

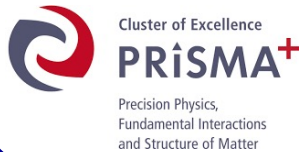


UNIVERSITY OF
LIVERPOOL

*3rd Liverpool Workshop on
Muon Precision Physics,
November 12-14, 2024*



Status of hadronic cross section experiments at low-energy e^+e^- colliders



*Achim Denig
Institute for Nuclear Physics
Johannes Gutenberg University Mainz*

Hadronic Cross Section and Hadronic Vacuum Polarization

Hadronic vacuum polarization

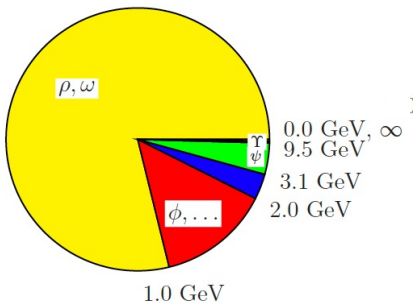
Anomalous magnetic moment of the muon $(g-2)_\mu$

Running electromagnetic fine structure constant

$$a_\mu^{HVP} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_{had}(s)$$

$$\alpha_{em}(M_Z^2) = \frac{1}{1 - \Delta\alpha(M_Z^2)};$$

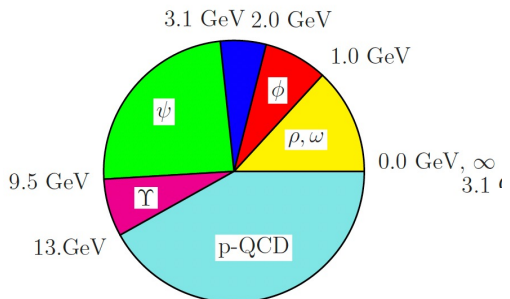
$$\Delta\alpha_{had}^{(5)}(M_Z^2) \sim \int_{4\pi^2}^{\infty} ds \frac{R_{had}(s)}{s(s - M_Z^2)}$$



→ relevant mass range < 2...3 GeV

$$\sigma_{had}(s) = \sigma_{tot}(e^+e^- \rightarrow \text{Hadrons})$$

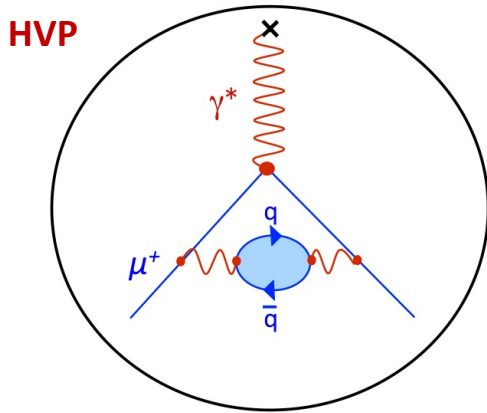
$$R_{had} = \frac{\sigma_{had}(s)}{\sigma_{ee \rightarrow \mu\mu}(s)}$$



→ relevant mass range < 13 GeV

Hadronic Vacuum Polarization (HVP) for $(g-2)_\mu$ from dispersive Analysis

$$a_\mu^{SM} = 11\,659\,181.0 (4.3) \times 10^{-10}$$



Estimate of $(g-2)$ Theory Initiative
based on dispersive approach
(including higher orders):

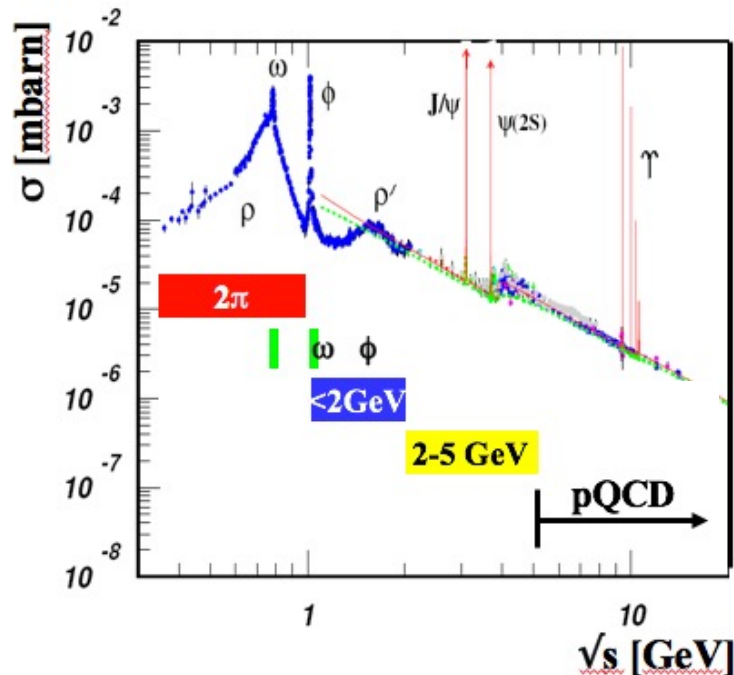
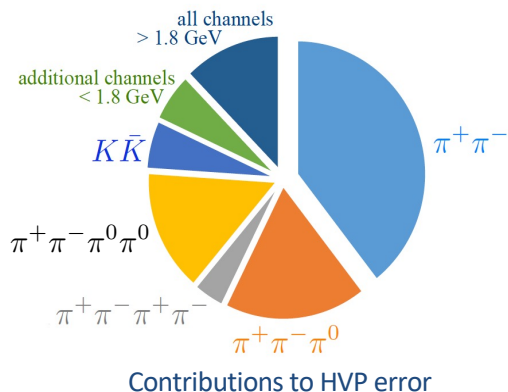
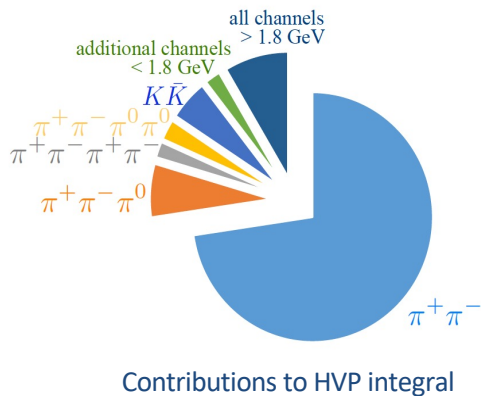
$$(693.1 \pm 4.0) \cdot 10^{-10}$$

was $(\cong 687 \dots 694 \pm 2.4 \dots 4.1) \cdot 10^{-10}$

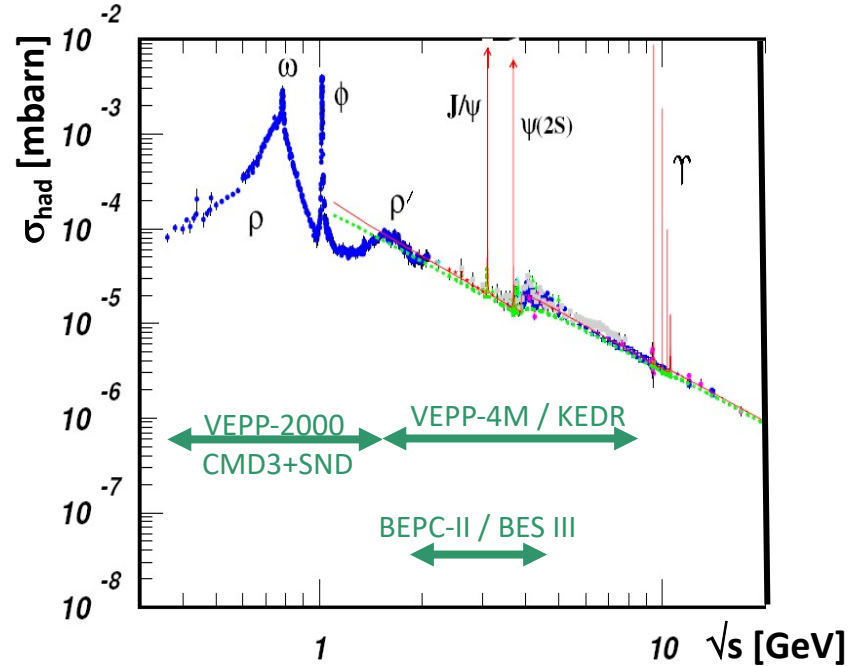
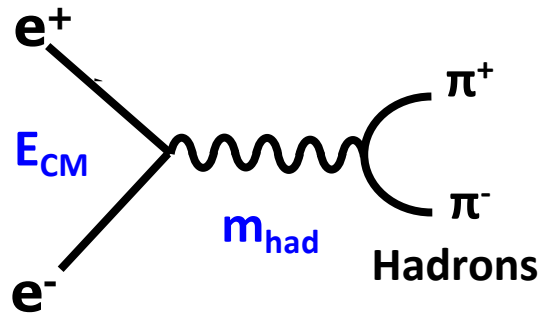
Hadronic Vacuum Polarization Contribution to $(g-2)_\mu$

