3rd Liverpool Workshop on Muon Precision Physics

RadioMonteCarLow 2

Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in $e^+ e^-$ collisions

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MPP24 / Liverpool / 14 Nov 2024

outline

- intro: what, why, when
- how: easily digestable technical remarks
- results: personal selection of results
- wrap up: so what and what next

what

- theoretical description for $e^+ e^- \to$ hadrons at low energies $\sqrt{s} \lesssim 1 - 2\,\mathrm{GeV}$

  making also use of radiative (return) processes (some call them ISR processes)

- MC comparisons for

$$e^+ e^- \to \mu^+ \mu^- \;(+\gamma)$$
$$e^+ e^- \to e^+ e^- \;(+\gamma)$$
$$e^+ e^- \to \pi^+ \pi^- \;(+\gamma)$$

- inspired by [0912.0749]

Eur. Phys. J. C (2010) 66: 585–686
DOI 10.1140/epjc/s10052-010-1251-4

THE EUROPEAN PHYSICAL JOURNAL C

Review

Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

- consolidate and implement the progress since 2010

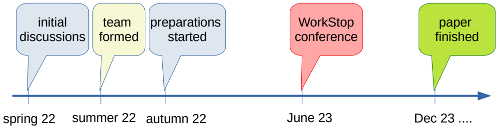
- in 2010 you got this guy  surely, this can be improved upon
- preserve, further develop, and make accessible some well established codes for low-energy $e^+ e^-$
- get new (preferably young) people to join with new ideas/approaches
- cross fertilisation from huge effort made for LHC
- open science approach: what is in which generator, where can I get it

  $\Rightarrow$ a public repository of all codes and all results

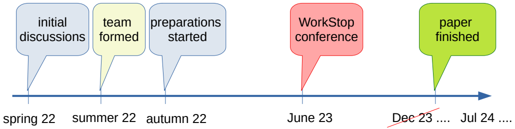
  $\Rightarrow$ `https://radiomontecarlow2.gitlab.io/`
- a community effort, hopefully continuing for many years to come

when

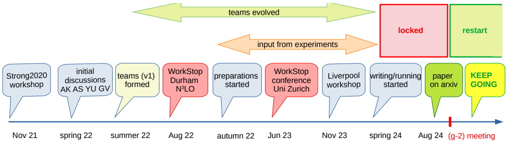
plan shown at MPP22



plan shown at MPP23



plan shown in June 2024 at MITP



when

Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

[2410.22882]

30 Oct 2024

RadioMonteCarLow 2 Working Group: Riccardo Aliberti[1], Paolo Beltrame[2], Ettore Budassi[3,4], Carlo M. Carloni Calame[4], Gilberto Colangelo[5], Lorenzo Cotrozzi[2], Achim Denig[1], Anna Driutti[6,7], Tim Engel[8], Lois Flower[2,9], Andrea Gurgone[3,4,6], Martin Hoferichter[5], Fedor Ignatov[2], Sophie Kollatzsch[10,11], Bastian Kubis[12], Andrzej Kupść[13,14,*], Fabian Lange[11,10], Alberto Lusiani[15,7], Stefan E. Müller[16], Jérémy Paltrinieri[2], Pau Petit Rosàs[2], Fulvio Piccinini[4], Alan Price[17], Lorenzo Punzi[7,15], Marco Rocco[10,18], Olga Shekhovtsova[19,20], Andrzej Siódmok[17], Adrian Signer[10,11,*], Giovanni Stagnitto[21], Peter Stoffer[10,11], Thomas Teubner[2], William J. Torres Bobadilla[2], Francesco P. Ucci[3,4], Yannick Ulrich[2,5,*], and Graziano Venanzoni[2,7,*]

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how

parts

WP1: QED for leptons at NNLO

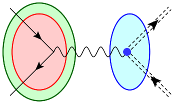
WP2: Form factor contributions at N^{3}LO

