UNIVERSITY OF LIVERPOOL

LEVERHULME TRUST

Phokhara Perspective

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University of Liverpool

Satellite RMCL2 WG meeting

15 November 2024

The Spine

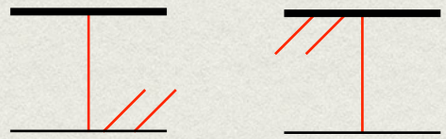
Towards $e^+e^- \to F^+F^-\gamma$ *@ NNLO*

Anatomy @ LO

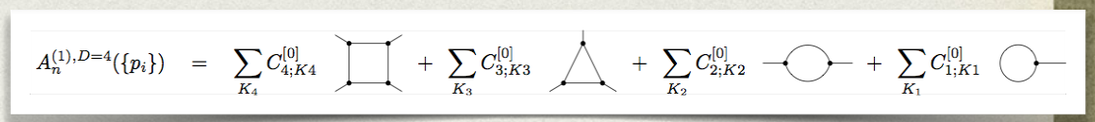
- Born matrix element
  tree-level & n-pt process

Anatomy @ NLO

- Real contribution tree-level (*n+1*)-particles



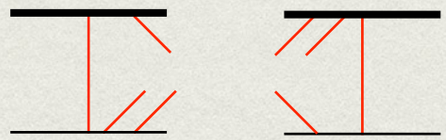
- Virtual Contribution one-loop (*n+1*)-particles



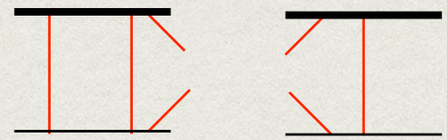
- ☑ Automated one-loop Feynman integral & phase-space evaluation
- ☑ IR subtraction schemes under control
- ☑ Efficient numerical evaluation (MC friendly)

Anatomy @ NNLO

- Real-Real contribution
  Tree-level (*n+2*)-particles



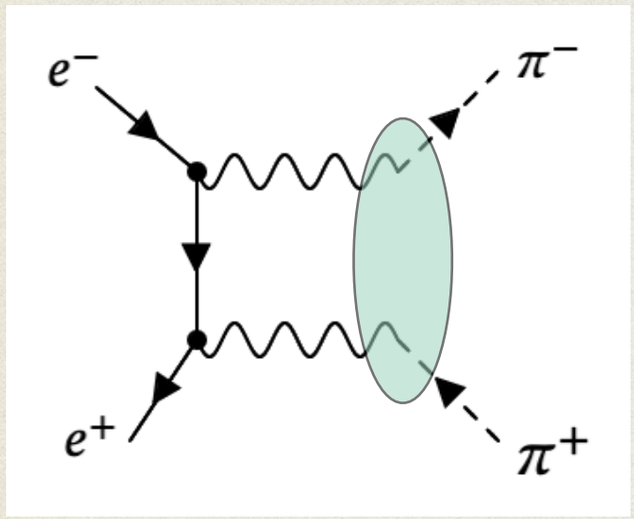
- Real-Virtual Contribution
  one-loop (*n+1*)-particles



- Virtual-Virtual Contribution
  two-loop *n*-particles

- ☐ Harder (but doable) phase-space integration
- ☐ Extend numerical evaluation of one-loop Feynman integrals
- ☐ Basis of two-loop Feynman integral not known

$$e^+e^- \to \pi^+\pi^-(\gamma)$$



Hadronic content

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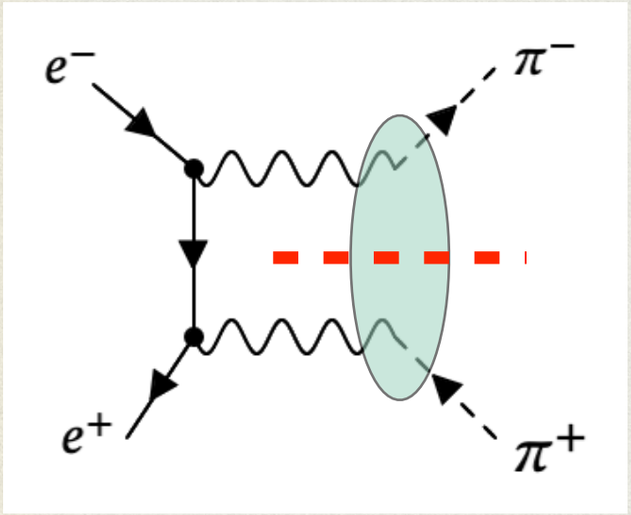


