



MCGPJ generator
&
Vacuum Polarisation

Fedor Ignatov
University of Liverpool

Radio MonteCarlo Low 2
Liverpool, 15 November 2024

Collinear jets limitation

Thanks to high statistics collected by CMD-3
It was observed a discrepancy in momentum distribution of experimental data vs theoretical spectra from MCGPJ

Important only for differential distributions in tails when two-photon kinematic selections play role.
Integrated cross section for scan scenario is unaffected at ~0.06%.

Comes from collinear jets approximation
photon jets angular distribution in one photon approximation (+ few other corrections):

$$f(c = \cos(\theta), x = \omega/E) \sim \frac{1}{pk} \frac{x(1-x)}{1+(1-x)^2} \frac{m^2}{(pk)^2} \sim \frac{1}{1-\beta c} - \frac{1-x}{1+(1-x)^2} * \frac{1-\beta^2}{(1-\beta c)^2}$$

MCGPJ Bhabha - jets with angles
 $\mu+\mu- / \pi+\pi-$ - in collinear SF approximation

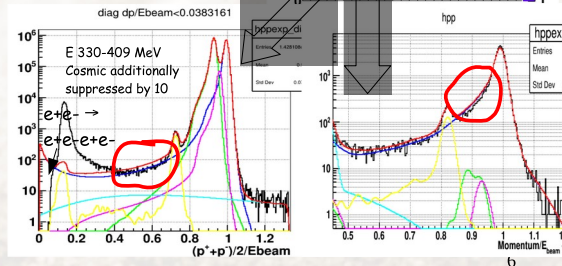
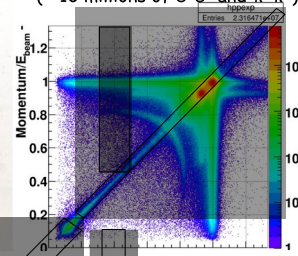
6 June 2024

Based on paper from 1997, since that:
Angle distribution of jets only for $e+e- \rightarrow e+e-$
GVMD & dispersive for the box diagram for $\pi+\pi-$
Limitations

Additive matching between NLO & Collinear structures (exact NLO in some phase space)
No resummation of ISR&FSR interference

No further development is expected

All events from RHO2013 scan (~10 millions of $e+e-$ and $\pi+\pi-$)



MITP, Mainz

Large angle QED processes at e^+e^- colliders at energies below 3 GeV

Andrej B. Arbuzov and Eduard A. Kuraev
Bogolubov Laboratory of Theoretical Physics, JINR, Dubna, 119880, Russia
Gennadi V. Fedotovich
Budker Institute for Nuclear Physics, Prospect Nauki, 11, Novosibirsk, 630090, Russia
Nikolay P. Merenkov
Kharkov Institute of Physics and Technology, Kharkov, 61108, Ukraine
Vladimir D. Ruzhik
Laboratory of Computing Techniques and Automation, JINR, Dubna, 119880, Russia
Luca Trentadue
Dipartimento di Fisica, Università di Parma and INFN, Gruppo Collegato di Parma, 43100 Parma, Italy

Monte-Carlo generator for e^+e^- annihilation into lepton and hadron pairs with precise radiative corrections

A.B. Arbuzov¹, G.V. Fedotovich^{2*}, F.V. Ignatov², E.A. Kuraev¹, A.L. Shildanov²
¹ Bogolubov Laboratory of Theoretical Physics, JINR, Dubna, 119880, Russia
² Budker Institute for Nuclear Physics, Prospect Lavrent'eva, 11, Novosibirsk, 630090, Russia

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Abstract. Recently, various cross sections of e^+e^- annihilation into hadrons were accurately measured in the energy range from 0.37 to 1.89 GeV with the CMD-2 detector at the VEPP-2M collider. In the 2^+ channel a systematic uncertainty of 0.05% has been achieved. Monte Carlo Generator Pava (MCGPJ) was developed to simulate events of Bhabha scattering as well as production of two charged pions, kaons and protons. Based on the formalism of structure functions, the leading logarithmic contributions related to the emission of photon jets in the collinear region are incorporated into the MC generator. Radiative corrections (RC) in the first order of α are accounted for exactly. The theoretical prediction of the cross sections with RC is estimated to be better than 0.2%. Numerous tests of the program as well as comparison with other MC generators and CMD-2 experimental data are presented.

