Charged Lepton Flavour Violation and Flavor Model Building

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$\mathscr{L}_4^{\mathrm{SM}}$: ***Accidental symmetries***

$\mathscr{L}_4^{\mathrm{SM}}$ sans Yukawa: $U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$

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$$-\mathscr{L}_4^{\text{Yuk}} = \bar{q} V^\dagger \hat{Y}^u \tilde{H} u + \bar{q} \hat{Y}^d H d + \bar{l} \hat{Y}^e H e$$

[$U(3)^5$ transformation and a singular value decomposition theorem]

$\mathscr{L}_4^{SM}$: $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

Lepton family number conservation

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$[U(3)^5$ transformation and a singular value decomposition theorem$]$

$\mathscr{L}_4^{SM}$: $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$

Lepton family number conservation

- Λ_{UV}^{-1} truncation at the $[\mathscr{L}^{\rm SMEFT}] \leq 4 \Longrightarrow$ **Exact** accidental symmetries
- Beyond this picture, νSM:
  $m_\nu \neq 0,\ U_{PMNS} \neq 1$ introduces lepton flavor violation (LFV)

cLFV

- Lepton family number violation in processes without neutrinos:
  $\mu \to e\gamma, \mu \to 3e, \mu N \to eN, \tau \to \mu\gamma, \ldots, K_L \to \mu e, \ldots, \ h \to \tau\mu, \ldots$
- Tiny breaking due to neutrinos in the minimal νSM realizations:
  $\mathscr{B}(\mu \to e\gamma) \sim 10^{-54}$ strong GIM suppression due to $\Delta m_\nu \ll m_W$
  $\Longrightarrow$ cLFV is a null test of the SM

cLFV

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No irreducible SM background

Advantages:

- Future observation $\implies$ unambiguous New Physics

Disadvantage :

- The $\mathbf{dim\,6}$ EFT effect scales as Λ^{-4} since $A_{SM} = 0$, while, for example in QFV, it is Λ^{-2}

cLFV

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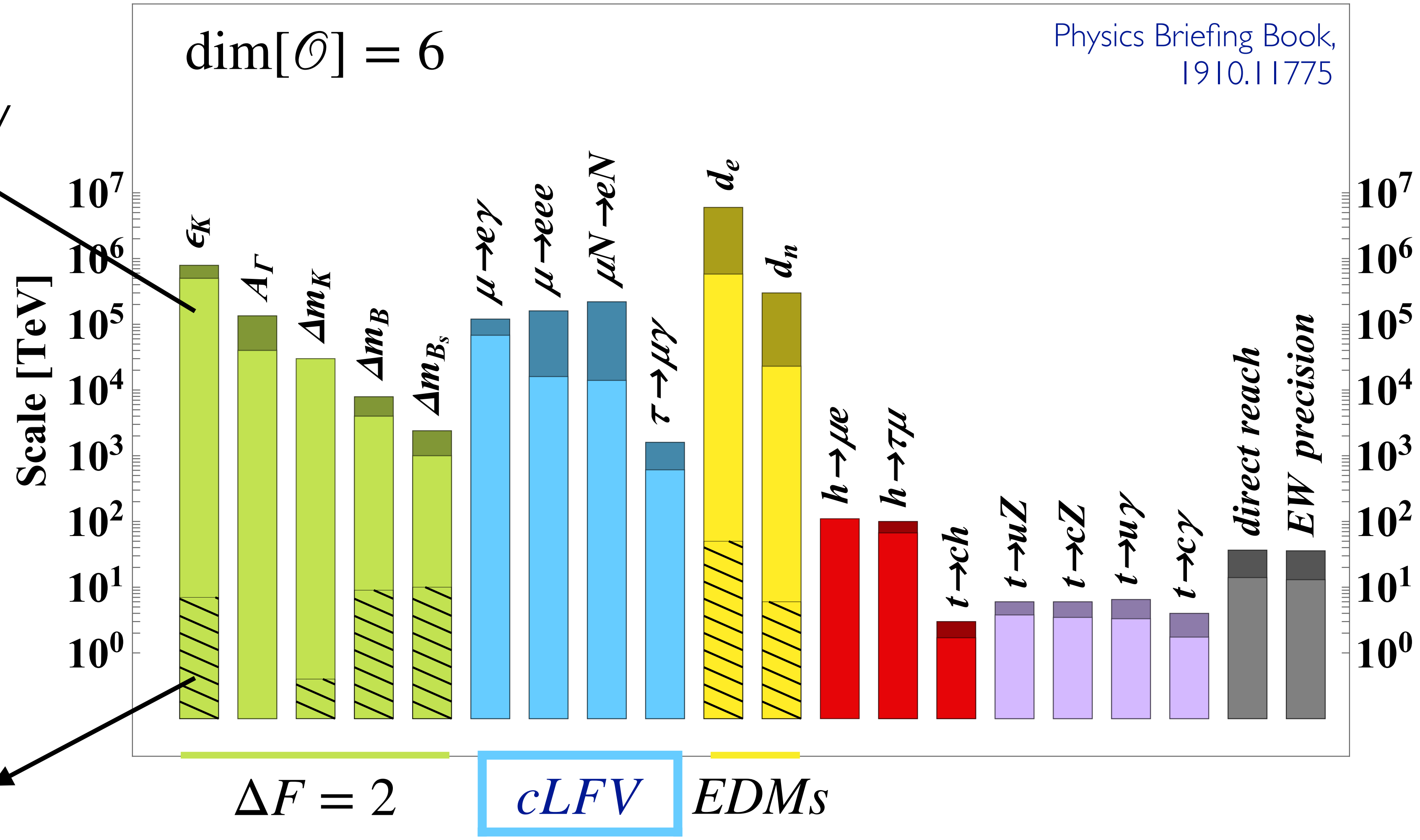
- cLFV already sets **stringent** constraints on BSM: Muon beams are so intense!

- Future experimental prospects are exciting! Davidson et al., 2209.00142
  *an order of magnitude on Λ is huge; think about the FCC

cLFV

- Unique probe of a wide range of BSM up to 10 PeV
- Dynamics of flavor **Today**

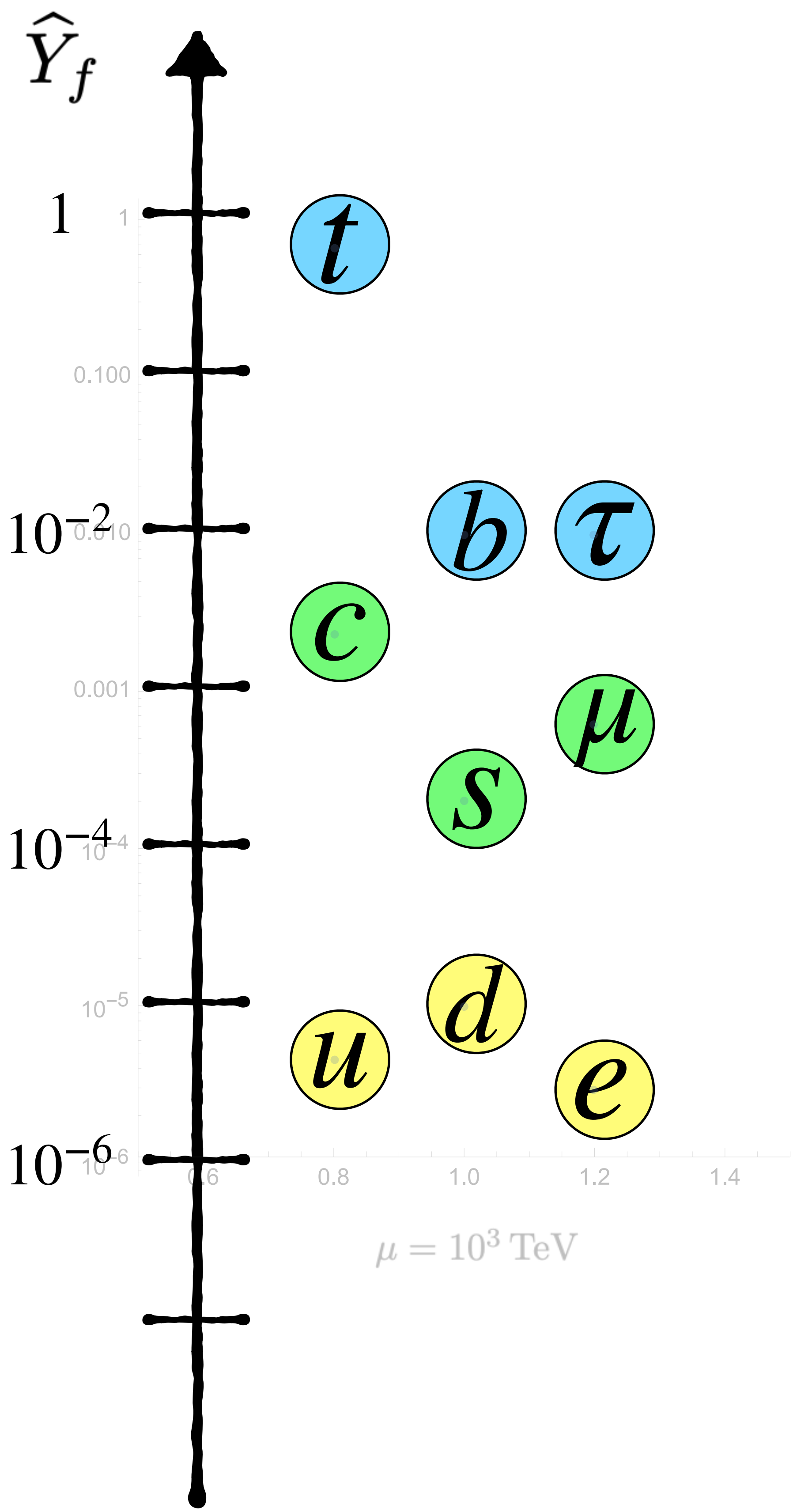
Flavour Anarchy



MFV

The Flavour Puzzle

Empirical



?

$$V_{CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.2^3 \\ 0.2 & 1 & 0.2^2 \\ 0.2^3 & 0.2^2 & 1 \end{pmatrix}$$

The Flavour Puzzle

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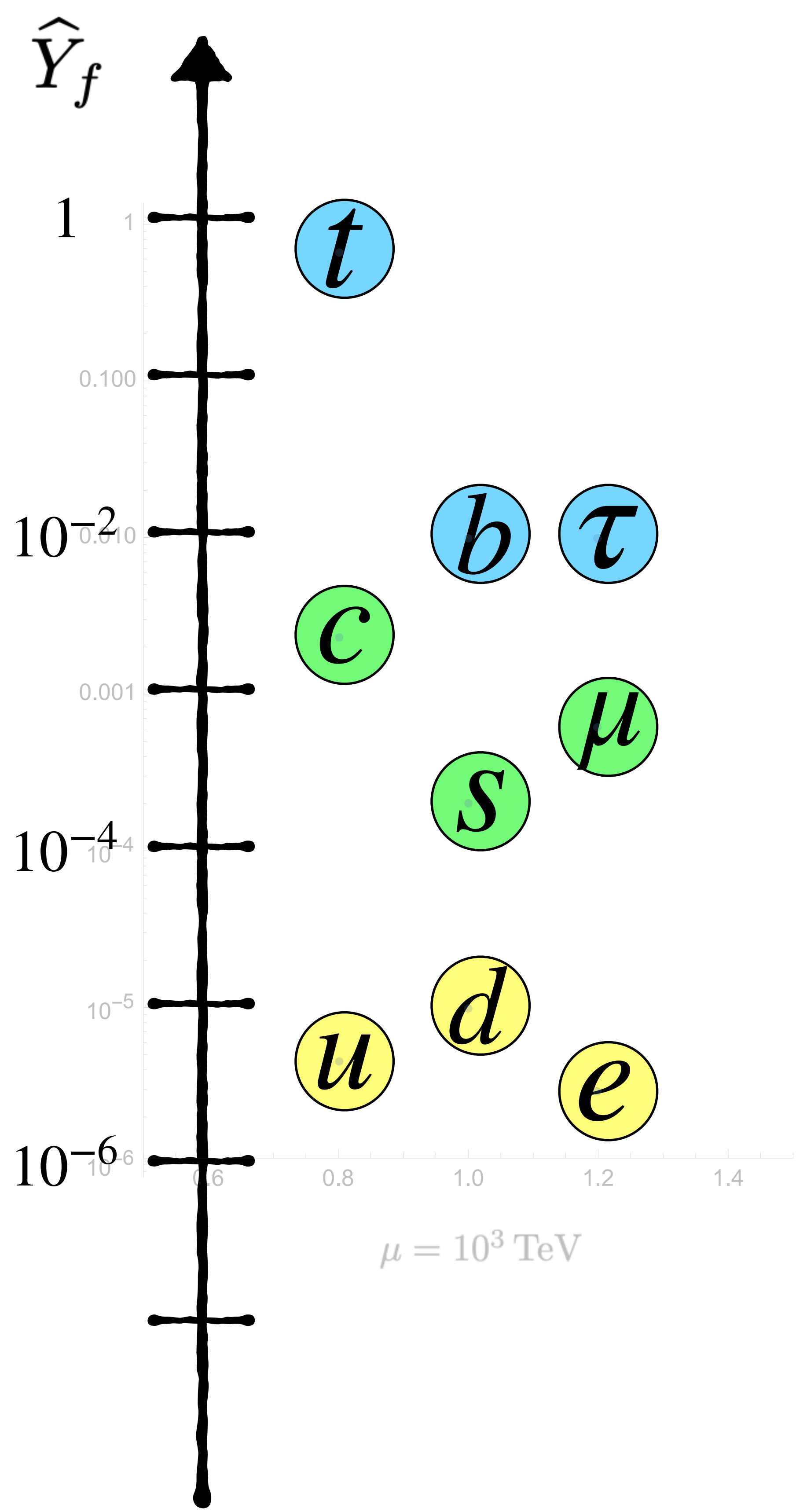