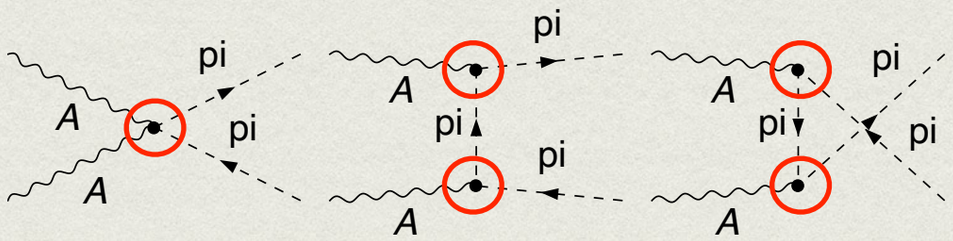
Hadronic content

$$A^{\mu\nu} = \sum_n F_n T_n^{\mu\nu}$$

$$\gamma(p_1)\,\gamma(p_2) \to \pi^-(p_3)\,\pi^+(p_4)$$

@tree-level :: sQED

$\left|F_\pi(???)\right|$ $\left|F_\pi(p_1^2)\right| \times \left|F_\pi(p_2^2)\right|$



$$\left|F_\pi(p_1^2)\right| \times \left|F_\pi(p_2^2)\right| = \left|F_\pi(???)\right|$$

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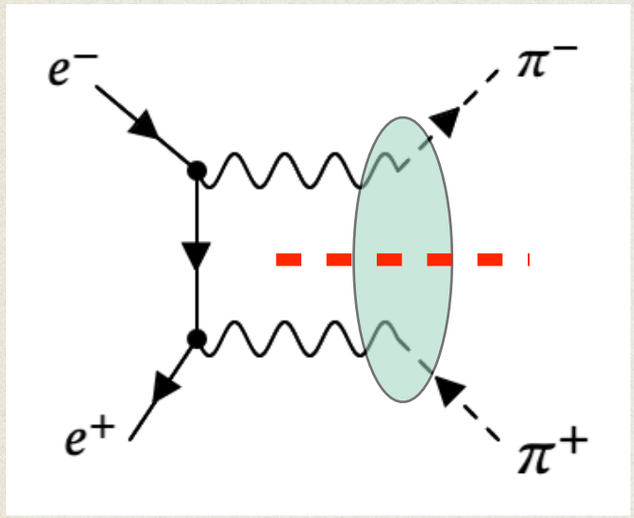

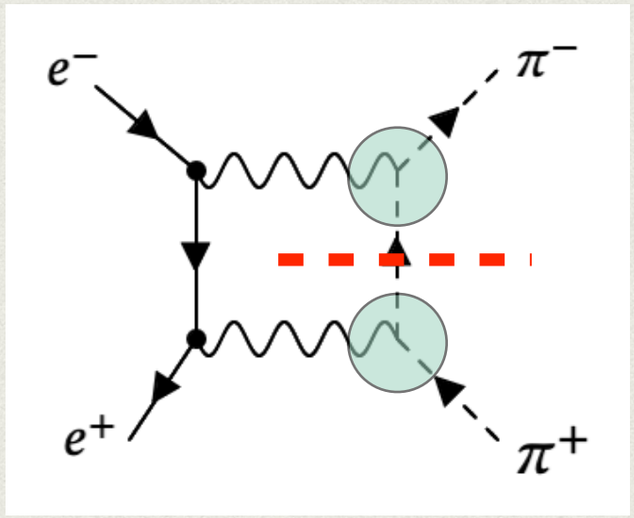

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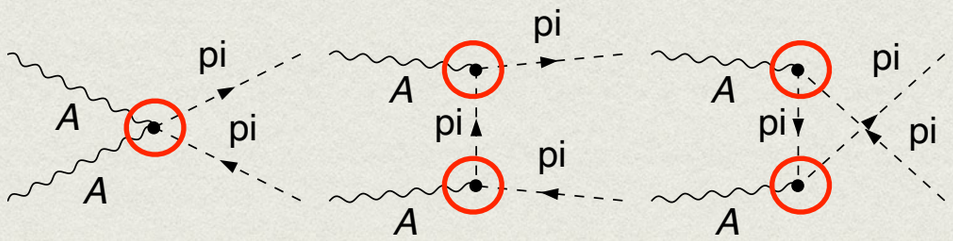

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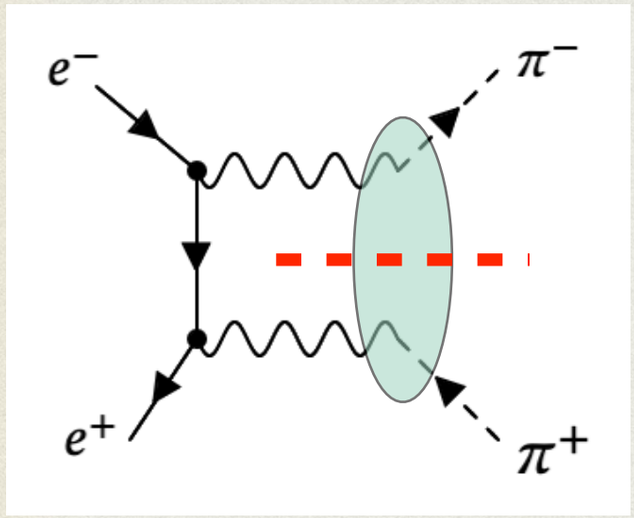