1 Introduction

The goal of the new JINR experiment [5] is to measure the anomalous magnetic moment of the muon with the relative accuracy ~0.25 ppm. To reduce the current systematic error of the hadronic contribution to a_μ^{had} at least to the same level, the theoretical prediction of the cross sections with radiative corrections (RC) should be better than 0.2%.

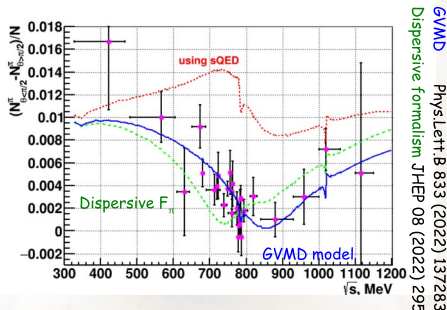
ABSTRACT: Processes of electron-positron annihilation into charged pions and kaons are considered. Radiative corrections are taken into account exactly in the first order and within the leading logarithmic approximation in higher orders. A combined approach for accounting exact calculations and electron structure functions is used. An accuracy of the calculation can be estimated about 0.2%.

KEYWORDS: 13.10.Cc, 13.10.Fg, 13.10.Hj, 13.10.Lb, 13.10.Lk, 13.10.Lm, 13.10.Ln, 13.10.Lo, 13.10.Lp, 13.10.Lq, 13.10.Lr, 13.10.Ls, 13.10.Lt, 13.10.Lu, 13.10.Lv, 13.10.Lw, 13.10.Lx, 13.10.Ly, 13.10.Lz, 13.10.Ma, 13.10.Mb, 13.10.Mc, 13.10.Md, 13.10.Me, 13.10.Mf, 13.10.Mg, 13.10.Mh, 13.10.Mi, 13.10.Mj, 13.10.Mk, 13.10.Ml, 13.10.Mm, 13.10.Mn, 13.10.Mo, 13.10.Mp, 13.10.Mq, 13.10.Mr, 13.10.Ms, 13.10.Mt, 13.10.Mu, 13.10.Mv, 13.10.Mw, 13.10.Mx, 13.10.My, 13.10.Mz, 13.10.Na, 13.10.Nb, 13.10.Nc, 13.10.Nd, 13.10.Ne, 13.10.Nf, 13.10.Ng, 13.10.Nh, 13.10.Ni, 13.10.Nj, 13.10.Nk, 13.10.Nl, 13.10.Nm, 13.10.Nn, 13.10.No, 13.10.Np, 13.10.Nq, 13.10.Nr, 13.10.Ns, 13.10.Nt, 13.10.Nu, 13.10.Nv, 13.10.Nw, 13.10.Nx, 13.10.Ny, 13.10.Nz, 13.10.Oa, 13.10.Ob, 13.10.Oc, 13.10.Od, 13.10.Oe, 13.10.Of, 13.10.Og, 13.10.Oh, 13.10.Oi, 13.10.Oj, 13.10.Ok, 13.10.OL, 13.10.Om, 13.10.On, 13.10.Oo, 13.10.Op, 13.10.Oq, 13.10.Or, 13.10.Os, 13.10.Ot, 13.10.Ou, 13.10.Ov, 13.10.Ow, 13.10.Ox, 13.10.Oy, 13.10.Oz, 13.10.Pa, 13.10.Pb, 13.10.Pc, 13.10.Pd, 13.10.Pe, 13.10.Pf, 13.10.Pg, 13.10.Ph, 13.10.Pi, 13.10.Pj, 13.10.Pk, 13.10.Pl, 13.10.Pm, 13.10.Pn, 13.10.Po, 13.10.Pp, 13.10.Pq, 13.10.Pr, 13.10.Ps, 13.10.Pt, 13.10.Pu, 13.10.Pv, 13.10.Pw, 13.10.Px, 13.10.Py, 13.10.Pz, 13.10.Qa, 13.10.Qb, 13.10.Qc, 13.10.Qd, 13.10.Qe, 13.10.Qf, 13.10.Qg, 13.10.Qh, 13.10.Qi, 13.10.Qj, 13.10.Qk, 13.10.QL, 13.10.Qm, 13.10.Qn, 13.10.Qo, 13.10.Qp, 13.10.Qq, 13.10.Qr, 13.10.Qs, 13.10.Qt, 13.10.Qu, 13.10.Qv, 13.10.Qw, 13.10.Qx, 13.10.Qy, 13.10.Qz, 13.10.Ra, 13.10.Rb, 