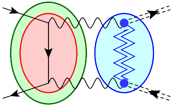
WP3: Processes with hadrons

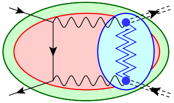
WP4: Parton showers / YFS

WP5: Experimental input

$e^+ e^- \to \pi^+ \pi^-$

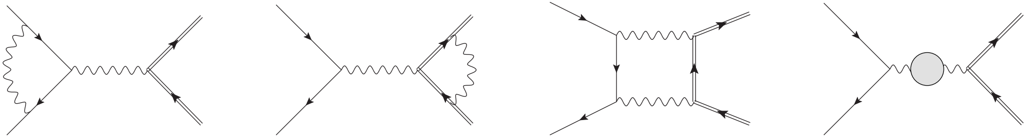






example: $e^+ e^- \to \mu^+ \mu^-$ at NLO, split into gauge invariant parts

for computational and conceptual reasons



initial-state (ISC) final-state (FSC) mixed corrections (H)VP corrections

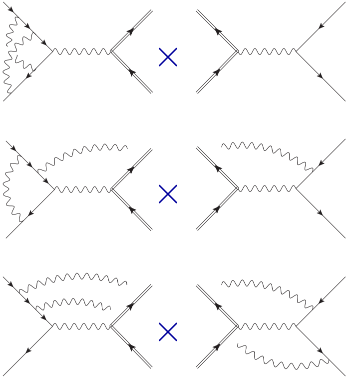
$$\mathcal{A}^{(1)}_{mm}(q_e\, q_m\, q_\ell^2) = \mathcal{A}^{(1)}_{mm}(q_e^3\, q_m) + \mathcal{A}^{(1)}_{mm}(q_e\, q_m^3) + \mathcal{A}^{(1)}_{mm}(q_e^2\, q_m^2) + \mathcal{A}^{(1)}_{mm}(q_e\, q_m\, \Pi^{(1)})$$

$$\mathrm{d}\sigma^{(1)}_{mm}(q_e^2\, q_m^2\, q_\ell^2) = \underbrace{\mathrm{d}\sigma^{(1)}_{mm}(q_e^4\, q_m^2)}_{\text{ISC}} + \underbrace{\mathrm{d}\sigma^{(1)}_{mm}(q_e^2\, q_m^4)}_{\text{FSC}} + \underbrace{\mathrm{d}\sigma^{(1)}_{mm}(q_e^3\, q_m^3)}_{\text{mixed}} + \underbrace{\mathrm{d}\sigma^{(1)}_{mm}(q_e^2\, q_m^2\, \Pi^{(1)})}_{\text{VPC}}$$

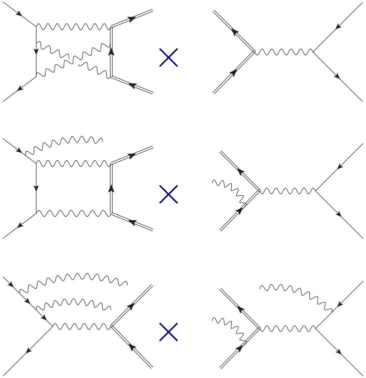
pure QED

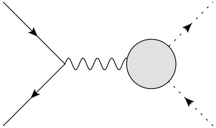
split for NNLO $2 \to 2$ (or NLO $2 \to 3$) corrections to cross section

initial-state corrections

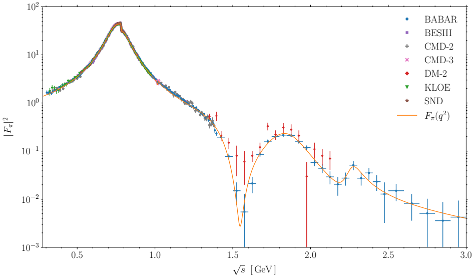


mixed corrections

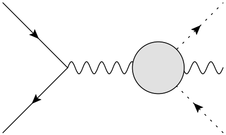




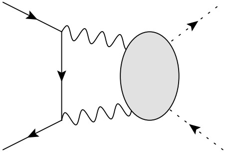
form factor ok



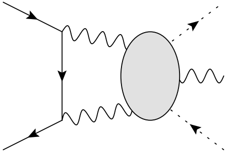
but



FSR $e^+ e^- \to \pi^+\pi^-\gamma$



NLO $e^+ e^- \to \pi^+\pi^-$



NLO $e^+ e^- \to \pi^+\pi^-\gamma$

Terminology for treatment of $\gamma\, \pi^+ \pi^-$ interaction

- **sQED:** no form factors, pointlike pion
  standard loop integrals
- **F$\times$sQED:** multiply sQED amplitude with form factor
  loop integrals not affected
- **FsQED:** include form factor in loops from pion pole
  loop integrals with dispersive approach
- **GVMD:** models pion form factor through Breit-Wigner propagators
  analytic loop integrals possible
- **full** hadronic matrix elements
  currently not in any MC code, just wishful thinking