?

$$V_{\rm CKM} \sim \begin{pmatrix} 1 & 0.2 & 0.2^3 \\ 0.2 & 1 & 0.2^2 \\ 0.2^3 & 0.2^2 & 1 \end{pmatrix}$$

$$-\mathcal{L}_{\rm SM} \supset \bar{q}_i Y_u^{ij} u_j \tilde{H} + \bar{q}_i Y_d^{ij} d_j H + \bar{\ell}_i Y_e^{ij} e_j H$$

SVD: $Y_f = L_f \widehat{Y}_f R_f^\dagger$

$$V_{\rm CKM} = L_u^\dagger L_d$$

- Small y_f — natural a la t' Hooft.
- Enter the theory in the same way. **Why hierarchies???**

The Flavour Puzzle

- Our goal: Explain the many hierarchies observed in masses and mixings from a few (or no) UV hierarchies.
- Tradeoff: **Simple UV realization** versus how well it produces the SM flavor

Approximate global U(2)

Barbieri et al; hep-ph/9512388, hep-ph/9605224, hep-ph/9610449, …

<u>Our revision</u>:

AG, Thomsen; <u>2309.11547</u>

Antusch, AG, Stefanek, Thomsen; <u>2311.09288</u>

AG, Thomsen, Tiblom; <u>2406.02687</u>

$\bar{f}^i_L \, Y^{ij} \, f^j_R$ Hierarchies from $\mathrm{U(2)_L}$

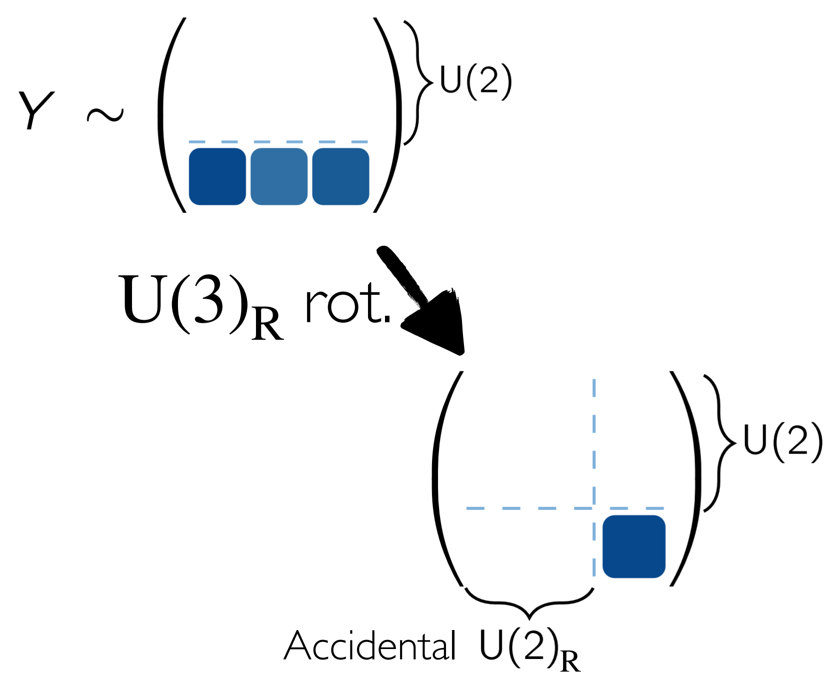
$$\mathrm{U(2)} \equiv \mathrm{SU(2)} \times \mathrm{U(1)} \qquad \text{IRREPs} \qquad \begin{bmatrix} f^1_L \\ f^2_L \end{bmatrix} \sim \mathbf{2}_{+1} \qquad f^3_L \, , f^i_R \sim \mathbf{1}_0$$

$\bar{f}_L^i \, Y^{ij} f_R^j$ **Hierarchies from** $\mathrm{U(2)_L}$

$$\mathrm{U(2)} \equiv \mathrm{SU(2)} \times \mathrm{U(1)} \qquad \text{IRREPs} \qquad \begin{bmatrix} f_L^1 \\ f_L^2 \end{bmatrix} \sim \mathbf{2}_{+1} \qquad f_L^3 \, , f_R^i \sim \mathbf{1}_0$$

Step A

Exact symmetry limit



$$m_3 \neq 0 \, , m_{1,2} = 0$$

$\bar{f}_L^i \, Y^{ij} \, f_R^j$ ***Hierarchies from*** $\mathrm{U(2)_L}$

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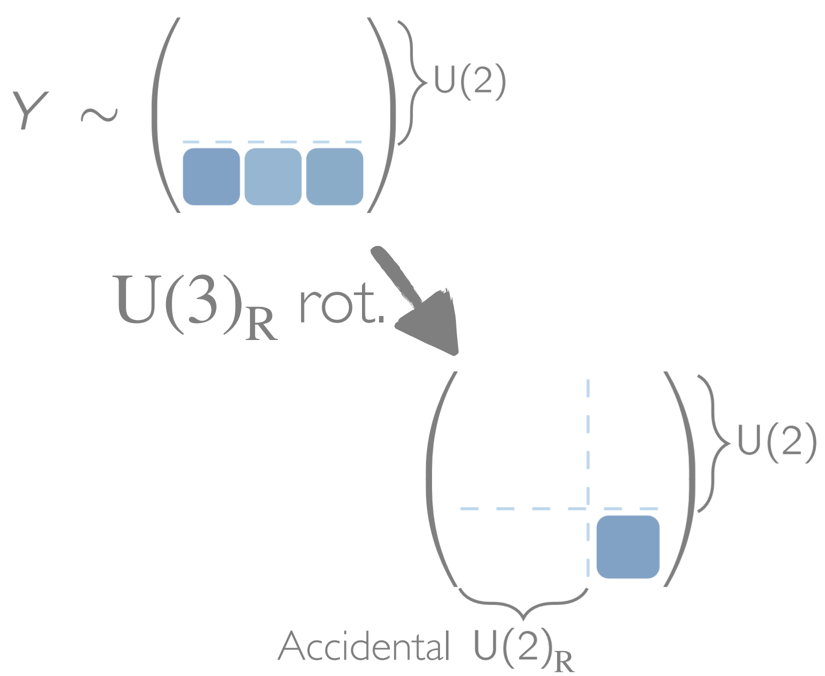

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Step B

Leading (small) breaking

$$V_2 = \begin{pmatrix} 0 \\ a \end{pmatrix} \sim \mathbf{2}_{+1}$$

$$\bar{f}_L V \sim \mathbf{1}_0$$

$$\mathrm{U(2)} \to \mathrm{U(1)}$$

$$1 \gg a > 0$$

$$m_3 \gg m_2 > 0 \, , m_1 = 0$$

$\bar{f}_L^i\, Y^{ij} f_R^j$ *Hierarchies from* $U(2)_L$

$U(2) \equiv SU(2) \times U(1)$ IRREPs $\begin{bmatrix} f_L^1 \\ f_L^2 \end{bmatrix} \sim \mathbf{2}_{+1}$ $f_L^3\,, f_R^i \sim \mathbf{1}_0$

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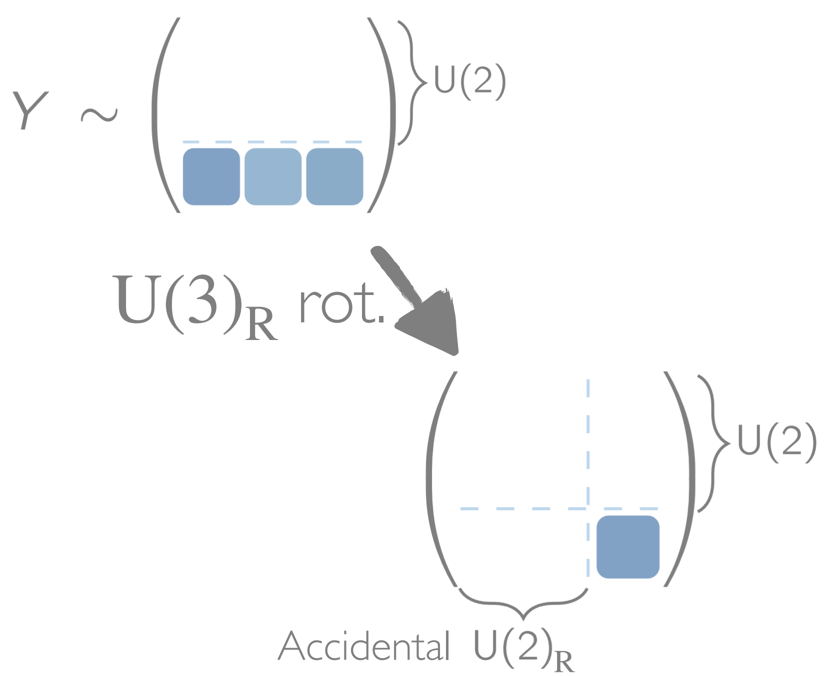

$m_3 \neq 0\,, m_{1,2} = 0$

Step B

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$\bar{f}_L V \sim \mathbf{1}_0$

$$U(2) \to U(1)$$

$$1 \gg a > 0$$

$m_3 \gg m_2 > 0\,, m_1 = 0$

Step C

Subleading breaking

$$V_1 = \begin{pmatrix} b \\ 0 \end{pmatrix} \sim \mathbf{2}_{+1}$$

$$\to 0$$

$$1 \gg a \gg b > 0$$

$m_3 \gg m_2 \gg m_1$

$U(2)_L$: *Singular value decomposition*

$$Y \equiv L_f \, \hat{Y} \, R_f^\dagger$$

$$Y \sim \begin{bmatrix} b & b & b \\ a & a & a \\ 1 & 1 & 1 \end{bmatrix}$$

$$1 \gg a \gg b$$

$U(2)_L$: *Singular value decomposition*

$$Y \equiv L_f \, \hat{Y} \, R_f^\dagger$$

$$Y \sim \begin{bmatrix} b & b & b \\ a & a & a \\ 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_f^{(0)} \sim \mathcal{O}(1) \text{ rot.}} Y^{(1)} \sim \begin{bmatrix} b & b & b \\ 0 & a & a \\ 0 & 0 & 1 \end{bmatrix}$$

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$\mathrm{U}(2)_L$: Singular value decomposition

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$1 \gg a \gg b$

Perturbative diagonalisation: $\mathrm{Y}^{(1)} = \mathrm{L}_f^{(0)} \, \hat{\mathrm{Y}} \, \mathrm{R}_f^{(1)\dagger}$

$$\hat{\mathrm{Y}} \sim \begin{bmatrix} b & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathrm{L}_f^{(0)} \sim \begin{bmatrix} 1 & b/a & b \\ & 1 & a \\ & & 1 \end{bmatrix}$$

Impose $\mathrm{U}(2)_{q+\ell}$:

AG, Thomsen; 2309.11547

Quarks

$$\begin{pmatrix} q_L^1 \\ q_L^2 \end{pmatrix} \sim \mathbf{2}_{+1}$$

all other singlets

→

- Both $\hat{\mathrm{Y}}_u$ and $\hat{\mathrm{Y}}_d$ hierarchical
- $V_{\mathrm{CKM}} \approx \mathrm{L}_u^{(0)\dagger} \mathrm{L}_d^{(0)}$ hierarchical

Imposing $\mathrm{U}(2)_q \implies$
$\mathrm{U}(2)_u \times \mathrm{U}(2)_d$ is
accidental at dim-4

Leptons

$$\begin{pmatrix} \ell_L^1 \\ \ell_L^2 \end{pmatrix} \sim \mathbf{2}_{+1}$$

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- Hierarchical $\hat{\mathrm{Y}}_e$

*needs a different structure in the neutrino sector to get large PMNS

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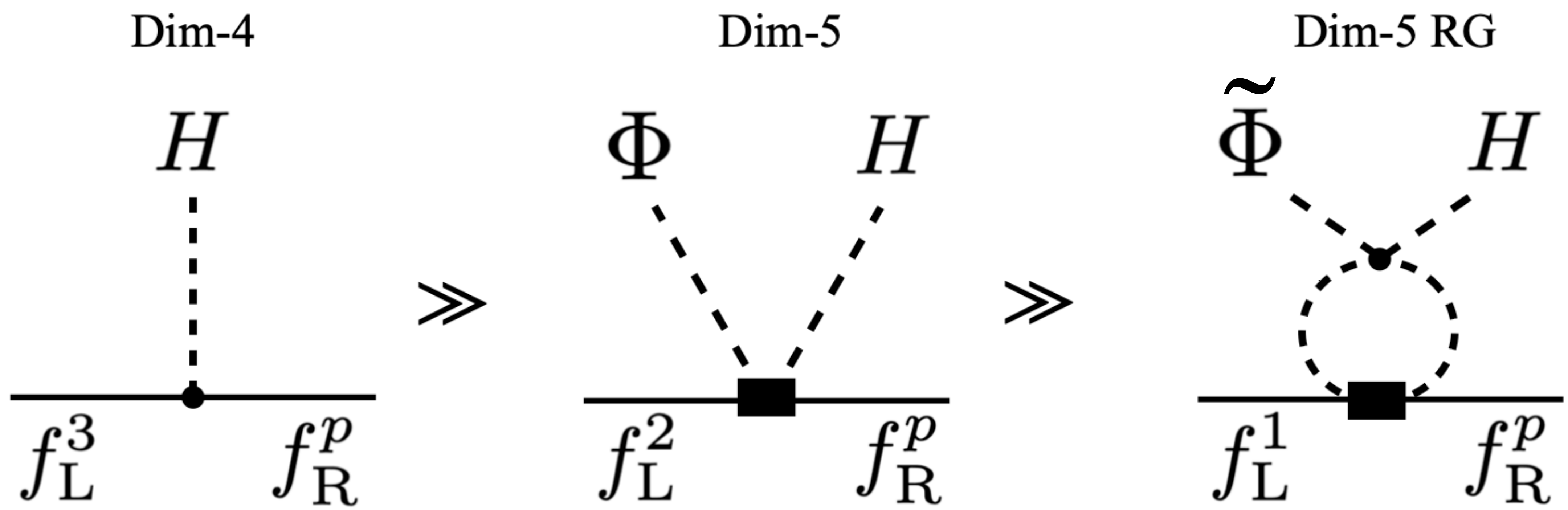
- For the variation $\mathrm{U}(2)_{q+e}$ or $\mathrm{U}(2)_{q+e^c+u^c}$ see backup and Antusch, AG, Stefanek, Thomsen; 2311.09288