13.10.Rc, 13.10.Rd, 13.10.Re, 13.10.Rf, 13.10.Rg, 13.10.Rh, 13.10.Ri, 13.10.Rj, 13.10.Rk, 13.10.Rl, 13.10.Rm, 13.10.Rn, 13.10.Ro, 13.10.Rp, 13.10.Rq, 13.10.Rr, 13.10.Rs, 13.10.Rt, 13.10.Ru, 13.10.Rv, 13.10.Rw, 13.10.Rx, 13.10.Ry, 13.10.Rz, 13.10.Sa, 13.10.Sb, 13.10.Sc, 13.10.Sd, 13.10.Se, 13.10.Sf, 13.10.Sg, 13.10.Sh, 13.10.Si, 13.10.Sj, 13.10.Sk, 13.10.Sl, 13.10.Sm, 13.10.Sn, 13.10.So, 13.10.Sp, 13.10.Sq, 13.10.Sr, 13.10.Ss, 13.10.St, 13.10.Su, 13.10.Sv, 13.10.Sw, 13.10.Sx, 13.10.Sy, 13.10.Sz, 13.10.Ta, 13.10.Tb, 13.10.Tc, 13.10.Td, 13.10.Te, 13.10.Tf, 13.10.Tg, 13.10.Th, 13.10.Ti, 13.10.Tj, 13.10.Tk, 13.10.Tl, 13.10.Tm, 13.10.Tn, 13.10.To, 13.10.Tp, 13.10.Tq, 13.10.Tr, 13.10.Ts, 13.10.Tt, 13.10.Tu, 13.10.Tv, 13.10.Tw, 13.10.Tx, 13.10.Ty, 13.10.Tz, 13.10.Ua, 13.10.Ub, 13.10.Uc, 13.10.Ud, 13.10.Ue, 13.10.Uf, 13.10.Ug, 13.10.Uh, 13.10.Ui, 13.10.Uj, 13.10.Uk, 13.10.UL, 13.10.Um, 13.10.Un, 13.10.Uo, 13.10.Up, 13.10.Uq, 13.10.Ur, 13.10.Us, 13.10.Ut, 13.10.Uu, 13.10.Uv, 13.10.Uw, 13.10.Ux, 13.10.Uy, 13.10.Uz, 13.10.Va, 13.10.Vb, 13.10.Vc, 13.10.Vd, 13.10.Ve, 13.10.Vf, 13.10.Vg, 13.10.Vh, 13.10.Vi, 13.10.Vj, 13.10.Vk, 13.10.Vl, 13.10.Vm, 13.10.Vn, 13.10.Vo, 13.10.Vp, 13.10.Vq, 13.10.Vr, 13.10.Vs, 13.10.Vt, 13.10.Vu, 13.10.Vv, 13.10.Vw, 13.10.Vx, 13.10.Vy, 13.10.Vz, 13.10.Wa, 13.10.Wb, 13.10.Wc, 13.10.Wd, 13.10.We, 13.10.Wf, 13.10.Wg, 13.10.Wh, 13.10.Wi, 13.10.Wj, 13.10.Wk, 13.10.Wl, 13.10.Wm, 13.10.Wn, 13.10.Wo, 13.10.Wp, 13.10.Wq, 13.10.Wr, 13.10.Ws, 13.10.Wt, 13.10.Wu, 13.10.Wv, 13.10.Ww, 13.10.Wx, 13.10.Wy, 13.10.Wz, 13.10.Xa, 13.10.Xb, 13.10.Xc, 13.10.Xd, 13.10.Xe, 13.10.Xf, 13.10.Xg, 13.10.Xh, 13.10.Xi, 13.10.Xj, 13.10.Xk, 13.10.Xl, 13.10.Xm, 13.10.Xn, 13.10.Xo, 13.10.Xp, 13.10.Xq, 13.10.Xr, 13.10.Xs, 13.10.Xt, 13.10.Xu, 13.10.Xv, 13.10.Xw, 13.10.Xx, 13.10.Xy, 13.10.Xz, 13.10.Ya, 13.10.Yb, 13.10.Yc, 13.10.Yd, 13.10.Ye, 13.10.Yf, 13.10.Yg, 13.10.Yh, 13.10.Yi, 13.10.Yj, 13.10.Yk, 13.10.Yl, 13.10.Ym, 13.10.Yn, 13.10.Yo, 13.10.Yp, 13.10.Yq, 13.10.Yr, 13.10.Ys, 13.10.Yt, 13.10.Yu, 13.10.Yv, 13.10.Yw, 13.10.Yx, 13.10.Yy, 13.10.Yz, 13.10.Za, 13.10.Zb, 13.10.Zc, 13.10.Zd, 13.10.Ze, 13.10.Zf, 13.10.Zg, 13.10.Zh, 13.10.Zi, 13.10.Zj, 13.10.Zk, 13.10.Zl, 13.10.Zm, 13.10.Zn, 13.10.Zo, 13.10.Zp, 13.10.Zq, 13.10.Zr, 13.10.Zs, 13.10.Zt, 13.10.Zu, 13.10.Zv, 13.10.Zw, 13.10.Zx, 13.10.Zy, 13.10.Zz

