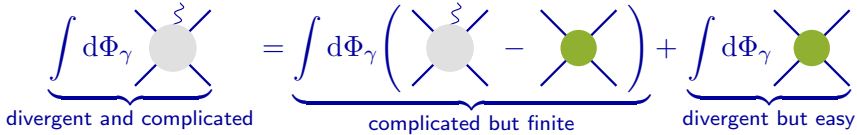
RadioMonteCarLow 2 Working Group Meeting

A mule never stops
– future plans for McMule –

Sophie Kollatzsch
for the McMule team

Monte **C**arlo for **MU**ons and other **LE**ptons

- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS$^\ell$ subtraction with massive fermions



- challenge virtual amplitudes with $m \neq 0 \implies$ massification (photonic)
- challenge numerical instabilities $\implies$ next-to-soft stabilisation + **O**pen**L**oops

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$$\mathcal{A}(m) = \left(\prod_j \sqrt{Z(m)}\right) \times \mathcal{A}(m=0) + \mathcal{O}(m) \quad \text{iff} \quad m^2 \ll \text{all other scales}$$

- challenge numerical instabilities $\implies$ next-to-soft stabilisation + **O**pen**L**oops

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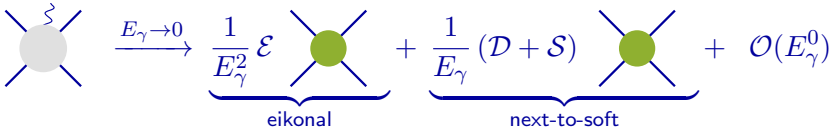

process#	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO
$\ell\mathrm{N} \to \ell\mathrm{N}$	P2, Muse, Prad, QWeak, ...	proton radius and weak charge	NNLO$(-)$
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO$-$
$e^-e^- \to e^-e^-$	Prad	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \gamma^*$			NNLO
$ee \to \ell\ell$	CMD+SND, BES, KLOE, ...	R-ratio	NNLO$+$
	Belle	τ properties & $\sin^2\theta_W$	
$ee \to \pi\pi$	CMD+SND, BES, KLOE, ...	R-ratio	NLO$+$
$ee \to \gamma\gamma$	KLOE	dark searches	NNLO$-$
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	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}eee$	Mu3e	background	NLO

in McMule (currently) $2 \to 2$ @ NNLO $\supset 2 \to 2\gamma$ @ NLO

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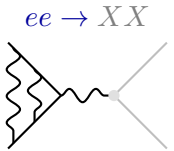
current state :: $ee \to \gamma^*$ @ NNLO $\supset$ $ee \to \gamma\gamma^*$ @ NLO

- universal framework for arbitrary currents $X \in \{\pi, {}^{12}C, p, {}^{2}H, \dots\}$
- full mass dependence



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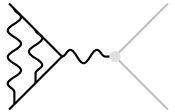
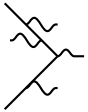


next step [$\sim$ 2025] :: $ee \to \gamma\gamma^*$ @ NNLO # not yet in McMule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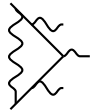
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[Badger et al 23]
+ massification

dispersive# &
hyperspherical#

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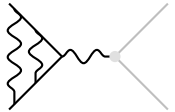

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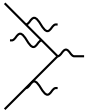
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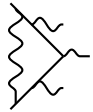
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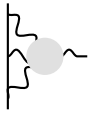
next-to-next step [$\sim$ 2026++] :: $ee \to \gamma^*$ @ N^{3}LO $\supset$ $ee \to \gamma\gamma^*$ @ NNLO

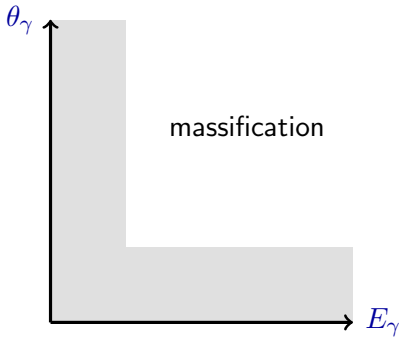

[Fael et al 22]

[Badger et al 23]
+ massification
+ jettification

light-by-light, ...

real-(virtual)^2 matrix element needs **massification**

$$\mathcal{M}_n(m) \xrightarrow{m\to 0} \mathcal{M}_n(0) \times Z \times Z \qquad \mathcal{M}^{(2)}_{n+1} \sim \frac{1}{E_\gamma^2}\frac{1}{(1-\beta\cos\theta_\gamma)}$$

- ($m^2 \ll$ all other scales) not valid everywhere



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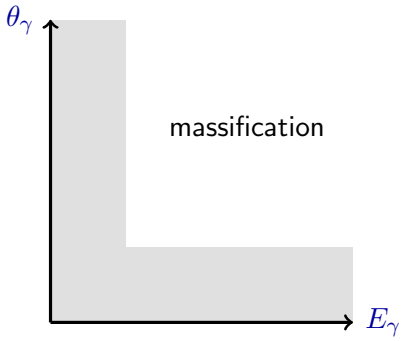

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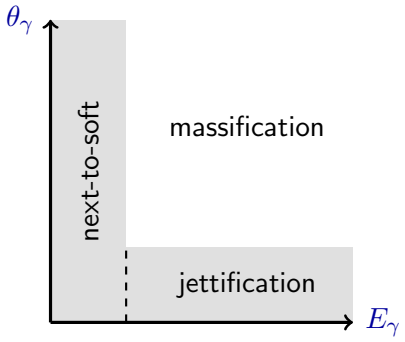

