Towards NNLO+ in Phokhara

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Muon Precision Physics workshop 2024

Jérémy Paltrinieri

Liverpool, 14/11/2024

Overview

1. PHOKHARA status

   State of the art, latest results

2. Towards NNLO theoretical predictions

   Anatomy of the NNLO calculation, milestones and plan

3. Resummation of soft photons effects

   YFS and CEEX formalisms, MC state of the art, extension of CEEX

PHOKHARA
radiative return at flavour factories

Motivation

- Importance of hadron cross-sections measurements in g-2 determination
- Radiative corrections studies must be undertaken in order to clear up the tensions between experiments
- From radiative return experiments like KLOE, this means taking care of NNLO and all-order exponentiation effects: many challenges!

See Aidan W.
and Achim D. talks

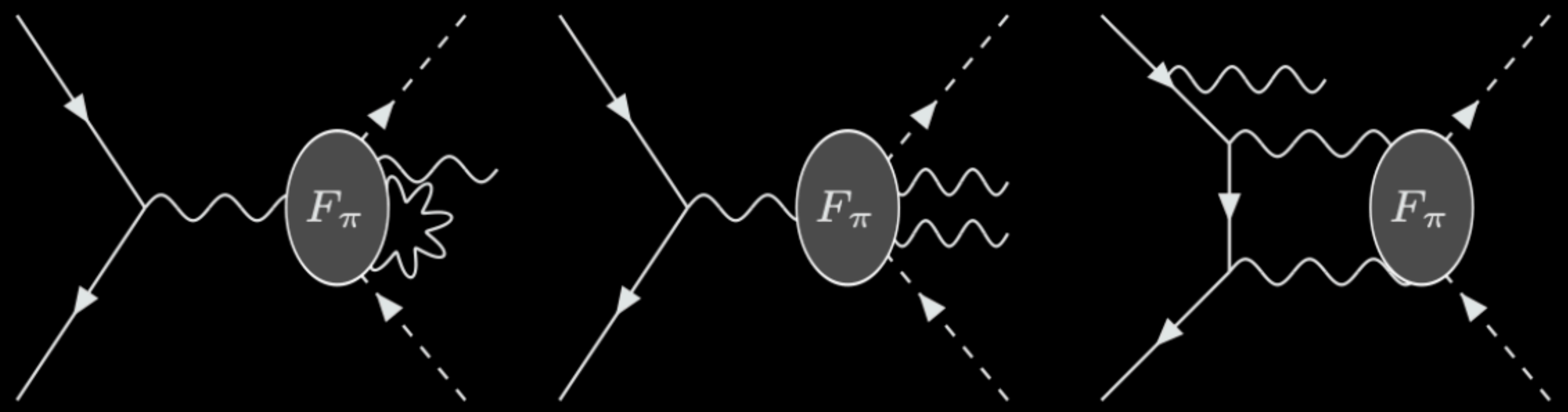
The Phokhara Monte Carlo Generator

- Phokhara simulates electron-positron annihilation into hadrons (and muons) at NLO accuracy. This includes virtual and soft photon corrections to one photon emission events and the emission of two real hard photons.
- Case of muons: complete NLO - [arXiv:1312.3610]
- Case of pions: FxsQED for the pion treatment - [Szymon Tracz PhD Thesis]
- Disclaimer: not discussing many other hadronic final states: 3π, 4π, NN, KK…

Status of Phokhara

- Current version: Phokhara v10.0: https://looptreeduality.csic.es/phokhara/ (October 2020)
- Last update: two remaining gauge-invariant contributions: FSRNLO & TVP
- Strong2020 paper [2410.22882] and website

See Adrian S. talk



Status of Phokhara

- Current version: Phokhara v10.0: https://looptreeduality.csic.es/phokhara/ (October 2020)
- Last update: [1903.10197]

ments. We calculate the last radiative corrections for the extraction of the pion form factor, which were believed to be potentially substantial enough to explain the data within the Standard Model. We find that the corrections are too small to diminish existing discrepancies in the determination of the pion form factor for different kinematical configurations of low-energy BABAR, BESS-III and KLOE experiments. Consequently, they cannot noticeably change the previous predictions for a_μ