sQED limitations

MITP, Mainz

Phys.Rev.Lett. 132, 231903 (2024)



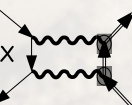
GVMD Phys.Lett.B 833 (2022) 137283
Dispersive formalism JHEP 08 (2022) 295

Implemented as correction to sQED:

$$d\sigma/dc = d\sigma_0/dc \times |F^2_\pi| \times (\delta_{sQED} + \delta_{FF})$$

$$\delta_{FF} \sim [F_{\pi}^{VMD}(q_1)F_{\pi}^{VMD}(q_2) - F_{\pi}^{VMD}(q)]/F_{\pi}^{VMD}(q) \times$$

δ_{FF} - IR finite, can be calculated separately as correction



MITP, Mainz

Radiative corrections for pion and kaon production at e^+e^- colliders of energies below 2 GeV

Andrej B. Arbuzov and Eduard A. Kuraev
Bogolubov Laboratory of Theoretical Physics, JINR, Dubna, 119880, Russia
Vladim A. Astashov and Gennadi V. Fedotovich
Budker Institute for Nuclear Physics, Prospect Nauki, 11, Novosibirsk, 630090, Russia
Andrey V. Fedorov
Laboratory of Computing Techniques and Automation, JINR, Dubna, 119880, Russia
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Institute of Physics and Technology, Kharkov, 61108, Ukraine

