Status and perspectives of tau physics

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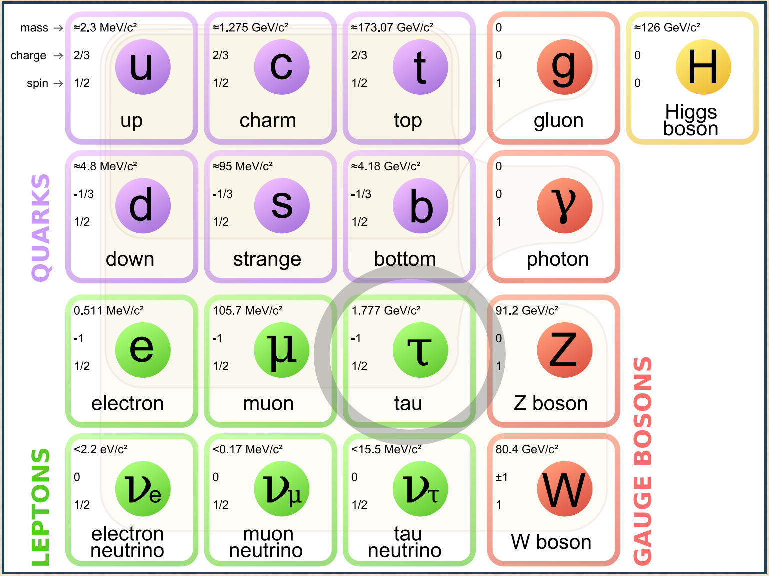
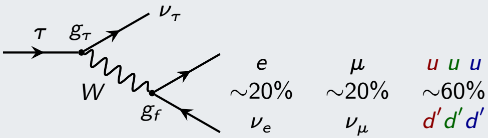
Tau Lepton in Standard Model

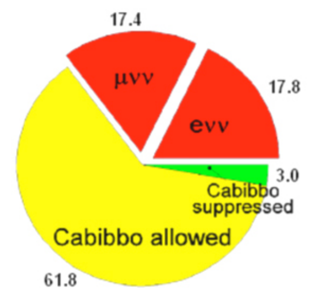


Tau decays in the Standard Model



$$|d'\rangle \approx V_{ud}\,|d\rangle + V_{us}\,|s\rangle \approx \cos\theta_C\,|d\rangle + \sin\theta_C\,|s\rangle$$

tau branching fractions



- $R_{\text{had}} = \mathcal{B}(\tau \to h)/\mathcal{B}(\tau \to e) = 3$ when neglecting phase space & QED/QCD corrections
- QCD corrections change R_{had} from $\sim$3 to $\sim$3.6
  $\Rightarrow \alpha_s(m_\tau)$ can be computed from measured R_{had}
- tau branching fraction to "strange", $\mathcal{B}(\tau \to s) \sim 3\%$ is Cabibbo-suppressed
  $\Rightarrow |V_{us}|/|V_{ud}|$ can be computed from $\mathcal{B}(\tau \to \text{strange})/\mathcal{B}(\tau \to \text{non-strange})$
  - must account for phase space corrections ($m_s > m_d$)
  - must include QCD & QED radiative corrections for precision measurements

Tau decays in the Standard Model

tau decay charged track multiplicity

	branching fraction (%)
1-prong	~ 85
3-prong	~ 15
5-prong	~ 0.1

Tau discovery

before tau discovery

- four leptons [$e, \mu(1936), \nu_e, \nu_\mu$] and three quarks [$u, d, s(1947)$]
- priority on finding 4th quark, with which:
  - $SU(2) \times U(1)$ gauge theories divergent triangle anomalies vanish
  - suppression of flavour changing neutral currrents with GIM mechanism

early tau searches

- in early 1960's searches started for next heavy lepton
- photo-production: $e + \text{nucleus} \to \gamma + X, \quad \gamma + \text{nucleus} \to \ell^+\ell^- + X'$
  - upper limits on m_ℓ in $[0.5, 1.0]\,\text{Gev}$ (SLAC, 1968)
- e^+e^- colliders, $e^+e^- \to \gamma^* \to \ell^+\ell^-$
  - exclude $m_\ell < 1.15\,\text{GeV}$ (ADONE, Frascati, 1974)

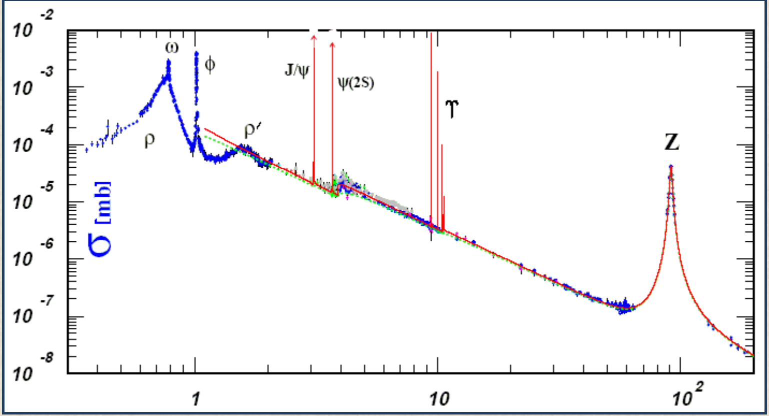
Tau discovery

MARK 1 at SLAC

- 1964 proposal of SPEAR e^+e^- collider, max CM energy 4.8 GeV, funded in 1970
- 1971 proposal of MARK 1 detector, 1971, Martin Perl *et al.*, proposal topics:
    - 1) Boson Form Factors, 2) Baryon Form Factors, 3) Inelastic Reactions
    - 4) Search for Heavy Leptons ($\tau_1 \to e\bar{\nu}\nu$, $\tau_2 \to \mu\bar{\nu}\nu$)
- theorists compute expected distributions (Tsai, Sakurai)
- 1974 beginning of data-taking
- 1975 evidence for “anomalous lepton production” (24 events)
- 1976 discovery “simplest hypothesis” . . . “production of a pair of heavy leptons”
- 1995 Martin Perl awarded with Nobel for tau discovery