The UV origin of approximate $\mathbf{U(2)}_L$

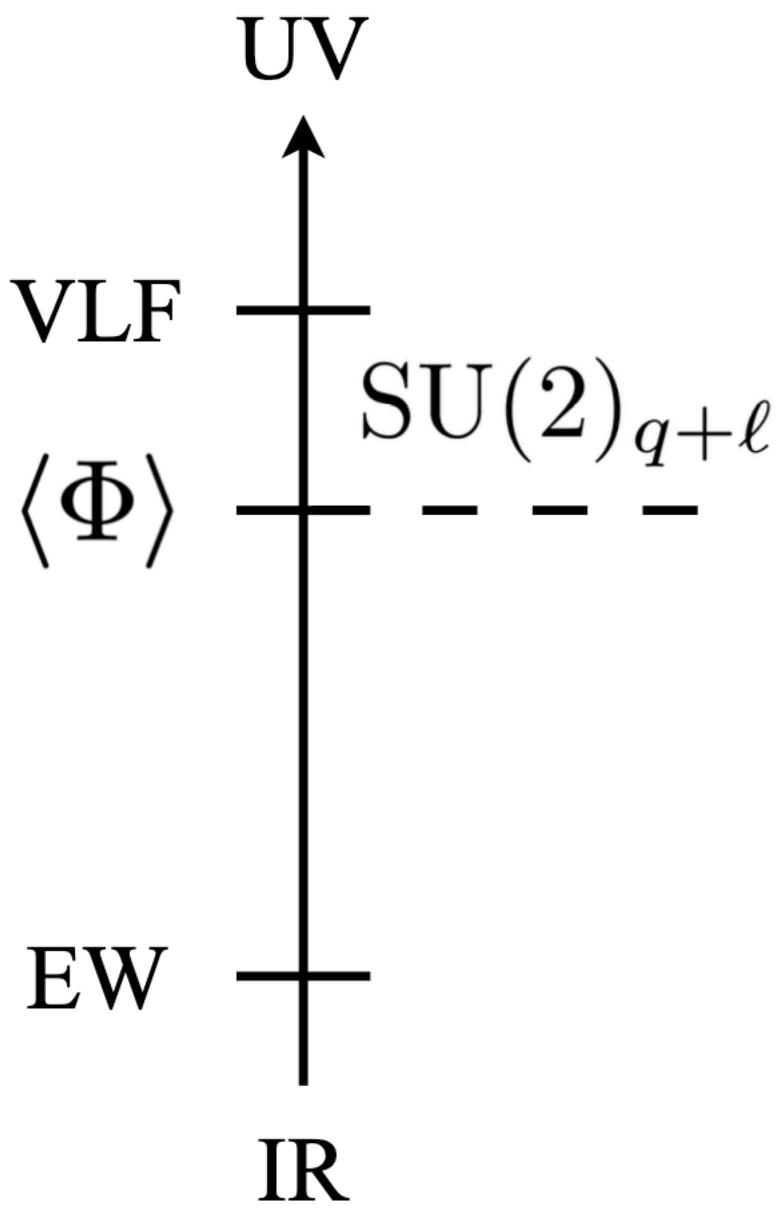
$\mathrm{SM} \times \mathrm{SU}(2)_{q+l}$ ***gauged***

AG, Thomsen; 2309.11547



- The SM-singlet scalar $\mathbf{\Phi} \sim \mathbf{2}$ of flavor: $\langle \Phi^\alpha \rangle = \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix}$ *2nd family; $\tilde{\Phi}^\alpha = \varepsilon^{\alpha\beta}\Phi^*_\beta$ *1st family

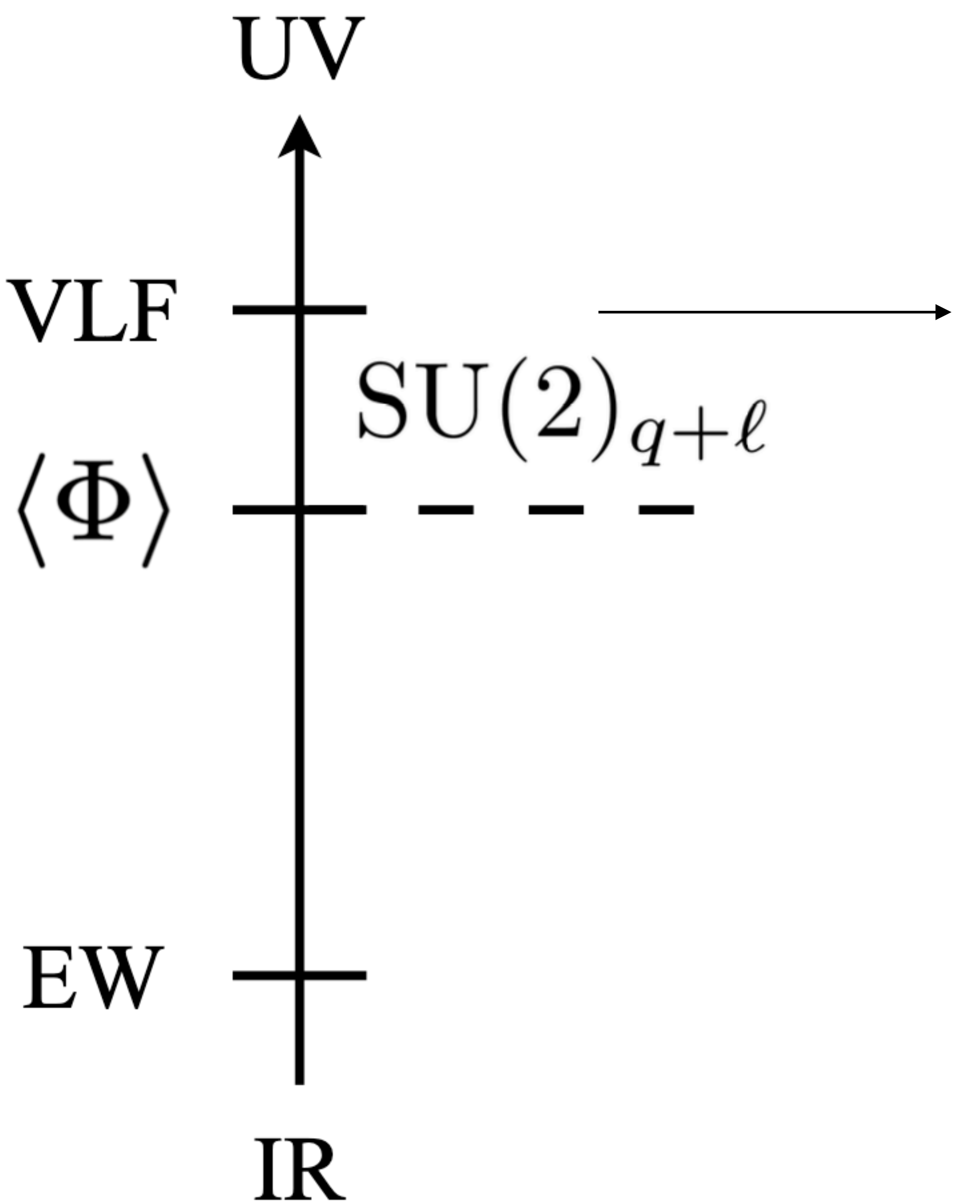
Gauged flavor



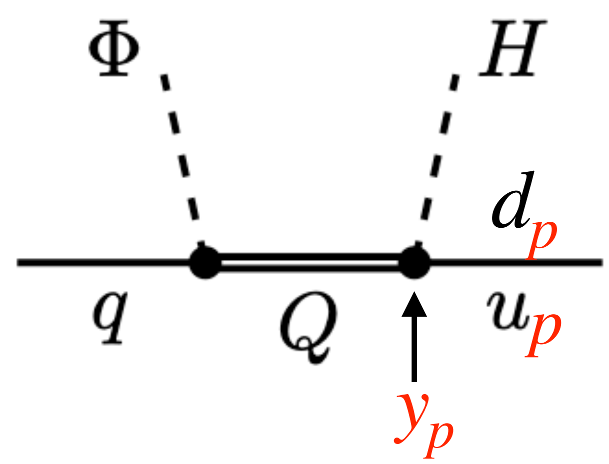
$$a = \frac{v_\Phi}{m_F}$$

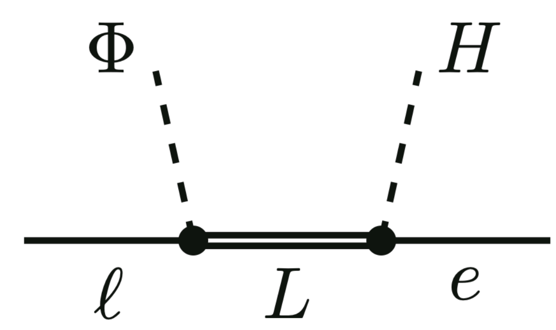
AG, Thomsen; 2309.11547

Gauged flavor



AG, Thomsen; 2309.11547





PS unification $m_Q = m_L$

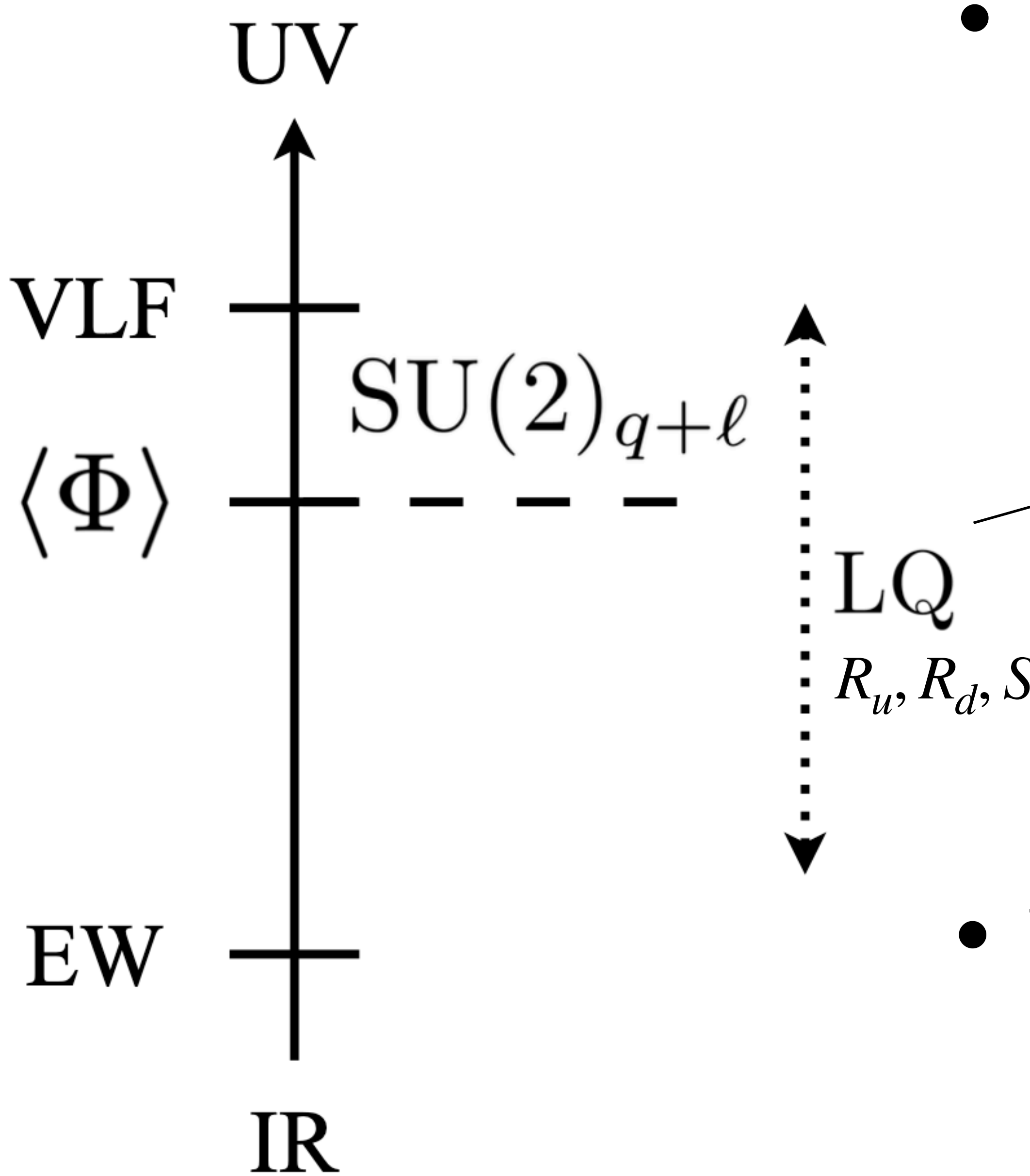
AG, Thomsen, Tiblom; 2406.02687

- A *single* VLQ $\Longrightarrow$ Y is **Rank 2**

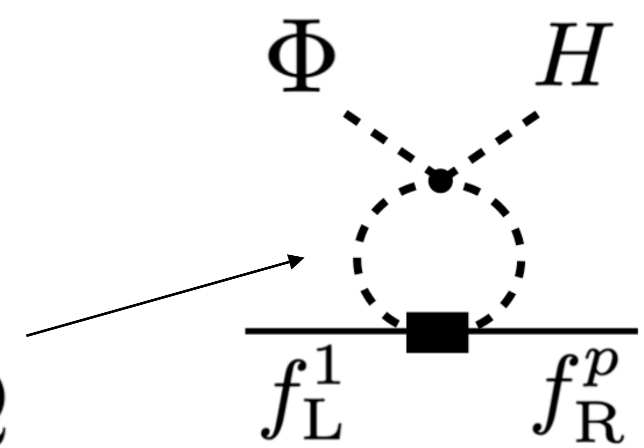
$$\mathrm{Y} \propto \begin{bmatrix} y^p \\ y^p \\ 1^p \end{bmatrix} \begin{matrix} \longleftarrow \tilde{\Phi} \\ \longleftarrow \Phi \\ \end{matrix}$$

- Accidental $\mathrm{U}(1)$:
  Massless 1st family!