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The MCGPJ generator is based on the papers from 1997
Theoretical support from Andrej Arbuzov and Eduard Kuraev (JINR)
From Novosibirsk it was lead by Gennadi Fedotovich (BINP)
The code implementation by Alexey Sibidanov for CMD-2 experiment
F. Ignatov: maintenance and etc at CMD-3

6 June 2024

Thanks to high statistics collected by CMD-3
It was observed a discrepancy in asymmetry from prediction
Integrated cross section for scan scenario is unaffected, but very important for study and control of systematics!
1% effect is disaster if we talk about ~0.1% precision

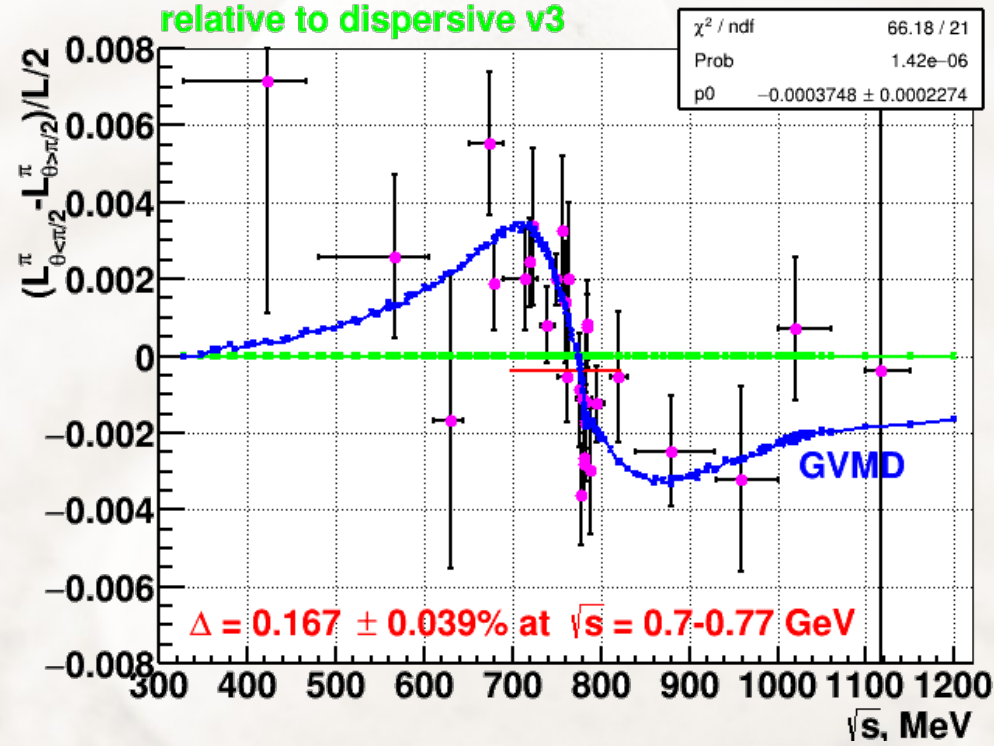
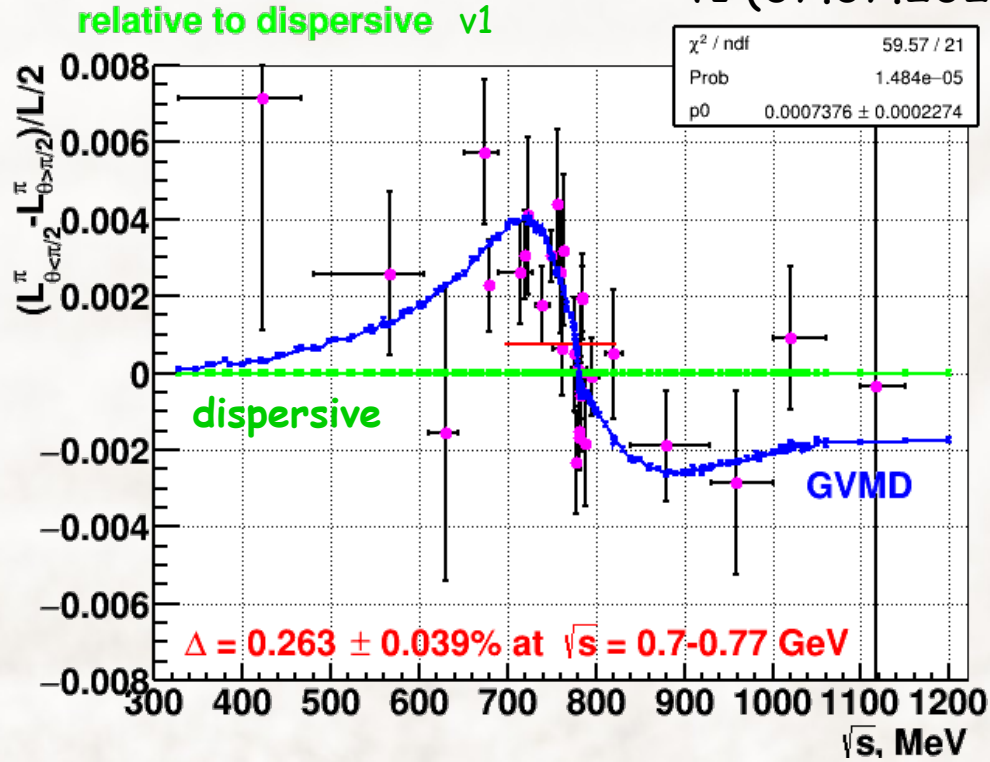
Comes from limitations of sQED approach
The theoretical model within GVMD was introduced, was confirmed by calculation in dispersive formalism