Anomalous magnetic moment of the muon $(g-2)_\mu$

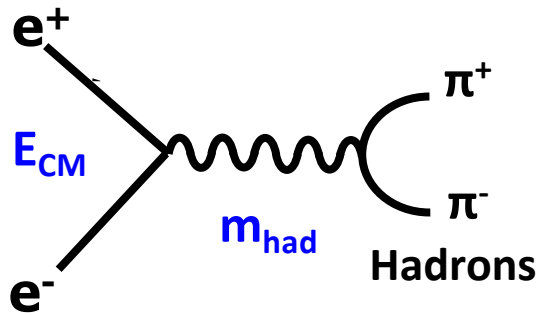
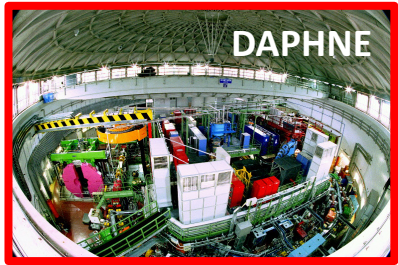
$$a_\mu^{HVP} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$



Measurements on R – Energy Scan vs. Initial State Radiation

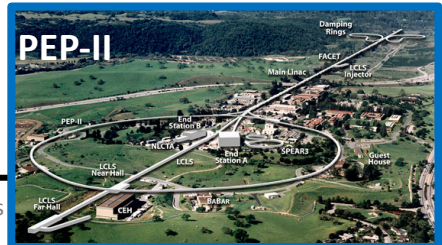
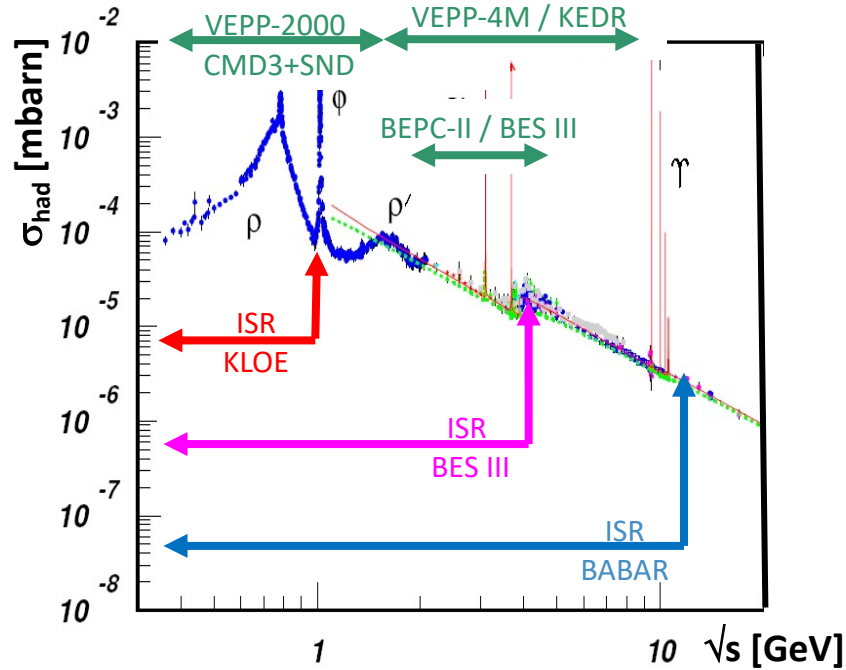


Measurements on R – Energy Scan vs. Initial State Radiation

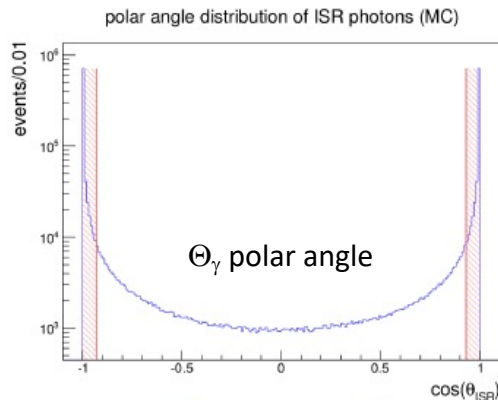


- No systematic variation of E_{beam}
- High statistics thanks to high luminosity
- Radiative corrections (H_{rad})

PHOKHARA event generator



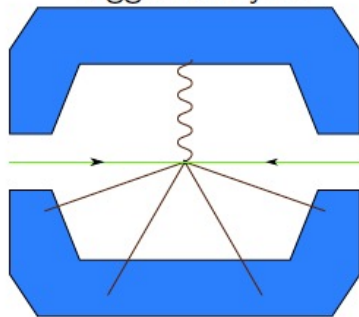
Initial State Radiation – tagged vs. untagged



Tagged analysis:
ISR photon measured in Calorimeter

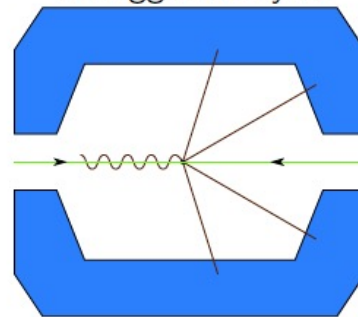
Untagged analysis:
No ISR detection; cut on missing momentum

Tagged analysis



- + exclusive reconstruction
- increased background
- reduced statistics
- + mass range $\sqrt{s'} < E_{CM}$

Untagged analysis



- + reduced background
- + very high statistics (x5)
- mass range $E_{th} < \sqrt{s'} < E_{CM}$
- KLOE: $E_{th} = \sim 0.6$ GeV
- BESIII: $E_{th} = \sim 1$ GeV
- BABAR: $E_{th} = \sim 3$ GeV

Two independent normalization methods:

1) normalization to L_{int} (obtained from Bhabha events) and H_{rad} ; subtraction of background ($\mu+\mu-\gamma$, ...)

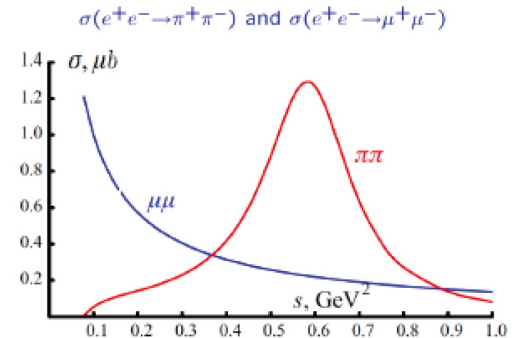
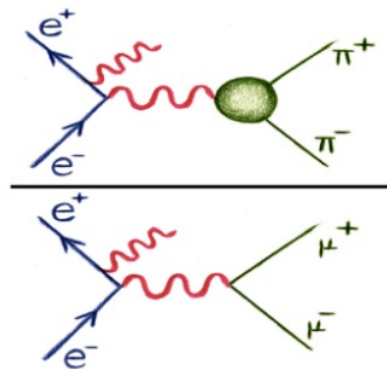
$$\sigma_{bare}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{N_{\pi\pi\gamma}/\epsilon_{exp}}{L_{int} \cdot H_{rad} \cdot \delta_{vac} \cdot (1 + \delta_{FSR})}$$

2) normalization to $\mu+\mu-\gamma$ events, i.e. R ratio ($\pi\pi\gamma/\mu\mu\gamma$)

→ L_{int} , H_{rad} , δ_{vac} cancel in ratio!