$X \in \{e, \mu, \pi\}$, many observables https://radiomontecarlow2.gitlab.io/

always cut on $p_\pm >$ something, selection of further acceptance cuts:

- CMD-like: $e^+ e^- \to X^+(p_+)\, X^-(p_-)$ with $\sqrt{s} = 0.7\,\mathrm{GeV}$

  cuts: $\supset$ $\big||\phi^+ - \phi^-| - \pi\big| < 0.15\,\mathrm{rad}$; $|\theta^+ + \theta^- - \pi| < 0.25\,\mathrm{rad}$;

- KLOE-like small angle (untagged): $e^+ e^- \to X^+ X^- \gamma$ with $\sqrt{s} = 1.02\,\mathrm{GeV}$

  cuts: $\supset$ range of $\theta_\pm$ and M_{XX}; set $\vec{p}_{\tilde{\gamma}} \equiv -(\vec{p}_+ + \vec{p}_-)$ and $\theta_{\tilde{\gamma}} \le 15^\circ$ or $\theta_{\tilde{\gamma}} > 165^\circ$

- KLOE-like large angle (tagged): $e^+ e^- \to X^+ X^- \gamma$ with $\sqrt{s} = 1.02\,\mathrm{GeV}$
- BESIII-like: $e^+ e^- \to X^+ X^- \gamma$ with $\sqrt{s} = 4\,\mathrm{GeV}$
- B: $e^+ e^- \to X^+ X^- \gamma$ with $\sqrt{s} = 10\,\mathrm{GeV}$

  cuts: $\supset$ range of $\theta_\pm$ and $\exists\gamma$ within range of θ_γ and $E_\gamma >$ something

Monte Carlo codes

the magnificent seven (and what they roughly contain)

- AfkQed: LO $2 \to 3$ + ISR collinear structures, FSR with Photos, $X \in \{\mu, \pi\}$
- BabaYaga@NLO: NLO $2 \to 2$ + parton shower, F×sQED* for π
- KKMC: YFS (CEEX) with $X = \mu$
- MCGPJ: NLO $2 \to 2$ + collinear structures for $X \in \{e, \mu, \pi\}$
- McMule: NNLO QED for $2 \to 2$ and NLO for $2 \to 3$ with $X \in \{e, \mu\}$, ISR for $X = \pi$
- Phokhara: NLO for $2 \to 3$ with $X \in \{\mu, \pi\}$, F×sQED for π
- Sherpa: YFS (EEX) $X \in \{\mu, \pi\}$ with matched NLO $2 \to 2$, sQED for π

possible further codes to be included in the future

* BabaYaga@NLO now has FsQED and GVMD for $e^+ e^- \to \pi^+ \pi^-$ (not used here)

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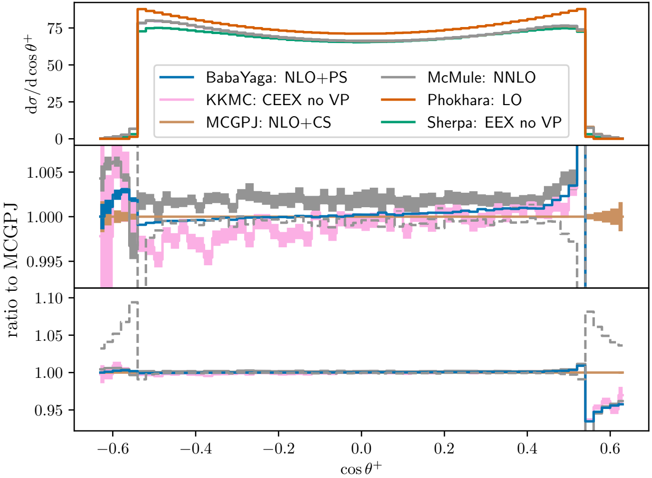
results