MCGPJ $\pi+\pi-$ - above sQED corrections can be used via pre-calculated tables $\delta_{FF}(s, \cos \theta)$ either from GVMD or dispersive paper

6 June 2024

Asymmetry in CMD3 $\pi^+\pi^-$ data

Dispersive calculation in arXiv:2207.03495 was updated thanks to BabaYaga & Yannick
v1 (07.07.2022) \rightarrow v4 (20.09.2024)



GVMD & dispersive disagree at $\sim 0.3\%$ -- GVMD & CMD-3 $\pi^+\pi^-$ agree very well

- Dispersive & data inconsistency, either :
- 1) systematic in data
 - 2) something caught by GVMD but not by dispersive
 - 3) better treatment of off-shell pion in box loop will change picture



Vacuum polarization

$$\Pi_h^{\mu\nu}(q) = \text{wavy line} \text{---} \text{blob} \text{---} \text{wavy line} = i e^2 \int d^4x e^{-iqx} \langle 0 | T \{ j_{\text{em}}^\mu(x) j_{\text{em}}^\nu(0) \} | 0 \rangle = \Pi_h(q^2) (g^{\mu\nu} q^2 - q^\mu q^\nu)$$

There is no any ambiguities,
the one-particle irreducible blob or the full photon propagator
is the fully defined physical tensor object

Vacuum polarization

Slide from 4th RMCL, Beijing, 2008

VP can be calculated from dispersion relation, based on analyticity and unitarity

$$P(s), \Pi(s) = \frac{s}{4\pi^2 \alpha} \left[PV \int_{4m_\pi^2}^{\infty} \frac{\sigma_{ee \rightarrow \gamma \rightarrow \text{everything}}^{\text{bare, dressed}}(s') ds'}{s-s'} - i \sigma_{ee \rightarrow \gamma \rightarrow \text{everything}}^{\text{bare, dressed}}(s) \right]$$

