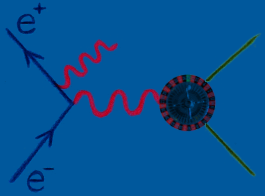
Radiative corrections in ISR measurements



S. E. Müller

Helmholtz-Zentrum Dresden-Rossendorf

Radio MonteCarLow2 Working Group Meeting, Liverpool

November 15, 2024

Introduction

Cross section in dispersion integral should be inclusive with respect to FSR:

Introduction

Cross section in dispersion integral should be inclusive with respect to FSR:

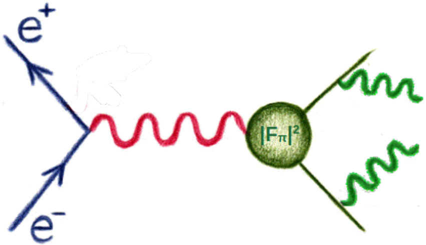
Definitions:

- Cross section $\sigma_{\pi\pi}$:
  FSR included, vacuum polarization removed (*bare* or *undressed*)
  $\rightarrow$ ready to plug into dispersion integral
- Pion form factor $|F_{2\pi}|^2$:
  FSR excluded, vacuum polarization included (*dressed*)
  $\rightarrow$ some kind of effective coupling constant (for fitting to obtain parameters of underlying mesons)

Introduction

Cross section in dispersion integral should be inclusive with respect to FSR:

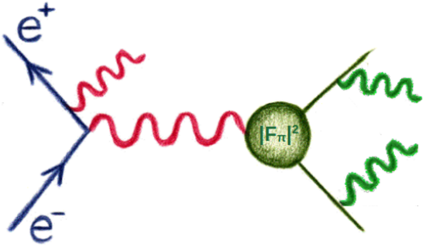
Do the measurement inclusive with respect to FSR:



Introduction

Cross section in dispersion integral should be inclusive with respect to FSR:

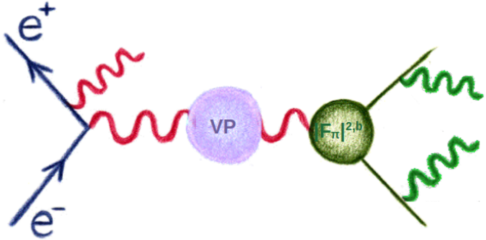
Include ISR photons at particle factories:



Introduction

Cross section in dispersion integral should be inclusive with respect to FSR:

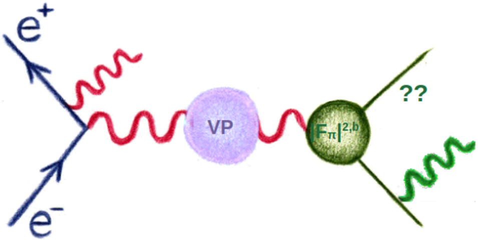
Internal photon is dressed by vacuum polarisation:



Introduction

Cross section in dispersion integral should be inclusive with respect to FSR:

Measurement misses out on a fraction of FSR:

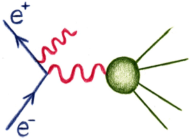


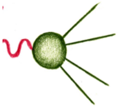
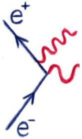
Radiative corrections: Radiator function

- Radiator function: Crucial ingredient for ISR analyses, quite complex analytic form. Extract radiator function from PHOKHARA MonteCarlo code:

$$H(s, M_{\pi\pi}^2) = s \cdot \frac{3M_{\pi\pi}^2}{\pi\alpha^2\beta^3} \cdot \left.\frac{\mathrm{d}\boldsymbol{\sigma}^{\mathrm{ISR}}_{\boldsymbol{\pi\pi\gamma(\gamma)}}}{\mathrm{d}\boldsymbol{M}^2_{\boldsymbol{\pi\pi}}}\right|_{|\boldsymbol{F}_{2\boldsymbol{\pi}}|^2=1}$$

$$\frac{\mathrm{d}\,\sigma(e^+e^- \to \mathrm{had} + \gamma)}{\mathrm{d}\,M^2_{\mathrm{had}}} = \frac{\sigma(e^+e^- \to \mathrm{had}, M^2_{\mathrm{had}})}{s} \times H(s, M^2_{\mathrm{had}})$$