Gauged flavor



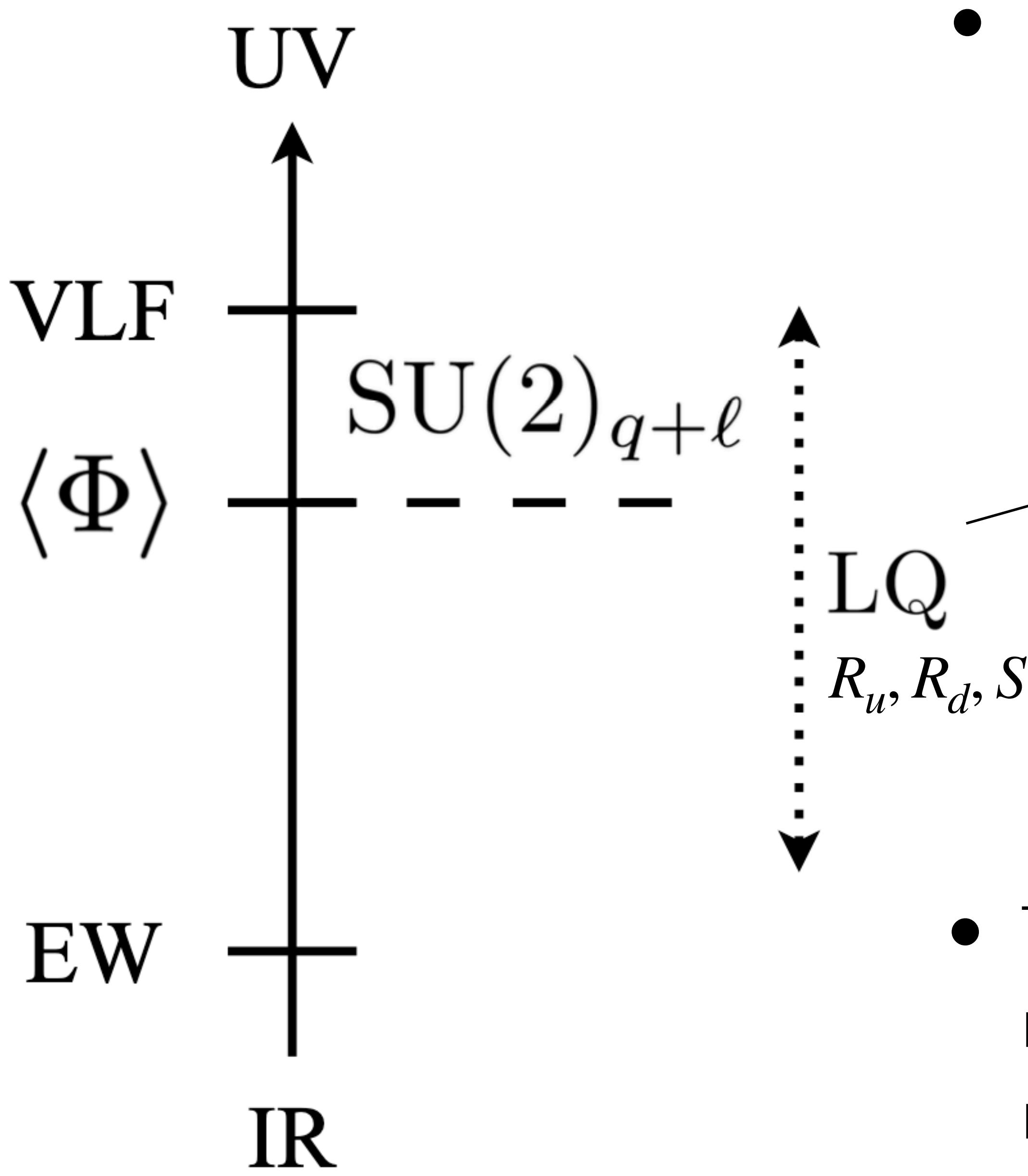
- Instead of new UV states, introduce IR states.



- The obtained Yukawas are mainly ***insensitive*** to their masses! $\sim \log m_F / m_S$

AG, Thomsen; 2309.11547

Gauged flavor



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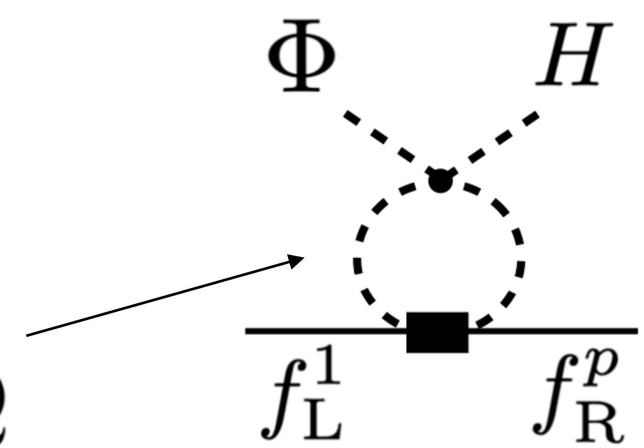

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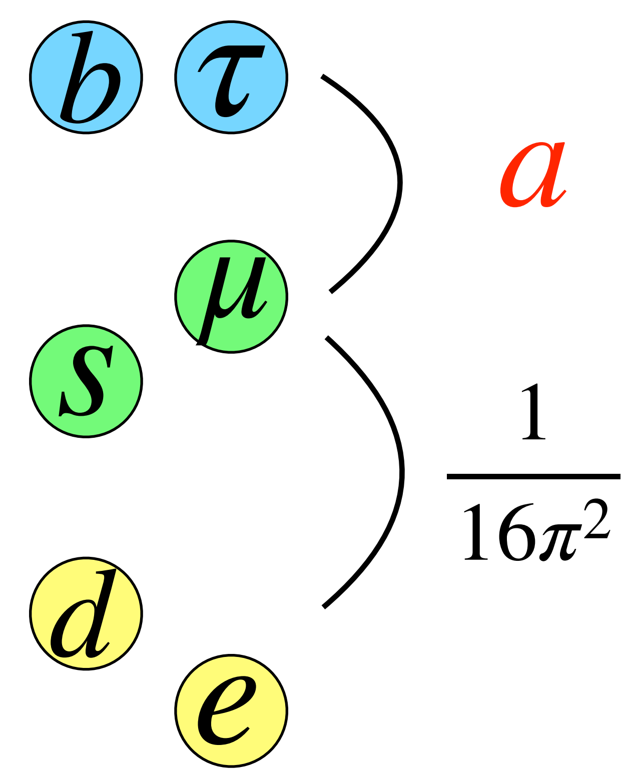


$$b \sim a/16\pi^2$$

A single parameter!

$$a = v_\Phi/m_F$$

AG, Thomsen; 2309.11547

Z' effects

- SSB of $SU(2)_{q+\ell}$ produced heavy, degenerate vector triplet.
- Integrating it out:

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{2v_\Phi^2}\left[\delta^\alpha{}_\delta \delta^\gamma{}_\beta - \frac{1}{2}\delta^\alpha{}_\beta \delta^\gamma{}_\delta\right]\left[(\bar{q}_\alpha \gamma^\mu q^\beta)(\bar{q}_\gamma \gamma^\mu q^\delta)\right.$$
$$\left. + 2(\bar{q}_\alpha \gamma^\mu q^\beta)(\bar{\ell}_\gamma \gamma^\mu \ell^\delta) + (\bar{\ell}_\alpha \gamma^\mu \ell^\beta)(\bar{\ell}_\gamma \gamma^\mu \ell^\delta)\right].$$

$$\alpha, \beta, \ldots \in \{1,\, 2\}$$

- Suppressed bounds from 4-quark and 4-lepton FCNCs — Darme et al; 2307.09595

Eg. $\mathcal{L}_{\text{LEFT}} \supset -\frac{1}{4v_\Phi^2} A_{sd}^2 (\bar{s}_{\text{L}} \gamma_\mu d_{\text{L}})^2$ $\qquad A_{f_p f'_r} = \left[L_f^\dagger \, \text{diag}(1,\, 1,\, 0) L_{f'}\right]_{pr}.$

- The strongest bounds involve 2q2l cLFV:

*complementary, can not be tuned away simultaneously

$$\text{BR}(K_L \to \mu^\pm e^\mp) = 5.9 \cdot 10^{-12} \left(\frac{300\,\text{TeV}}{v_\Phi}\right)^4 \left|A_{se} A_{\mu d} + A_{de} A_{\mu s}\right|^2.$$

$$\text{CR}(\mu\text{Au} \to e\text{Au}) = 2 \cdot 10^{-11} \cdot \left(\frac{300\,\text{TeV}}{v_\Phi}\right)^4 \times \left|1.01\, s_{2\ell} - 0.25\, c_{2\ell}\right|^2.$$

$$v_\Phi \gtrsim 300\ \text{TeV}$$

*Future MU2E and COMET will improve by an order of magnitude!

R_u leptoquark

- Induces chirality-enhanced dipoles at one-loop:

$$C_{e\gamma}^{pr} = -\frac{1}{16\pi^2}\frac{(L_e^{3p})^* \kappa_u^3 x_{3u} \kappa_e^r}{M_{R_u}^2} \log\frac{M_{R_u}^2}{m_t^2}$$

- The strongest bounds:

$\mu \to e\gamma$

$$|\kappa_u^3 x_{3u} \kappa_e^1| \frac{|L_e^{32}|}{0.1}\left(\frac{500\,\mathrm{TeV}}{M_{R_u}}\right)^2 \frac{\log\frac{M_{R_u}}{m_t}}{8} < 0.017$$

eEDM

$$\frac{|x_{3u}\,\mathrm{Im}((L_e^{31})^*\kappa_u^1\kappa_e^1)|}{10^{-3}}\left(\frac{500\,\mathrm{TeV}}{M_{R_u}}\right)^2 \frac{\log\frac{M_{R_u}}{m_t}}{8} < 4\cdot 10^{-3}$$

$M_{R_u} \gtrsim 500\,\mathrm{TeV}$ when couplings $\mathcal{O}(0.3)$

PS unification

- All five scalar fields of the model fit into just two irreps!

$$H, R_u, R_d \subseteq \Sigma_H \sim (\mathbf{15},\ \mathbf{2},\ 1/2,\ \mathbf{1})$$

$$\Phi, S \ \subseteq\ \Sigma_\Phi \ \sim\ (\mathbf{15},\ \mathbf{1},\ 0,\ \mathbf{2})$$

$\mathrm{SU}(4) \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{\mathrm{R}} \times \mathrm{SU}(2)_{q+\ell}$

- Explains why $M_Q = M_L$. They unify into a single VLF!

$$\Psi_{\mathrm{L,R}} \sim (\mathbf{4},\ \mathbf{2},\ 0,\ \mathbf{1})$$

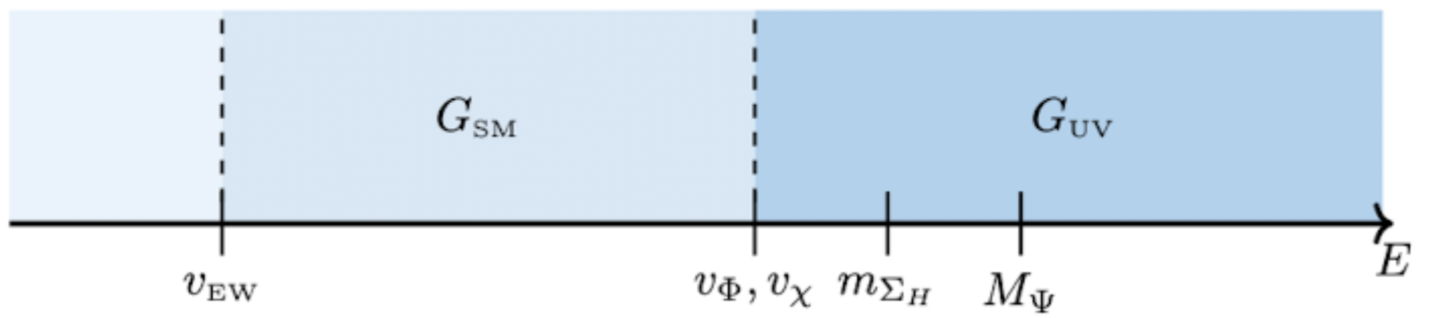
- Can one fit masses and mixings? Predictions from unification?