CMD-like $e^+ e^- \to \mu^+ \mu^-$

scattering angle θ^+

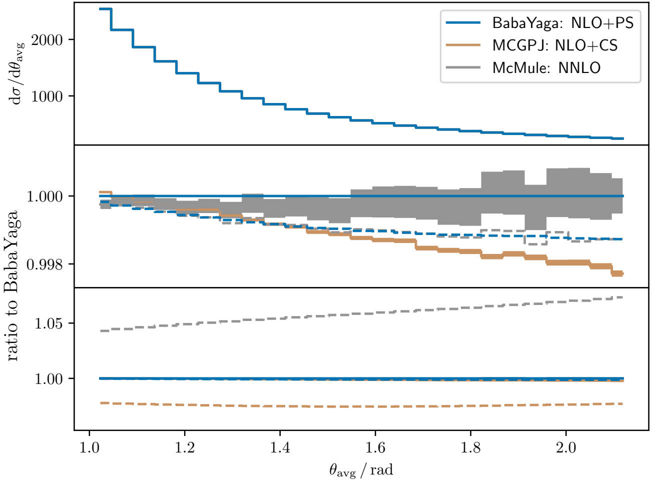


dashed McMule: NLO

- for 'well behaved' obs. (not vanishing at LO) all within $\sim 0.2\%$
- outside this region larger deviations
- CS+PS mostly through one additional emission
- NLO VPC $\simeq 0$, accidental
- at NNLO, VP as important as rest

CMD-like $e^+ e^- \to e^+ e^-$

$$\theta_{\text{avg}} \equiv (\theta^- - \theta^+ + \pi)/2$$



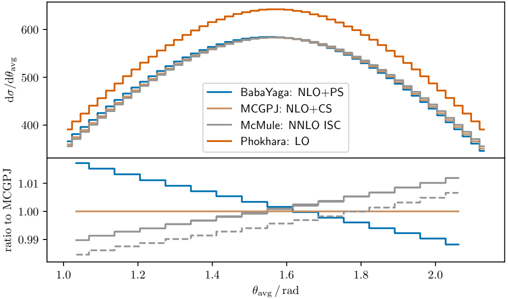
- for 'well behaved' obs. (not vanishing at LO) all within ~ 0.2 %
- CS and PS have different sign w.r.t. NLO

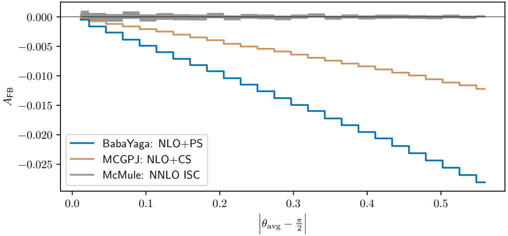
  dashed McMule and BabaYaga@NLO: NLO
- band: only MC error
- NLO VPC not small (t channel)

  dashed McMule: LO

  dashed MCGPJ NLO+CS but no VP

CMD-like $e^+ e^- \to \pi^+ \pi^-$





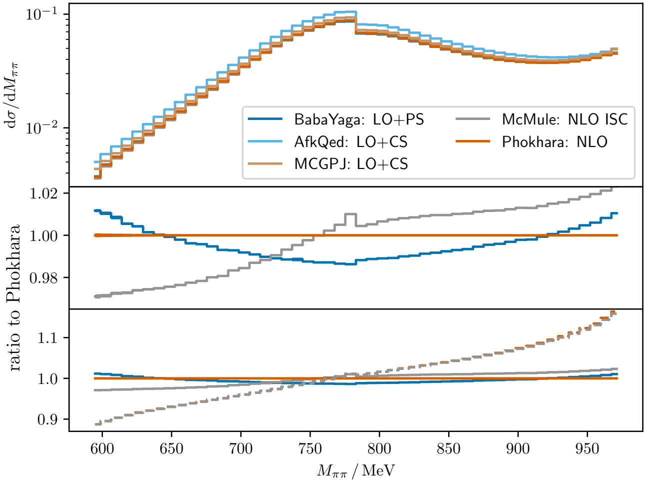
- at LO or N^{x}LO ISC only
  θ_{avg} distribution symmetric w.r.t. $\pi/2$
  dashed McMule: NLO ISC
- asymmetry through ISR-FSR interference
- forward-backward asymmetry