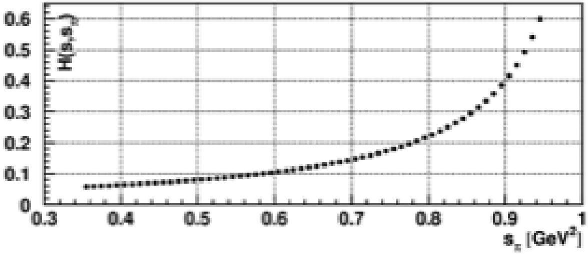
=

×


measured cross section — resulting cross section — radiator function

Radiative corrections: Radiator function

- Radiator function: Crucial ingredient for ISR analyses, quite complex analytic form. Extract radiator function from PHOKHARA MonteCarlo code:

$$H(s, M_{\pi\pi}^2) = s \cdot \frac{3M_{\pi\pi}^2}{\pi\alpha^2\beta^3} \cdot \left.\frac{d\boldsymbol{\sigma}^{ISR}_{\boldsymbol{\pi\pi\gamma(\gamma)}}}{d\boldsymbol{M}^2_{\boldsymbol{\pi\pi}}}\right|_{|\boldsymbol{F}_{2\boldsymbol{\pi}}|^2=1}$$

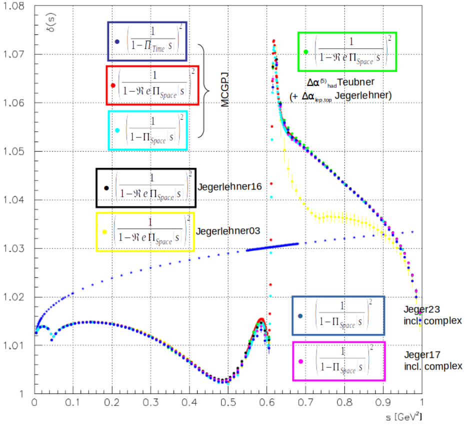


Theoretical precision due to missing terms: 0.5%

Radiative corrections: Vacuum Polarisation

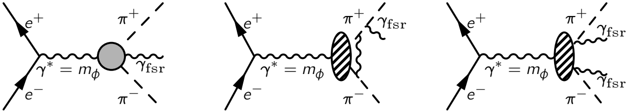
- Vacuum polarization: Cross section in dispersion integral needs to be undressed from vacuum polarization effects:

$$\sigma_0(s) = \sigma_{\text{obs}}(s)\left(\frac{\alpha(0)}{\alpha(s)}\right)^2 \equiv \sigma_{\text{obs}}(s)/\delta(s)$$



Final State Radiation - categories

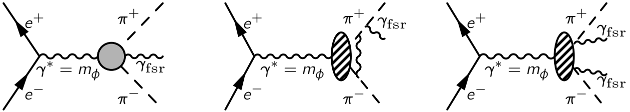
FSR-only: Contributions in which there is only FSR, no ISR:



For these contributions, $s_{\gamma^*} = s_{ee} = M_\phi^2$ (in the case of KLOE08/12), and thus they are outside the s_π range of interest. They are proportional to the value of $|F_\pi(s_{ee})|^2$. Suppressed if $s_{ee} >>$ than the s_π range of interest.

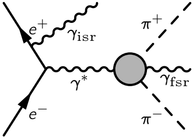
Final State Radiation - categories

FSR-only: Contributions in which there is only FSR, no ISR:



For these contributions, $s_{\gamma^*} = s_{ee} = M_\phi^2$ (in the case of KLOE08/12), and thus they are outside the s_π range of interest. They are proportional to the value of $|F_\pi(s_{ee})|^2$. Suppressed if $s_{ee} >> $ than the s_π range of interest.

mixed FSR: Contributions in which there is both FSR and ISR:

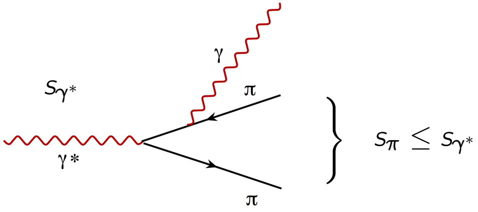


For these contributions, $s_\pi \leq s_{\gamma^*}$ - this contribution should be kept, but events sit at the wrong s_π in the spectrum $\rightarrow$ "Unshifting" method