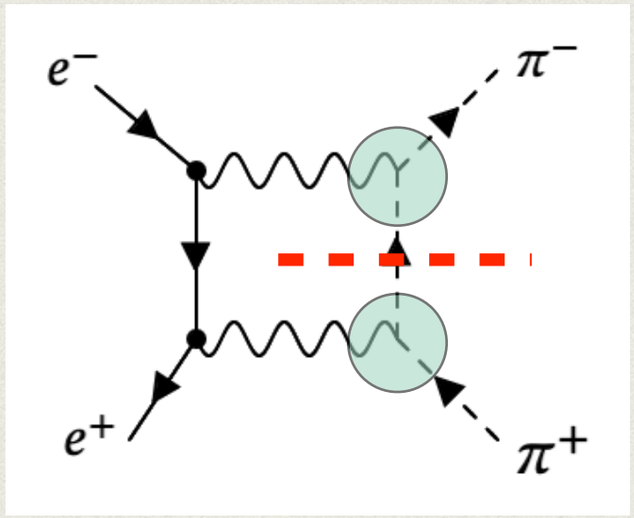

Hadronic content

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Adopt a parametrisation

$$F(q^2) = \sum_{v=1}^{n} a_v \frac{\Lambda_n}{\Lambda_n - q^2}$$

with $\Lambda_n = m_v^2 - i m_v \Gamma_{v'}$

and $\sum_v a_v = 1$

Generalised vector dominance model (GVMD)

[Lee, Ignatov (2022)]
[Colangelo, Hoferichter, Monnard, Ruiz de Elvira (2022)]
[Budassi et al (2024)]

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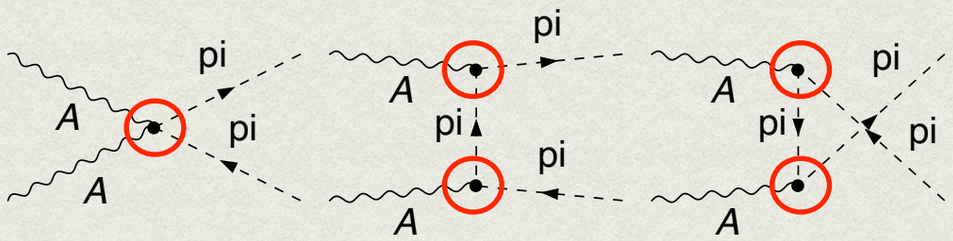

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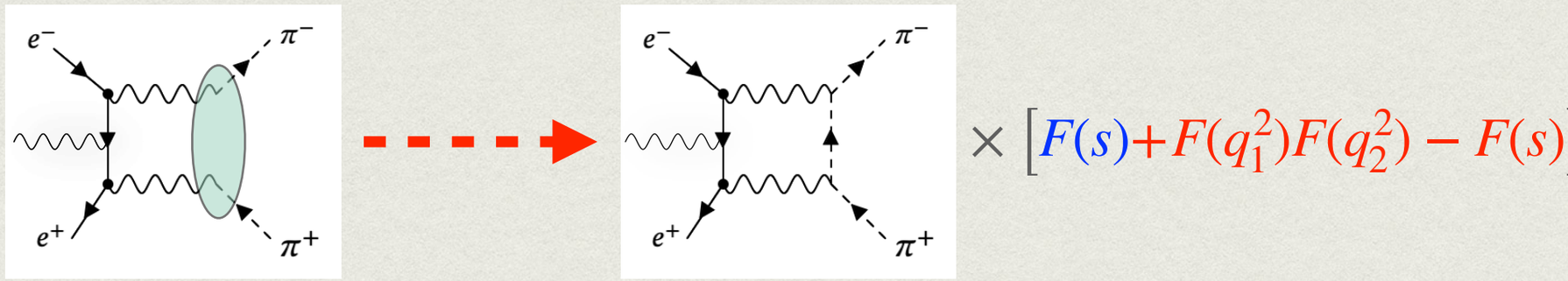
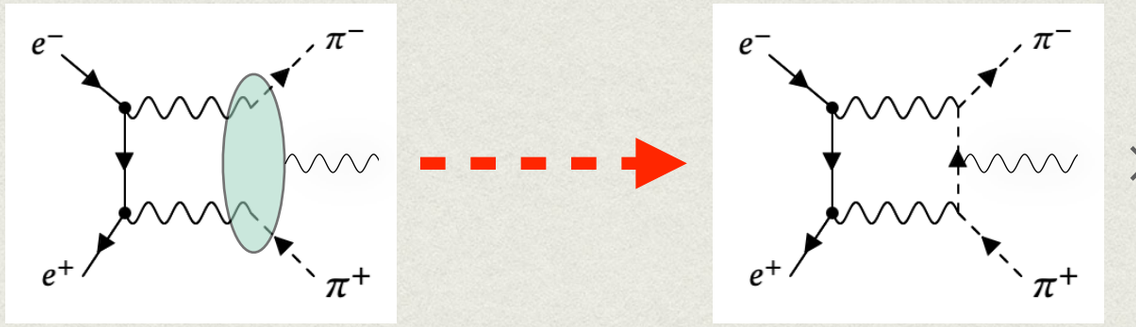
$e^+e^- \to \pi^+\pi^-(\gamma)$