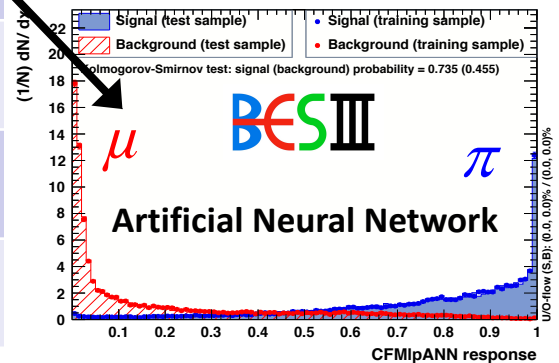
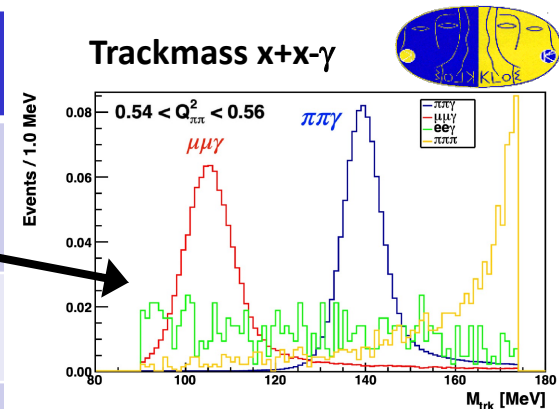
→ requires high statistics of $\mu+\mu-\gamma$

$$R = \frac{N_{\pi^+\pi^-}}{N_{\mu^+\mu^-}} \cdot \frac{\epsilon_{\mu^+\mu^-} \cdot (1 + \delta_{\mu^+\mu^-}^{FSR})}{\epsilon_{\pi^+\pi^-} \cdot (1 + \delta_{\pi^+\pi^-}^{FSR})}$$



Overview Experiments

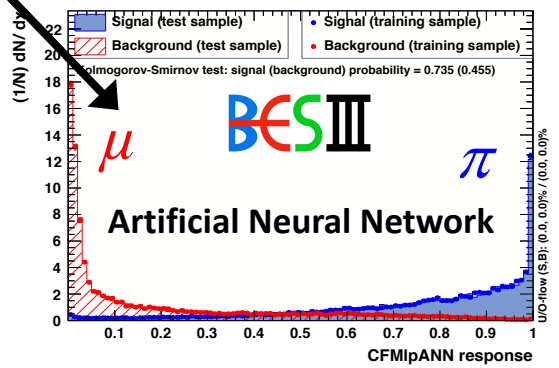
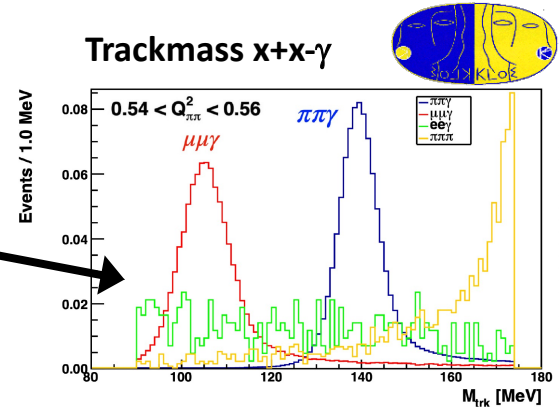
Experiment	Published Method	Normalization	Separation $\pi - \mu$
KLOE $\sqrt{s} \sim 1$ GeV	ISR untagged ISR tagged ISR untagged	Luminosity + H_{rad} Luminosity + H_{rad} $\mu + \mu - \gamma$	Kinematics Track Kinematics Track Kinematics Track
BABAR $\sqrt{s} \sim 10$ GeV	ISR tagged	$\mu + \mu - \gamma$	Particle ID
BESIII $\sqrt{s} \sim 4$ GeV	ISR tagged	Luminosity + H_{rad}	Particle ID (ML)
BELLE-II $\sqrt{s} \sim 10$ GeV			
CMD-2/CMD-3	Scan $< \sim 1$ GeV	$e + e -$	Kinematics Track Kinematics EMC
SND	Scan $< \sim 1$ GeV	$e + e -$	Kinematics EMC



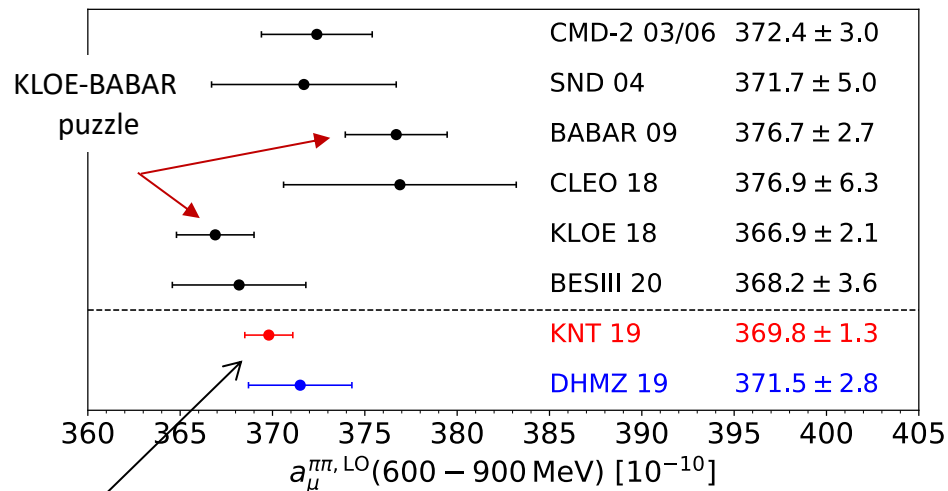
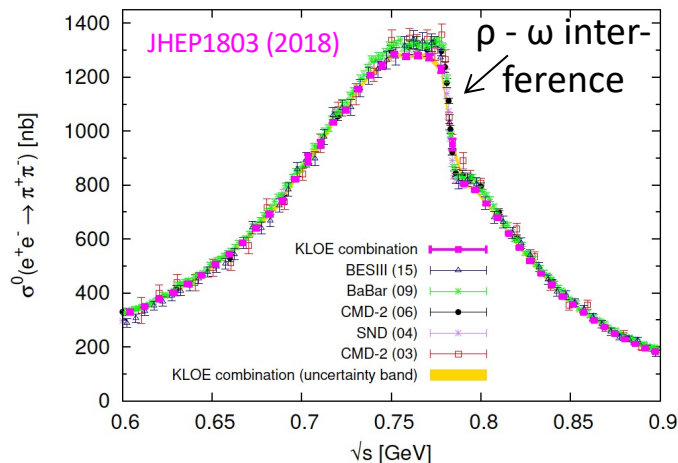
Overview Experiments

Diverse set of methods, cms energies, normalizations, selections

Experiment	Publication	Normalization	Separation $\pi - \mu$
KLOE	ISR tagged	Luminosity + H_{rad}	Kinematics Track Kinematics Track Kinematics Track
BES-III	ISR tagged	Luminosity + H_{rad}	Particle ID
BELLE-II $\sqrt{s} \sim 10$ GeV	ISR tagged	Luminosity + H_{rad}	Particle ID (ML)
CMD-2/CMD-3	Scan $< \sim 1$ GeV	e^+e^-	Kinematics Track Kinematics EMC
SND	Scan $< \sim 1$ GeV	e^+e^-	Kinematics EMC



Most relevant Channel: $e^+e^- \rightarrow \pi^+\pi^-$ (until 2023)



Systematic Uncertainties on $\rho(770)$ peak

- ISR BABAR 0.5%
- ISR KLOE 0.6%
- ISR BESIII 0.9%
- Energy Scan CMD-2 0.8%*

* limited in addition by statistics

Most recent evaluations of HVP:

- Davier, Höcker, Malaescu, Zhang (DHMZ)
 - averaging via 2nd ord. polynomial interpolation
 - systematic correlat. propagated via pseudo-data (MC)
- Keshavarzi, Nomura, Teubner (KNT)
 - data subjected to a clustering procedure
 - fit over all data sets taking into account correlations

2020 Whitepaper Estimate of HVP

Merging of **KNT**, **DHMZ** estimates + input from **ChPT/dispersive fits**: **CHKS** for 2π , 3π channels;

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62 :-)
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, ∞] GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP,LO}}$	694.0(1.0)(3.5)(1.6)(0.1) $_{\psi(0.7)}\text{pQCD}$	692.8(2.4)	1.2 reasonable agreement

$$\rightarrow a_\mu^{\text{HVP,LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{pQCD}} = 693.1(4.0) \times 10^{-10} \text{ Whitepaper estimate}$$

experimental uncertainties:
dominated by 2π uncertainty

KLOE/BABAR tension:
leaving out KLOE or
BABAR, respectively

energy region [1.8;3.7] GeV; usage of pQCD by
DHMZ, while KNT follows data-driven approach

2020 Whitepaper Estimate of HVP

Merging of **KNT, DHMZ** estimates with **ChPT/dispersive fits: CHHK** for 2π , 3π channels;

	DHMZ19	KNT19	Difference
	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62 :-)
	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, ∞] GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP,LO}}$	694.0(1.0)(3.5)(1.6)(0.1) _{$\psi(0.7)$} _{pQCD}	692.8(2.4)	1.2 reasonable agreement