Validation of Phokhara

$e^+e^- \to \mu^+\mu^-\gamma$

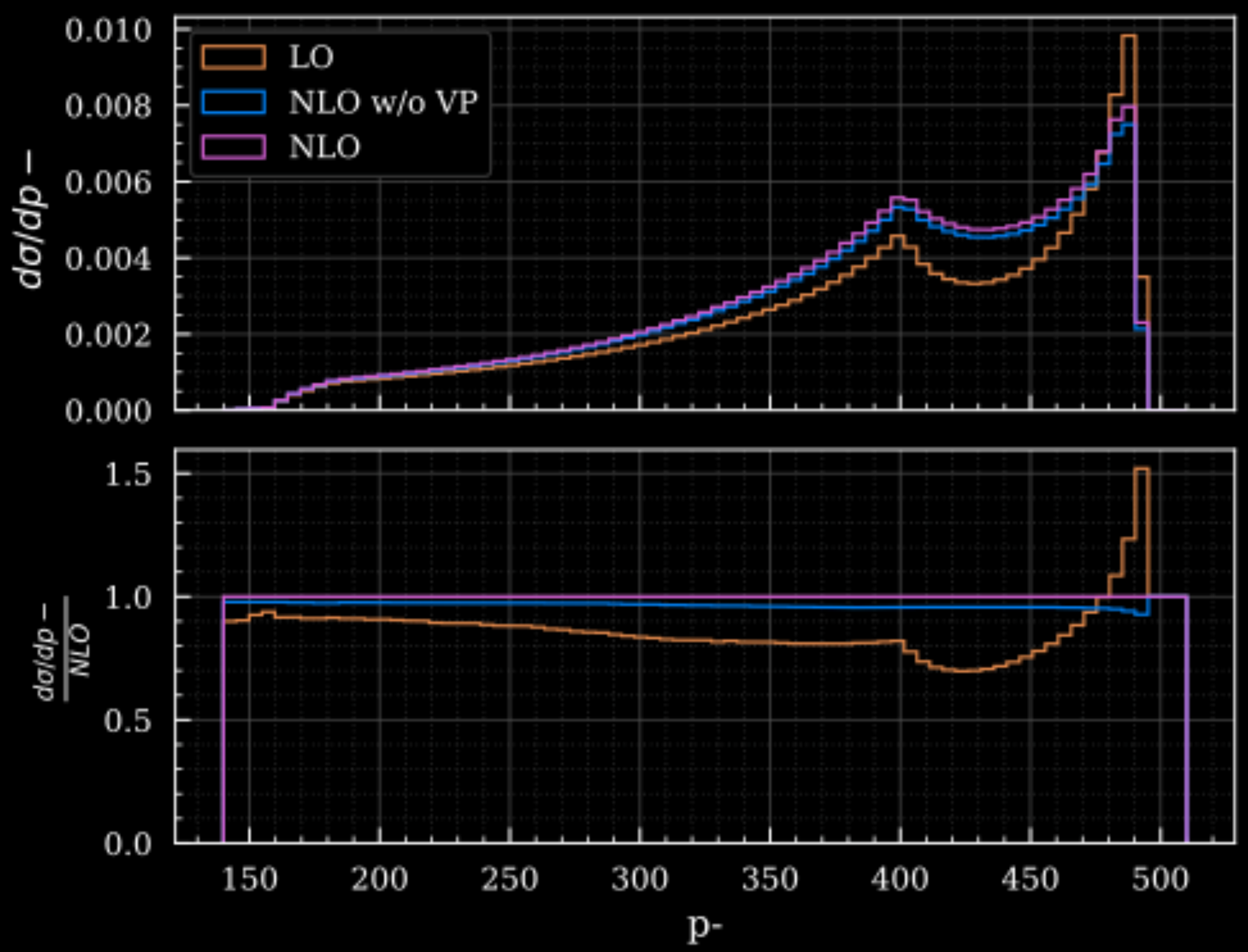
- Evaluation of 5 and 6-point amplitudes (real and virtual)
- Tools from the Amplitude community

  Construction of Amplitudes (Qgraph/FeynArts,…), numerical evaluation of integrals (QCDLoop,…) and amplitudes (OpenLoops,…)

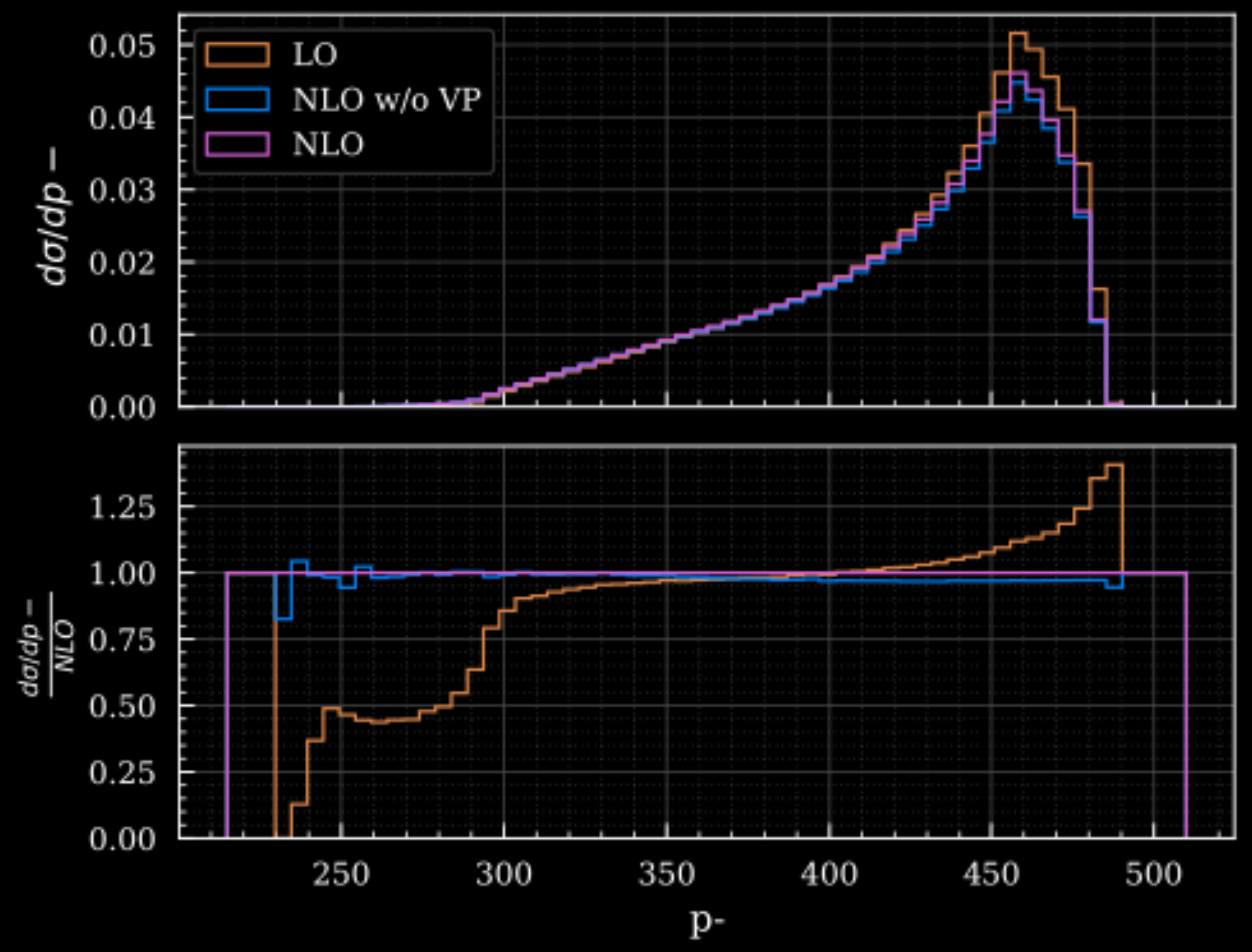
- Analytic validation of $\mathscr{A}^{\text{NLO}}\left(e^+e^- \to \mu^+\mu^-\gamma\right) = \frac{c_{-1}}{\epsilon} + c_0$ in dimensional regularisation: $D = 4 - 2\epsilon$.