In collaboration w/: ***Pau Petit-Rosas, Daniel Melo Porras,***
Olga Shekhovtsova, Stefan Müller, Fedor Ignatov

Initial & Final State radiation @ NLO



$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$



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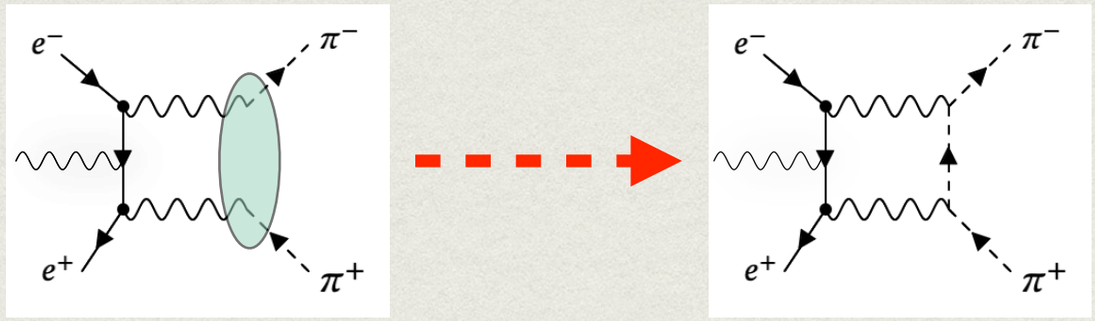

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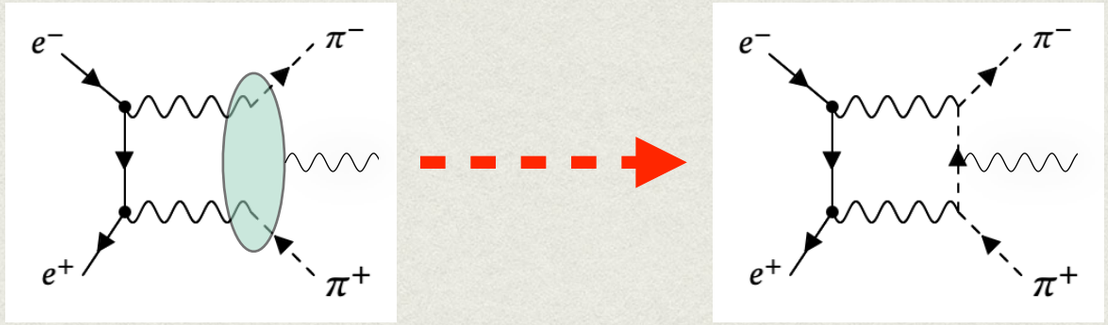
- Semi-analytic computation (Pau's talk)
- In-house implementation of FSR
- To be included in Phokhara-X



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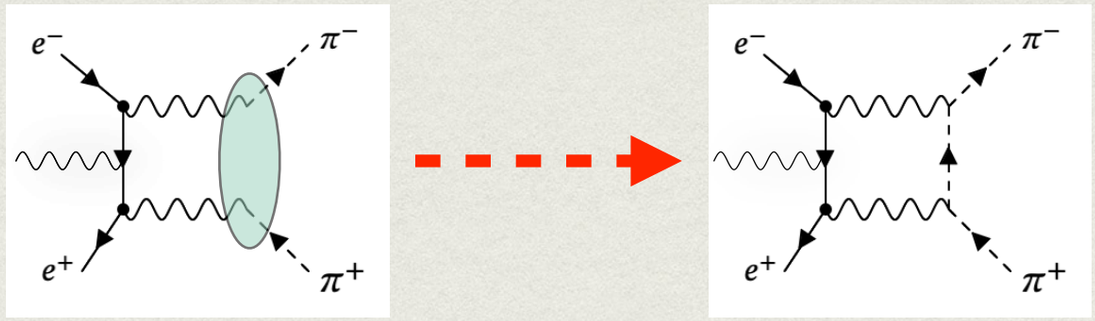

