Hadronic Vacuum Polarisation Contributions to Muon $g-2$: Status of the Dispersive Approach

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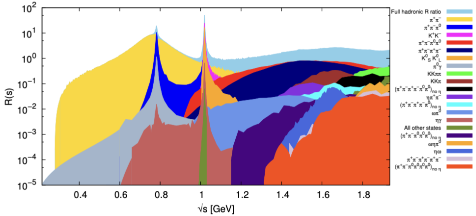
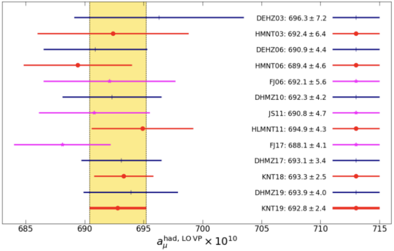
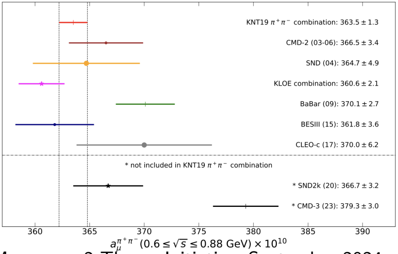


- Dispersive calculation has been used to provide the HVP input to a_μ calculation for > 20 years.
- Values stable despite BaBar-KLOE tension.
- Larger CMD-3 measurement confuses the picture.
- Goals:
  1. Understand the tensions between dispersive, lattice and experiment.
  2. **Produce an accurate and meaningful prediction for a_μ^{HVP}.**

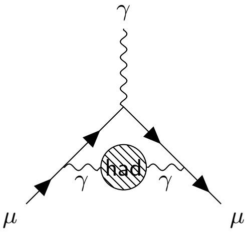




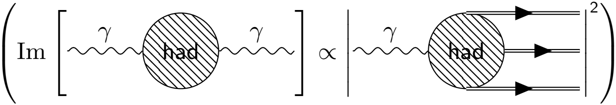
A. Keshavarzi, Seventh Plenary Meeting of the Muon $g-2$ Theory Initiative, September 2024

Dispersion Integral

- Problem: QCD is non-perturbative at low energies so HVP of photon cannot be calculated in a loop expansion.
- Solution: *dispersion integral* over the $e^+e^- \to$ *hadrons* cross section.



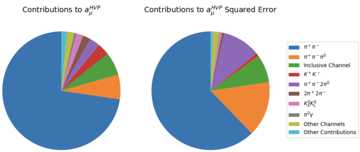
$\propto \Pi^{had.}_{\mu\nu}(q^2)$	*by definition*
$\propto \Pi^{had.}(q^2)$	*due to gauge invariance*
$\propto \int \frac{\mathrm{Im}\left\{\Pi^{had.}(s)\right\}}{s\left(s-q^2-i\varepsilon\right)} ds$	*by analyticity and Cauchy's theorem*
$\propto \int \frac{\sigma^0_{had.}(s)}{s-q^2-i\varepsilon} ds$	*by unitarity* $\implies$ *the Optical Theorem*

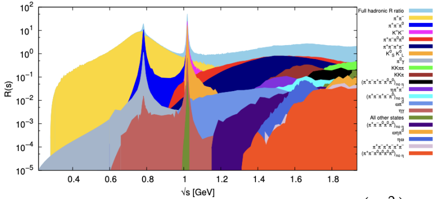


- For > 40 years, low energy $e^+e^- \to$ *hadrons* data have been collected.

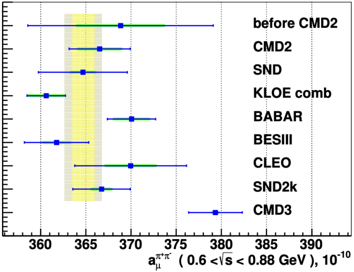
Data Combination

- Non trivial processing required of the $e^+e^- \to \textit{hadrons}$ data:
  - Bare cross section requirement necessitates knowledge of radiative corrections.
  - Consistent and precise combination procedure required.
- More than 50 channels, multiple datasets in most.