Validation of Phokhara

$e^+e^- \to \mu^+\mu^-\gamma$



Large angle



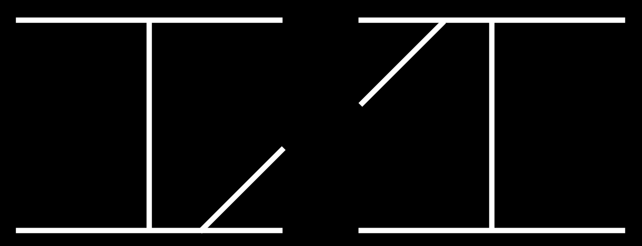
Small angle

William @KEK2024

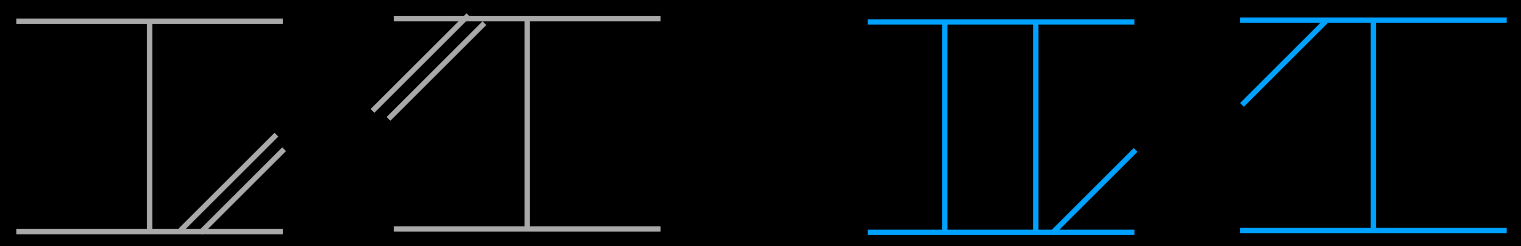
Towards NNLO

Anatomy of the fixed order calculation

- LO: Born Level



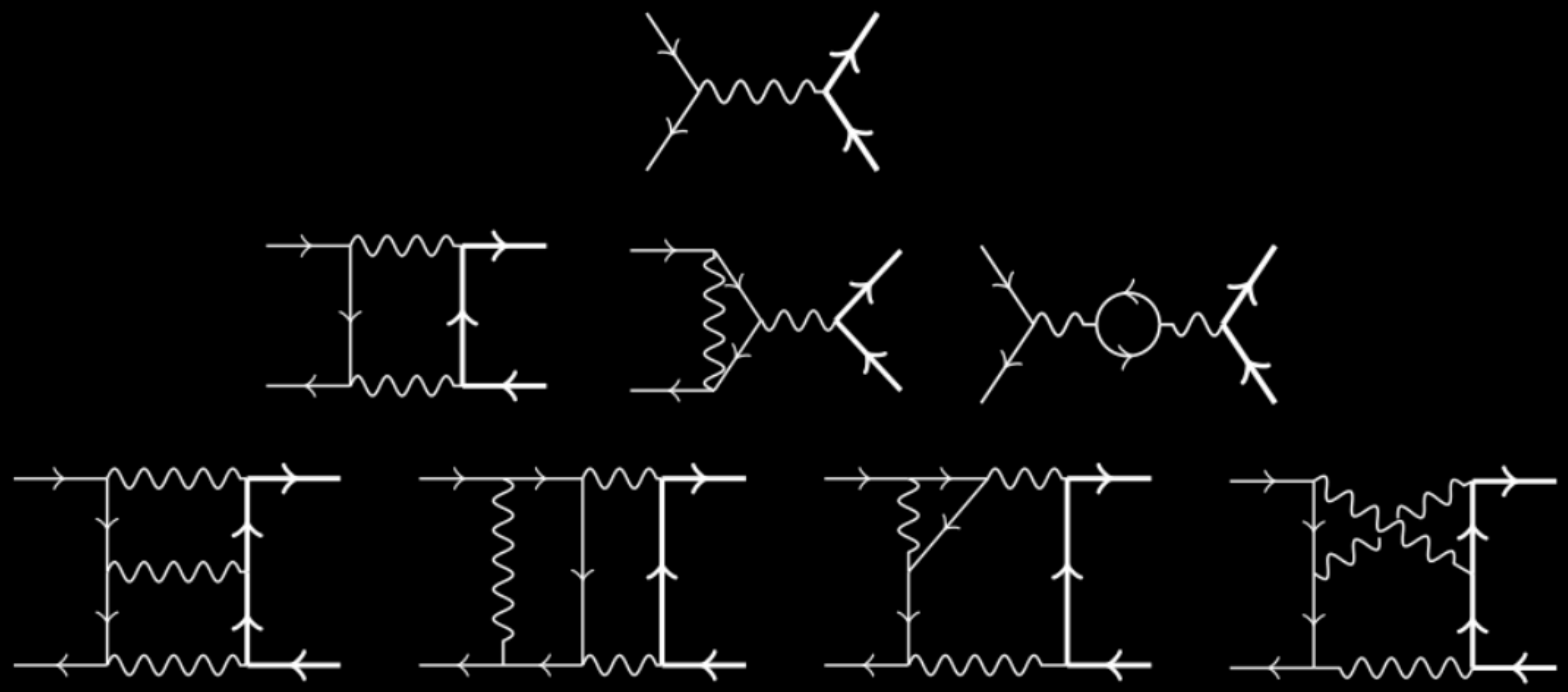
- NLO: Real + Virtual



- NNLO: Double Real + Virtual-Real + Double Virtual

Towards NNLO

- Important achievement: NNLO for electron-muon scattering: [2106.13179]
- Massless electron: 4-point process depends on three scales s, t, u
- Analytic evaluation of loop interferences
- Needed for muon-e experiment
- Massification was achieved in [2212.06481]