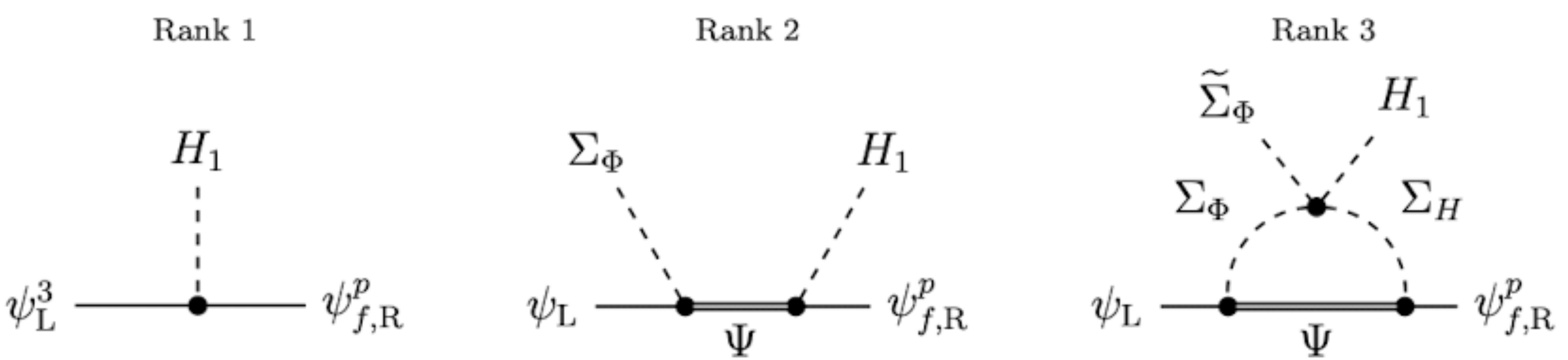
AG, Thomsen, Tiblom; 2406.02687

PS unification

AG, Thomsen, Tiblom; 2406.02687

Field	SU(4)	$SU(2)_L$	$U(1)_R$	$SU(2)_{q+\ell}$
ψ_L	**4**	**2**	0	**2**
ψ_L^3	**4**	**2**	0	**1**
$\psi_{u,R}^p$	**4**	**1**	1/2	**1**
$\psi_{d,R}^p$	**4**	**1**	-1/2	**1**
$\Psi_{L,R}$	**4**	**2**	0	**1**
χ	**4**	**1**	1/2	**1**
H_1	**1**	**2**	1/2	**1**
Σ_H	**15**	**2**	1/2	**1**
Σ_Φ	**15**	**1**	0	**2**





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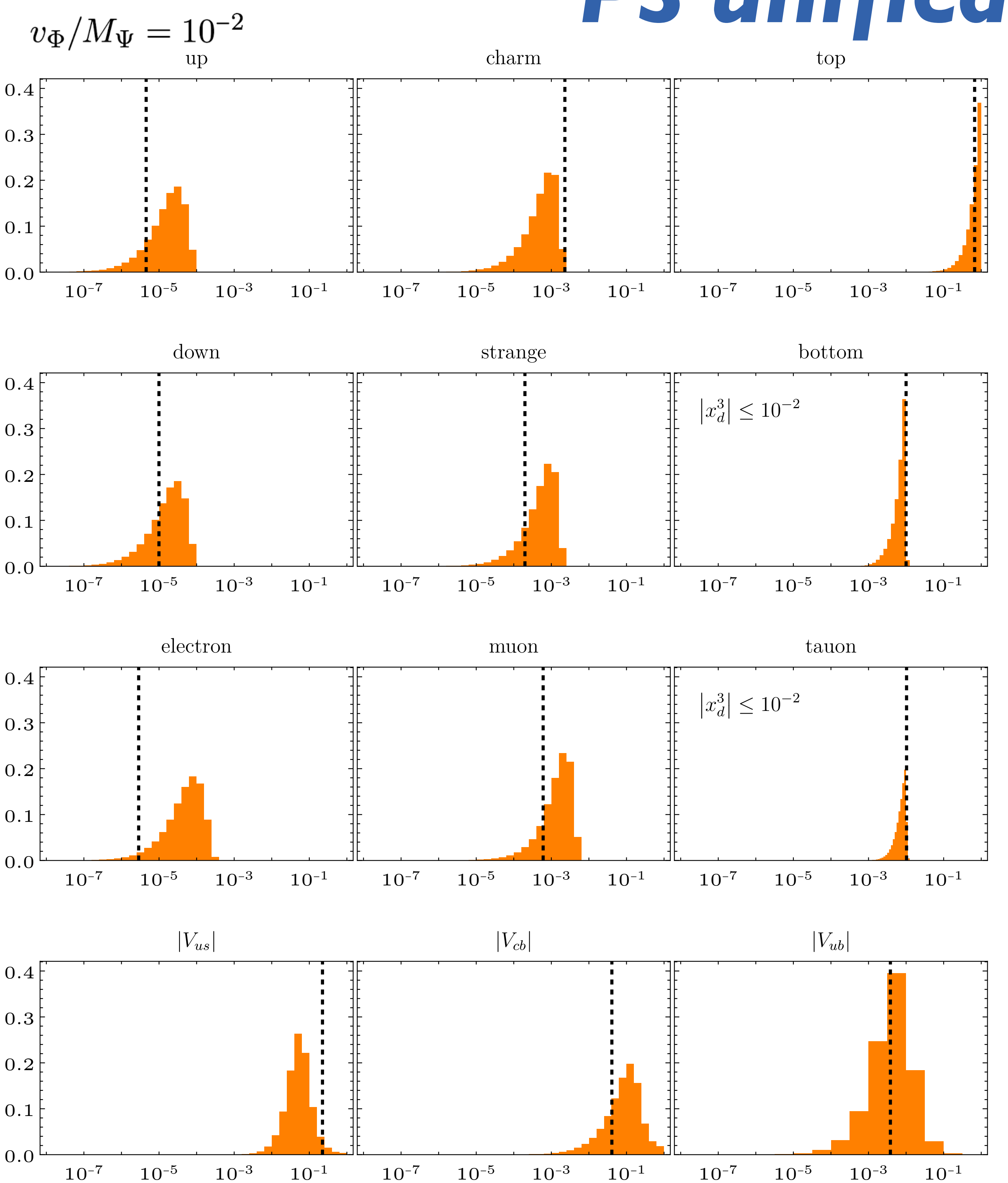
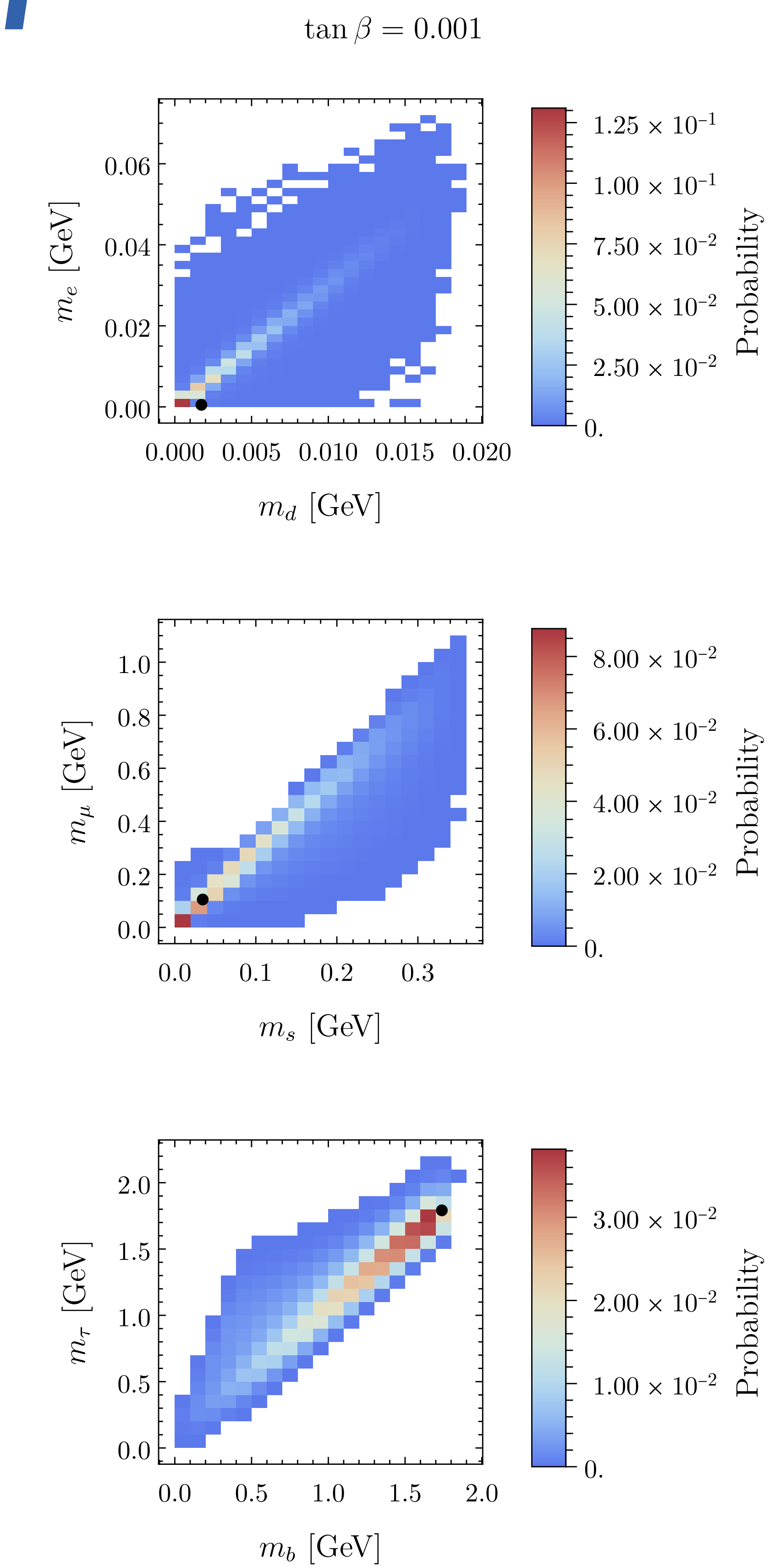


Figure 3. Histogram showing the probability of obtaining the correct order of magnitude for the SM flavor parameters when the UV parameters take on random numbers drawn from a flat distribution with the magnitude ≤ 1. The black lines display the running SM values at the renormalization scale 1 PeV. See Section 4.2 for details.



AG, Thomsen, Tiblom; 2406.02687

Phenomenology

AG, Thomsen, Tiblom; 2406.02687

- Integrating out vector leptoquark and flavored Z's

$$\mathcal{L}_{\text{SMEFT}} \supset -\frac{1}{v_\chi^2}\left|\bar{q}^3\gamma_\mu\ell^3 + \bar{q}_\alpha\gamma_\mu\ell^\alpha + \bar{d}^p\gamma_\mu e^p\right|^2 - \frac{1}{v_\Phi^2}\left|\bar{q}_\alpha\gamma_\mu t^\alpha_{\alpha\beta} q^\beta + \bar{\ell}_\alpha\gamma_\mu t^\alpha_{\alpha\beta}\ell^\beta\right|^2$$

- Leading bounds from $K_L \to \mu e$ and $\mu \to e$ conversion, $v_\chi \gtrsim 3$ PeV

- Mu2e and COMET will push the bounds to $v_\chi \gtrsim 10$ PeV range

Conclusions

- I presented simple UV models based on a single SU(2) gauged flavor symmetries that explain the gross feature of observed flavor hierarchies.
- cLFV provides the most stringent present bounds and the most promising future probe

Antusch, AG, Stefanek, Thomsen; 2311.09288

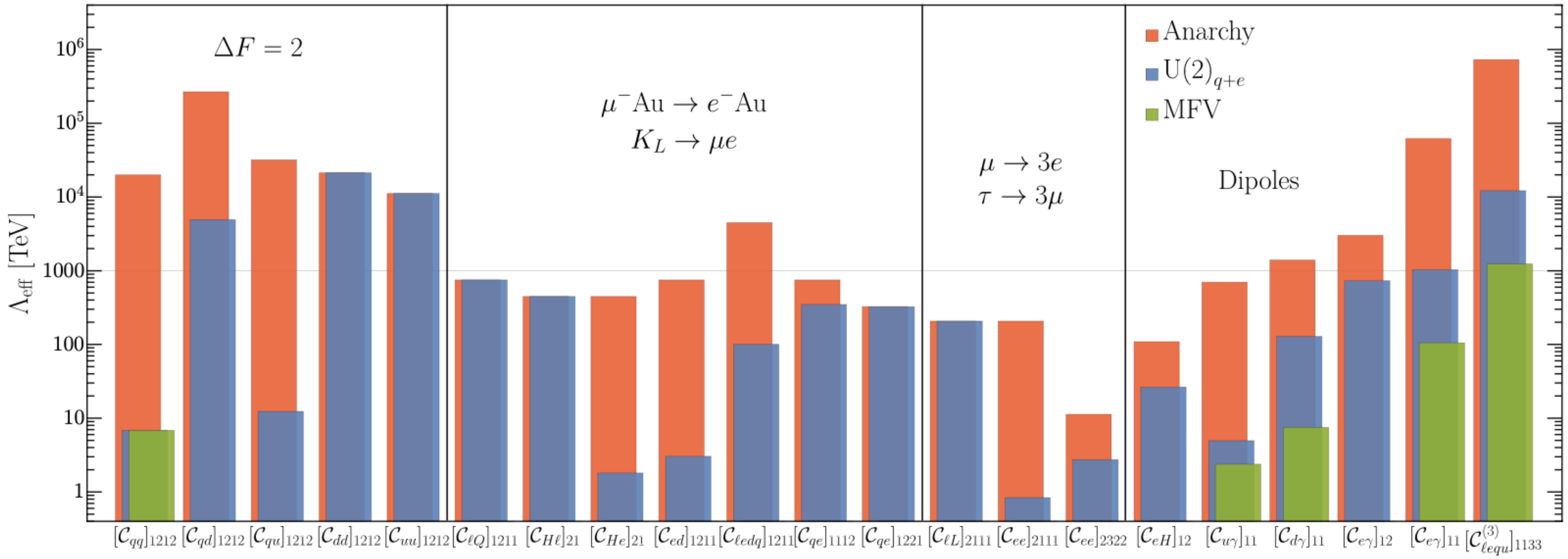


FIG. 1. Comparative constraints on SMEFT operators from flavor and CP violation: Minimally-broken $\mathrm{U}(2)_{q+e}$ (Blue), MFV (Green), Flavor Anarchy (Red). Here, $Q = q, u, d$ and $L = \ell, e$. See Section 3 for details.