Unshifting

Correction for shift in s_π due to FSR events

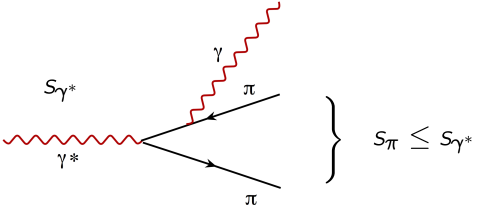
The presence of FSR shifts the observed value of s_π (evaluated from the 2 pion tracks' momenta) away from the invariant mass squared of the virtual photon s_{γ^*}:



Unshifting

Correction for shift in s_π due to FSR events

The presence of FSR shifts the observed value of s_π (evaluated from the 2 pion tracks' momenta) away from the invariant mass squared of the virtual photon s_{γ^*}:



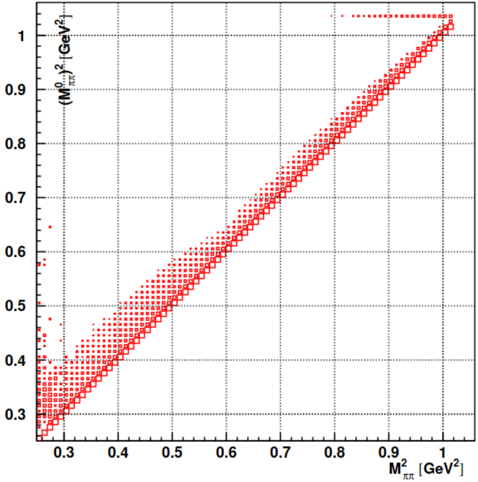
Redistribute events to obtain "unshifted" distribution:

$$N_i^{s_{\gamma^*}} = \sum_{j=1}^{n} P(N_i^{s_{\gamma^*}} | N_j^{s_\pi}) \cdot N_j^{s_\pi}$$

$P(N_i^{s_{\gamma^*}} | N_j^{s_\pi})$ obtained from MC (Phokhara_omega for events with 1γ ISR, 1γ FSR or 1γ ISR+1γ FSR; no interference)

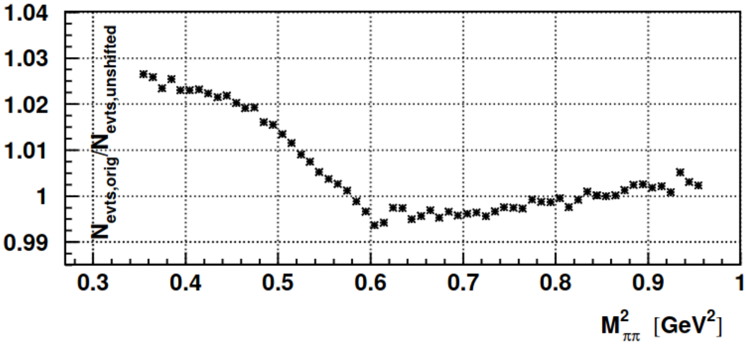
Unshifting

Probability matrix (from KLOE Note 221 for KLOE08 analysis):



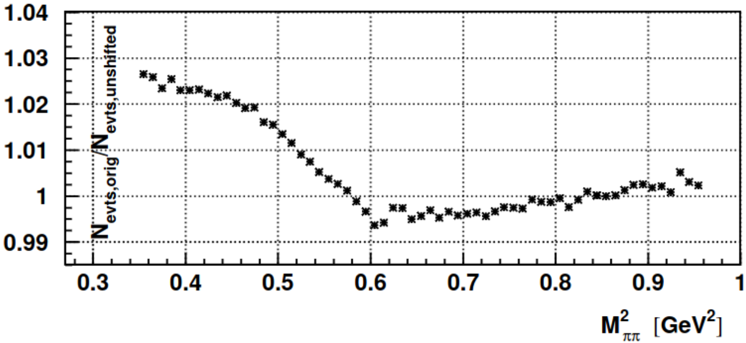
Unshifting

Effect of the unshifting procedure (from KLOE Note 221 for KLOE08 analysis):



Unshifting

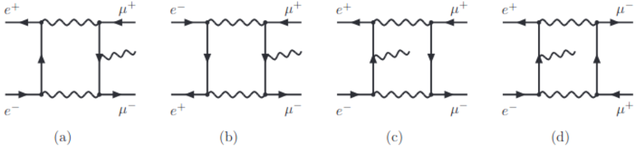
Effect of the unshifting procedure (from KLOE Note 221 for KLOE08 analysis):



Then estimate correction to add back fraction of FSR from MC generator taken out by selection cuts.