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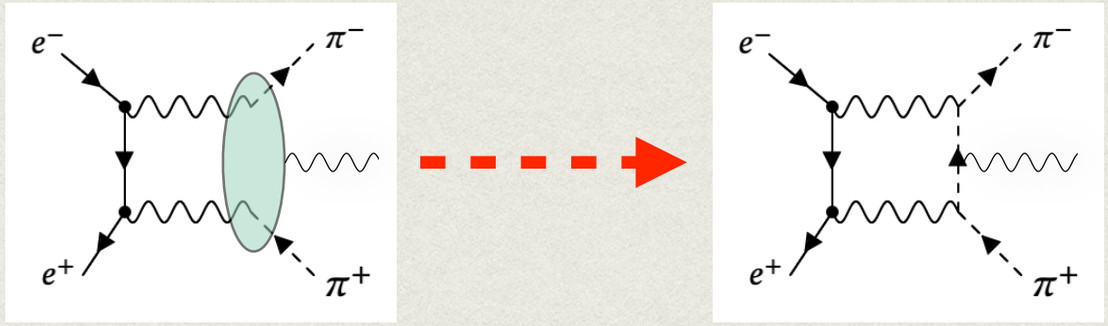

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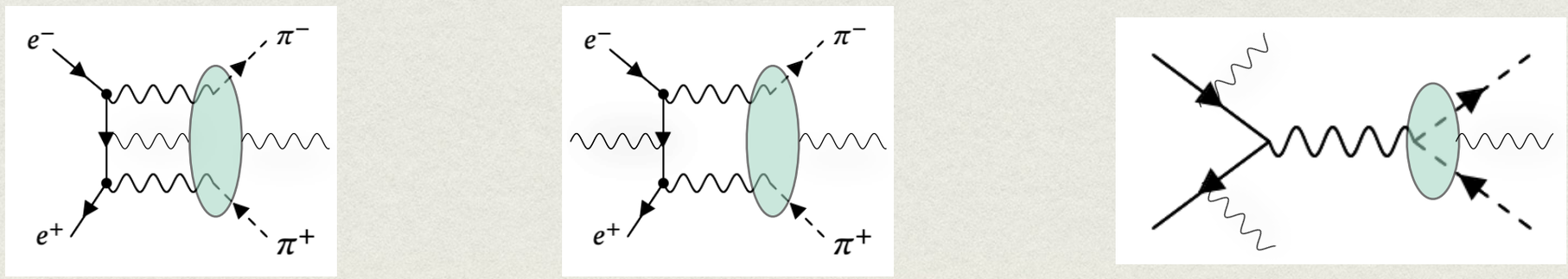
Same treatment at NNLO (VV, VR, RR) ?



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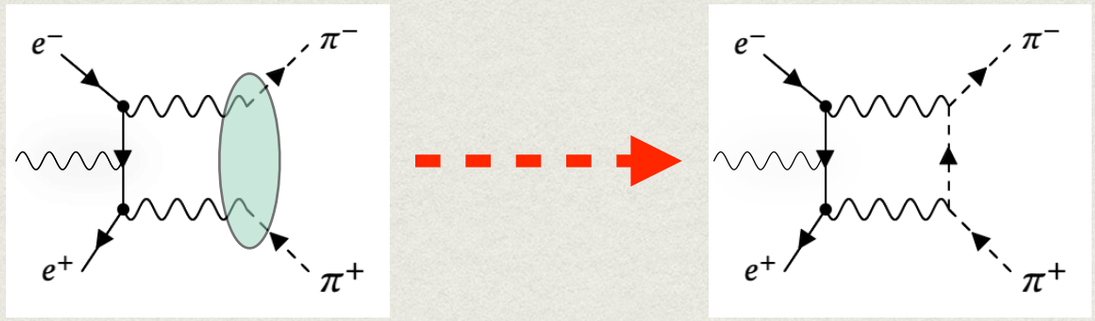