Tau discovery

Total e^+e^- cross section

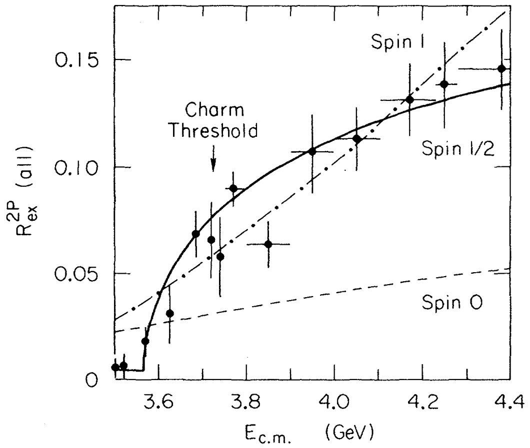


difficulties of tau discovery

- production threshold close to (then poorly known) open charm threshold
  - J/ψ discovered jus before tau, in 1974
- open charm $\Rightarrow$ increase in track multiplicity and strange mesons, peaks in $K\pi$, $K\pi\pi$
- $\tau^+\tau^-$ $\Rightarrow$ low multiplicity, Cabibbo suppression of strange mesons, leptonic decays

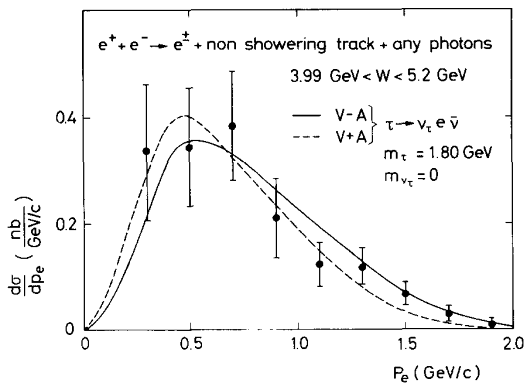
Tau spin, chirality

tau spin [DELCO / SPEAR / SLAC, 1977]



- tau production cross-section favors spin 1/2

tau chirality [DASP / DORIS / DESY, 1979]



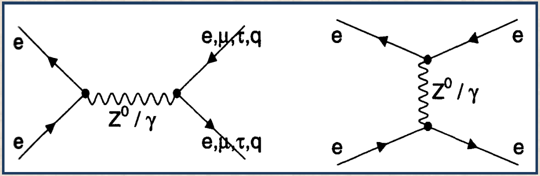
- data distribution favors $V-A$

Koide formula

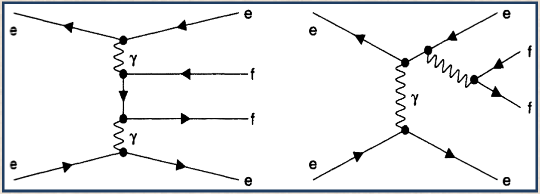
$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3},$$ using PDG 2024 averages, Koide formula residual $= -2 \pm 5\,\text{ppm}$

[Yoshio Koide, 1981]

Tau selection at e^+e^- colliders



- s and t channel e^+e^- scattering



- two-photon process, radiative Bhabha

Backgrounds

- $\mu^+\mu^-$: collinear, zero aplanarity, half beam energy, momentum $\sim E_{\text{CM}}/2$
- e^+e^-: collinear, zero aplanarity, half beam energy, momentum $\sim E_{\text{CM}}/2$, cross-section peaked at small angles between the incoming and outcoming e^-
- $q\bar{q}$: large average number of charged tracks, few leptons and neutrinos
- two-photon: zero aplanarity, zero net transverse momentum, large missing energy
- $\mu^+\mu^-\gamma$ and $e^+e^-\gamma$ with undetected γ have missing mass $m_{\text{miss}} = |P^\mu_{\text{CM}} - P^\mu_{\text{RECO}}| = 0$
  on the other hand, tau pairs have $m_{\text{miss}} > 0$ because of $\geq$2 missing neutrinos

