this work, Equation (34)
strange $00-28$	$53.19(13)(16)[21]$	this work, Equation (37)
charm $00-28$	$14.64(24)(28)[37]$	this work, Equation (40)
light qed	$-1.57(42)(35)$	[5], Table 15 corrected in Equation (45)
light sib	$6.60(63)(53)$	[5], Table 15
disconnected qed	$-0.58(14)(10)$	[5], Table 15
disconnected sib	$-4.67(54)(69)$	[5], Table 15
disconnected charm	$0.0(1)$	[31], Section 4 in Supp. Mat.
strange qed	$-0.0136(86)(76)$	[5], Table 15
charm qed	$0.0182(36)$	[43]
bottom	$0.271(37)$	[44]
tail from data-driven $28-\infty$	$27.59(17)(9)[26]$	this work, Equation (50)
total	$714.1(2.2)(2.5)[3.3]$	

$$a_\mu^{\text{LO-HVP}} \times 10^{10} = 714.1(2.2)(2.5)[3.3] \quad [0.46\%]$$



BMW-DMZ '24 vs g-2 experiment



Indicates Standard Model confirmed to 0.37 ppm!

Podcast (generated by AI) on the current status of muon g-2:

https://drive.google.com/file/d/1aAi9CWSPVEYv2SMMxuGQT3l3KmEKGwKu/view?usp=d

Further connections

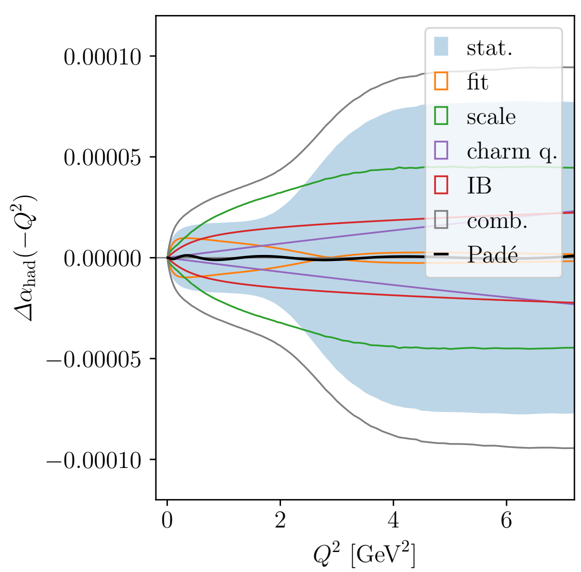
Hadronic running of α_{em} from the lattice

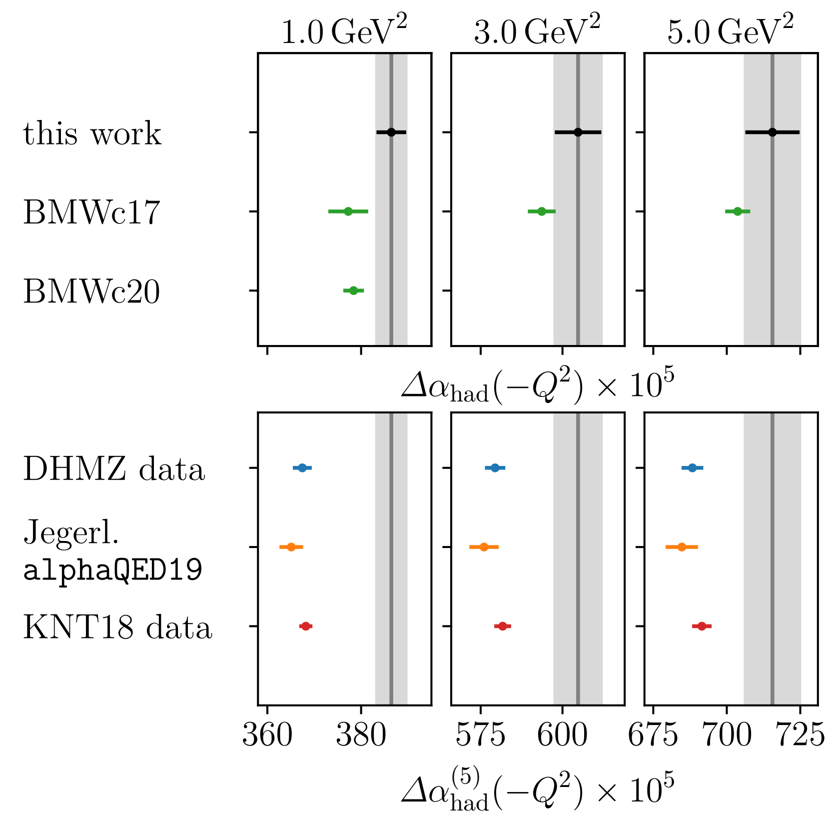
Lattice result for the hadronic running of α