A. Keshavarzi et al, The muon $g-2$ and $\alpha\left(M_Z^2\right)$: a new data-based analysis, Figure 7 (arXiv:1802.02995)

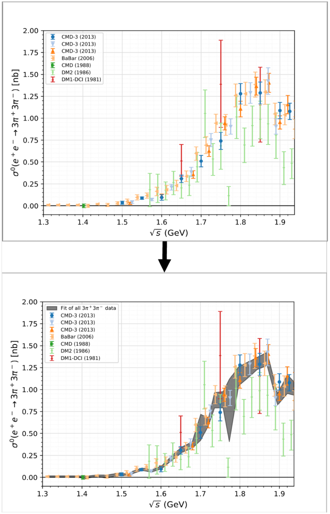


F. Ignatov et al, Measurement of the $e^+e^- \to \pi^+\pi^-$ cross section from threshold to 1.2 GeV with the CMD-3 detector, Figure 36 (arXiv:2302.08834)

KNTW Fit

- Use all information (e.g. covariance matrices) provided by experiments.
- Assume complete correlation of *all* systematics where no further information is provided.
- Dynamically cluster data within channels to prevent over-fitting.
- Iterated covariance matrix fit procedure to remove d'Agostoni bias.

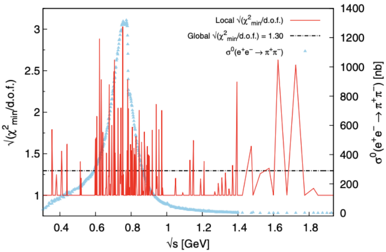
$\Longrightarrow\ a_\mu^{LO\ HVP}[KNT19] = (692.79 \pm 2.42) \times 10^{-10}$



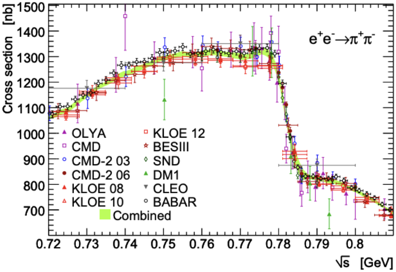
DHMZ Fit

- Data interpolated on quadratic splines to set binning, finer on resonances.
- Uncertainties generated using pseudo-experiments.
- Correlations between datasets and channels simulated using local averaging regions.
- Use of PDG-style local χ^2 error inflation (also in KNTW) in tense bins.
- Closure test applied in 2π channel.

$$\implies a_\mu^{LO\ HVP}[DHMZ19] = (694.0 \pm 4.0) \times 10^{-10}$$

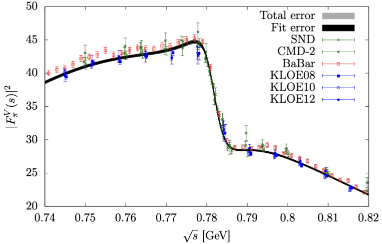


A. Keshavarzi et al, The muon $g-2$ and $\alpha\left(M_Z^2\right)$: a new data-based analysis, Figure 7 (arXiv:1802.02995)

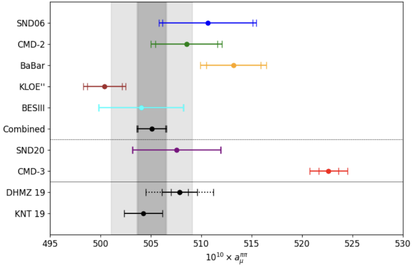


M. Davier et al, A new evaluation of the hadronic vacuum polarisation contributions to the muon anomalous magnetic moment and to $\alpha\left(M_Z^2\right)$, Figure 1 (arXiv:1908.00921)

- The shape of the pion vector form factor is constrained by unitarity and analyticity $\Longrightarrow$ a theory motivated fit for $a_\mu\left(\pi^+\pi^-\right)$.
- Some data in KLOE and BESIII cause massive tension in the fit so are removed.
- Theoretically motivated to have no zeros in pion VFF and the data are supportive.
- Leads to a result consistent with KNTW and DHMZ results depsite greatly differing method.



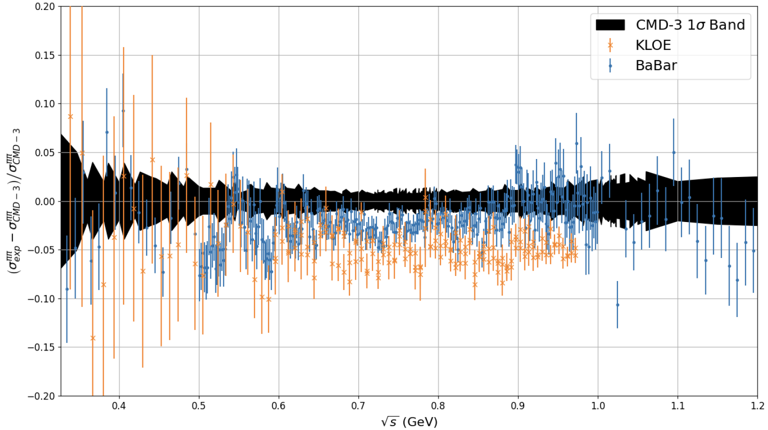
G. Colangelo et al, Two-pion contribution to hadronic vacuum polarization, Figure 10 (arXiv:1810.00007)



T. Leplumey, Seventh Plenary Meeting of the Muon $g-2$ Theory Initiative, September 2024

Data Tensions

- Particularly on the ρ, CMD-3 2π significantly in excess of all previous data.