- ($m^2 \ll$ all other scales) not valid everywhere
- detected photon ($ee \to \gamma\gamma^*$ @ NNLO [ISC]): region excluded by cuts
- inclusive process ($ee \to \gamma^*$ @ N^{3}LO): switch to expansion
- next-to-soft ✓
- jettification: massive J unknown at 2 loop

$$\mathcal{M}_{n+1}(m) \xrightarrow[\theta_\gamma\to 0]{m\to 0} \mathcal{M}_n(0) \times Z \times J$$

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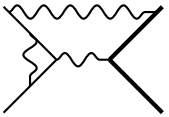
- massification (m_e) for mixed, full mass dependence everything else



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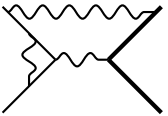

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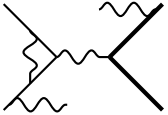

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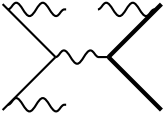

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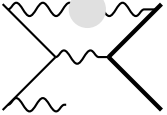
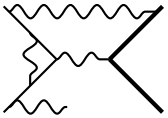
(not ideal)



dispersive# &
hyperspherical#



$pp \to 2j + \gamma$ from
[Badger et al 23]# + massification (m_e & m_μ) $\mathcal{O}(10\%)$ on NNLO

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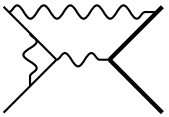

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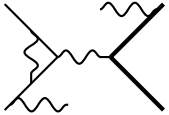

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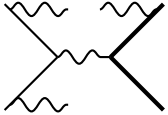

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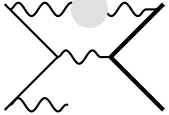

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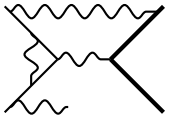

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next-to-next step [?] :: $ee \to \mu\mu$ @ N^{3}LO $(e\mu \to e\mu)$ [see talk by Marco B.]

previously :: ISC via $ee \to \gamma^*$ @ NNLO

- below state of the art

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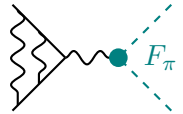
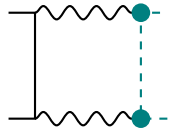
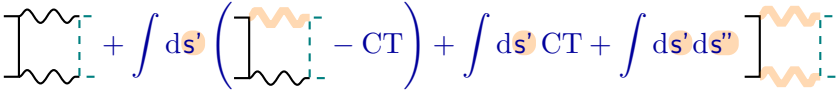




in testing [$\sim$ 2024] :: $ee \to \pi\pi$ @ NLO :: [Colangelo et al 22] in dim. regularisation

$$\frac{F_\pi(k^2)}{k^2} = \frac{1}{k^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}s' \, \frac{\mathrm{Im}F_\pi(s')}{s'(k^2 - s')}$$

$$\Longrightarrow$$



- expansion for large $s'(s'')$ implemented via EFT methods

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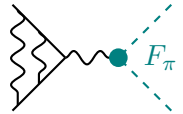

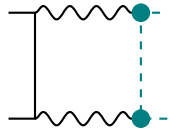

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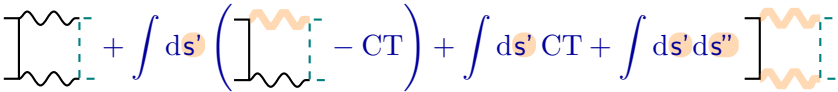
$$\Longrightarrow$$  $$+ \int \mathrm{d}s' \left($$ $$- \mathrm{CT}\right) + \int \mathrm{d}s' \, \mathrm{CT} + \int \mathrm{d}s' \mathrm{d}s''$$

- expansion for large $s'(s'')$ implemented via EFT methods

next step [?] :: $ee \to \pi\pi\gamma$ @ NLO . . . final state radiation $\Longrightarrow$ 'disperon' from OL

previously :: ISC via $ee \to \gamma^*$ @ NNLO

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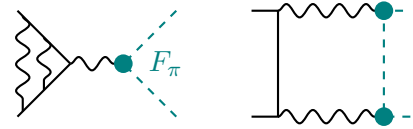

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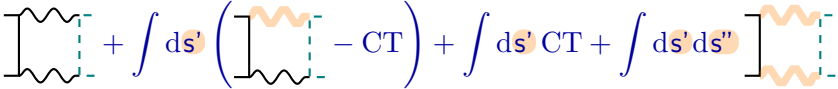
$$\Longrightarrow$$  $$+ \int \mathrm{d}s' \left($$ $$- \mathrm{CT}\right) + \int \mathrm{d}s' \, \mathrm{CT} + \int \mathrm{d}s' \mathrm{d}s''$$

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next step [?] :: $ee \to \pi\pi\gamma$ @ NLO . . . final state radiation $\Longrightarrow$ 'disperon' from OL

next-to-next step [?] :: $ee \to \pi\pi$ @ NLO with full Compton tensor [Hoferichter et al 19]

yesterday

- $ee \to \pi\pi$ @ NLO :: tests

now

- $ee \to \gamma\gamma^*$ @ NNLO :: tests
- $ee \to \mu\mu\gamma$ @ NNLO :: start tests in parallel

in our mind

- McMule @ higher energies ::
  numerical instability for real-real in $ee \to ee$ @ B-like
  $\implies$ collinear subtraction? [Dittmaier et al 08]
- $ee \to ee$ @ NNLO without massification [Delto et al 23]
- $ee \to \mu\mu$ @ N^{3}LO [see talk by Marco B.]

later

- $ee \to \pi\pi(\gamma)$ @ NLO
- $ee \to \gamma^*$ @ N^{3}LO

general

- event generation
- soft resummation
- electroweak
- polarisation
- ...

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