[Cè et al., arXiv:2203.08676]

Starting point: Results for $\Delta\alpha_{\mathrm{had}}(-Q^2)$ for Euclidean momenta $0 \leq Q^2 \leq 7\,\mathrm{GeV}^2$ **[T. San José, TUE 17:10]**

Rational approximation:



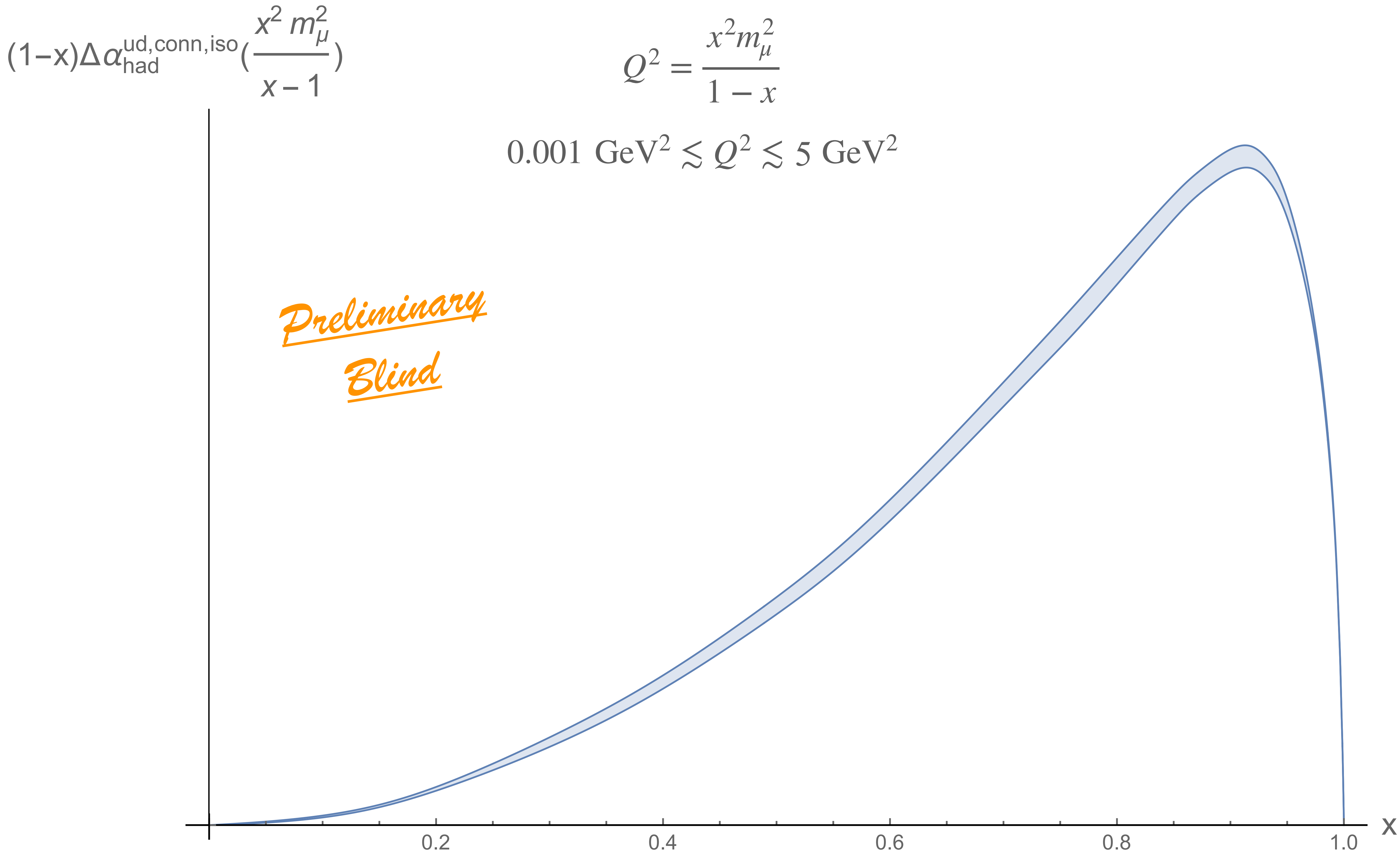


- Mainz/CLS and BMWc (2017) differ by 2–3% at the level of 1–2σ
- Tension between Mainz/CLS and phenomenology by $\sim 3\sigma$ for $Q^2 \gtrsim 3\,\mathrm{GeV}^2$
- Tension increases to $\gtrsim 5\sigma$ for $Q^2 \lesssim 2\,\mathrm{GeV}^2$ (smaller statistical error due to ansatz for continuum extrapolation)

Systematic uncertainties from fit ansatz, scale setting, charm quenching, isospin-breaking and missing bottom quark contribution (five flavour theory) included in error budget