Thank you

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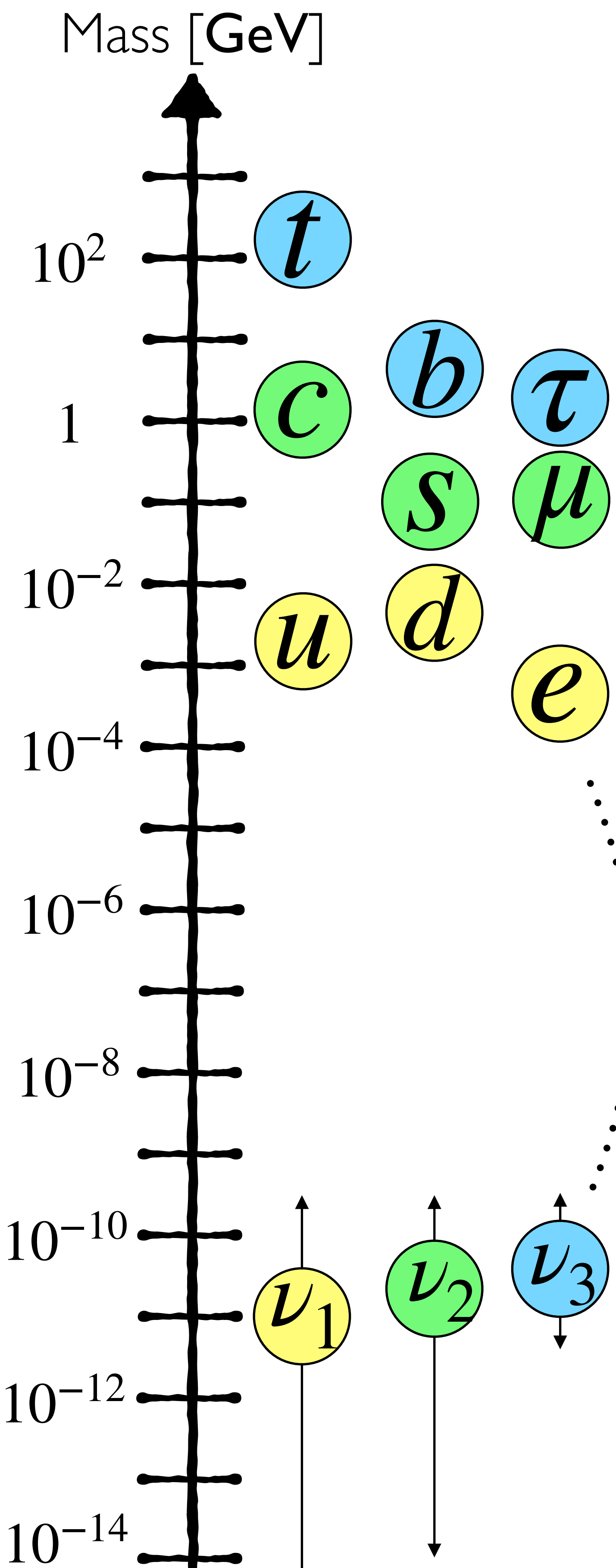
Backup

The Flavour Puzzle

Empirical



<u>The neutrino sector is different</u>

$$-\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{\Lambda_\nu} \ell_i Y_\nu^{ij} \ell_j H H$$

1) High-scale Λ_ν predicts a mass gap!

2) Large/Anarchic mixing!

$$V_{\text{PMNS}} \sim \begin{pmatrix} 0.8 & 0.6 & 0.15 \\ 0.4 & 0.6 & 0.7 \\ 0.4 & 0.6 & 0.7 \end{pmatrix}$$

The success of the SM(EFT)?

$U(2)_R$?

$$\begin{bmatrix} f_R^1 \\ f_R^2 \end{bmatrix} \sim \mathbf{2}_{+1} \qquad f_R^3, f_L^i \sim \mathbf{1}_0$$

$$Y \sim \begin{bmatrix} b & a & 1 \\ b & a & 1 \\ b & a & 1 \end{bmatrix} \xrightarrow{L_f^{(0)} \sim \mathcal{O}(1) \text{ rot.}} Y^{(1)} \sim \begin{bmatrix} b & 0 & 0 \\ b & a & 0 \\ b & a & 1 \end{bmatrix}$$

$1 \gg a \gg b$

Perturbative diagonalisation: $Y^{(1)} = L_f^{(1)} \hat{Y} R_f^{(0)\dagger}$

$$\hat{Y} \sim \begin{bmatrix} b & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad R_f^{(0)} \sim \begin{bmatrix} 1 & b/a & b \\ & 1 & a \\ & & 1 \end{bmatrix}$$

Quarks

Impose $\mathrm{U}(2)_q$: $\begin{pmatrix} q_L^1 \\ q_L^2 \end{pmatrix} \sim \mathbf{2}_{+1}$ all other singlets

→

- Both $\hat{\mathrm{Y}}_u$ and $\hat{\mathrm{Y}}_d$ hierarchical
- $V_{\mathrm{CKM}} \approx \mathrm{L}_u^{(0)\dagger}\mathrm{L}_d^{(0)}$ hierarchical

Imposing $\mathrm{U}(2)_q \implies$

$\mathrm{U}(2)_u \times \mathrm{U}(2)_d$ is *accidental* at dim-4

Quarks

Impose $\mathrm{U}(2)_q$: $\begin{pmatrix} q_L^1 \\ q_L^2 \end{pmatrix} \sim \mathbf{2}_{+1}$ all other singlets

⟶

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- $V_{\mathrm{CKM}} \approx \mathrm{L}_u^{(0)\dagger} \mathrm{L}_d^{(0)}$ hierarchical

Imposing $\mathrm{U}(2)_q \implies$ $\mathrm{U}(2)_u \times \mathrm{U}(2)_d$ is *accidental* at dim-4

Leptons

Impose $\mathrm{U}(2)_e$: $\begin{pmatrix} e_R^1 \\ e_R^2 \end{pmatrix} \sim \mathbf{2}_{+1}$ all other singlets

⟶

- Hierarchical $\hat{\mathrm{Y}}_e$ and $L_l^{(0)} \sim \mathcal{O}(1)$.
- No selection rules on the dim-5 Weinberg operator!
  PMNS $\sim \mathcal{O}(1)$

A single $\mathrm{U}(2)$ *to rule them all?*

$$\mathrm{U}(2)_{q+e}$$

U(2) Is Right for Leptons and Left for Quarks

Stefan Antusch (Basel U.), Admir Greljo (Basel U.), Ben A. Stefanek (King's Coll. London), Anders Eller Thomsen (Bern U. and U. Bern, AEC) (Nov 15, 2023)

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- Nine hierarchies in terms of two small parameters:

$$1 \gg a \gg b \gg a^2 \implies \begin{array}{l} y_f^3 \gg y_f^2 \gg y_f^1 \ (\times\ 3 \text{ for } f = u, d, e) \\ 1 \gg |V_{us}| \gg |V_{cb}| \gg |V_{ub}| \end{array}$$

Phenomenology

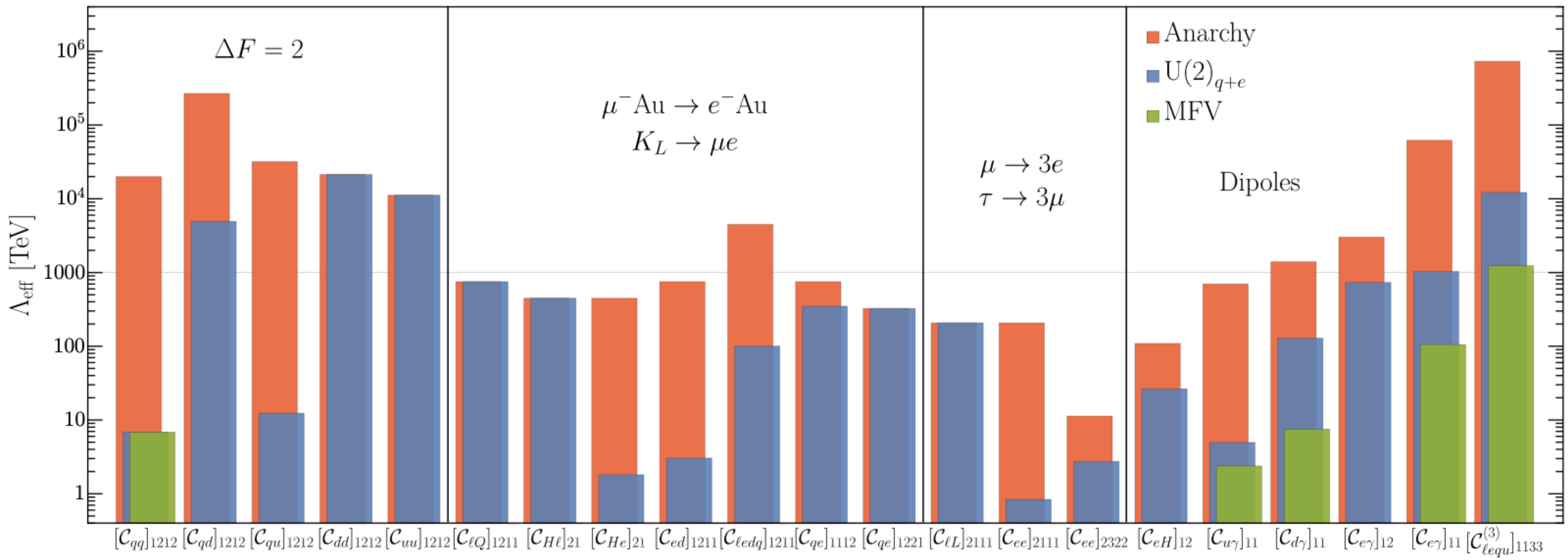
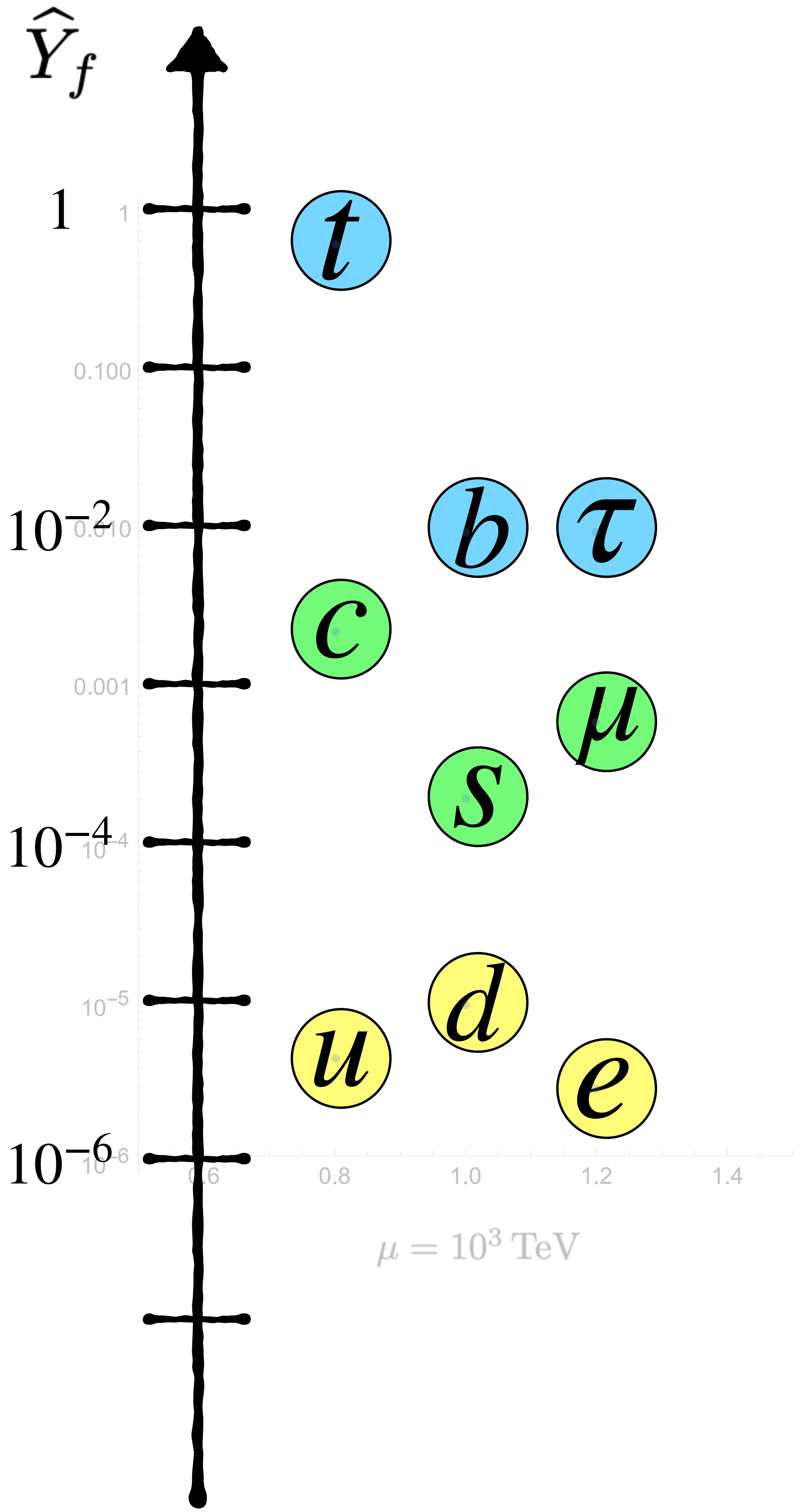


FIG. 1. Comparative constraints on SMEFT operators from flavor and CP violation: Minimally-broken $\mathrm{U}(2)_{q+e}$ (Blue), MFV (Green), Flavor Anarchy (Red). Here, $Q = q, u, d$ and $L = \ell, e$. See Section 3 for details.