$$A_{FB} = \frac{\frac{d\sigma}{d\theta_{avg}}(\theta_{avg} > \frac{\pi}{2}) - \frac{d\sigma}{d\theta_{avg}}(\theta_{avg} < \frac{\pi}{2})}{\frac{d\sigma}{d\theta_{avg}}(\theta_{avg} > \frac{\pi}{2}) + \frac{d\sigma}{d\theta_{avg}}(\theta_{avg} < \frac{\pi}{2})}$$

sensitive test for $\gamma\ \pi^+\pi^-$ treatment

KLOE-like small angle $e^+ e^- \to \pi^+ \pi^- \gamma$

invariant mass $M_{\pi\pi}$



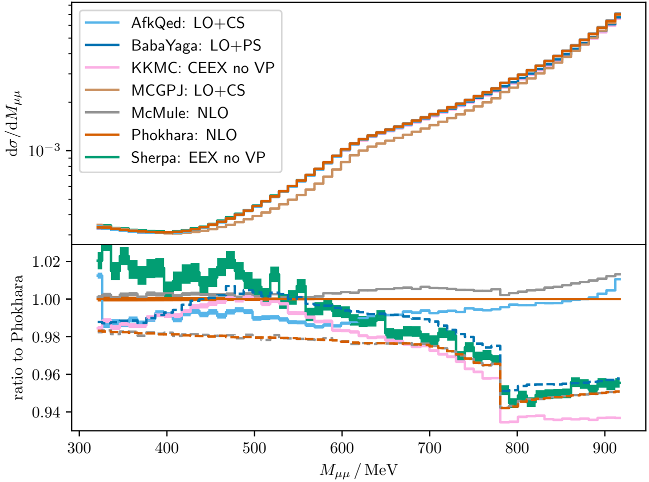
- Phokhara territory (full NLO)
- NLO corrections are large $\sim 10\%$
- at LO, ISC only agrees extremely well with full result

  dashed McMule: LO ISC

  dashed Phokhara: LO full
- but at NLO there is a 2% difference !

KLOE-like large angle $e^+ e^- \to \mu^+ \mu^- \gamma$

invariant mass $M_{\mu\mu}$



- Phokhara territory (NLO)
- difference to McMule: VPC, resummed vs. single insertion $\sim 1\%$
- (C)EEX within $2-3\%$ of NLO (no VP)