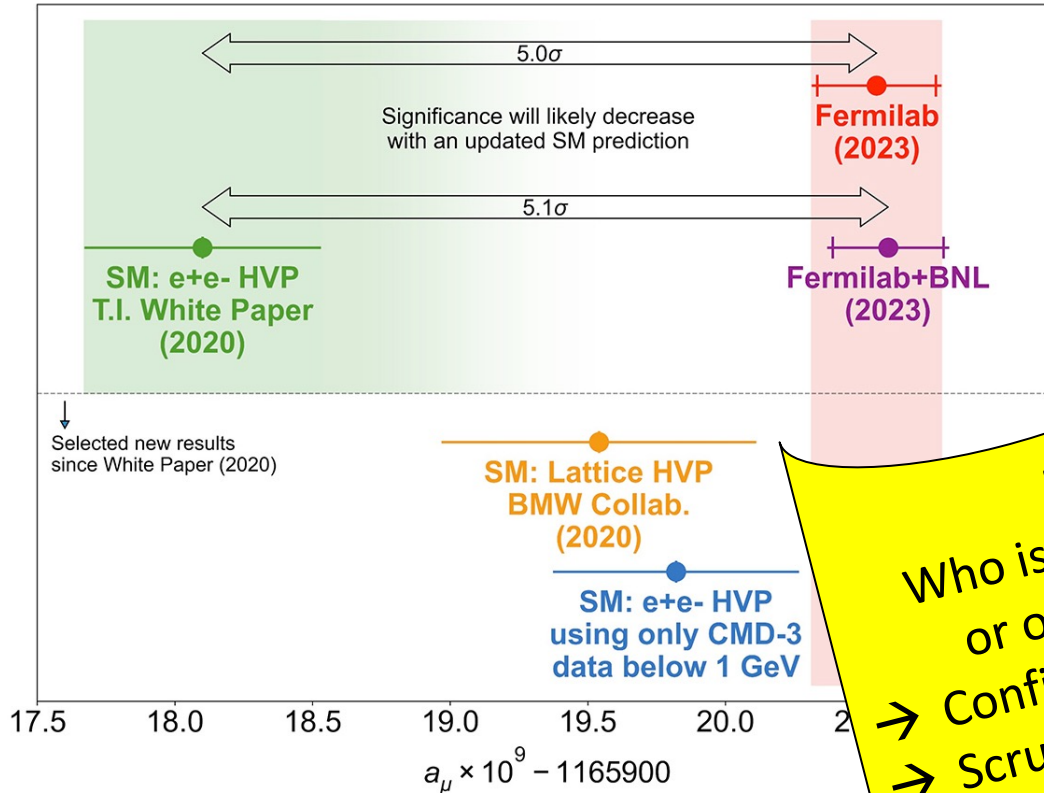
$\rightarrow a_\mu^{\text{HVP,LO}} = 693.1(2.8)_{\text{exp}}(2.8)_{\text{sys}}(0.7)_{\text{pQCD}} = 693.1(4.0) \times 10^{-10}$ Whitepaper estimate

experimental uncertainties:
dominated by 2π uncertainty

KLOE/BABAR tension:
leaving out KLOE or
BABAR, respectively

energy region [1.8;3.7] GeV; usage of pQCD by
DHMZ, while KNT follows data-driven approach

SM – Theory vs. Experiment: $(g-2)_\mu$



Post-2020 Whitepaper:

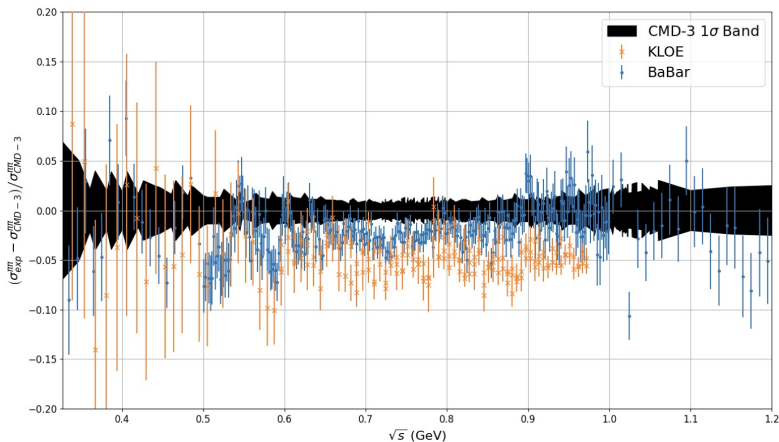
- BMW Lattice QCD HVP
- CMD-3 data on $\pi^+\pi^-$

Big debate now:
Who is right? CMD-3 and BMW
or older e+e- experiments?
→ Confirmation of BMW needed
→ Scrutiny of CMD-3 & old e+e- data

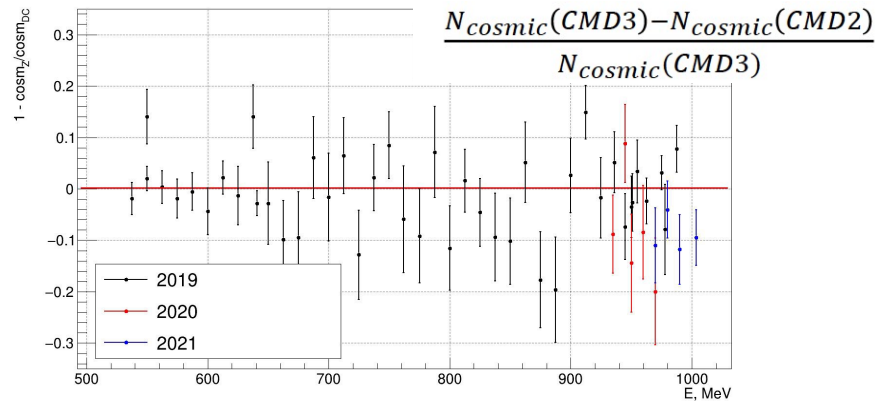
- New result from CMD-3 collaboration @ VEPP-2000 collider in Novosibirsk
 - Energy scan (from threshold up to 1.2 GeV) method, no ISR!
 - Form factor extraction via selection of $\pi\pi/e\bar{e}$ ratio
 - **Highest statistics data sample** of all experiments, systematic uncertainty 0.7% on ρ peak
- **Significant deviation from previous ISR and energy scan experiments (CMD-2)! Why?**

$$|F_\pi|^2 = \left(\frac{N_{\pi^+\pi^-}}{N_{e^+e^-}} - \Delta^{bg} \right) \cdot \frac{\sigma_{e^+e^-}^0 \cdot (1 + \delta_{e^+e^-}) \cdot \varepsilon_{e^+e^-}}{\sigma_{\pi^+\pi^-}^0 \cdot (1 + \delta_{\pi^+\pi^-}) \cdot \varepsilon_{\pi^+\pi^-}}$$

Courtesy Aidan Wright



Background from cosmic ray events
as a possible explanation for CMD-2/CMD-3 difference?

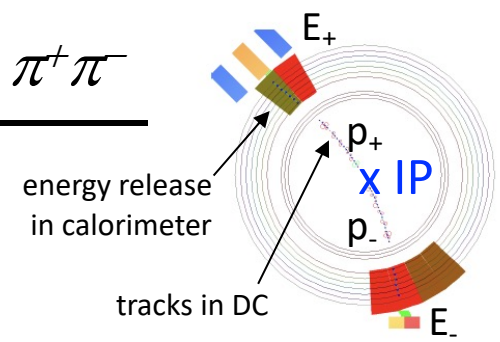


NEW

2023 Shock: CMD-3 @ Novosibirsk $e^+e^- \rightarrow \pi^+\pi^-$

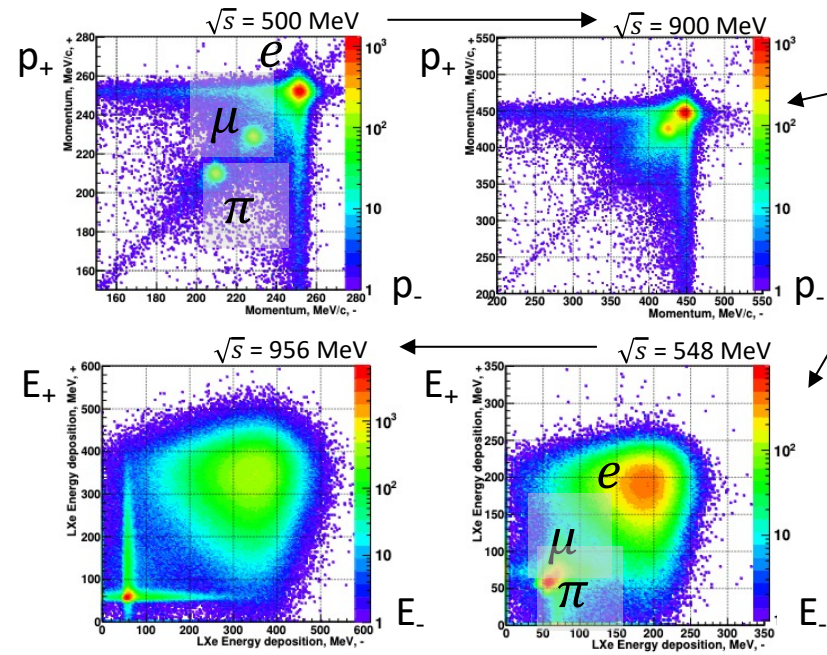
Scrutiny of CMD-3 result within the Theory Initiative