Tau selection at e^+e^- colliders

typical tau selection at LEP

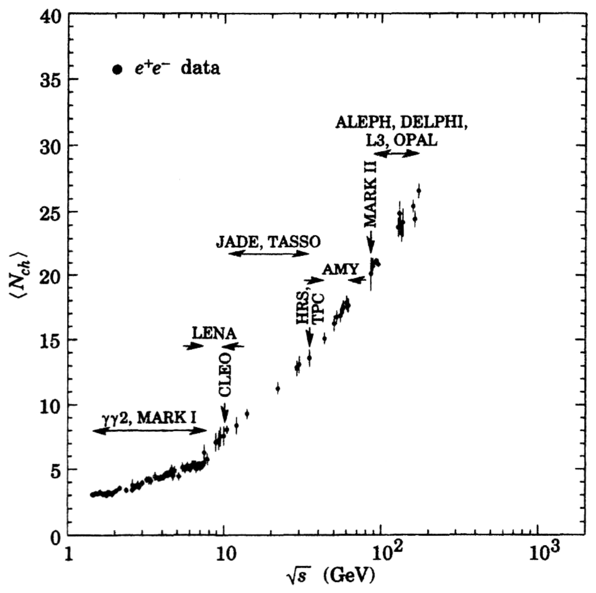
- events are clearly organized in two back-to-back cones
- high efficiency and purity tau selection possible even in single cone
- require few charged tracks (1-1, 1-3, 1-5, 3-3)
- track enercy and momentum lower than $E_{\mathrm{CM}}/2$ to suppress e^+e^-, $\mu^+\mu^-$
- total reconstructed energy $< E_{\mathrm{CM}}$ because of tau decay neutrinos
- missing mass $>$ threshold to suppress $e^+e^-\gamma$, $\mu^+\mu^-\gamma$ with undetected γ
- $\dfrac{p_T^{\mathrm{event}}}{E_{\mathrm{CM}} - p_1 - p_2} >$ threshold to suppress $\gamma\gamma$ events with $p_T^{\mathrm{event}} \sim 0$, large E_{missing}

typical tau selection at the *B*-factories

- hadronic background is difficult to suppress
- tau pair events are still organized in two separate hemispheres
- require tau pair topologies: 1-1, 1-3, 1-5
- require a single lepton and nothing else in one hemishpere
- require hadronic system with low invariant mass in one hemisphere

Tau selection at e^+e^- colliders

hadronic track multiplicity grows with E_{CM}



Brief history of tau physics

- 1976 discovery, mass, spin, chirality, SM 3rd generation lepton
- 1976-1989 low statistics, difficult measurements at SLAC & DESY esperiments
  - one-prong paradox ($B_1 > \sum B_{1,i}$ at 4-5 σ)
  - hints of lepton universality violation
- 1982-2000 ARGUS & CLEO symmetric B-factories measurements
- 1990-1995 high statistics precise measurements at LEP, precise tau mass by BES(1996)
  - one-prong paradox solved with larger statistics and better quality measurements
  - understood $\leq$1992 tau lifetime PDG averages spoiled by uncorrected biases
  - Standard Model lepton universality precisely confirmed
  - calculation of $\alpha_s(m_\tau)$ using tau hadronic decays
- 2000-now BaBar & Belle asymmetric B factories high-stat. measurements, BES III & KEDR tau mass
  - searches for lepton-flavour-violating tau decays
  - measurements of high-multiplicity and small branching fractions
  - improved tau lifetime by Belle
- 2019-now Belle II lowish statistics but refined and precise measurements
  - tau mass, $\mathcal{B}(\tau \to \mu\bar{\nu}\nu)/\mathcal{B}(\tau \to e\bar{\nu}\nu)$
- future
  - Belle II operation towards 50 ab^{-1}total integrated luminosity
  - projects STCF, FCC-cc(Z), CEPC

Future tau lepton measurements

tau pairs yields at past, present and future e^+e^- colliders

facility	Z	$\tau^+\tau^-$	$\tau^+\tau^-$ relative sample size	
	[million]	[million]		
LEP	25	0.84		
BABAR	-	$0.5\cdot10^{3}$		
Belle	-	$1.0\cdot10^{3}$		
Belle II	-	$45\cdot10^{3}$		
STCF at 4.26 GeV, 10 years	-	$35\cdot10^{3}$		
CEPC	$4\cdot10^{6}$	$135\cdot10^{3}$	$1.6\cdot10^{5}\times$LEP	$3.0\times$Belle II
FCC-ee	$6\cdot10^{6}$	$200\cdot10^{3}$	$2.4\cdot10^{5}\times$LEP	$4.5\times$Belle II

experimental conditions of tau pairs much better at Z peak compared to lower energies

- better momentum resolution and vertexing because less multiple scattering with higher track momenta
- better higher momentum muon id (much lower pion-to-muon misidentification)
- much better $\tau^+\tau^-$ separation from $q\bar{q}$ background because of higher $q\bar{q}$ multiplicity at Z peak
- LHC produces more tau leptons, but with much less favourable experimental conditions

Lepton flavour universality (LFU)

- Standard Model predicts universal couplings for leptons (except Higgs Yukawa couplings)
- most tested:
  charged-weak-current couplings universality
- precise SM predictions of coupling ratios
  ⇒ can search for small NP deviations
  ⇒ can obtain constraints on NP models
- lepton universality tests related to
  - B anomalies (R_D, R_{D^*}, R_K, R^*_K, $\dots$)
    - [Feruglio, Paradisi, Pattori JHEP 09 (2017) 061],
      [Allwicher, Isidori, Selimovic, 2021], [Allwicher, Isidori, Lizana, Selimovic, Stefanek, 2023]
  - Cabibbo angle anomaly
    - [Coutinho, Crivellin, Manzari, PRL 125 (2020) 071802],
      [Crivellin, Hoferichter, PRL 125 (2020) 111801]