- SMEFT as a proxy for short-distance physics: $\mathrm{U}(2) \Longrightarrow$ selection rules.
- A pattern of deviations emerges, distinct from MFV and anarchy.
- cLFV plays a prominent role! Exciting prospects.

Refining the picture



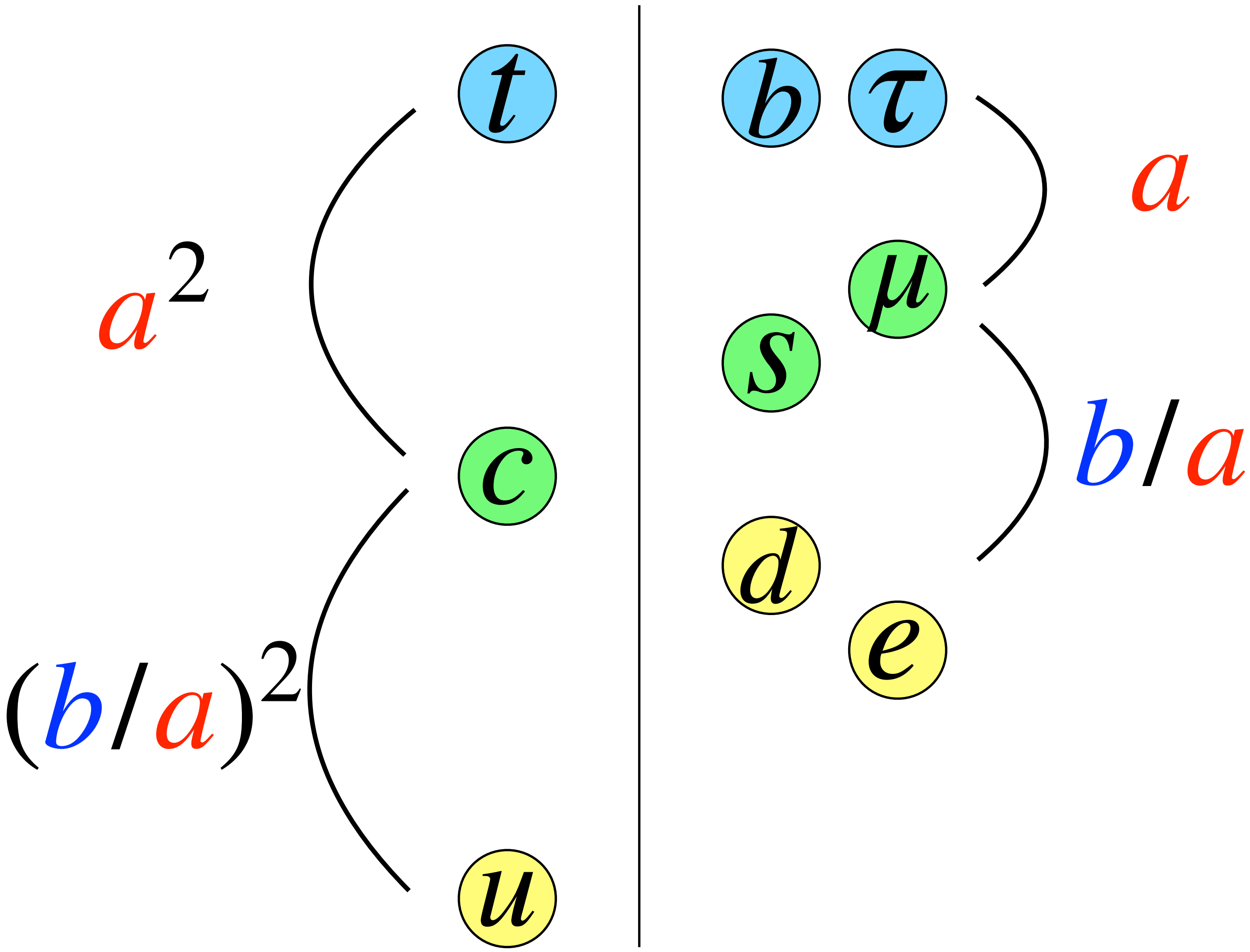
- What about $y_b\,, y_\tau \sim 10^{-2}$?
- d^i & e^i spectrum seems ***compressed*** compared with u^i.

$\mathrm{U}(2)_{q+e^c+u^c}$

- Up-quarks also charged under the $\mathrm{U}(2)$:

$$Y_u = \begin{pmatrix} z_{u1} b^2 & z_{u2} ab & z_{u3} b \\ y_{u1} ab & y_{u2} a^2 & y_{u3} a \\ x_{u1} b & x_{u2} a & x_{u3} \end{pmatrix}$$

- Double **suppression** in the up-quark spectrum!



$$\mathbf{U(2)_{q+e^c+u^c} \times \mathbb{Z}_2}$$

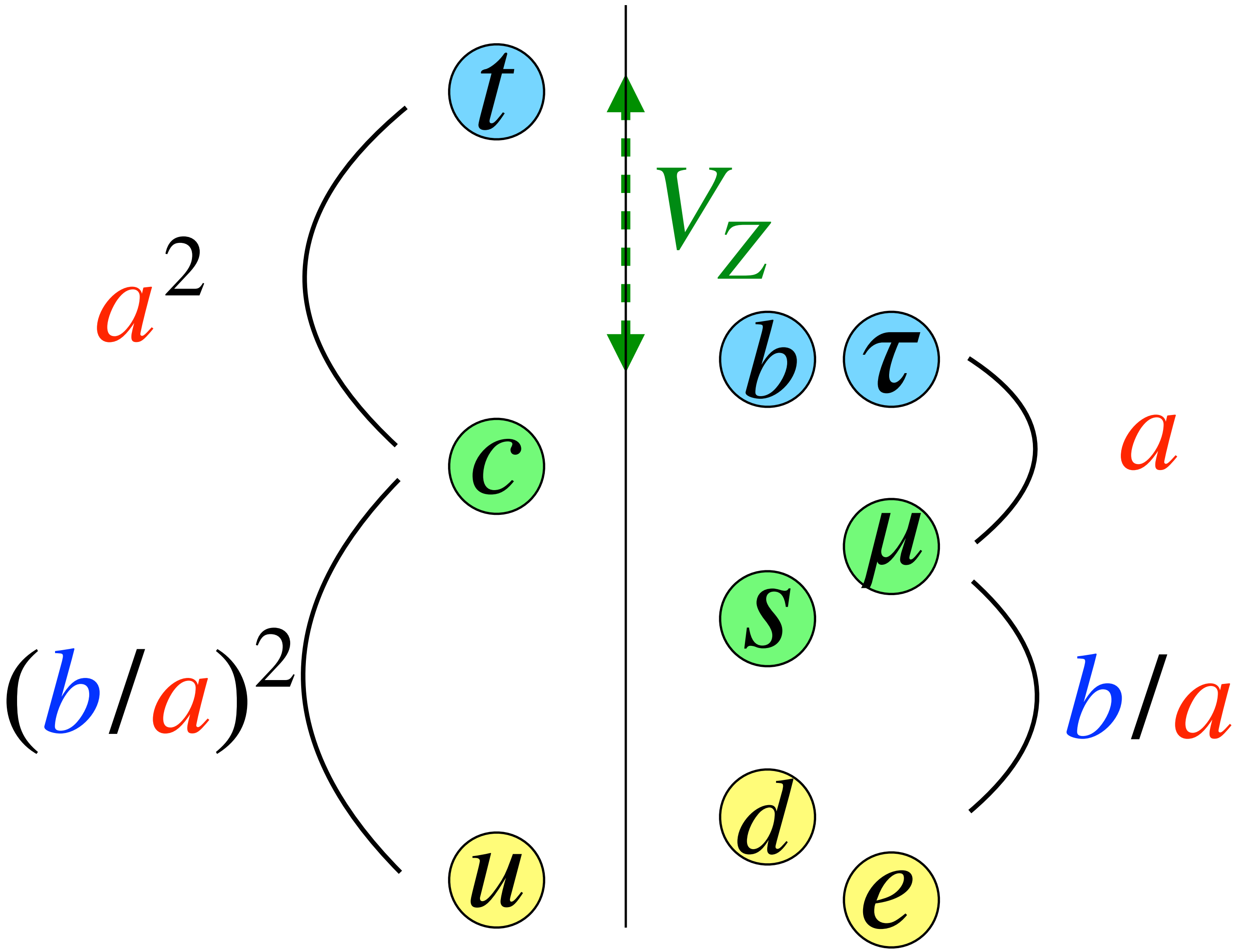
- l_L^i, d_R^i are Z_2-odd

$$Y_d = V_Z \begin{pmatrix} z_{d1}b & z_{d2}b & z_{d3}b \\ & y_{d2}a & y_{d3}a \\ & & x_{d3} \end{pmatrix}$$

$$Y_e = V_Z \begin{pmatrix} z_{\ell 1}b & & \\ z_{\ell 2}b & y_{\ell 2}a & \\ z_{\ell 3}b & y_{\ell 3}a & x_{\ell 3} \end{pmatrix}$$

- V_Z — Z_2 spurion
- 2HDM-II $\tan^{-1}\beta$ (SUSY?)

$$\langle H_u \rangle \gg \langle H_d \rangle$$



We recently achieved similar texture with Z_8 FN

AG, Smolkovic, Valenti; 2407.02998 (Froggatt-Nielsen ALP)

$\mathrm{U}(2)_{q+e^c+u^c} \times \mathbb{Z}_2$

Fixing three spurions,

$$(V_Z, a, b) = (0.01, 0.03, 0.002)$$

predicts the order of magnitudes for all flavor parameters (neutrinos++).

Fit of $\mathcal{O}(1)$ parameters:

$$\begin{aligned}
z_{\ell 1} &= 0.14 & y_{\ell 2} &= 2.0 & x_{\ell 3} &= 1.0 \\
z_{u1} &= 1.1 & y_{u2} &= 2.5 & x_{u3} &= 0.67 \\
z_{d1} &= 0.50 & y_{d2} &= 0.66 & x_{d3} &= 1.0 \\
z_{d2} &= 2.2 e^{i\alpha} & z_{d3} &= 1.8 e^{i(\beta-1.2)} & y_{d3} &= 1.3 e^{i(\beta-\alpha)}
\end{aligned} \tag{A9}$$

Antusch, AG, Stefanek, Thomsen; 2311.09288

$$\mathbf{U(2)_{q+e^c+u^c} \times \mathbb{Z}_2}$$

Q: Why do q, u, e feel U(2) flavor but l, d don't?

A: SU(5) GUT…

$$\bar{\mathbf{5}} \to (\bar{3},1)_{\frac{1}{3}} \oplus (1,2)_{-\frac{1}{2}}$$ d^c and ℓ

$$\mathbf{10} \to (3,2)_{\frac{1}{6}} \oplus (\bar{3},1)_{-\frac{2}{3}} \oplus (1,1)_1$$ q, u^c and e^c

$$\mathrm{U}(2)_{10} \equiv \mathrm{U}(2)_{q+e^c+u^c}$$

Revisiting SUSY GUT predictions, [wip]

The UV origin of $\mathrm{U}(2)$

- Gauge the $\mathbf{SU(2)}$ part!