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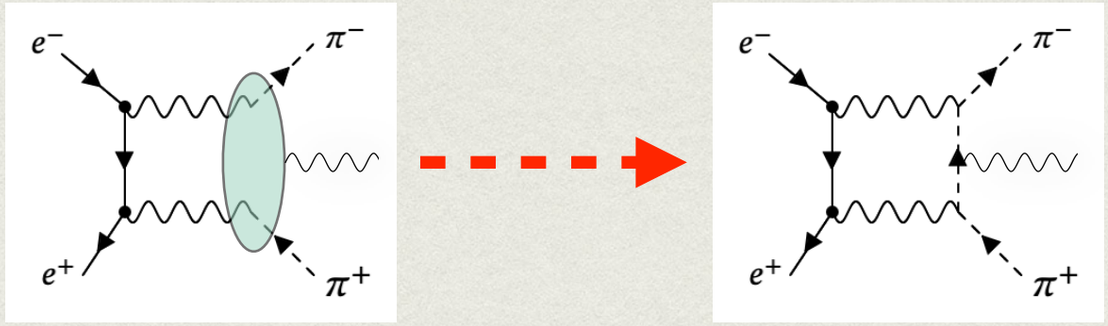
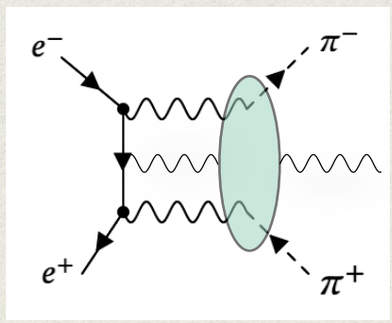
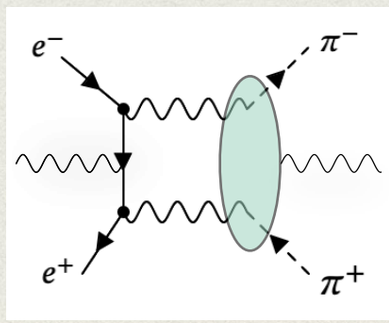


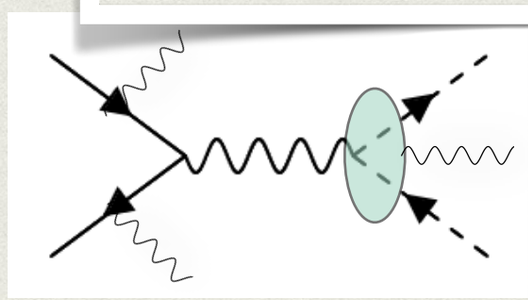
$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

Same treatment at NNLO (VV, VR, RR) ?

- Fully numerical computation
- Use Collier
- Focus on RR & RV (at the moment)