Lepton universality tests with tau leptonic branching fractions

Standard Model predictions for leptons $\mathcal{L}, \ell = e, \mu, \tau$

[Marciano, 1988], [Pich, Precision Tau Physics, 2014]

$$\Gamma[\mathcal{L} \to \nu_{\mathcal{L}} \ell \bar{\nu}_\ell(\gamma)] = \Gamma_{\mathcal{L}\ell} = \Gamma_{\mathcal{L}} \mathcal{B}_{\mathcal{L}\ell} = \frac{\mathcal{B}_{\mathcal{L}\ell}}{\tau_{\mathcal{L}}} = \frac{G_{\mathcal{L}} G_\ell m_{\mathcal{L}}^5}{192\pi^3} f_{\mathcal{L}\ell} R_W^{\mathcal{L}\ell} R_\gamma^{\mathcal{L}}$$

$$G_{\mathcal{L}} = \frac{g_{\mathcal{L}}^2}{4\sqrt{2} M_W^2} \; ; \qquad f_{\mathcal{L}\ell} = f\left(m_\ell^2/m_{\mathcal{L}}^2\right) \; ; \qquad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$R_W^{\mathcal{L}\ell} = 1 + \frac{3}{5}\frac{m_{\mathcal{L}}^2}{M_W^2} + \frac{9}{5}\frac{m_\ell^2}{M_W^2} \; ; \qquad R_\gamma^{\mathcal{L}} = 1 + \frac{\alpha(m_{\mathcal{L}})}{2\pi}\left(\frac{25}{4} - \pi^2\right) .$$

[HFLAV 2023 report, in preparation]

$$\left(\frac{g_\tau}{g_\mu}\right) = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^{\mu e}}{\tau_\tau m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^{\tau e}}} = 1.0016 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau e}}{\mathcal{B}_{\tau e}^{\text{SM}}}}$$

$$\left(\frac{g_\tau}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\mu e}} \frac{\tau_\mu m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^{\mu e}}{\tau_\tau m_\tau^5 f_{\tau \mu} R_\gamma^\tau R_W^{\tau \mu}}} = 1.0018 \pm 0.0014 = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau \mu}^{\text{SM}}}}$$

$$\left(\frac{g_\mu}{g_e}\right) = \sqrt{\frac{\mathcal{B}_{\tau \mu}}{\mathcal{B}_{\tau e}} \frac{f_{\tau e}}{f_{\tau \mu}}} = 1.0002 \pm 0.0011$$

Lepton universality tests with tau compared with other measurements

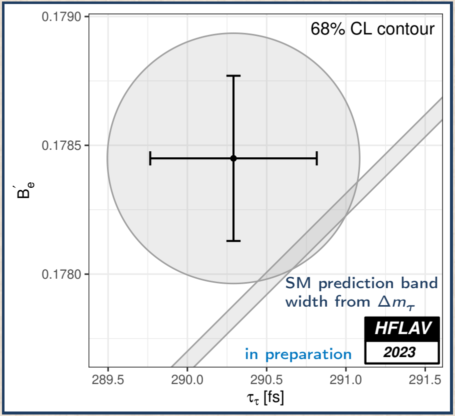
2013 [A.Pich, Precision Tau Physics (2014)]

$\lvert g_\mu/g_e \rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$ 1.0018(14)	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$ 1.0021(16)	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$ 0.9978(20)	$\Gamma_{K\to\pi\mu}/\Gamma_{K\tau\to\pi e}$ 1.0010(25)	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$ 0.996(10)
$\lvert g_\tau/g_\mu \rvert$	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$ 1.0011(15)	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$ 0.9962(27)	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$ 0.9858(70)		$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$ 1.034(13)
$\lvert g_\tau/g_e \rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$ 1.0030(15)				$\Gamma_{W\to\tau}/\Gamma_{W\to e}$ 1.031(13)

2024 [V.Cirigliano *et al.*, 2022] [HFLAV 2023 report, in preparation] [PDG 2024]

$\lvert g_\mu/g_e \rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\tau\to e}$ 1.002(11)	$\Gamma_{\pi\to\mu}/\Gamma_{\pi\to e}$ 1.0010(9)	$\Gamma_{K\to\mu}/\Gamma_{K\to e}$ 0.9978(18)	$\Gamma_{K\to\pi\mu}/\Gamma_{K\tau\to\pi e}$ 1.0009(18)	$\Gamma_{W\to\mu}/\Gamma_{W\to e}$ 1.001(3)
$\lvert g_\tau/g_\mu \rvert$	$\Gamma_{\tau\to e}/\Gamma_{\mu\to e}$ 1.0016(14)	$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$ 0.9958(38)	$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$ 0.9856(75)		$\Gamma_{W\to\tau}/\Gamma_{W\to\mu}$ 1.007(10)
$\lvert g_\tau/g_e \rvert$	$\Gamma_{\tau\to\mu}/\Gamma_{\mu\to e}$ 1.0018(14)				$\Gamma_{W\to\tau}/\Gamma_{W\to e}$ 1.008(10)

Canonical tau lepton universality test plot



[HFLAV 2023 report, in preparation]