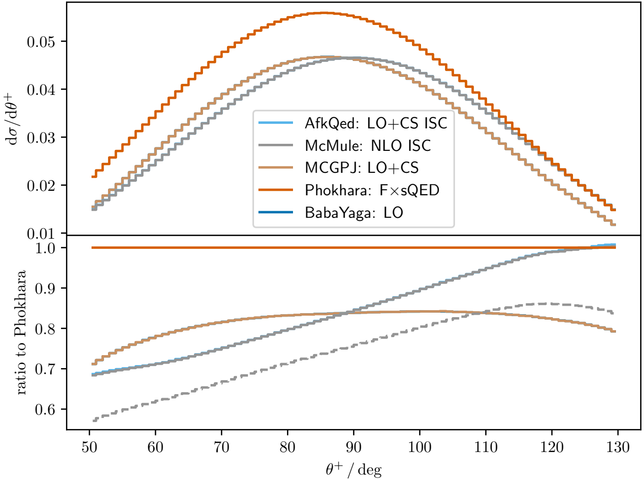
  dashed McMule: NLO no VP

  dashed Phokhara: NLO no VP

  dashed BabaYaga: NLO no HVP
- VPC $\sim 2-5\%$

KLOE-like large angle $e^+ e^- \to \pi^+ \pi^- \gamma$

scattering angle θ^+

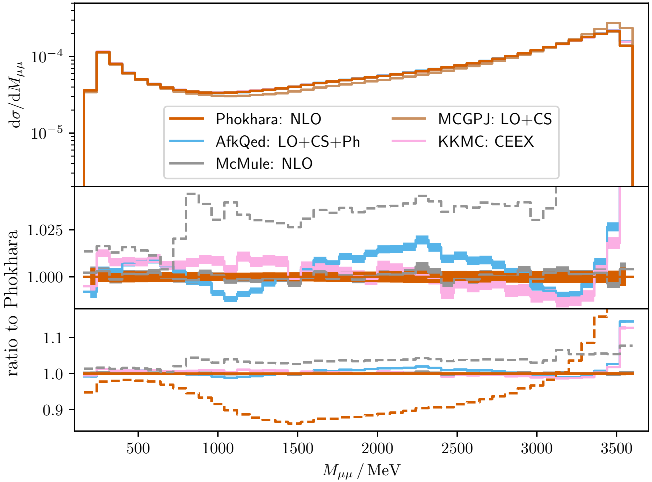


- MCGPJ $\simeq$ Phokhara LO
- AfkQed $\simeq$ McMule NLO
- FSR huge at LO $10 - 20\%$

  dashed McMule: LO (ISC only)
- even larger at NLO $20 - 40\%$
- reliable FSC implementation crucial

BESIII-like $e^+ e^- \to \mu^+ \mu^- \gamma$

invariant mass $M_{\mu\mu}$, without VPC



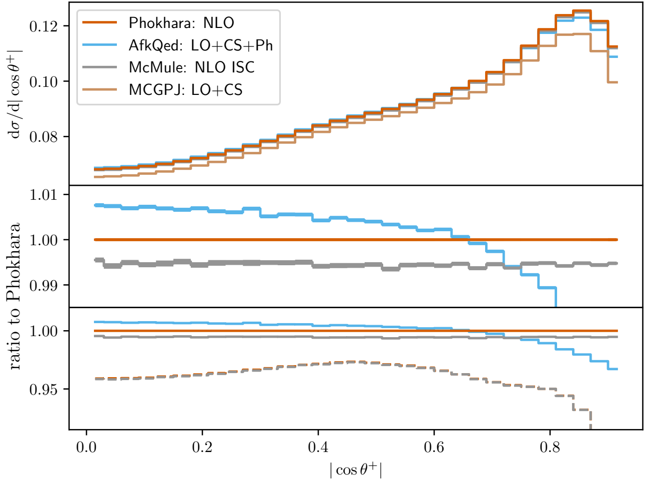
- technical comparison
- VPC $\gtrsim 3\%$

  dashed McMule: NLO with VPC
- NLO corrections large $\pm 10\%$

  dashed Phokhara: LO
- non-VP part within 2% except at large end of distribution

BESIII-like $e^+ e^- \to \pi^+ \pi^- \gamma$

scattering angle θ^+



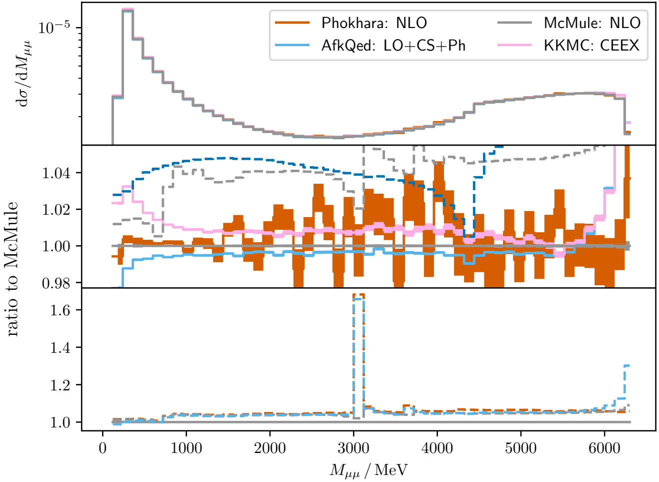
- at LO, FSR $\lesssim 0.1\%$
  form factor suppression

  dashed Phokhara: LO full

  dashed McMule: LO ISC only
- at NLO, FSR $\sim 0.5\%$
- big differences between AfkQed and NLO for $|\cos\theta^+| \to 1$
- multiple photon emission? NNLO ISC might answer