See M. Bonetti's talk for N3LO

Different MC generators

Different MC take into account different effects beyond FO:

- Collinear structure
- Parton shower
- YFS, CEEX

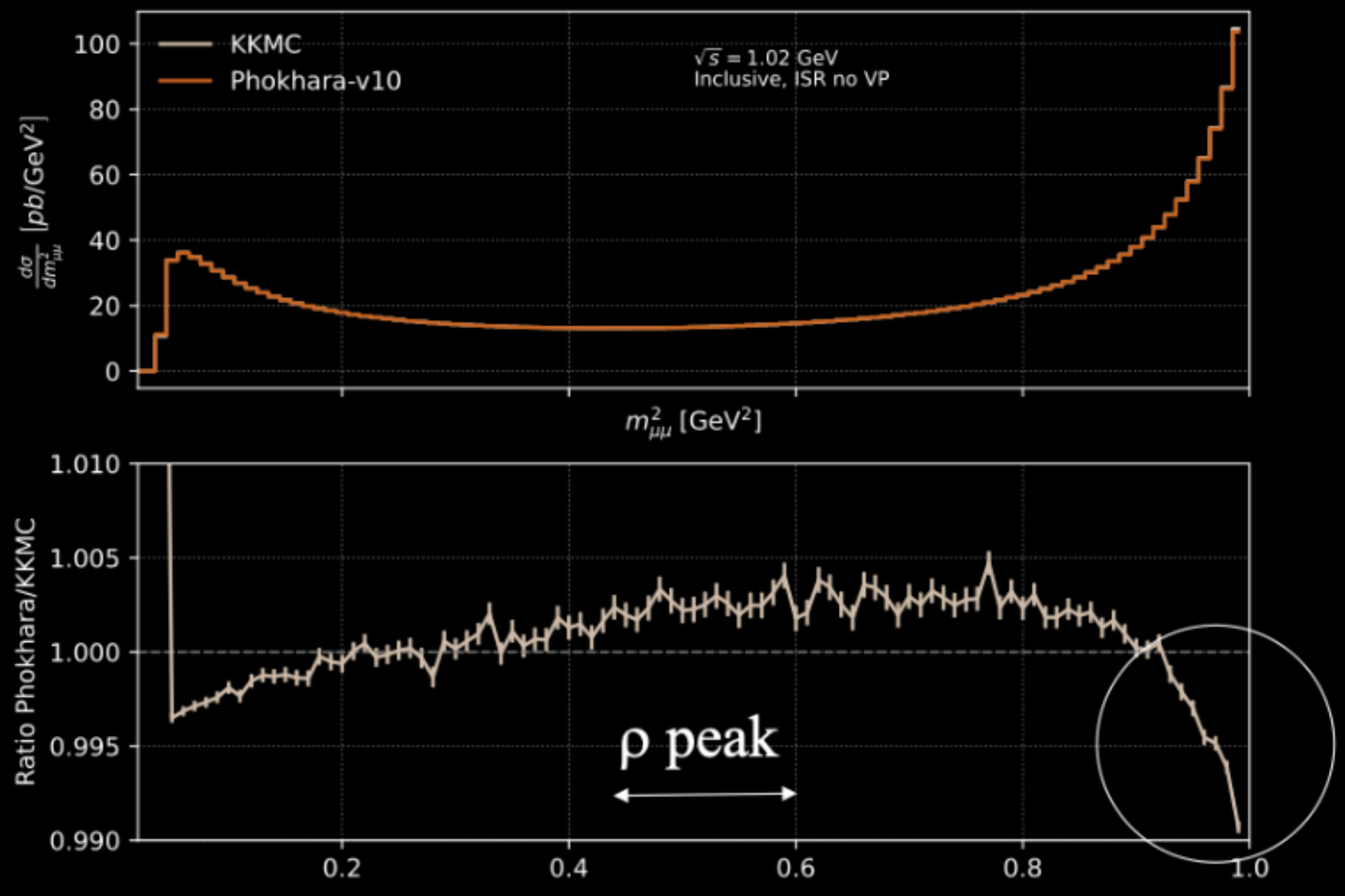
Yannick @KEK2024

code		$ee \to \mu\mu$
AfkQed		
	$+\gamma$	LO+CS
BabaYaga@NLO		NLO+PS
	$+\gamma$	LO+PS
KKMC		CEEX
	$+\gamma$	CEEX
MCGPJ		NLO+CS
	$+\gamma$	LO+CS
McMule		NNLO
	$+\gamma$	NLO
Phokhara		
	$+\gamma$	NLO
Sherpa		NLO+YFS
	$+\gamma$	NLO

Soft photon resummation

- Sheds another light on missing higher-order effects
- YFS formalism: resummation of soft photons effects
- 2-loop integrals are notoriously hard: approximations provide insight!