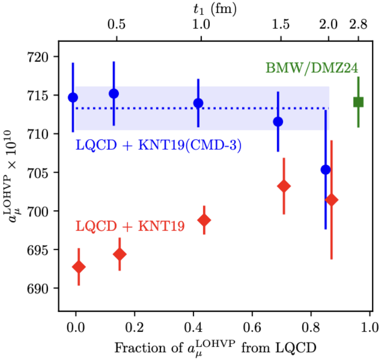


- A potential KLOE-CMD-3 systematic is too large for a_μ^{HVP} to be useable.
- Similarity at low energy used to motivate hybrid approaches...
- Belle-II see a similar excess on the 3π resonances but there are potential issues with the data.

Lattice Hybrids

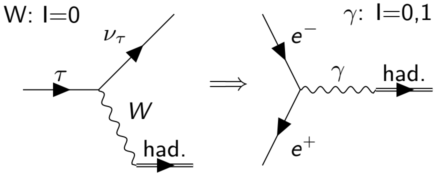
- Lattice "long distance windows" typically have a large uncertainty.
- Hybrid approach: at some Euclidean time t switch from lattice input to dispersive input.
- BMWc-DHMZ hybrid: KLOE + BaBar + CMD-3 + τ, $t_{hybrid} = 2.8$ fm.

- C.T.H. Davies et al (arXiv:2410.23832v1) investigate the effect of t_{hybrid} on a_μ^{HVP}.
- KNT19 data as input, with KNT19(CMD-3) *replacing* $\pi^+\pi^-$ data with only CMD-3 data.
- CMD-3 consistent with lattice; KLOE and BaBar only above 2 fm.
- Reinforces the need to understand why CMD-3 is discrepant.

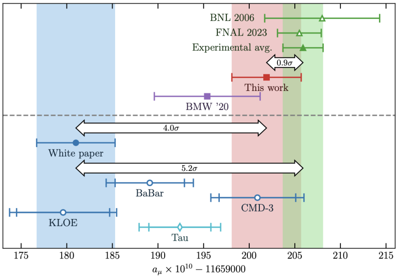


C.T.H. Davies et al, Utility of a hybrid approach to the hadronic vacuum polarisation contribution to the muon anomalous magnetic moment, Figure 4. (arXiv:2410.23832)

Tau Data



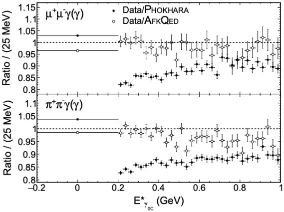
"... at the required precision to match the e^+e^- data, the present understanding of the IB [isospin breaking] corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals." - TI White Paper, 2020



A. Boccaletti et al, High precision calculation of the hadronic vacuum polarisation contribution to the muon anomaly, Figure 3 (arXiv:2407.10913).

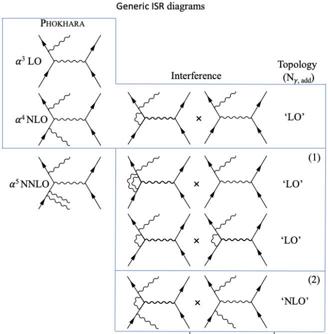
- Historically data from hadronic tau decays used to supplement lacking or low accuracy cross section data.
- Poor understanding of the scale of systematic uncertainties associated with IB corrections meant these data was no longer to be included.
- More accurate calculations of IB corrections are in process, supplemented by lattice QCD and ChPT.
- DHMZ argue for the re-inclusion of τ data due to the existence of greater discrepancies among the cross section datasets.

- BaBar study of additional radiation in $e^+e^- \to \pi^+\pi^-$ finds a possible NLO excess in PHOKHARA and a potentially significant NNLO contribution.
- Raises concerns about the KLOE and BESIII radiative correction systematics.