$(g_\tau/g_{e\mu}) = 1.0017 \pm 0.0013$

$[g_{e\mu} = g_e = g_\mu$ assuming $g_e = g_\mu]$

$\Delta(g_\tau/g_{e\mu})$ contributions

input	Δinput	$\Delta(g_\tau/g_{e\mu})$
$\mathcal{B}'_{\tau\to e}$	0.180%	0.090%
τ_τ	0.181%	0.090%
m_τ	0.005%	0.012%
total		0.128%

best measurements

$\mathcal{B}'_{\tau\to e}$	ALEPH
τ_τ	Belle
m_τ	Belle II

- $\mathcal{B}'(\tau \to e\bar{\nu}\nu) =$ average of $\begin{cases} \mathcal{B}(\tau \to e\bar{\nu}\nu) \\ \mathcal{B}(\tau \to \mu\bar{\nu}\nu) \cdot \frac{f_{\tau e} R_W^{\tau e}}{f_{\tau\mu} R_W^{\tau\mu}} \end{cases}$
- $\dfrac{\mathcal{B}'(\tau \to e\bar{\nu}\nu)\tau_\mu}{\mathcal{B}(\mu \to e\bar{\nu}\nu)\tau_\tau} = \dfrac{g_\tau^2}{g_{e\mu}^2} \dfrac{m_\tau^5 f_{\tau e} R_\gamma^\tau R_W^{\tau e}}{m_\mu^5 f_{\mu e} R_\gamma^\mu R_W^{\mu e}}$
- $\left(\dfrac{g_\tau}{g_{e\mu}}\right)^2 = \dfrac{\mathcal{B}'(\tau \to e\bar{\nu}\nu)}{\mathcal{B}(\mu \to e\bar{\nu}\nu)} \dfrac{\tau_\mu}{\tau_\tau} \dfrac{m_\mu^5}{m_\tau^5} \dfrac{f_{\mu e} R_\gamma^\mu R_W^{\mu e}}{f_{\tau e} R_\gamma^\tau R_W^{\tau e}}$

Tau-muon universality using tau hadronic branching fractions

[HFLAV 2023 report, in preparation]

$$\left(\frac{g_\tau}{g_\mu}\right)^2_h = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\bar{\nu}_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta R_{\tau/h}) m_\tau^3 \tau_\tau} \left(\frac{1-m_\mu^2/m_h^2}{1-m_h^2/m_\tau^2}\right)^2 ,$$

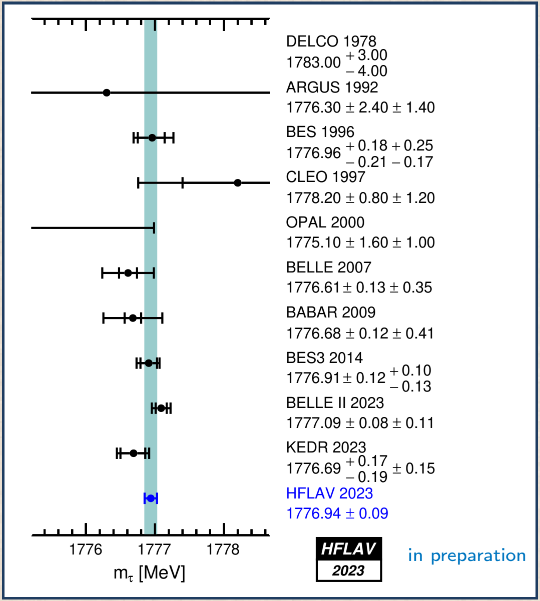
- $h = \pi, K$
- $\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$ [PhysRevD.104.L091502 (2021)]
- $\delta R_{\tau/K} = (0.97 \pm 0.58)\%$ [PhysRevD.104.L091502 (2021)]
- $\mathcal{B}(\tau \to \pi\nu_\tau)$ $\mathcal{B}(\tau \to K\nu_\tau)$ [HFLAV 2023 report, in preparation]
- $\mathcal{B}(\pi \to \mu\bar{\nu}_\mu)$, $\mathcal{B}(K \to \mu\bar{\nu}_\mu)$ [PDG 2023]

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.996 \pm 0.004 , \qquad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.986 \pm 0.008 .$$

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0011 \pm 0.0014 , \quad \text{average of} \quad \left(\frac{g_\tau}{g_\mu}\right)_\tau, \left(\frac{g_\tau}{g_\mu}\right)_\pi, \left(\frac{g_\tau}{g_\mu}\right)_K$$

- assuming uncorrelated $\delta R_{\tau/\pi}$ and $\delta R_{\tau/K}$

Tau mass measurements



threshold scan technique

- measure onset of $\sigma(e^+e^- \to \tau^+\tau^-)(\sqrt{s})$
- DELCO, BES (92, 96), KEDR and BES III
- require precise E_{beam} using
    - either resonant depolarization of polarized $e^\pm$
    - or laser-beam Compton scattering

pseudo-mass technique

- select $e^+e^- \to \tau^+\tau^-$ events
- fit pseudo-mass using visible tau-decay products
    - empirical function, potential systematic
- ARGUS, OPAL, Belle, *BABAR*, Belle II (2023)
- require
    - good momentum scale calibration

Threshold scan technique: tau pair cross-section vs. energy

$\sigma_{e^+e^-\to\tau^+\tau^-}(E)$

at lowest order

$$\sigma_{\tau\tau} = \frac{4\pi\alpha^2}{3s}\beta\left(\frac{3-\beta^2}{2}\right), \quad \beta = \frac{p_\tau}{E_\tau}, \quad s = E^2$$