Hadronic running of α_{em} from the lattice

talk by DG @ MUonE Workshop 2024



Isospin-breaking corrections in τ-decays

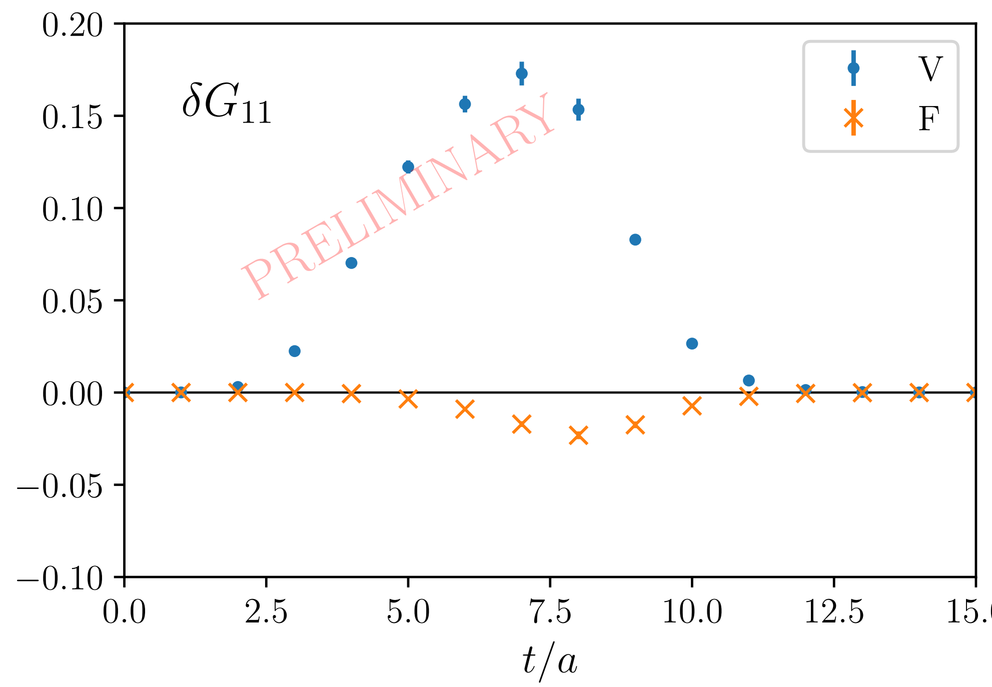
talk by M. Bruno @ Muon g-2 TI 2022

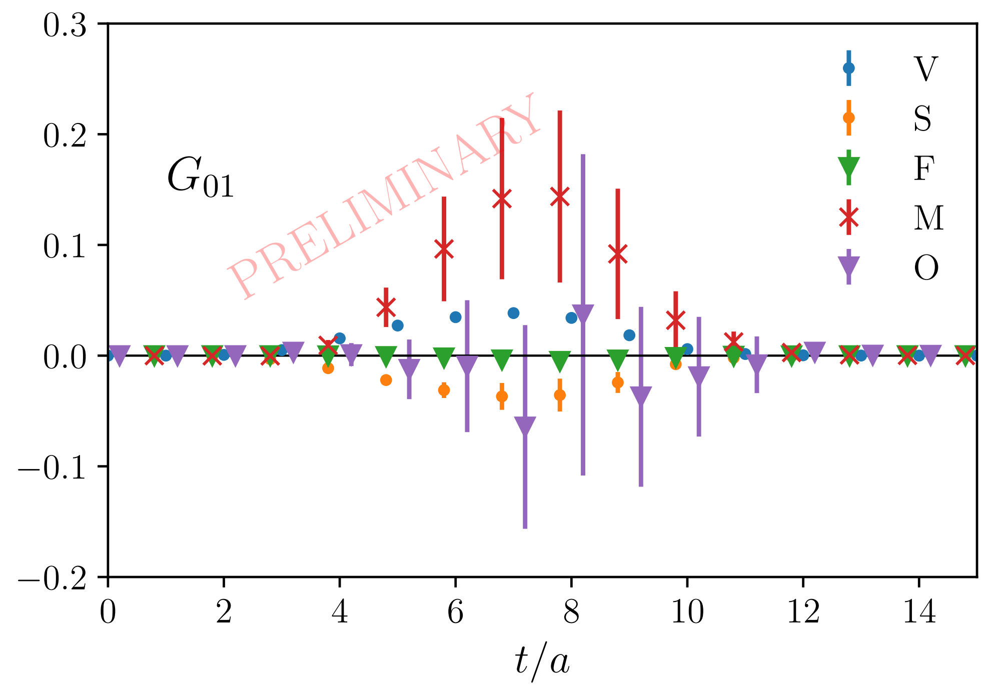
Results - Preliminary

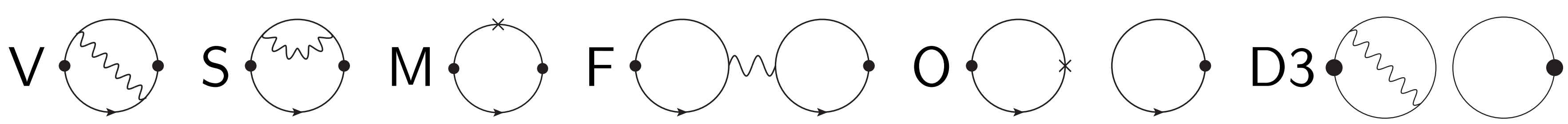
Preliminary from 48I ensemble

phys. pions, $a^{-1} \simeq 1.73$ GeV, 17 configs

cross-checks of code, data, analysis still missing

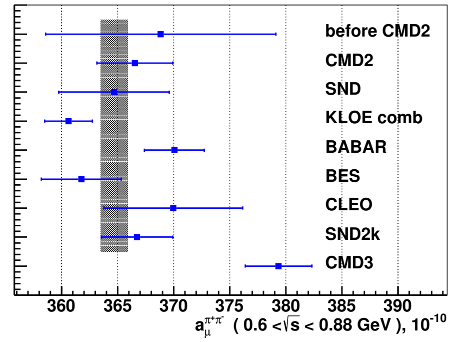






Summary and Outlook

- Tremendous progress in lattice calculations of HVP (and HLbL!) contributions
- New BMW-DMZ calculation to 0.46% w/ fully blinded analysis, confirming the SM to 0.37 ppm. It needs confirmation by other groups
- Good agreement between lattice calculations for various windows
- An update of the White Paper is aimed for the beginning of 2025
- Awaiting Fermilab ~ 0.1 ppm measurement of a_μ in 2025 and J-PARC entirely new method measurement
- Awaiting new BaBar, KLOE, BESIII, Belle II, CMD3, SND2 data/analysis to clarify tensions in $\pi^+\pi^-$