J.P. Lees et al, Measurement of additional radiation in the initial-state-radiation processes $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$ at BABAR, Figure 3(b) (arXiv:2308.05233).

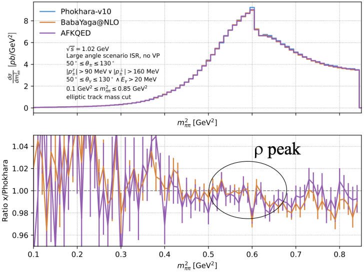
- DHMZ define two scenarios; NNLO dominated by:
  1. Diagrams "(1)" and the excess in PHOKHARA is a generator issue.
  2. Diagrams "(2)" and the data deficit comes from virtual NNLO.



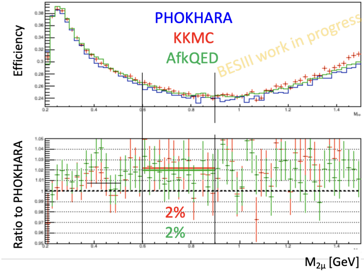
M. Davier et al, Tensions in $e^+e^- \to \pi^+\pi^-(\gamma)$ measurements: the new landscape of data-driven hadronic vacuum polarization predictions for the muon g – 2, Figure 6 (arXiv:2312.02053).

Radiative Corrections - Experimental Impact

- This has potential consequences:
  1. KLOE08,10 over -1% on the ρ; BESIII -3.2%.
  2. KLOE08 -0.8% on the ρ; BESIII unbiased.
- KLOE investigation: strong agreement of PHOKHARA with KKMC, McMule; difference with AFKQED $< 1\%$ on the ρ.
- BESIII investigation: inclusive of higher order radiation and results consistent. Agreement at $\sim 1\%$ on the ρ.
- Both strongly disfavour DHMZ scenario 1 (hence not a full explanation).
- Both still work in progress; detector effects.



G. Venanzoni, Seventh Plenary Meeting of the Muon $g-2$ Theory Initiative, September 2024



A. Denig, Seventh Plenary Meeting of the Muon $g-2$ Theory Initiative, September 2024

- Already noted profound differences between combination procedures; leading to not insignificant differences between results.
- Choices to be made:
  - Rebinning of data.
  - Additional constraints.
  - Use of correlations.
  - Interpolation and integration.

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
$[3.7, \infty)$ GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

T. Aoyama et al, The anomalous magnetic moment of the muon in the Standard Model, Table 5 (arXiv:2006.04822)

- This analysis needs to be accurate $\implies$ unbiased $\implies$ blind.

Blinding - Details

- Blinding requirements:
  - Cannot introduce biases to the data.
  - Cannot blind publicly available data.
  - Cannot infer blinding offsets from results.
  - Cannot interfere with combination and fit.

A. Keshavarzi et al, Muon $g-2$: blinding for data-driven hadronic vacuum polarization, arXiv:2409.02827. Accepted into PRD.

- Therefore:
  - Blind scale but blind shape only with a weakly varying kernel.
  - Blind only the outputs: integrals ($X = 0$) and plots ($X = 1$).
  - Blind integrals and plots with different kernels.

$$\sigma^{blind}_{had,(i,X)}(s) = a_i b_{(i,X)} \left(s + s_{0,(i,X)}\right)^{c_{(i,X)}} \sigma_{had,i-m}(s)$$

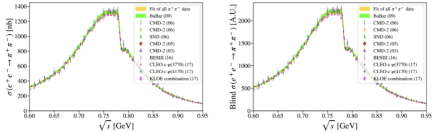
- At the first stage of blinding, channel numbers will be mapped by a random offset m (modulo 100) and different seeds generated for each channel.
- At the *relative unblinding* stage, all channel numbers will be know and the seeds will be common to all channels.

Seed	a_i	$b_{(i,X)}$	$c_{(i,X)}$	$s_{0,(i,X)}$
Value	$a = \pm 1$	$0.1 \leq b \leq 0.9$ $1.1 \leq b \leq 10$	$0.01 \leq \lvert c \rvert \leq 0.05$	$-0.01 \leq s_0 \leq 1$
Comment	Integral only	Avoid no scaling	Avoid no distortion	Avoid knowledge at $s = 1$ GeV

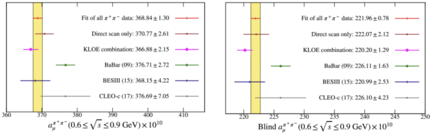
Blinding - Implementation

- Seeds stored in a compiled Python script and known by an external blinder (Mark Lancaster).
- Must be correctly input to produce unblinded results.