$\mathbf{SU(2)}_{q+l}$

anomaly-free

AG, Thomsen; 2309.11547

AG, Thomsen, Tiblom; 2406.02687

*Neutrinos need an elaborate structure

$\mathbf{SU(2)}_{q+e}$

anomalons

Antusch, AG, Stefanek, Thomsen; 2311.09288

$\mathbf{SU(2)}_{q+e^c+u^c}$

anomaly-free

wip

The Model

- Rank 1

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{q+\ell}$
q_L^α	**3**	**2**	1/6	**2**
q_L^3	**3**	**2**	1/6	**1**
u_R^p	**3**	**1**	2/3	**1**
d_R^p	**3**	**1**	-1/3	**1**
ℓ_L^α	**1**	**2**	-1/2	**2**
ℓ_L^3	**1**	**2**	-1/2	**1**
e_R^p	**1**	**1**	-1	**1**
H	**1**	**2**	1/2	**1**
Φ	**1**	**1**	0	**2**

$$\mathcal{L} \supset -x_u^p\, \bar{q}^3 \widetilde{H} u^p - x_d^p\, \bar{q}^3 H d^p - x_e^p\, \bar{\ell}^3 H e^p + \text{H.c.}$$

$\widetilde{H}^i = \varepsilon^{ij} H_j^*$ $\qquad x_f^p = (0,\, 0,\, x_{f3}), \qquad x_{f3} \in \mathbb{R}_0^+$

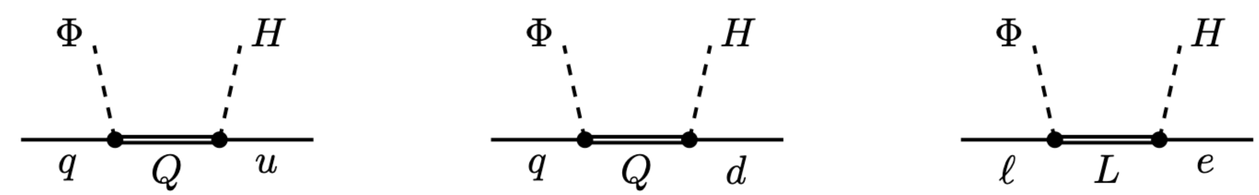
- Rank 2

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{q+\ell}$
$Q_{L,R}$	**3**	**2**	1/6	**1**
$L_{L,R}$	**1**	**2**	-1/2	**1**

$$\mathcal{L} \supset + \left(y_q \Phi^\alpha + \tilde{y}_q \widetilde{\Phi}^\alpha\right) \bar{q}_\alpha Q + \left(y_\ell \Phi^\alpha + \tilde{y}_\ell \widetilde{\Phi}^\alpha\right) \bar{\ell}_\alpha L - y_u^p\, \overline{Q} \widetilde{H} u^p - y_d^p\, \overline{Q} H d^p - y_e^p\, \overline{L} H e^p + \text{H.c.}\ .$$

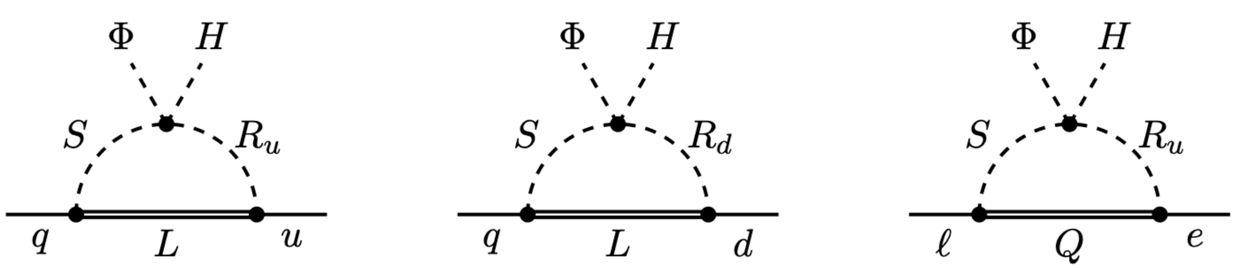
$\widetilde{\Phi}^\alpha = \varepsilon^{\alpha\beta} \Phi_\beta^*$ $\qquad y_f^p = (0,\, y_{f2},\, y_{f3}), \qquad \tilde{y}_q = 0\,,$

$y_{f2},\, y_{d3},\, y_{e3},\, y_q,\, y_\ell,\, \tilde{y}_\ell \in \mathbb{R}_0^+, \qquad y_{u3} \in \mathbb{C}$



- Rank 3

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$SU(2)_{q+\ell}$
R_u	**3**	**2**	7/6	**1**
R_d	**3**	**2**	1/6	**1**
S	**3**	**1**	2/3	**2**



$$\mathcal{L} \supset -\, z_u^p\, \overline{L} u^p \widetilde{R}_u - z_d^p\, \overline{L} d^p \widetilde{R}_d - z_e^p\, \overline{Q} e^p R_u - z_q\, \bar{q}_\alpha L S^\alpha - z_\ell\, \bar{\ell}_\alpha Q \widetilde{S}^\alpha + \text{H.c.}\ ,$$

$$V \supset (\lambda_u \Phi^\alpha + \tilde{\lambda}_u \widetilde{\Phi}^\alpha) S_\alpha^* R_u H^* + (\lambda_d \Phi^\alpha + \tilde{\lambda}_d \widetilde{\Phi}^\alpha) S_\alpha^* R_d \widetilde{H}^* + \text{H.c.}$$

$z_f^p = (z_{f1},\, z_{f2},\, z_{f3}), \qquad z_{f1},\, z_q,\, \tilde{\lambda}_u,\, \tilde{\lambda}_d \in \mathbb{R}_0^+,$

$z_\ell,\, z_{f2},\, z_{f3},\, \lambda_u,\, \lambda_d,\, \kappa_f^p \in \mathbb{C}.$

accidental $U(1)_B \times U(1)_L$ global symmetry

Producing SM flavor parameters

The quark–Higgs coupling matrices are

$$Y_{u(d)} = \begin{pmatrix} b_q \tilde{\lambda}_{u(d)} z_{u(d)} \\ a_q y_{u(d)} + b_q \lambda_{u(d)} z_{u(d)} \\ x_{u(d)} \end{pmatrix},$$

where

$$a_q = \frac{y_q v_\Phi}{M_Q}, \qquad b_q = \frac{z_q}{16\pi^2} \frac{v_\Phi}{M_L} \left(\log \frac{M_L^2}{\mu^2} - 1 \right)$$

- Singular value decomposition, ***perturbatively***:

$$Y_f = L_f \widehat{Y}_f R_f^\dagger$$

$$L_d \simeq \begin{pmatrix} 1 & \frac{b_q \tilde{\lambda}_d z_{d2}}{a_q y_{d2}} & \frac{b_q \tilde{\lambda}_d z_{d3}}{x_{d3}} \\ -\frac{b_q \tilde{\lambda}_d z_{d2}^*}{a_q y_{d2}} & 1 & \frac{a_q y_{d3}}{x_{d3}} \\ \frac{b_q \tilde{\lambda}_d}{x_{d3}} \left[\frac{y_{d3} z_{d2}^*}{y_{d2}} - z_{d3}^* \right] & -\frac{a_q y_{d3}}{x_{d3}} & 1 \end{pmatrix}$$

- Masses:

$$\widehat{Y}_u \simeq \mathrm{diag}\left(b_q \tilde{\lambda}_u z_{u1},\, a_q y_{u2},\, x_{u3}\right),$$
$$\widehat{Y}_d \simeq \mathrm{diag}\left(b_q \tilde{\lambda}_d z_{d1},\, a_q y_{d2},\, x_{d3}\right).$$

$$\widehat{Y}_e \simeq \mathrm{diag}\left(b_\ell \tilde{\lambda}_e z_{e1},\, A_\ell y_{e2},\, x_{e3}\right)$$

- CKM:

$$V_{\rm CKM} = L_u^\dagger L_d \simeq \begin{pmatrix} 1 & \left[\frac{m_d z_{d2}}{m_s z_{d1}} - \frac{m_u z_{u2}}{m_c z_{u1}}\right] & \left[\frac{m_d z_{d3}}{m_b z_{d1}} - \frac{m_s m_u}{m_b m_c} \frac{y_{d3} z_{u2}}{y_{d2} z_{u1}}\right] \\ \left[\frac{m_u z_{u2}^*}{m_c z_{u1}} - \frac{m_d z_{d2}^*}{m_s z_{d1}}\right] & 1 & \left[\frac{m_s y_{d3}}{m_b y_{d2}} - \frac{m_c y_{u3}}{m_t y_{u2}}\right] \\ \left[\frac{m_d}{m_b} \frac{y_{d3} z_{d2}^*}{y_{d2} z_{d1}} - \frac{m_d z_{d3}^*}{m_b z_{d1}} - \frac{m_d m_c}{m_s m_t} \frac{z_{d2}^* y_{u3}^*}{z_{d1} y_{u2}}\right] & \left[\frac{m_c y_{u3}^*}{m_t y_{u2}} - \frac{m_s y_{d3}}{m_b y_{d2}}\right] & 1 \end{pmatrix}$$

Numerical benchmark

- The observed Yukawas at $\mu = 1$ PeV

$$
\begin{aligned}
(y_u, y_c, y_t)_{\rm SM} &= (4.54 \cdot 10^{-6}, 2.29 \cdot 10^{-3}, 0.667), \\
(y_d, y_s, y_b)_{\rm SM} &= (9.95 \cdot 10^{-6}, 1.98 \cdot 10^{-4}, 0.0100), \\
(y_e, y_\mu, y_\tau)_{\rm SM} &= (2.87 \cdot 10^{-6}, 6.05 \cdot 10^{-4}, 0.0103).
\end{aligned}
$$

- The CKM taken from PDG

$$
\lambda = 0.22500 \pm 0.00067\,, \qquad A = 0.826^{+0.018}_{-0.015}\,,
$$
$$
\bar\rho = 0.159 \pm 0.010\,, \qquad \bar\eta = 0.348 \pm 0.010\,.
$$

$$
y_{d3} = 0.16\,, \quad z_{d2} = 0.95 e^{i\alpha}\,, \quad z_{d3} = 0.77 e^{i(\alpha - 1.20)}
$$

- Assume

$$
M_Q = M_L = 100\, v_\Phi
$$

$$
\begin{aligned}
a_q &= 2.5 \cdot 10^{-3}, & b_q &= 1.6 \cdot 10^{-4}, \\
A_\ell &= 2 \cdot 10^{-3}, & b_\ell &= 1.6 \cdot 10^{-4}.
\end{aligned}
$$

$$
y_q = 0.25\,, \quad z_q = 0.3\,, \quad y_\ell = 0.2\,, \quad \tilde y_\ell = 0\,,
$$
$$
z_\ell = 0.1\,, \quad \tilde\lambda_u = \tilde\lambda_d = 0.3\,, \quad \lambda_u = 0.2\,.
$$

$$
\begin{aligned}
(z_{u1}, y_{u2}, x_{u3}) &= (0.097, 0.91, 0.67), \\
(z_{d1}, y_{d2}, x_{d3}) &= (0.21, 0.079, 0.010), \\
(z_{e1}, y_{e2}, x_{e3}) &= (0.092, 0.30, 0.010).
\end{aligned}
$$

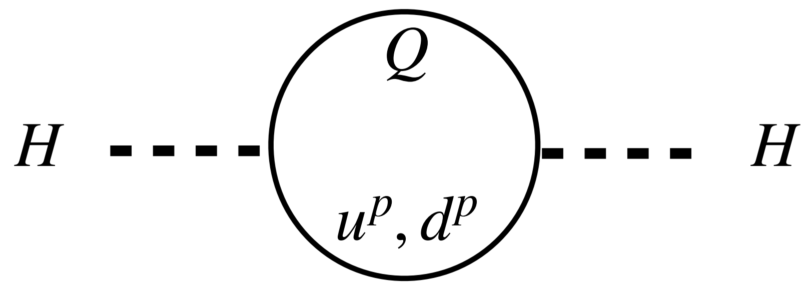
- All marginal couplings (but two) within a factor of ~3 around 0.3.
- Two (accidentally) smaller parameters contributing to ***Tau*** and ***Bottom*** Yukawas ~ 0.01
- The CKM is dominated by the down-type contributions, as the hierarchy in the down quark sector is compressed compared to the up quark sector.

Phenomenology

- Decoupling limit exits: Take the new mass thresholds substantially heavy while keeping $v_\Phi/M_{Q,L}$ fixed and $M_{S,R_u,R_d} \lesssim M_{Q,L}$.

- The low-scale variant of the model is interesting for experiments.

- Finite Higgs naturalness provides another motivation for low-scale $M_{Q,L}$



- Q1: What are the bounds on the new masses given the current data?

- Q2: Which observables and deviation patterns should be prioritized?

Discussion

Q: How to fit neutrinos?

- Add 3 RHN and do high-scale seesaw:

$$m_{\nu_{\rm L}} \simeq -M_D M_{\rm R}^{-1} M_D^{\intercal} \simeq U^{\intercal} \widehat{m}_{\nu_{\rm L}} U$$

- The model predicts hierarchical M_D. Large PMNS require M_R to also be hierarchical to "undo" the hierarchy in M_D. :(

A possible resolution comes from a mechanism to generate anarchic Y_ν. To this end, we can extend the field content with a single vector-like fermion representation $N_{\rm L,R} \sim (\mathbf{1}, \mathbf{1}, 0, \mathbf{2})$. When the mass of this field is comparable to v_Φ, marginal interactions $\bar{\ell}_\alpha \widetilde{H} N^\alpha$ and $\overline{N}_\alpha \overset{(\sim)}{\Phi}{}^\alpha \nu_p$ wash out the hierarchy in Y_ν. In this case, the required Majorana mass matrix $M_{\rm R}$ is also anarchic. This is an elegant solution, provided one accepts the coincidence of scales $M_N \sim v_\Phi$.

- Alternative: $SU(2)_{q_L+e_R}$