RadioMonteCarLow 2

$ee \to \pi\pi\gamma$ the Bern/PSI/McMule perspective

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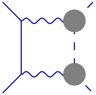
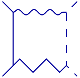
this is not my own work

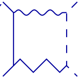
- the methods are from [Colangelo, Hoferichter, Monnard, Ruiz de Elvira 22]
- this is a FsQED calculation (not F$\times$sQED)
- we re-calculated this & implemented in McMule (see also Sophie's talk)
- I am not an expert on pions.
- this was suggested as a sensible first step, not the be all, end all

- we calculate the pion pole $\mathcal{A} =$ 
- we include form factors $F_\pi(k^2)$ but the pion remains a pion
- basic idea: dispersive treatment $$\frac{F_\pi(k^2)}{k^2} = \frac{1}{k^2} - \frac{1}{\pi}\int_{4m_\pi^2}^\infty \mathrm{d}s' \frac{\Im F_\pi(s')}{s'(k^2-s)}$$
- then do the loop integral with a massive particle ('disperon')
- be careful at $s' = s$ (threshold singularity)
- numerically integrate over s'

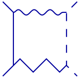
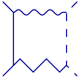
$\mathcal{A}^{(pd)} \supset \int_{4m_\pi^2}^{\infty} \mathrm{d}s'$  $\sim \int_{4m_\pi^2}^{\infty} \mathrm{d}s' \frac{\Im F_\pi(s')}{s'} \Big(\frac{s}{s'-s}\Big)^{1+2\epsilon}$

different options to calculate the principal value

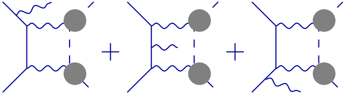
- counter-term for the amplitude [McMule]

  $\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \Bigg($  $- \frac{1}{s'} \Big(\frac{s}{s'-s}\Big)^{1+2\epsilon} f(s,t)\Bigg) + \int_{4m_\pi^2}^{\infty} \mathrm{d}s' \frac{1}{s'} \Big(\frac{s}{s'-s}\Big)^{1+2\epsilon} f(s,t)$

- subtract $\Im F_\pi(s)$ [Colangelo et al. 22] (m_γ), [Cottini, Holz] (DIMREG), [Budassi et al.] (both)

  $\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \frac{\Im F_\pi(s') - \Im F_\pi(s)}{s'}$  $+ \int_{4m_\pi^2}^{\infty} \mathrm{d}s' \frac{\Im F_\pi(s)}{s'}$ 

all three methods agree after numerical integration!

- let's begin with the $q_e^3 q_\pi^2$ term $\mathcal{A} \supset$ 
- this should be fairly doable using eg. LHC tech
  (amplitude from OpenLoops, threshold subtraction using S(C)ET, high-energy expansion using EFT)
- not necessarily the most important bit (C-odd) but a good first step
- conceptually well-defined

next-to-next steps

- similar tech could be used for $q_e^2 q_\pi^3$
- unclear (at least to me) if that's sufficiently well defined to be helpful
- unclear (at least to me) what a two-loop version of $ee \to \pi\pi$ would look like
  ($ee \to \pi\pi\gamma$ seems very distant from where I stand)
- unclear (at least to me) if FsQED is a good approximation here
- comparing FsQED and F$\times$sQED would still be interesting
- for $ee \to \pi\pi$, take Compton tensor of [Hoferichter, Stoffer 19] (dispersive description of two-pion rescattering)
- we need a full hadronic description of this