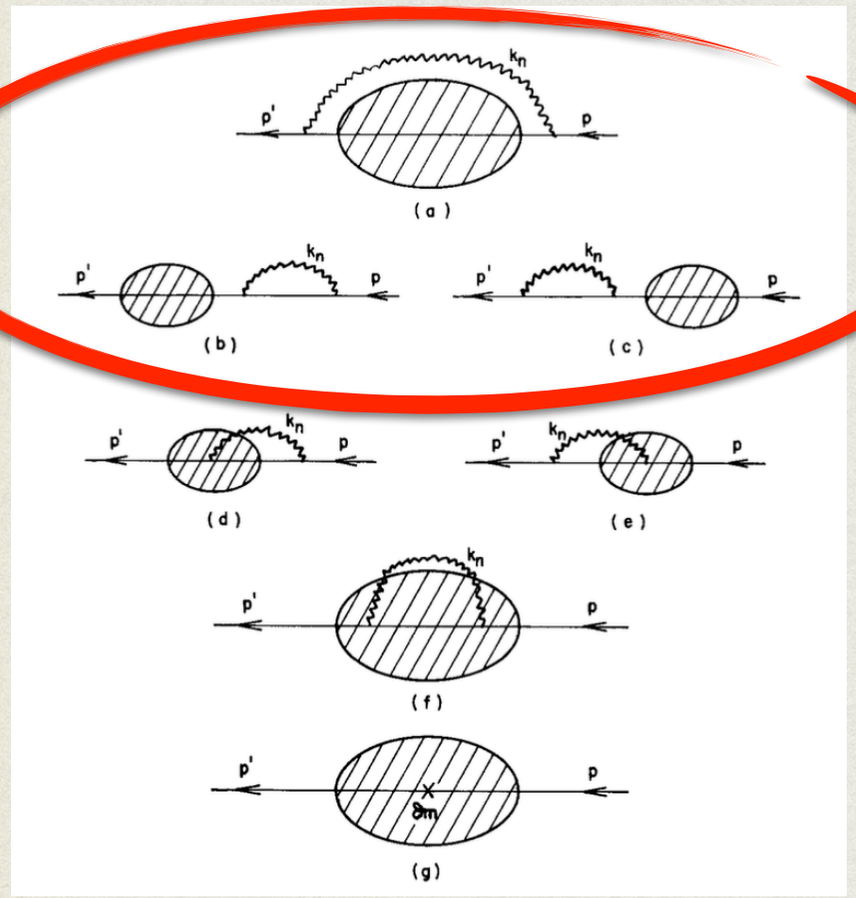


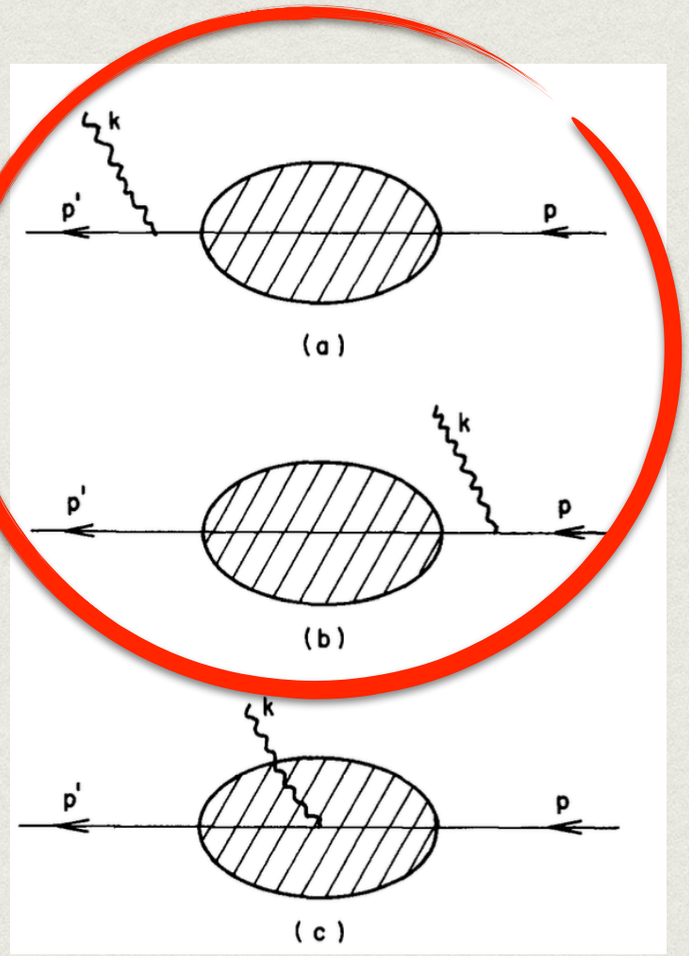


YFS/CEEX

In collaboration w/: ***Tom Dave, Jeremy Paltrinieri,*** *Thomas Teubner, Lois Flower*

Sources of IR singularities





$$\mathscr{A}^{n_\gamma} = \exp(\alpha B)\,\mathscr{R}^{n_\gamma}$$

$$\mathscr{R}^{n_\gamma} = \sum_{k=1}^{\infty} \left(\frac{\alpha}{\pi}\right)^k \mathscr{R}_k^{n_\gamma}$$

$$\tilde{\mathscr{A}}^{n_\gamma} = \exp\left(\alpha \tilde{B}\right)\tilde{\mathscr{R}}^{n_\gamma}$$

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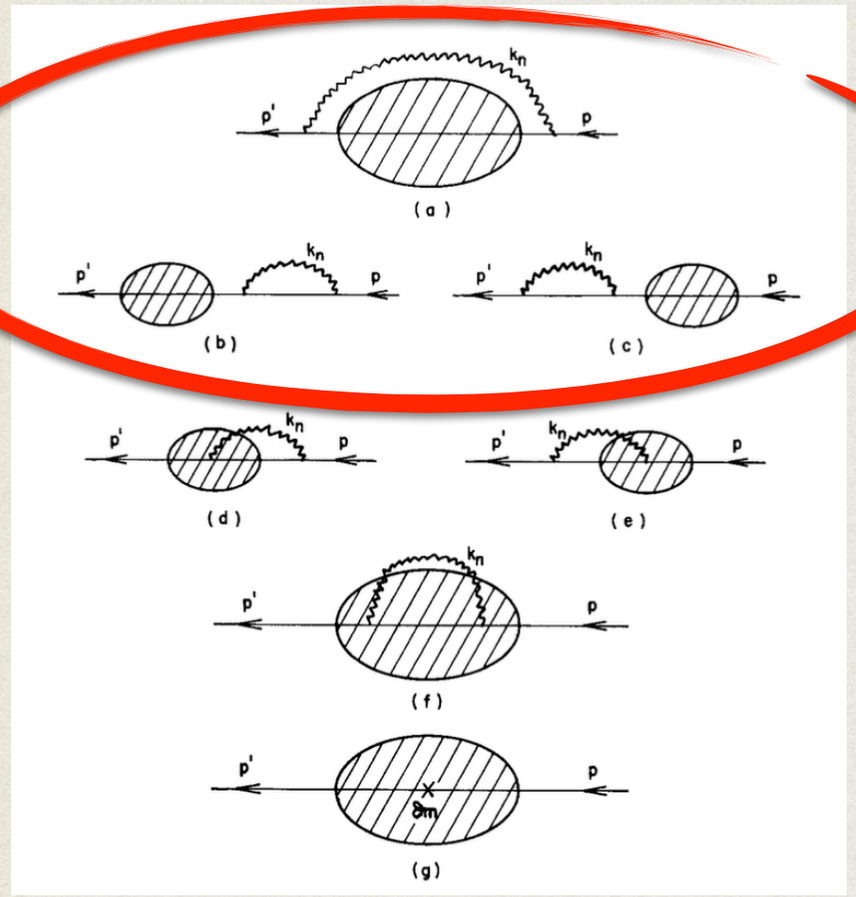