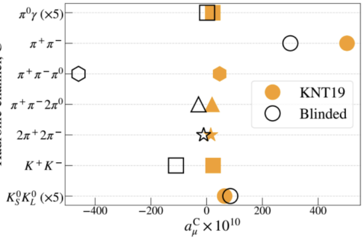
KNT19 Example



(a) The ρ resonance of the $\pi^+\pi^-$ channel.



(b) Comparison of values $a_\mu^{\pi^+\pi^-}$ in the range $0.6 \le \sqrt{s} \le 0.9$ GeV.



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Upcoming Analyses

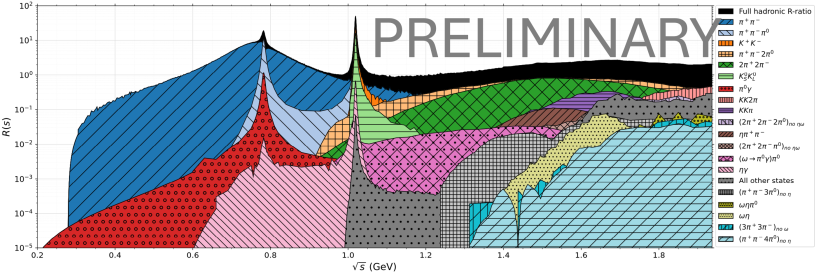
- New cross section data could prove crucial to understanding the discrepancy in the 2π channel.
- Measurements of high multiplicity channels important to improve large uncertainties or replace estimated channels.

Experiment	Projected Precision	Current Precision	Timeline	Comments
BaBar	$< 0.5\%$	0.5%	2025	Dataset luminosity doubled + new analysis method.
Belle-II	$< 0.5\%$	N/A	Late 2025/26	All 427 fb^{-1} of run-1 data.
BESIII	0.7%(0.5%)	0.9%	2025(28)	Also aiming to measure important 4π and $2K$ channels.
CMD-3 (> 1 GeV)	$1-2\%$	N/A	Unclear	New measurement < 1 GeV planned along with $n\pi$.
KLOE	0.4%	0.8% (KLOE12)	2026	-
SND	$0.6-0.7\%$	0.8%	2025/26	-

- Future measurements of $g-2$ (J-PARC, MUonE) will increase the precision of the experimental result.

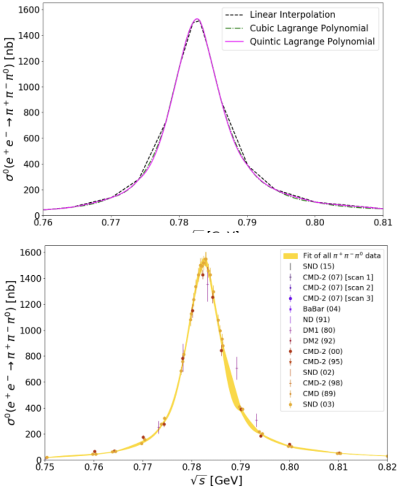
KNTW Updates

- Conversion of FORTRAN to Python is nearly complete:
  - Text files replaced by relational database.
  - Approaching end of conversion: new analysis can begin soon.
  - Blinding built in from the start.
- Interactive plotting software introduced for easy visualisation.
- All data inputs ($\sim$ 280 datasets) cross checked from papers and (minor) corrections made where necessary.



KNTW Plan

- KNTW to begin a new analysis for inclusion in **White Paper III** (no dispersive in WPII).
- Planning to:
  - Examine and modernise the VP routine.
  - Refine FSR treatments and extend to more channels.
  - Investigate alternatives to the clustering procedure (i.e. spline interpolation).
  - Investigate the effects of varying which systematics we correlate and the correlation coefficients.
- Creation of a new interface to view, integrate and plot the data in our database.



A. Keshavarzi et al, The $g-2$ of charged leptons, $\alpha\left(M_Z^2\right)$ and the hyperfine splitting of muonium, Figure 4 (arXiv:1911.00367)

Conclusions

- Not presently at a point in our analysis where we could provide a number we think is representative of the situation.
- Dispersive method retains value, but has puzzles to solve.
- A number of outstanding questions need value before a "safe" dispersive calculation of $a_\mu^{LO\,HVP}$ can be provided.
- A lot depends on ongoing analyses and unpublished data.
- Of paramount importance: no jumping to conclusions about muon $g-2$.