- Very open replies by F. Ignatov \rightarrow no major showstopper observed
- Very powerful analysis with many and impressive internal cross checks
- Monte-Carlo generator for energy scan cannot be independently verified

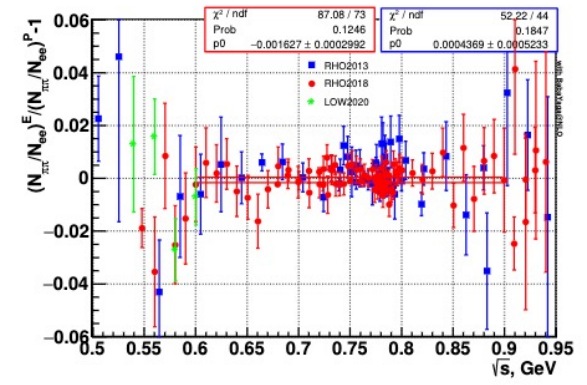


Most impressive feature: $\pi\pi/ee$ ratio determined independently by two complementary methods

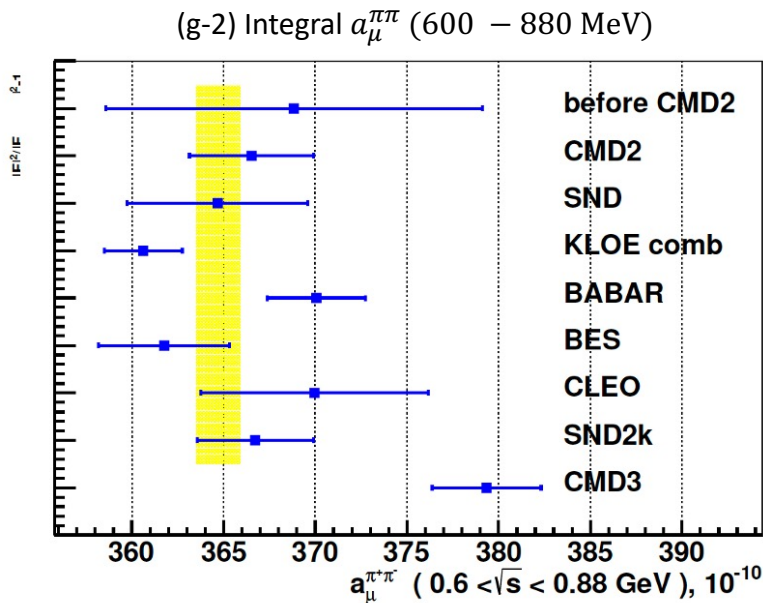
- Momentum based
- Calorimetric



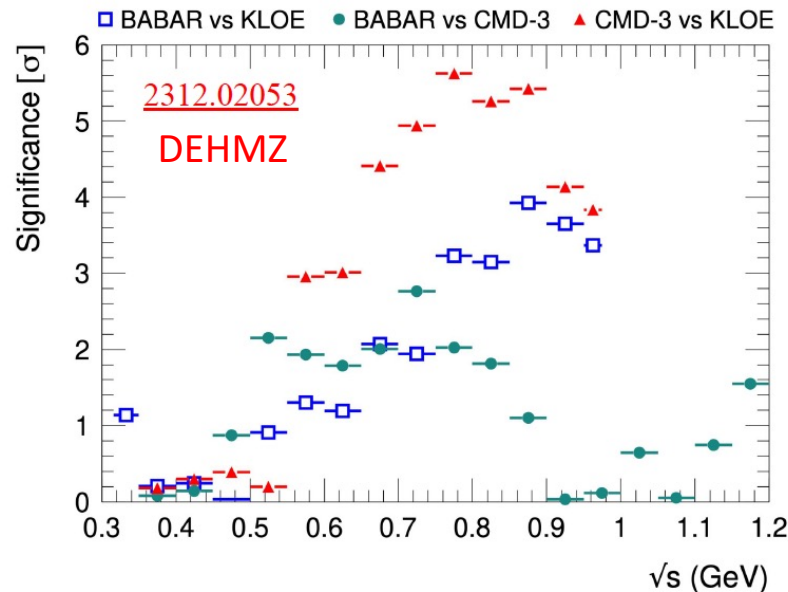
agreement $\sim 0.2\%$ around rho peak



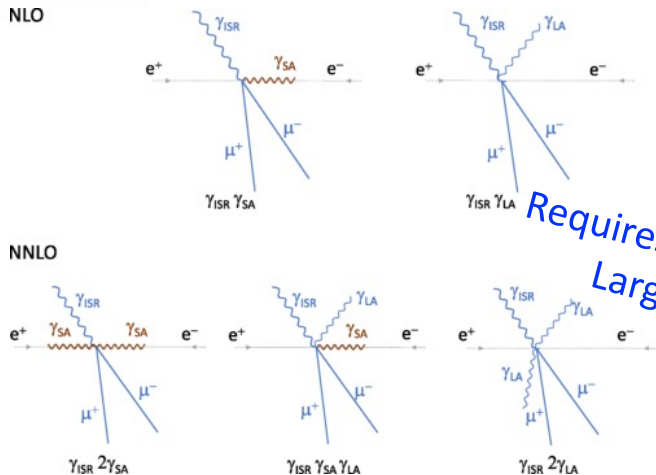
CMD-3 Compatibility with other Experiments for HVP Integral



Deviation between data sets (in statistical significance)

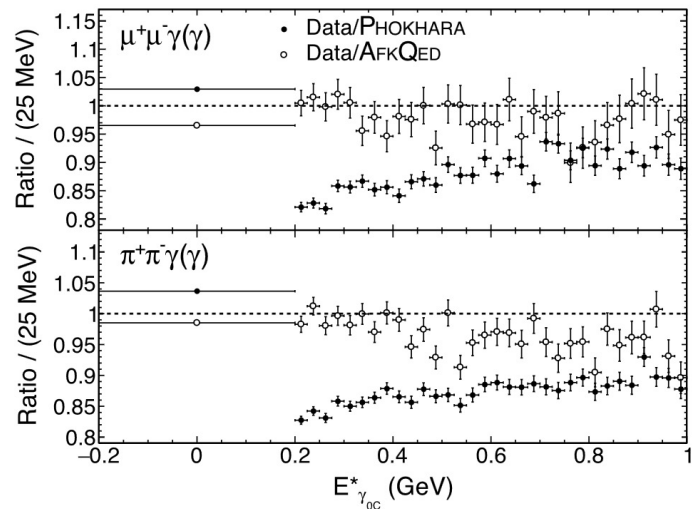


→ Significant deviation from previous ISR and energy scan experiments (CMD-2)! Why?