Caveat: Higher order box-diagrams

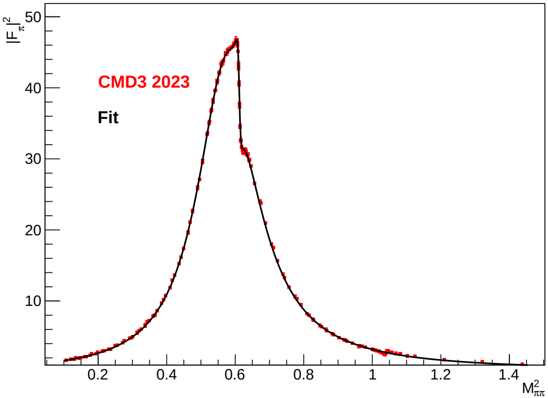
PHOKHARA10 now contains the NLO contributions for $\mu\mu\gamma$ and $\pi\pi\gamma$ which include the diagrams with pentagram topology (arXiv:1312.3610):



- Factorization ansatz does not work any more for radiator function and vacuum polarisation
  - if the box-contributions are not too large, subtract from spectrum and then assume factorization still holds
- Event-by-event distinction of ISR and FSR-photons probably not possible
  - This is needed to “unshift” events with both an ISR and FSR photons
  - “PHOKHARA Omega”

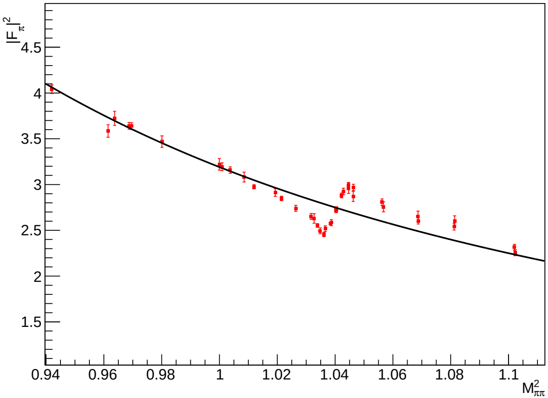
Estimation of the FSR-only contribution

The contribution with only final state radiation in the data depends directly on the value of the pion form factor at the e^+e^- collision energy of the collider. CMD-3 data shows the interference wiggle around the ϕ mass:



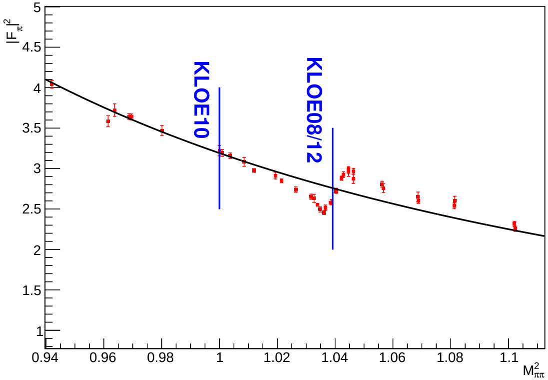
Estimation of the FSR-only contribution

The contribution with only final state radiation in the data depends directly on the value of the pion form factor at the e^+e^- collision energy of the collider. CMD-3 data shows the interference wiggle around the ϕ mass:



Estimation of the FSR-only contribution

The contribution with only final state radiation in the data depends directly on the value of the pion form factor at the e^+e^- collision energy of the collider. CMD-3 data shows the interference wiggle around the ϕ mass:



For KLOE2010, this was checked at 1 GeV using different pion form factor parameterizations and accounted for in the systematic uncertainty of FSR (see KLOE Note 225, Sec. 4.12.1). A 5% change was found at 1 GeV, at the ϕ peak we have $\pm$10%, depending where the collision energy sits.

Summary

- Radiative corrections are a crucial ingredient of ISR analyses
  - Radiator function
  - Vacuum Polarization
  - Final state radiation
- FSR more relevant for tagged photon / “large angle” analyses
  - While photons from ISR are emitted preferably at small angles respect to the beams, FSR photons are more isotropically distributed
- More complete description of processes in Monte Carlo codes gives better precision
  - But may need us to get rid of “old habits”
- I did not discuss contribution of ϕ-meson decays like $\phi \to f_o\gamma \to \pi\pi\gamma$
  - Interference with FSR
  - Work done by Olga Shekhovtsova to include these channels in PHOKHARA
  - Suppressed in “small angle” analysis with untagged photons in KLOE
- I skipped Bhabha luminosity and muon normalization...