$$P(s) = P_{\text{leptons}}(s) + P_{\text{hadrons}}(s)$$

Using "bare" cross sections

"dressed" cross sections

"Polarization operator"
one particle irreducible blob

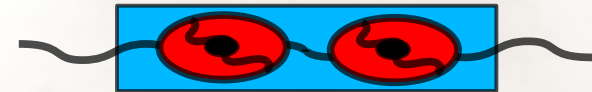


$$\frac{1}{1 - P(s)} = 1 + P + P^2 + \dots = 1 + \Pi(s)$$

- x "undress" experimental cross sections
- x "nontrivial" task to correct physical parameters of resonances

results in

"Self-energy function of photon"
Full photon propagator

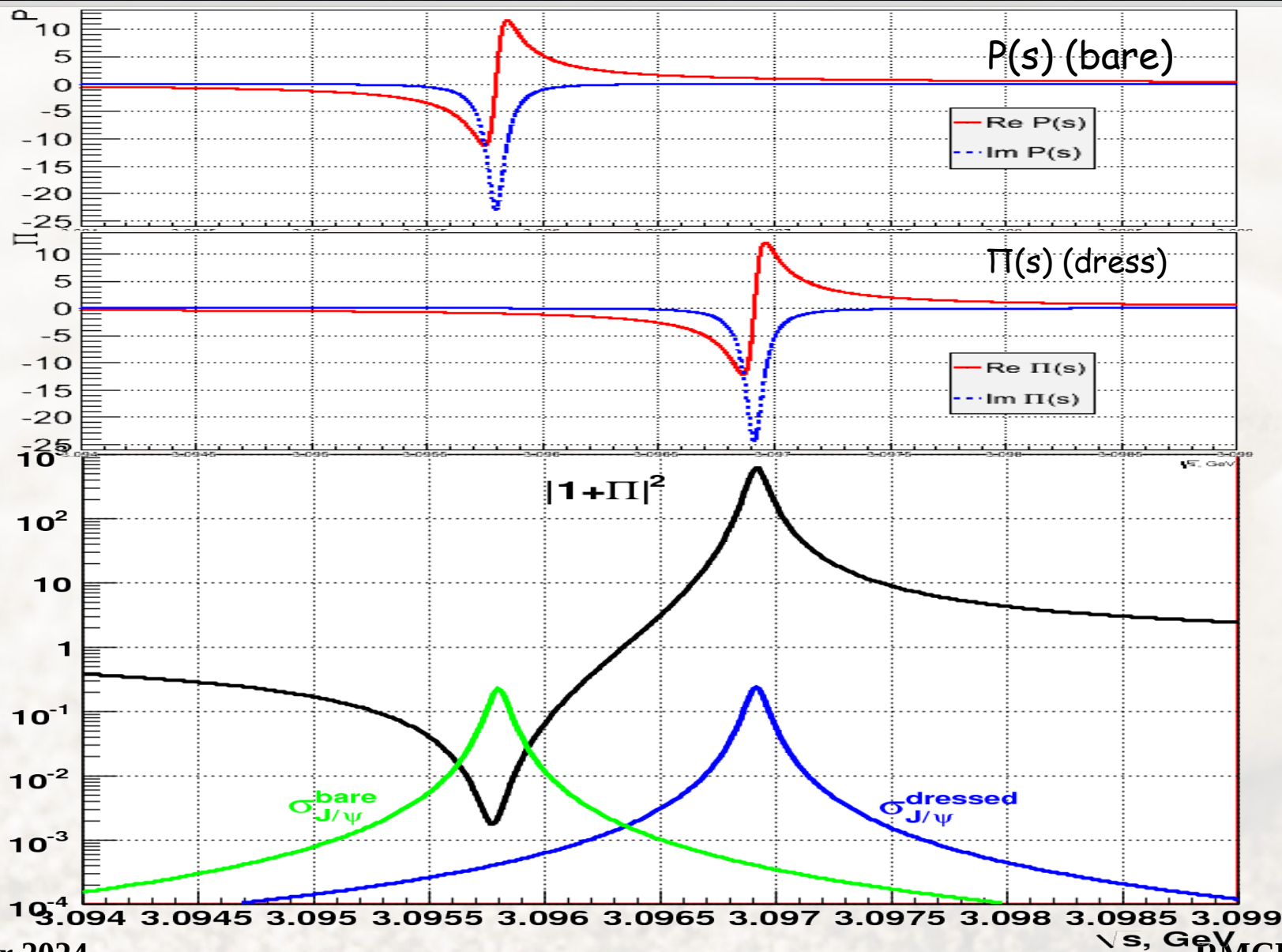


This require to use either

- x "dress" leptonic part and pQCD
- x "dress" some of experimental data

Iterations are necessary in all cases

VP at J/Psi



Whatever prescription of the VP will be chosen, the final objects, like

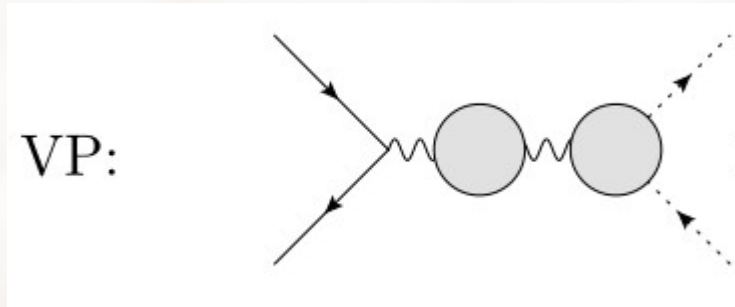