Requires 1 photon at large angle
Large smearing effects

Phys. Rev. D 108, L111103



Detailed study of NLO and NNLO radiative corrections

- Kinematic fits for $\pi^+\pi^-\gamma_{ISR,LA}(\gamma)$, $\mu^+\mu^-\gamma_{ISR,LA}(\gamma)$
- Comparison with PHOKHARA (NLO full correction) and AfkQED (collinear approximation beyond LO) generators

- NNLO radiation observed at 3.5% level (missing in PHOKHARA)
- Phokhara prediction for small angle ISR photons at NLO too high by ~25% (AfkQED fits better to data)

- BABAR:**
- rather inclusive selection and therefore weak dependence from PHOKHARA
 - small effect on published BABAR result due to PHOKHARA NLO limitations
 - however: in original BABAR 2π paper 2% correction applied to AfkQED due to statement that PHOKHARA provides better NLO correction → claim: only valid for acceptance

- KLOE/BESIII:**
- less inclusive selection regarding NLO
 - claim: large effects due PHOKHARA NLO limitations of up to 3.2% in the case of BESIII

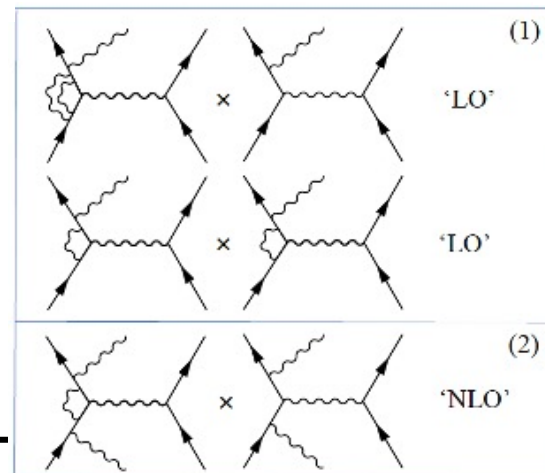
However, scenarios need to be taken into account:

1: NNLO interference terms (1) dominate → large effects

2: NNLO interference terms (2) dominate

→ significantly reduced effects on experimental analyses

So far no explicit calculation of these NNLO interference effects



KLOE / BESIII Response to PHOKHARA Shortcomings

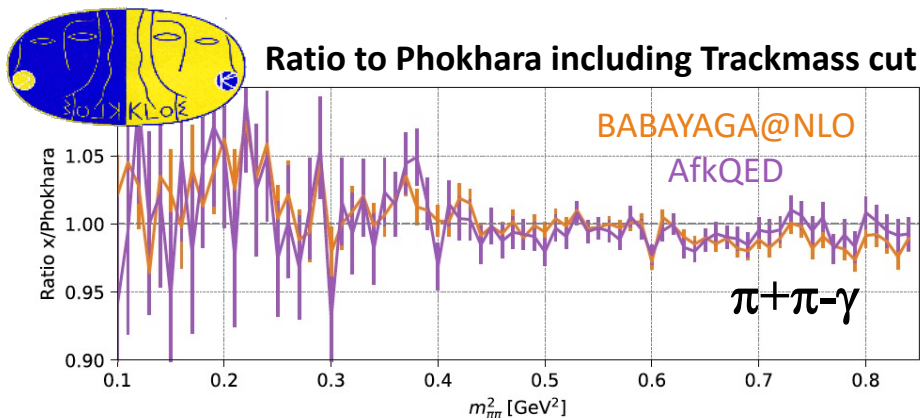
20

Investigation of kinematic cuts, which are sensitive to NLO corrections: Trackmass (KLOE), χ^2 (BESIII)

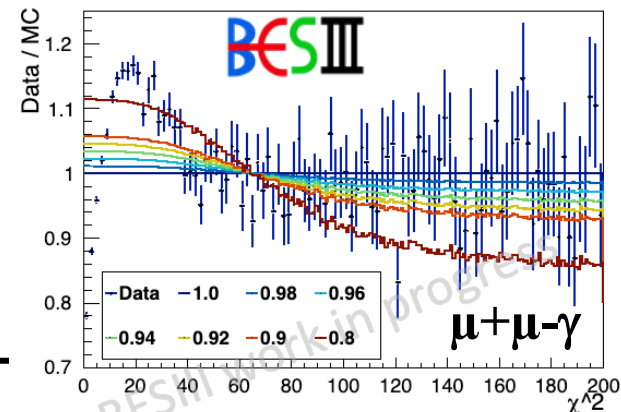
Radio-MonteCarlow Initiative with detailed comparisons

- KLOE has presented a **good agreement between various MC generators** for realistic acceptance cuts and also in the case of the kinematic trackmass cut for KLOE-10
- BESIII has carried out a **full detector simulations for various MC generators** and a **data-PHOKHARA comparison for $e^+e^- \rightarrow \mu^+\mu^-\gamma$ in the χ^2 distribution**; furthermore it has been demonstrated that the published analysis is **largely inclusive in higher order corrections**

→ **scenario 2 from DHLM23 paper strongly preferred**



Data-Phokhara comparison for χ^2 distribution



KLOE / BESIII Response to PHOKHARA Shortcomings

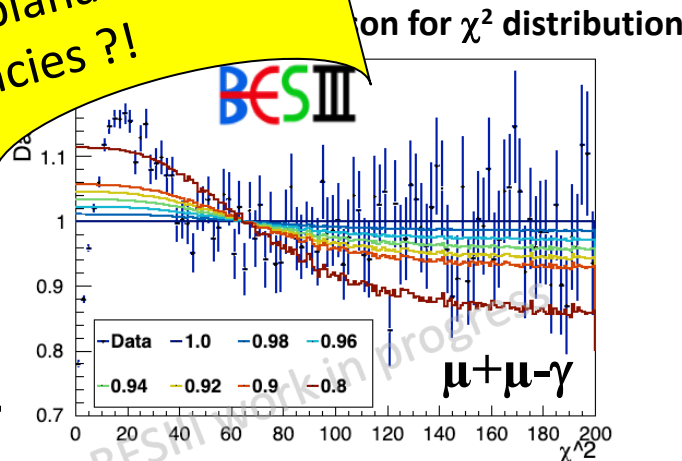
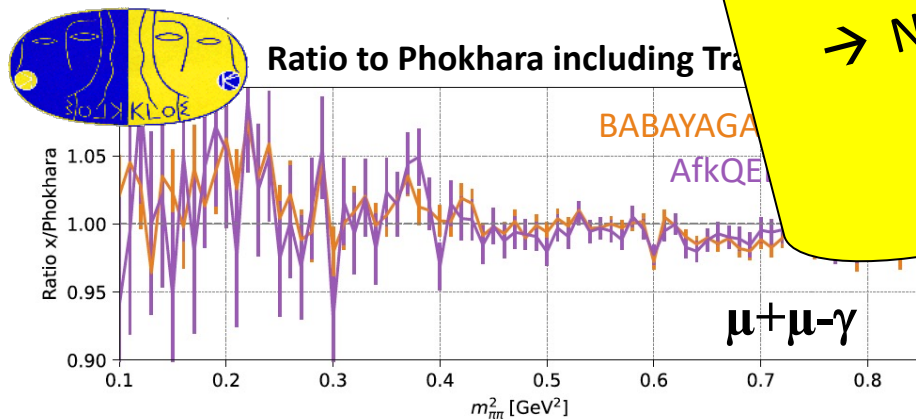
Investigation of kinematic cuts, which are sensitive to NLO corrections: Trackmass (KLOE), χ^2 (BESIII)

Radio-MonteCarlow Initiative with detailed comparisons

- KLOE has presented a **good agreement between various MC generators** and also in the case of the kinematic trackmass cut for KLOE-10
- BESIII has carried out a **full detector simulations for various kinematic cuts** and a **comparison for $e^+e^- \rightarrow \mu^+\mu^-\gamma$ in the χ^2 cut**. The published analysis is largely inclusive in hadronic decays.

→ **scenario 2 from DHLM223 paper strongly**

News from 2024 KEK workshop:
 Phokhara limitations likely not impacting KLOE/BESIII
 → No viable explanation for discrepancies ?!



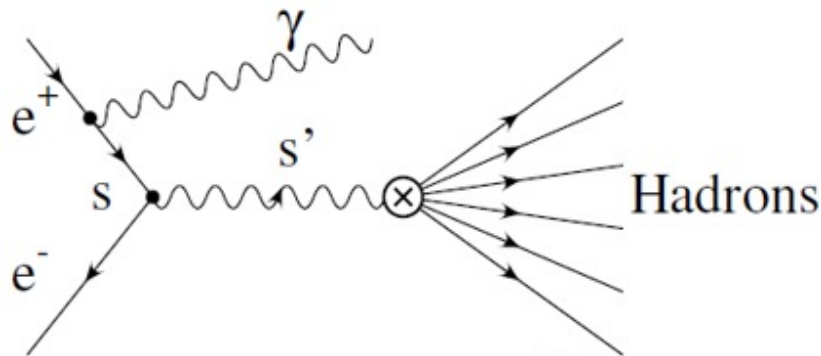
Overview Experiments – Past and Future

Experiment	Published Method	Normalization	Separation $\pi - \mu - e$	Future	
KLOE	ISR untagged ISR tagged ISR untagged	Luminosity Luminosity $\mu + \mu - \gamma$	Kinematics Track Kinematics Track Kinematics Track	ISR untagged $\mu + \mu - \gamma$ statistics x 7	0.4%
BABAR	ISR tagged	$\mu + \mu - \gamma$	Particle ID	ISR tagged, separation by polar angle, statistics x 2	<0.5%
BESIII	ISR tagged	Luminosity	Particle ID (ML)	ISR tagged, $\mu + \mu - \gamma$, statistics x 7, 1C kin. fit	0.5%
BELLE-II				ISR tagged, $\mu + \mu - \gamma$, Particle ID	0.5%
			Kinematics Track Kinematics EMC	overall improvements	0.3%
	energy scan	$e + e -$	Kinematics EMC	overall improvements ML for $\pi - e$ separation	0.6%