including radiative corrections, beam energy spread

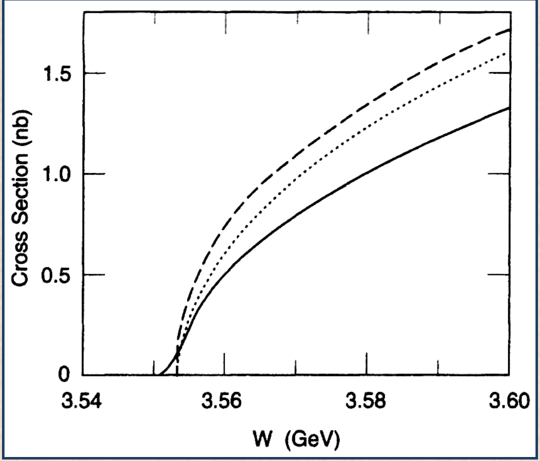
$$\sigma(E, m_\tau, \sigma_E) = \frac{1}{\sqrt{2\pi}\sigma_E}\int_{2m_\tau}^{\infty} dE' e^{\frac{-(E-E')^2}{2(\sigma_E)^2}} \int_0^{1-\frac{4m^2}{E'^2}} dx F_I(x, E')\tilde{\sigma}(E'\sqrt{1-x}, m_\tau),$$

$$\tilde{\sigma}(E) = \frac{4\pi\alpha^2}{3E^2}\beta\left(\frac{3-\beta^2}{2}\right)\frac{F_C(\beta)F_\tau(\beta)}{[1-\prod(E)]^2}$$

- $F_\tau(\beta)$ final state radiation from τ
- $F_C(\beta)$ Coulomb $\tau^+\tau^-$ interation in final state
- $\prod(E)$ QED corrections to photon propagator
- $F_I(x, E')$ initial state radiation probability
- σ_E beam energy spread

Threshold scan technique: tau pair cross-section vs. energy

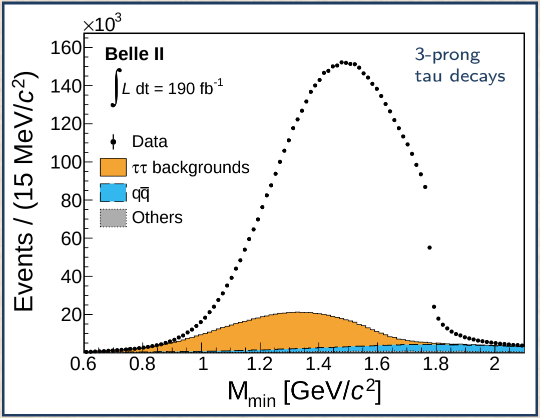
$\sigma_{e^+e^- \to \tau^+\tau^-}(E)$



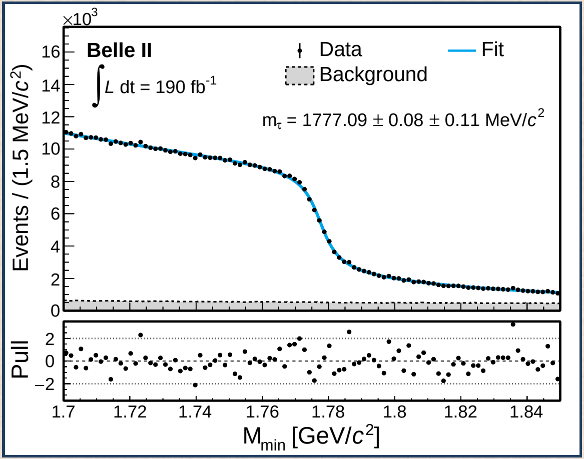
- dotted curve: lowest order prediction
- dashed curve: Coulomb correction and final state radiation
- solid curve: all effects

Tau mass measurement with pseudo-mass method

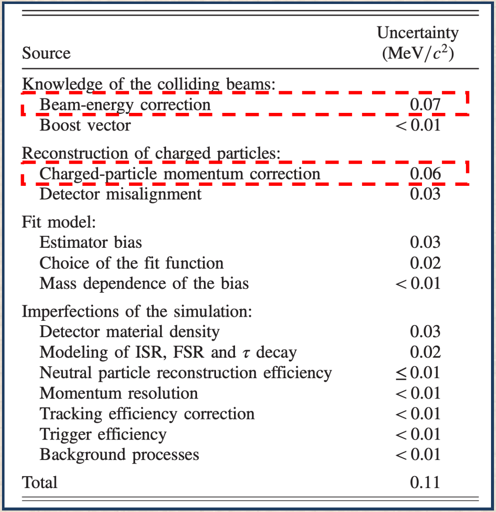
- $m_\tau = \sqrt{(P_h^\mu + P_\nu^\mu)^2} = \sqrt{P_h^{\mu 2} + P_\nu^{\mu 2} + 2(E_h E_\nu - \vec{p}_h \vec{p}_\nu)} \quad E_\nu = \frac{E_{\text{CM}}}{2} - E_h, \quad P_\nu^{\mu 2} = m_\nu = 0, \quad P_h^{\mu 2} = m_h,$
- $m_\tau = \sqrt{m_h^2 + 2[E_h(E_{\text{CM}}/2 - E_h) - p_h(E_{\text{CM}}/2 - E_h)\cos\theta_{h\nu}]} = \sqrt{m_h^2 + 2(E_{\text{CM}}/2 - E_h^*)(E_h^* - p_h^* \cos\theta_{h\nu}^*)}$
- mass function of unknown $\theta_{h\nu}$ – get minimum possible mass by setting $\theta_{h\nu} = 0$
- $m_\tau^{\text{pseudo}} = \sqrt{m_h^2 + 2(E_{\text{CM}}/2 - E_h^*)(E_h^* - p_h^*)}$