B factory $e^+ e^- \to \mu^+ \mu^- \gamma$

invariant mass $M_{\mu\mu}$, without VPC



- agreement within 2%, except for large $M_{\mu\mu}$
- VPC as important as photonic NLO

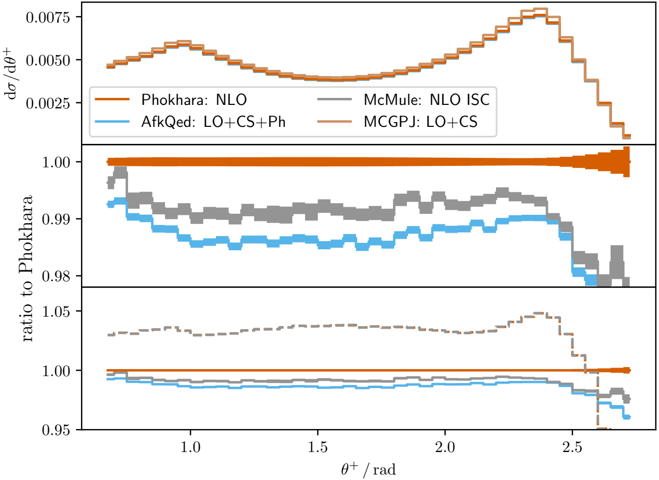
  dashed BabaYaga@NLO: LO

  dashed McMule: NLO with VPC
- J/ψ resonance
  VPC goes crazy
  resum vs. single insertion

  dashed lines: include VPC

B factory $e^+ e^- \to \pi^+ \pi^- \gamma$

scattering angle θ^+



- at LO, FSR $\lesssim 0.01\%$
  form factor suppression

  dashed Phokhara: LO full

  dashed McMule: LO ISC only
- at NLO, FSR $1 - 2\%$!!
- difficult to claim we have this under control at the 1% level

wrap up

generic observations and ...

- so far, this was mostly a theory game
- we have $2 \to 2$ at NNLO or NLO+(YFS/PS)
  and $2 \to 3$ at NLO or LO+(YFS/PS)
- comparisons without VP are technically interesting, but have no phenomenological value
- details of VP treatement at least a $\sim 1\%$ effect, larger with resonances
- impact of FSR at LO from huge to completely negligible
- even if completely negligible at LO, still a $\sim 1\%$ effect at NLO

... what next

my personal opinion

- NNLO $e^+ e^- \to \gamma\, \gamma^*$, doable (read will be done)
- NNLO $e^+ e^- \to \mu^+ \mu^- \gamma$, doable with approximations (mass effects)
- NNNLO $e^+ e^- \to \gamma^*$, sort of doable
- NNNLO $2 \to 2$, tough (maybe in time for MUonE ...)
- combine NNLO with CS/PS/YFS
- for $2 \to 3$ processes with $\pi^\pm$, MC treatement of FSR needs to be improved
- reconsider inclusion of VPC
- more processes, e.g. luminosity processes, 3π, ...
- think about a meaningful theory error estimate
- more direct connection with experimental data

Phase II $\longrightarrow$ we will start this Friday !

thank you

Radiative corrections and Monte Carlo tools for
low-energy hadronic cross sections in e^+e^- collisions

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to everyone who contributed and to the

Monte Carlo responsible

AfkQed	P. Beltrame, L. Cotrozzi	with external help from V. Druzhinin
BabaYaga@NLO	C. Carloni Calame, A. Gurgone	
KKMC	J. Paltrinieri, A. Siódomok	
MCGPJ	F. Ignatov	
McMule	S. Kollatzsch, M. Rocco	
Phokhara	P. Petit Rosàs, W. Torres Bobadilla	with external help from H. Czyż
Sherpa	A. Price, L. Flower	

in particular