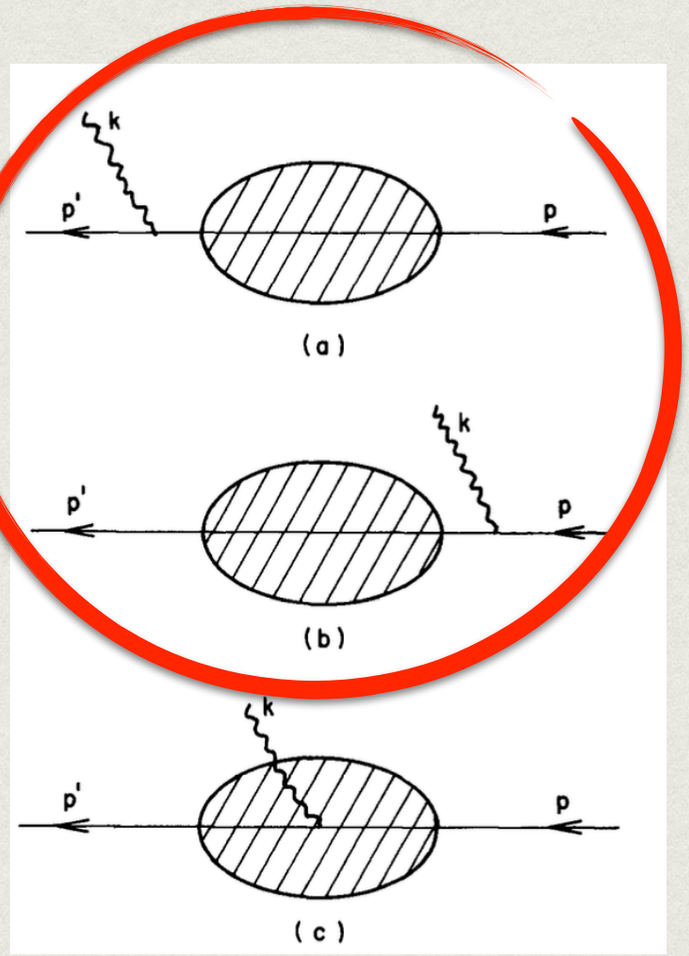

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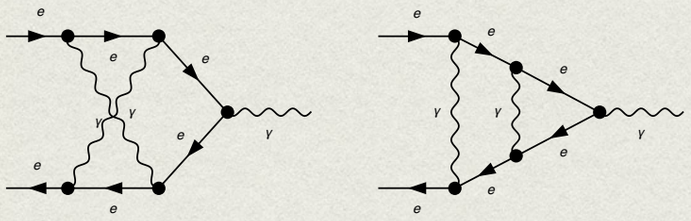
- Semi-analytic computation (Jeremy's talk)
- To be included in Phokhara-X
- Implementation in DimReg

Fixed NNLO calculation

In collaboration w/: ***Everyone!***

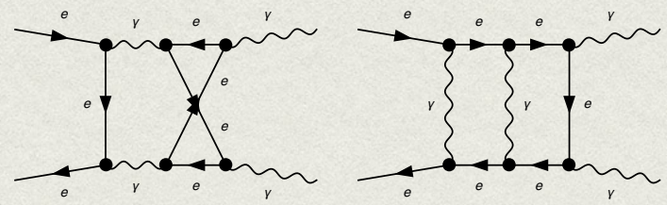
$$f^+f^- \to \gamma^* \to F^+F^- + \gamma$$



Easy (m_f^2, s)

- Trivial calculation (Pau's talk)

$$f^+f^- \to \gamma\,\gamma^* \to F^+F^-$$



Normal (s, t, m_e^2, q^2)

- Preliminary results for planar contributions w/o closed fermion loops.

$$f^+f^- \to F^+F^-\,\gamma$$

Hard $(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_f^2, m_F^2)$

- Very very hard!

Conclusions

We are working on:

- Improvement of the pion form factor
  From F × sQED to GVMD

- Get NLO+ from all possible viewpoints!
  Tutti frutti recipe!
  1) Exponentiation
  2) “honest” fixed NNLO calculation

- Extensions of Phokhara:
  1) Proof of concept calculations
  2) Validation w/ other tools (when available)
  3) Deliver implementation on Phokhara