YFS exponentiation



Graziano

@KEK2024

YFS exponentiation

[Yennie, Frauschi, Suura Annals Phys. 13 (1961) 379-452]

- Insight on the structure of IR divergences in QED at all-order

  Separation of divergences at the cross-section level in an exponential factor

- No collinear singularities

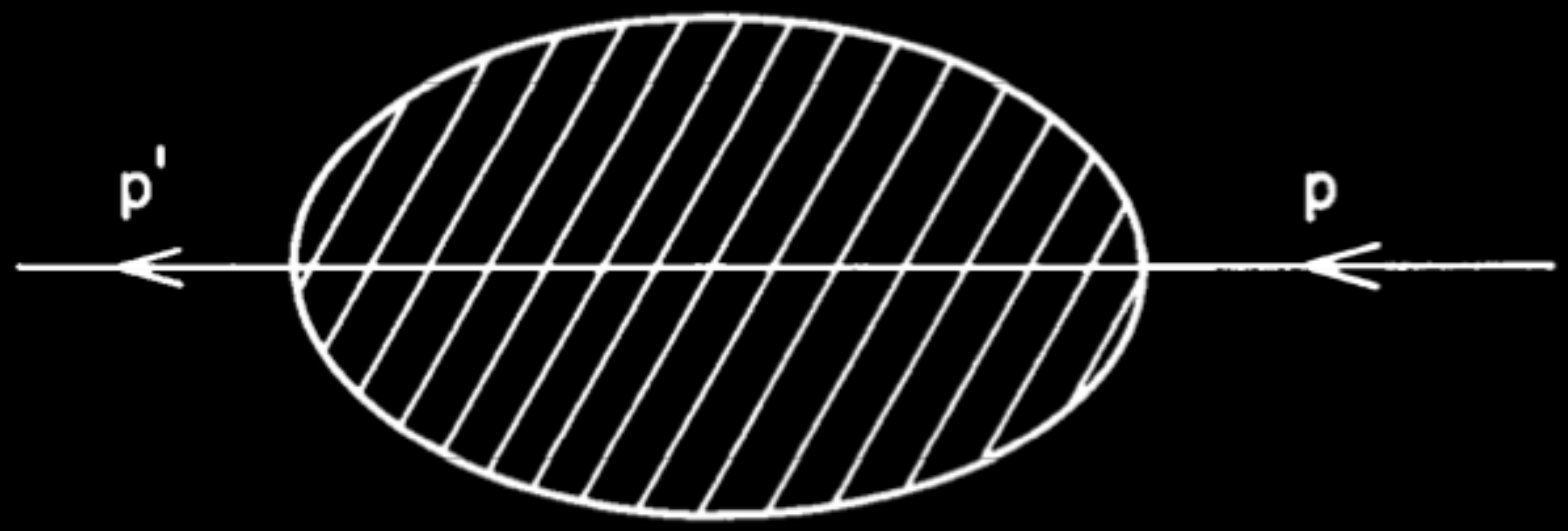
  Dependence on all fermion/hadron masses must be taken into account

- All divergences are associated with emission off external legs

YFS exponentiation

Virtual contributions

Fermion line where *n* virtual photons are produced (no real emissions):



YFS exponentiation

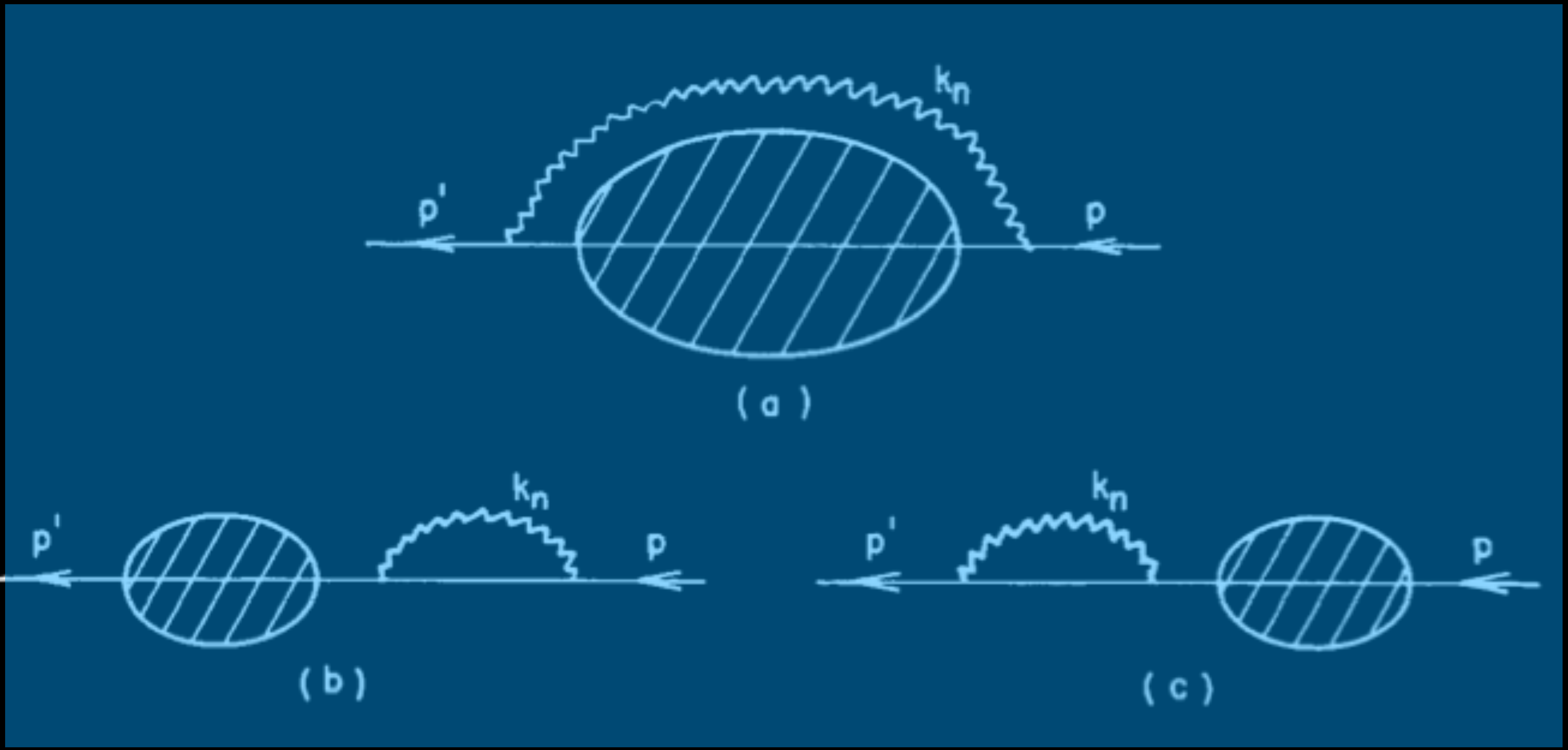
Virtual contributions

Add another virtual photon:

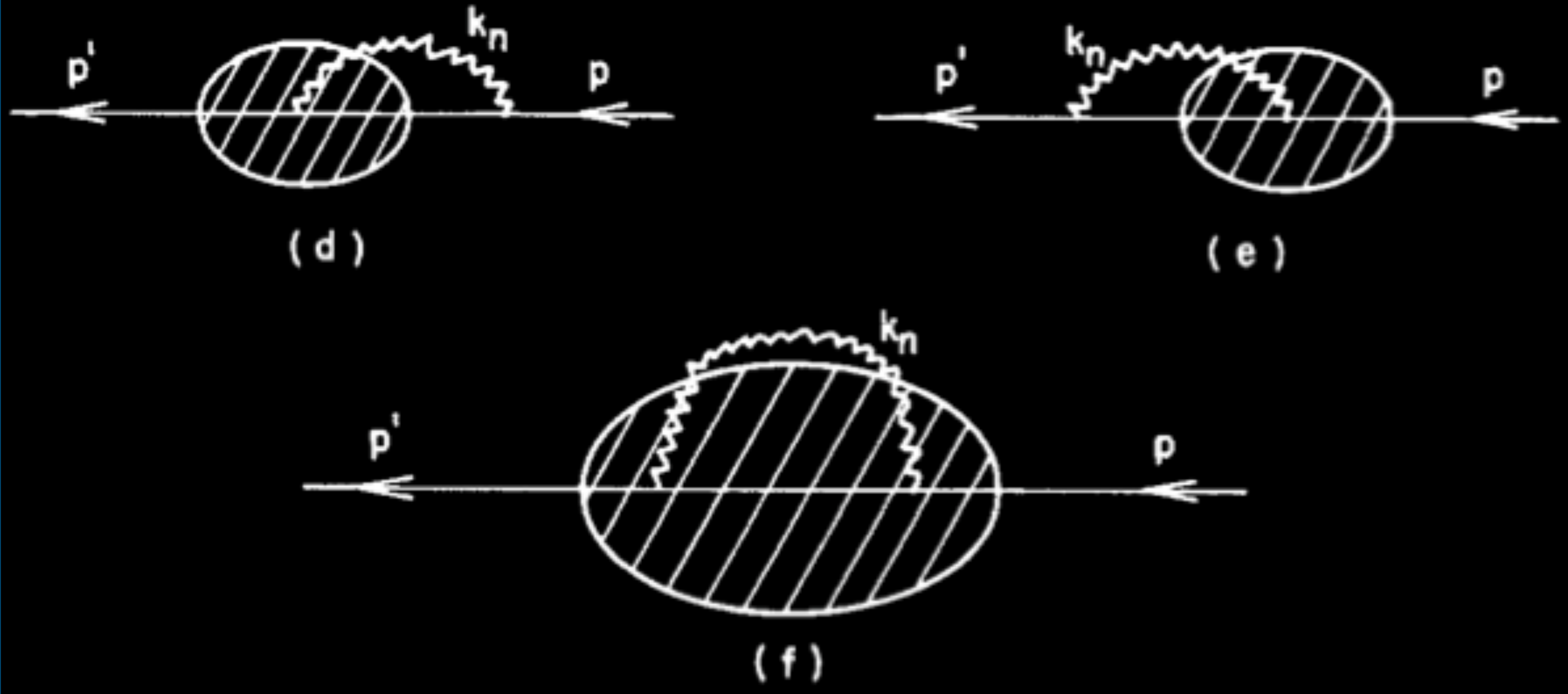
YFS exponentiation

Virtual contributions

IR divergent pieces:



Finite (when combined in a GI way)



YFS exponentiation

Virtual contributions

Mathematically, this implies:

$$\mathscr{M}_0^{n_\gamma} = \sum_{r=0}^{n_\gamma} M_0^{n_\gamma - r} \frac{(\alpha B)^r}{r!} \text{ which implies } \mathscr{M} = \exp(\alpha B) \sum_{n_\gamma} M_0^{n_\gamma}$$