$$d\sigma_{mm}(q_e^2 q_m^2 \Pi) = \frac{d\sigma_{mm}(q_e^2 q_m^2)}{|1 - \Pi^{\text{ren}}(s)|^2}.$$

Should represent the physically observable resonance cross sections.

NSK VP via resummed form $1/(1-P(s))$ provides this, others VP compilations do not

VP usage

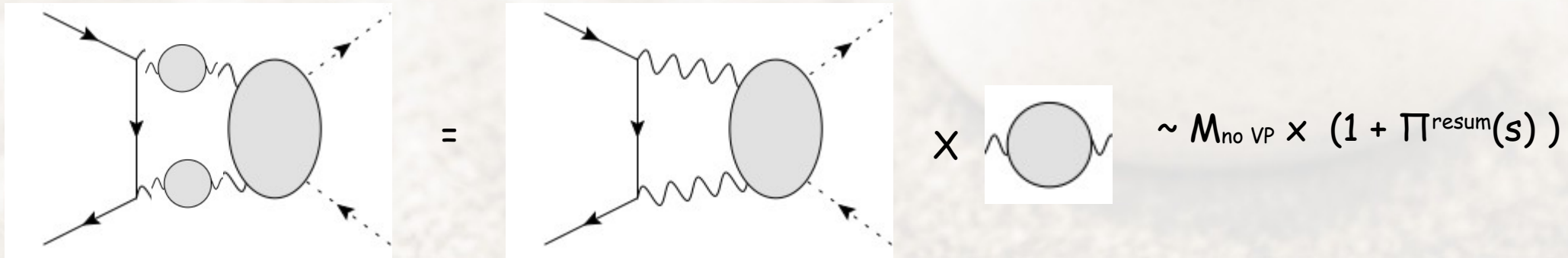
It seems, the most accurate approach is to use resummed version of VP whenever possible



$$x(1 + P(s) + P^2(s) + \dots) = \frac{1}{1 - P(s)} = 1 + \Pi^{\text{resummed}}(s)$$

Simplifications via order by order calculations, like $\sigma \sim |M|^2 \times |1 + \Pi(s)|^2 \sim |M|^2 \times (1 + 2 \cdot \text{Re } \Pi(s))$ doesn't work on narrow resonances

Probably right now, the common approximation of VP usage in the more complicated cases is much similar like FxS QED for $\pi^+\pi^-$:



Way to improve