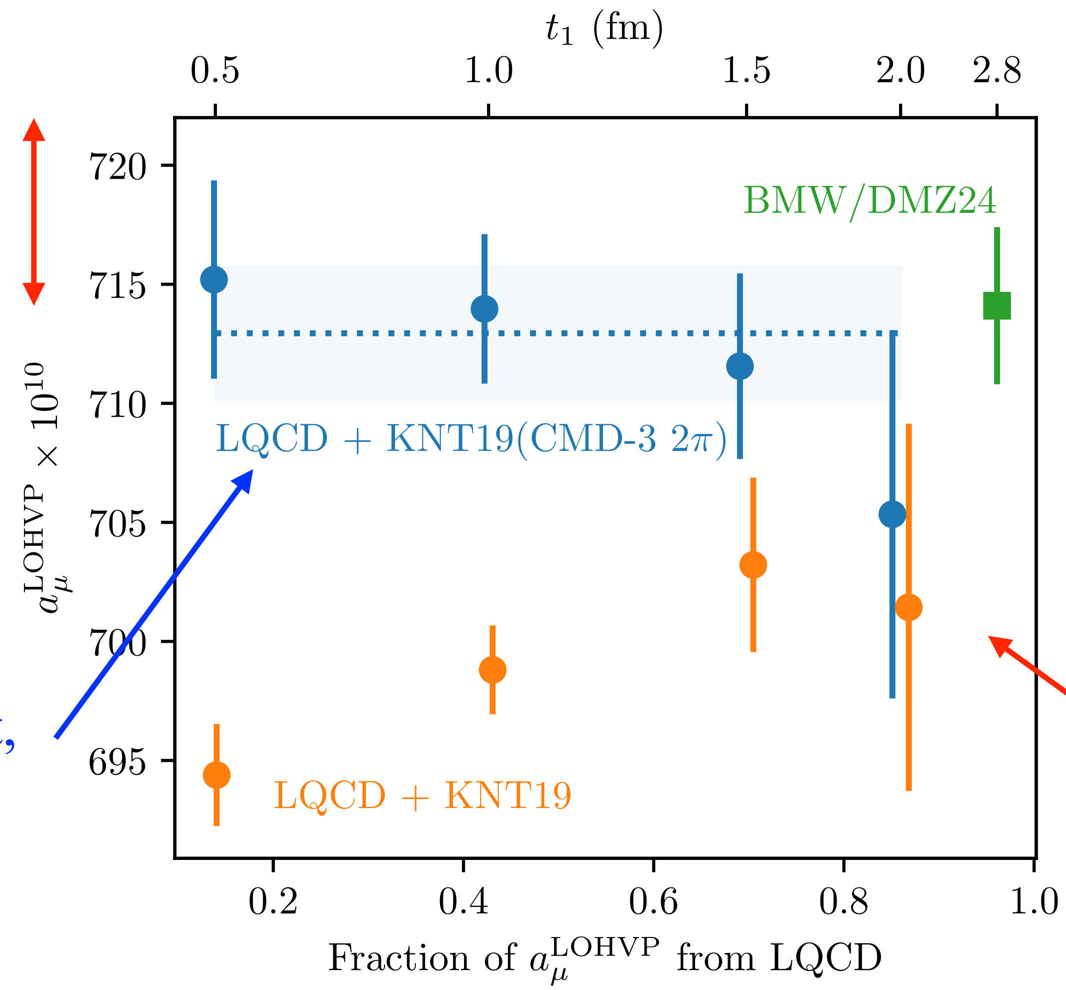


- $\mu e \to \mu e$ experiment MUonE very important for experimental cross-check and complementarity w/ LQCD talk by C. Davies @ Lattice 2024

Pragmatic hybrid strategy for further full HVP results

Thanks to A. Keshavarzi and P. Lepage

No new physics in muon g-2



Fit to a constant, χ^2/dof =0.6

Use LQCD in one-sided time window up to t_1 .
Add in data-driven result for t_1 to ∞.

Totals should agree for different t_1
- test of validity of data-driven (and LQCD)
- choose smallest error or fit to a constant

Using 2019 FHM LQCD results for one-sided windows (2207.04765):
- totals are flat in t_1 for CMD3 2π
- total w. CMD-3 agrees with BMW/DMZ '24 for all values of t_1
- newer lattice data have much better uncertainties for $t_1 \gtrsim 2$fm

Smaller t_1 : reduces lattice stat. and finite vol. error but increases input from data-driven tail

Hybrid strategy best to optimise uncertainty on total HVP?

Larger t_1 : CMD3/KNT19 tension falls: <0.3% total HVP for $t_1 \geq 2.5$ fm