IR divergent

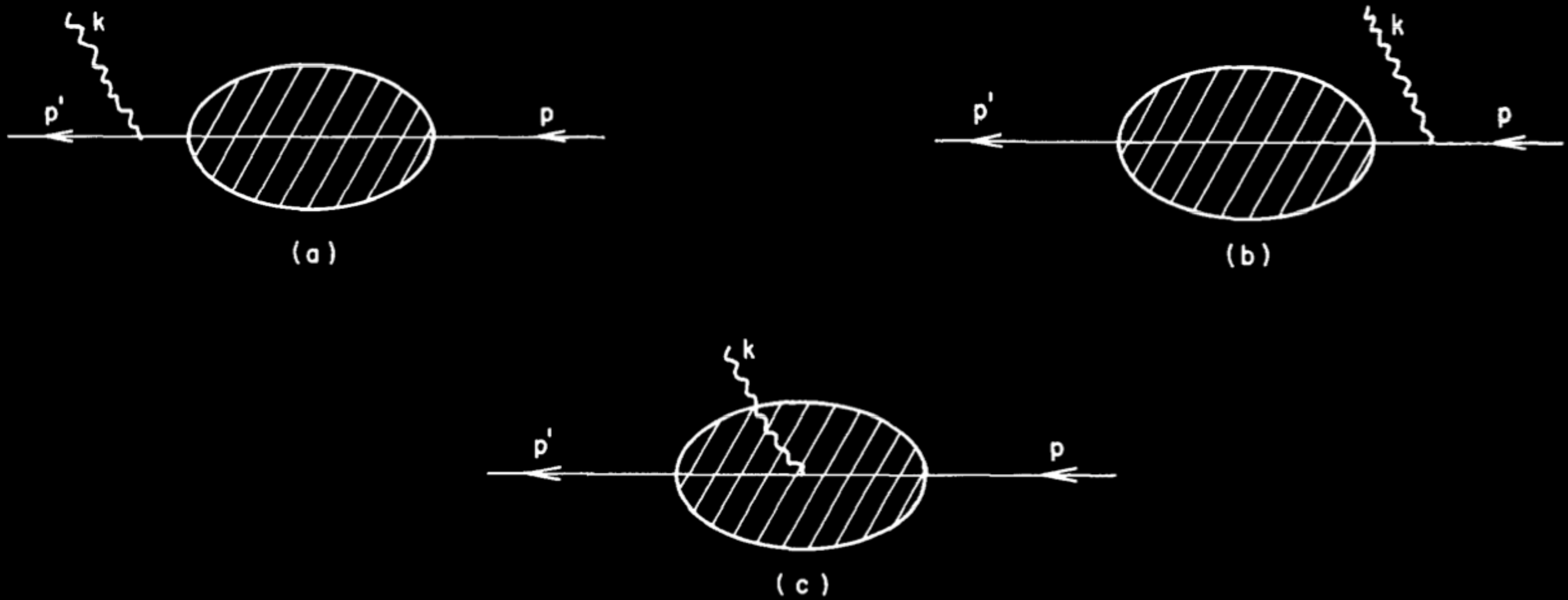
Virtual IR safe

B contains the IR divergences associated to the diagrams I showed before

Note: this is at the amplitude level!

YFS exponentiation

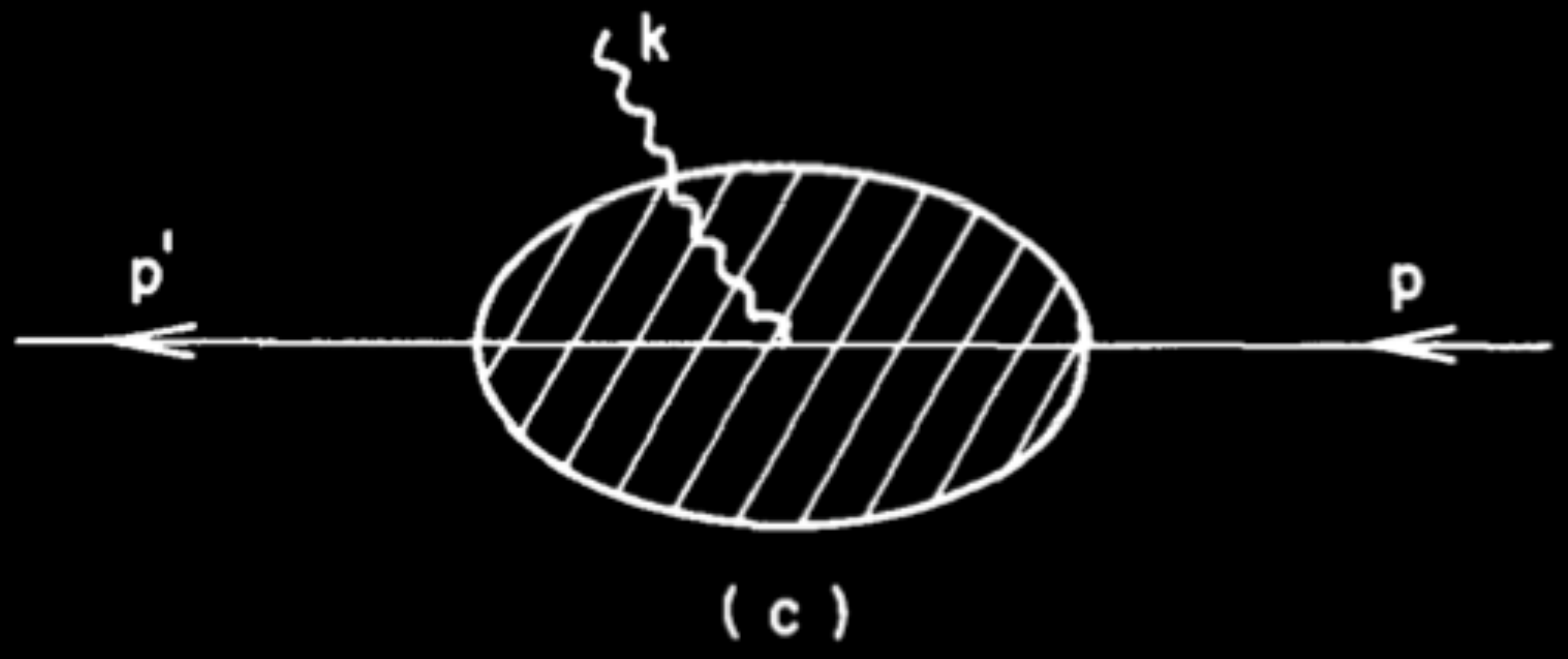
Real emissions



YFS exponentiation

Real emissions

IR divergent pieces



Finite

YFS exponentiation

Real emissions

Mathematically, this implies:

$$|M_1|^2 = \tilde{\beta}_1 + \alpha\tilde{S}|M_0|^2$$

"1 real photon = finite + eikonal times x Born"

$$|M_2|^2 = \tilde{\beta}_2 + \alpha\tilde{S}|M_1|^2 + (\alpha\tilde{S})^2|M_0|^2$$

Iterate this!

S contains the IR divergences associated to the diagrams I showed before

It is an on-shell Eikonal factor. Note: this is now done at the amplitude squared level.