Tau mass Belle II 2023 measurement



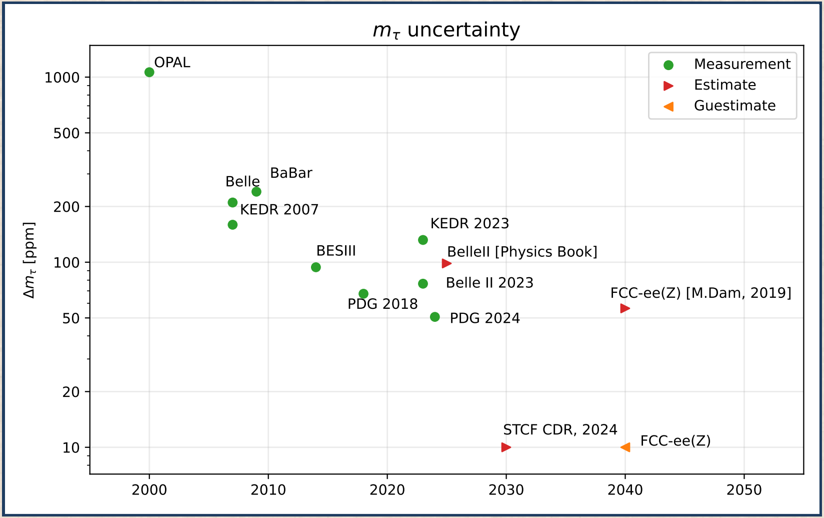
- empirical pseudo-mass distribution



Source	Uncertainty (MeV/c^2)
Knowledge of the colliding beams:	
Beam-energy correction	0.07
Boost vector	< 0.01
Reconstruction of charged particles:	
Charged-particle momentum correction	0.06
Detector misalignment	0.03
Fit model:	
Estimator bias	0.03
Choice of the fit function	0.02
Mass dependence of the bias	< 0.01
Imperfections of the simulation:	
Detector material density	0.03
Modeling of ISR, FSR and τ decay	0.02
Neutral particle reconstruction efficiency	≤ 0.01
Momentum resolution	< 0.01
Tracking efficiency correction	< 0.01
Trigger efficiency	< 0.01
Background processes	< 0.01
Total	0.11

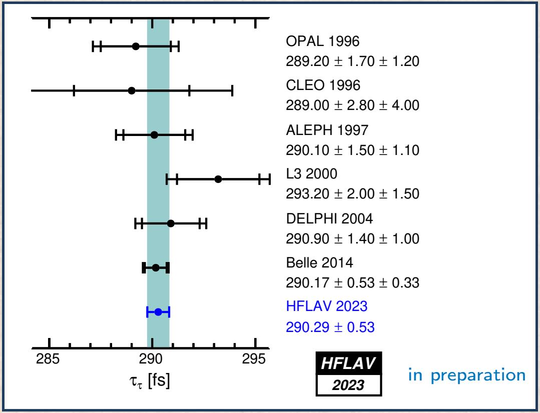
[Belle II, PRD 108 (2023) 032006]

Tau mass measurement prospects



FCC-ee(Z) with $6{\cdot}10^{12}$ Z

Tau lifetime measurements



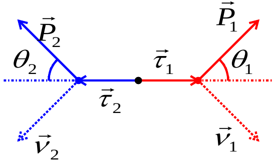
Belle 2014 tau lifetime measurement

analysis

- select 3-prong vs. 3-prong events
- kinematically reconstruct the $\tau^+\tau^-$ direction in the center-of-mass frame
- reconstruct signed decay length in lab frame
- fit decay length distribution, accounting for resolution
- compute tau lifetime using tau average velocity (momentum)
  - initial and final state radiation reduce average tau momentum

Belle 2014 tau lifetime: method

- In the CM frame for the reaction $e^+e^- \to \tau^+\tau^- \to 3\pi\nu\, 3\pi\nu$ flight directions of τ⁺ and τ⁻ are back-to-back.
- Energy of each τ-lepton is $\sqrt{s}/2$.
- Each τ-lepton is decayed into $\tau \to 3\pi\nu$; mass of τ-lepton is taken from PDG; neutrino mass assumed to be zero.



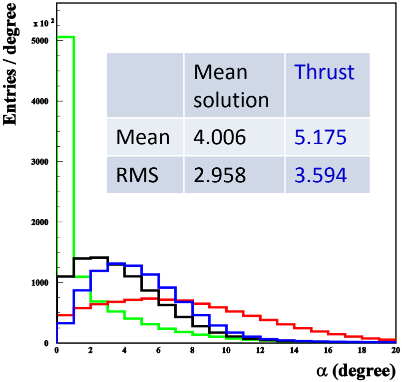
- $$\cos\theta = \frac{2E_\tau E_x - m_\tau^2 - m_x^2}{2P_\tau P_x} = \frac{2E_\tau E_x - m_\tau^2 - m_x^2}{2\sqrt{(E_\tau^2 - m_\tau^2)}P_x}$$
- $$\begin{cases} (\vec{P}_1 \cdot \vec{n}_+) = xP_{x1} + yP_{y1} + zP_{z1} = |P_1|\cos\theta_1 \\ (\vec{P}_2 \cdot \vec{n}_+) = xP_{x2} + yP_{y2} + zP_{z2} = -|P_2|\cos\theta_2 \\ (\vec{n}_+)^2 = x^2 + y^2 + z^2 = 1 \end{cases}$$

$\vec{n}_+$ is the unit vector in the direction of the positive τ-lepton

- Two solutions of quadratic equation are possible τ-lepton flight directions.