New analyses in preparation:
 New MC generators, new techniques,
 awareness to (N)NLO issues, ...



$$s' = m_{\text{had}}^2 = s - 2E_{\gamma} \sqrt{s}$$



Inclusive R- Measurement via Initial State Radiation

NEW

PhD N.J.P. Berger (2006, Stanford)
PhD project, Th. Lenz (JGU Mainz)

R_{incl} Measurement BESIII (2022)

Phys. Rev. Lett. 128 (2022) 062004

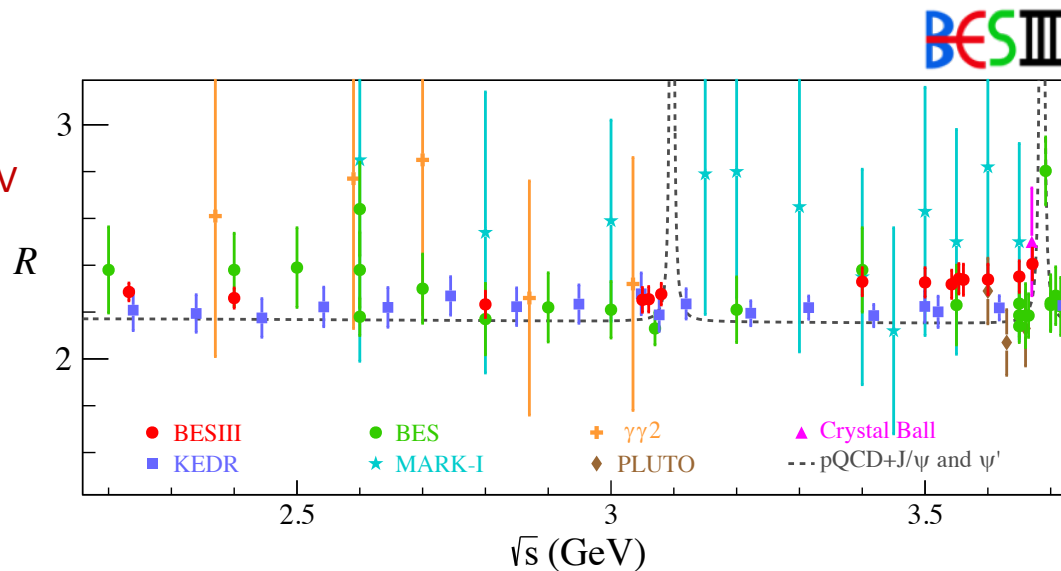
$$R_{had}(s) = \frac{1}{\sigma_{\mu^+\mu^-}} \cdot \frac{N_{had} - N_{bkg}}{\mathcal{L} \cdot \epsilon_{had} \cdot (1 + \delta)}$$

Analysis strategy: select all events with ≥ 2 tracks

- Reject back-to-back 2-prong events (Bhabha, $\mu^+\mu^-$)
- Remaining background from ISR and QED events subtracted from MC

- Energy range covered: $2.2 < \sqrt{s} < 3.7$ GeV
- Statistical uncertainty $< 0.5\%$
Systematic uncertainty $< 2.6\%$ below 3.1 GeV
 $\sim 3.0\%$ above
- Above 3.4 GeV deviation observed with:
 - KEDR/Novosibirsk on the level of 1.9σ
 - pQCD theory on the level of 2.7σ

World's most precise R_{incl} measurement !
Some deviations from pQCD seen ?!
Much more data will be published shortly !

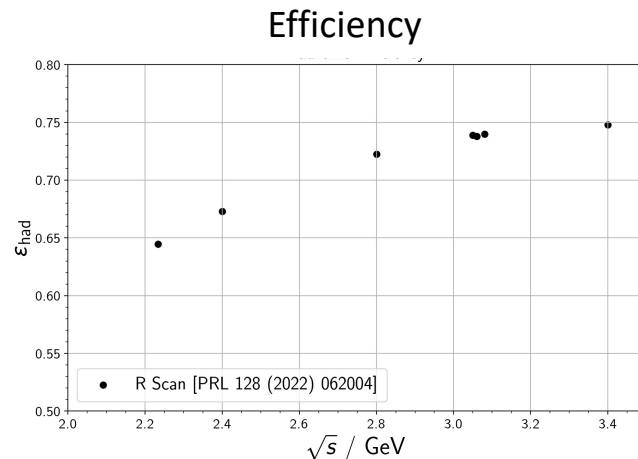


Messages learnt from Inclusive R Measurement

- Selection requires ≥ 2 tracks, which are not back-to-back
- Detector acceptance starts **above 21°**

→ For low-multiplicity final hadronic states ($\pi^+\pi^-$, $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^0\pi^0$, ...), the probability to be not selected large relatively large

→ Total event efficiency at 60% 70% level

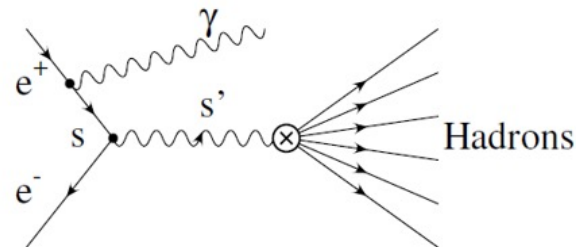


For the determination of the event efficiency, a precise MC generator for $e^+e^- \rightarrow \text{Hadrons}$ is needed (possible model dependence difficult to estimate)



Inclusive ISR with detection of ISR photon only

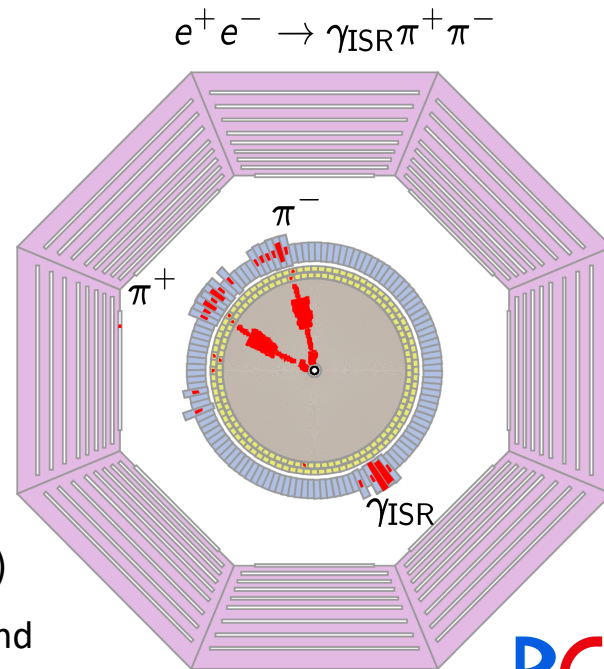
$$s' = m_{\text{had}}^2 = s - 2E_\gamma \sqrt{s}$$



New Inclusive Approach using ISR

Event selection:

- **Select 1 high-energetic photon > 1.2 GeV \equiv ISR photon at large polar angle $[\cos\Theta_{ISR}] < 0.8$**
 → Restricts hadronic mass spectrum < 2.7 GeV
 - **Require (for time being) ≥ 1 charged track in the event**
 → Does currently not include fully neutral states (e.g. $e^+e^- \rightarrow \pi^0\gamma$)
- ISR boost confines particles into narrow cone
 → Very high detection efficiency
 - Less reliant on description of hadronic MC
 → ISR description in MC under control
 - Single measurement down to threshold (does not need scan)
 - Measurement fully inclusive for Final State Radiation (FSR) and higher order corrections of ISR
 - In principle able to measure fully neutral channels



BESIII

New Inclusive Approach using ISR: Efficiency

Event selection:

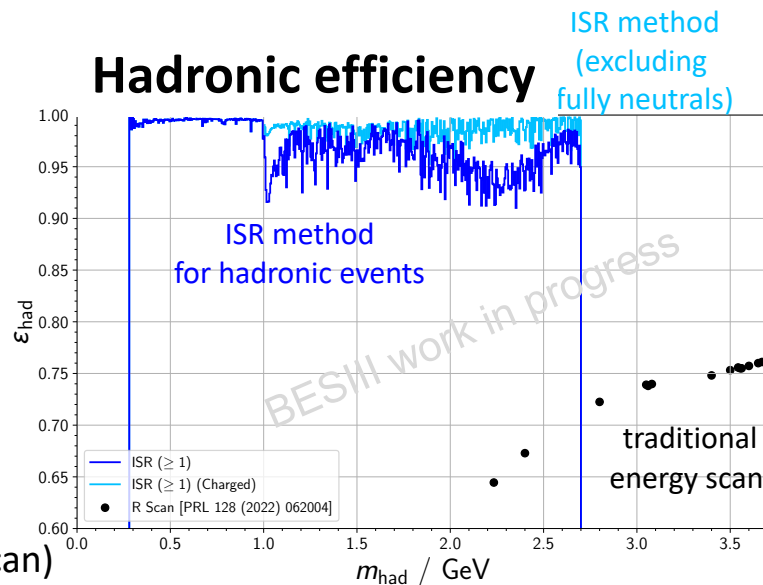
- **Select 1 high-energetic photon > 1.2 GeV \equiv ISR photon at large polar angle $[\cos\Theta_{ISR}] < 0.8$**

→ Restricts hadronic mass spectrum < 2.7 GeV

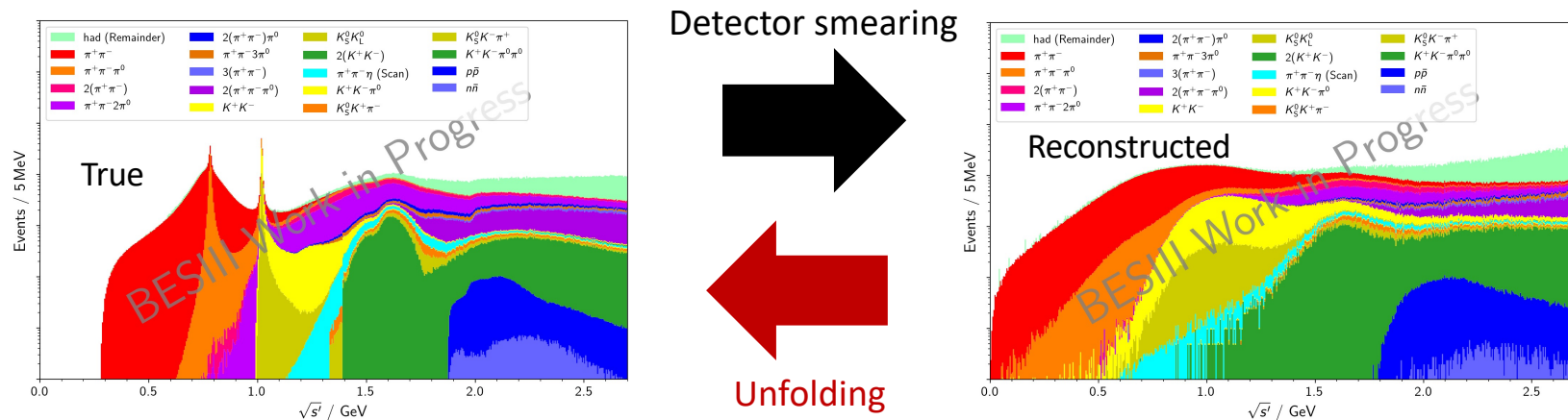
- **Require (for time being) ≥ 1 charged track in the event**

→ Does currently not include fully neutral states (e.g. $e^+e^- \rightarrow \pi^0\gamma$)

- ISR boost confines particles into narrow cone
→ Very high detection efficiency
- Less reliant on description of hadronic MC
→ ISR description in MC under control
- Single measurement down to threshold (does not need scan)
- Measurement fully inclusive for Final State Radiation (FSR) and higher order corrections of ISR
- In principle able to measure fully neutral channels



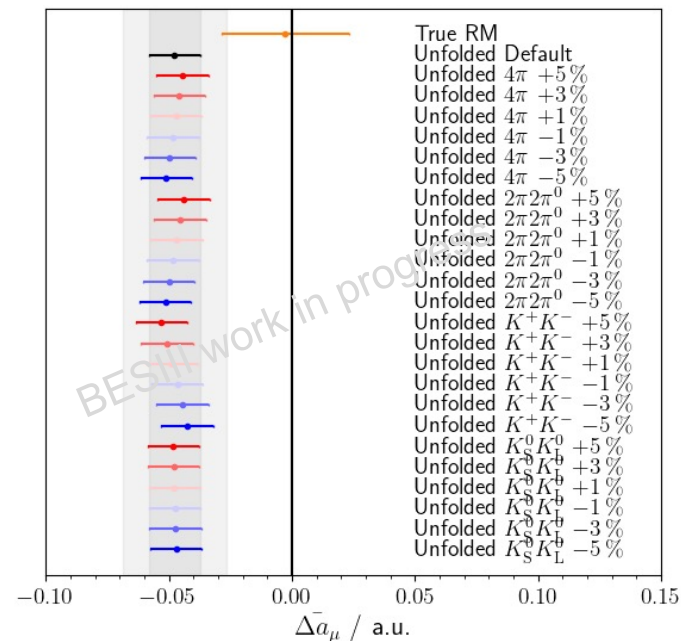
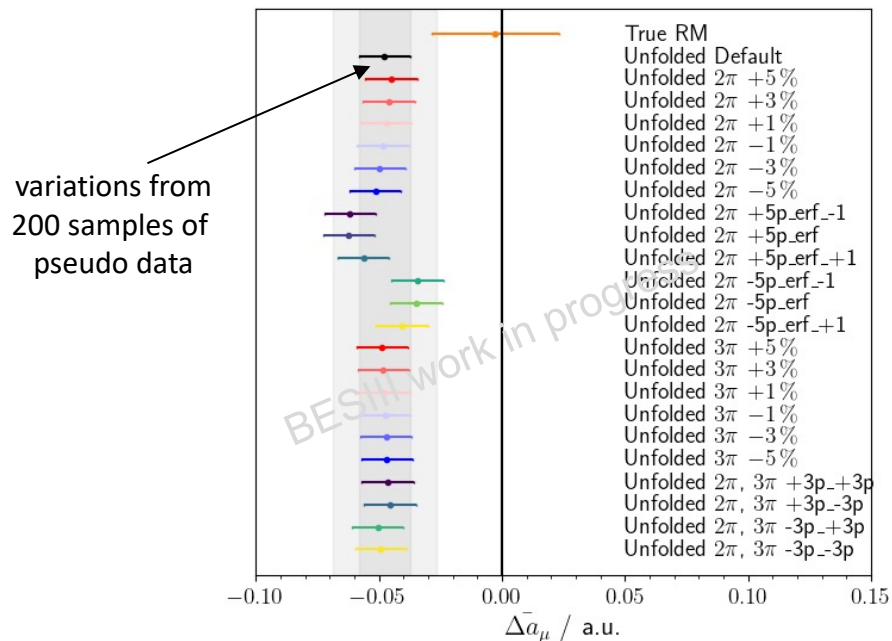
Unfolding from Detector Mass Resolution



- Large smearing introduced by limited detector resolution
- Application of unfolding algorithms to recover the *true* spectrum
- Requires Monte-Carlo program to construct unfolding matrix – Response Matrix (RM)
- Systematically testing the bias in the unfolding procedure due to wrong input Monte-Carlo Pseudo Data (PD)

Unfolding from Detector Mass Resolution

- More than 50 cross section variations in input MC tested (e.g. up to $\pm 5\%$ variation of 2π cross section)
 → **Very stable result for unfolded spectrum** → **variation well within percent level (=precision goal)**



- With larger data sets also conversion events might be used to significantly improve mass resolution**



Conclusions

Conclusions

- New Lattice as well as CMD-3 results challenging old e^+e^- data
 - difference in $\pi^+\pi^-$ between CMD-3 and other expts. to be understood
 - radiative corrections are a key issue → RadioMonteCarlow initiative!
- Have not covered other hadronic channels beyond $\pi^+\pi^-$ - puzzles there as well
- Luckily, new e^+e^- data at the horizon
 - BABAR with fit to angular distributions for $\pi/\mu/e$ separation
 - KLOE with full KLOE statistics
 - BESIII with 20/fb data sample (normalization to $\mu\mu$), new ideas R_{incl} via ISR
 - BELLE-II has joined the team of ISR experiments
 - further cross checks by CMD-3 and new SND data from energy scan



Thank you !