YFS exponentiation

Master Formula

Combine all the ingredients:

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{2\alpha(B+\tilde{B}(\Omega))}}{n_\gamma!} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i)\Theta(k_i,\Omega)\right]\left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma}\frac{\tilde{\beta}_1(k_j)}{\tilde{S}(k_j)} + \cdots\right)$$

Form factor   Real phase-space integration with Eikonals   Regulated matrix elements

YFS exponentiation

Master Formula

$$d\sigma = \sum_{n_\gamma=0}^{\infty} \frac{e^{2\alpha(B+\tilde{B}(\Omega))}}{n_\gamma !} d\Phi_Q \left[\prod_{i=1}^{n_\gamma} d\Phi_i^\gamma \tilde{S}(k_i) \Theta(k_i, \Omega) \right] \left(\tilde{\beta}_0 + \sum_{j=1}^{n_\gamma} \frac{\beta_1(k_j)}{\tilde{S}(k_j)} + \cdots \right)$$

Implemented in Sherpa [2203.10948] and KKMC [0006359]

YFS Resummation for Future Lepton-Lepton Colliders in SHERPA

F. Krauss[a], A. Price[b], and M. Schönherr[a]

Coherent Exclusive Exponentiation For Precision Monte Carlo Calculations[†]

S. Jadach[a,b], B.F.L. Ward[a,c] *and* Z. Wąs[b,d]

YFS exponentiation

Plan for PHOKHARA

Implement CEEX in Phokhara for $e^+e^- \to \mu^+\mu^-\gamma$ with better $\tilde{\beta}$

Reformulate the formalism in dim-reg

instead of using a finite photon mass

See [0108255]

A Practical Approach for Exponentiation of QED Corrections in Arbitrary Processes

GIAMPIERO PASSARINO*

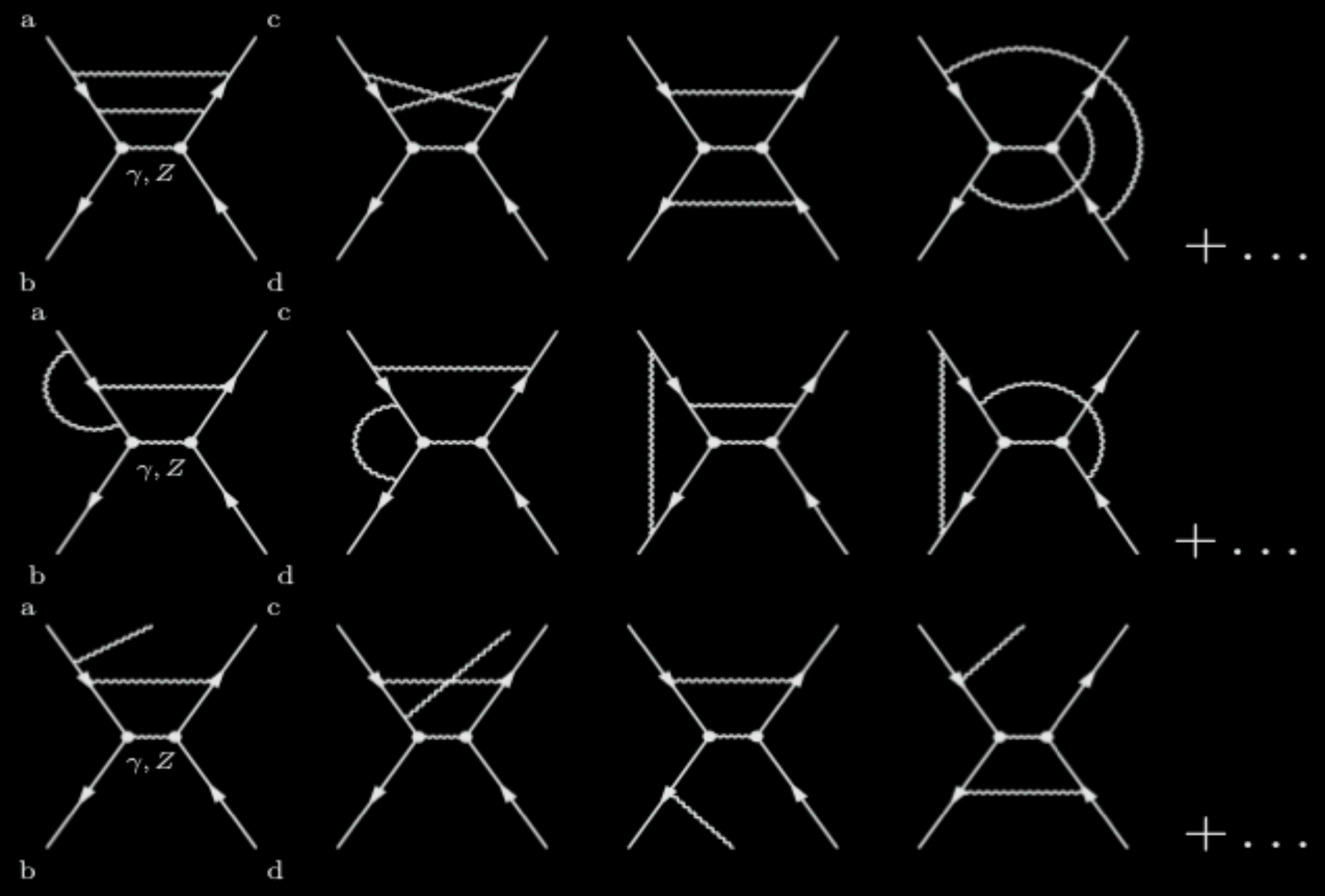


Figure 5: Missing second order diagrams.

Conclusion

- Work was done to verify the results of Phokhara and for proof of concept
- Work is undergoing to reach NNLO theoretical predictions for radiative return processes with muons
- Work on loop integrals can be used in the CEEX resummation procedure to provide better IR safe matrix elements $\tilde{\beta}$

Thanks!