[M.Shapkin, Tau2014]

Belle 2014 tau lifetime: tau lepton direction resolution



	Mean solution	Thrust
Mean	4.006	5.175
RMS	2.958	3.594

Angle between reconstructed and true τ-direction for ττ Monte Carlo events.

Monte Carlo samples:

— Mean solution

$$\vec{n} = \frac{\vec{n}_1 + \vec{n}_2}{2}$$

— n_1 - True solution

— n_2 - Wrong solution

— Thrust direction as τ-direction

MC simulation predicts no bias from this choice

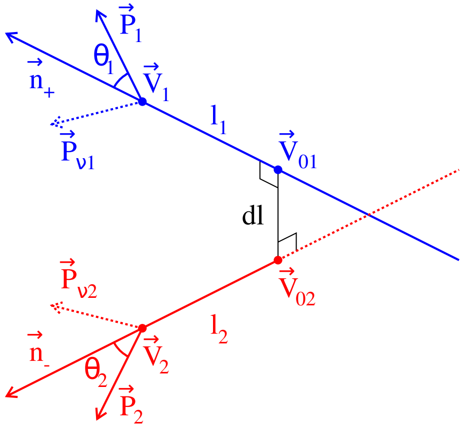
[M.Shapkin, Tau2014]

Belle 2014 tau lifetime: lifetime reconstruction

tau lifetime reconstruction

- compute $\tau^{\pm}$ flight direction in CM frame
- compute $\tau^{\pm}$ 4-momenta using m_{τ}, E_{τ}^{CM}
- boost tau 4-momenta to lab frame
  (boost determined by asymmetric beam energies)
- each 4-momentum anchored to its 3-prong vertex to compute tau paths (straight line approx.)
- origin = 3D closest approach of tau paths
- get 2 decay lengths from origin to 3-prong vertices
- $c\tau = \dfrac{l_{\tau}^{\text{lab}}}{(\beta\gamma)_{\tau}^{\text{lab}}}$

lab-frame tau momenta

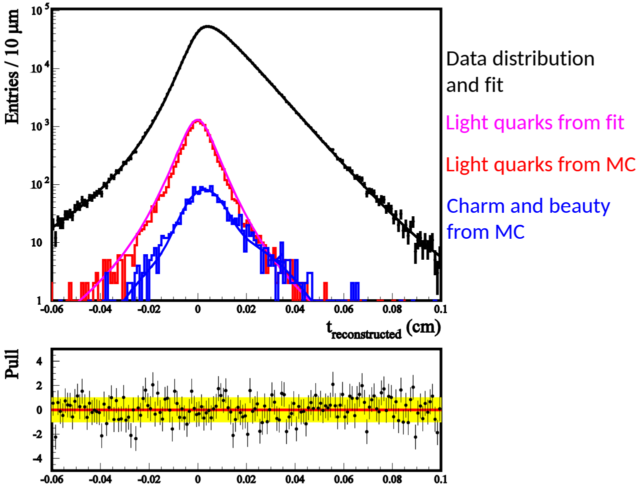


Belle 2014 tau lifetime: lifetime fit

tau lifetime fit model

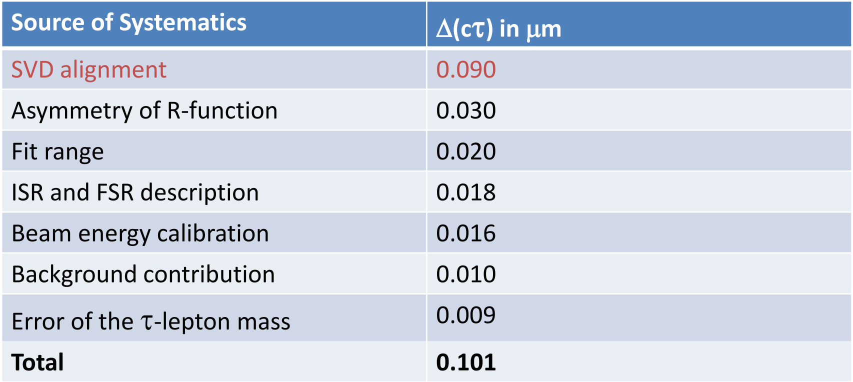
- reconstructed lifetime components:
  - tau: resolution-convoluted $e^{-t/\tau}$
  - *uds*: resolution-convoluted $\delta(t)$
  - *bc*: estimated with simulation
- resolution from simulation with parameters fitted to data
- bkg normalizations from simulation

lifetime fit



[M.Shapkin, Tau2014]

Belle 2014 tau lifetime: systematics

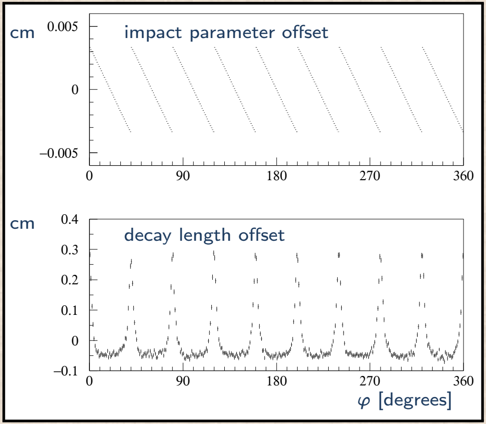


Source of Systematics	$\Delta(c\tau)$ in µm
SVD alignment	0.090
Asymmetry of R-function	0.030
Fit range	0.020
ISR and FSR description	0.018
Beam energy calibration	0.016
Background contribution	0.010
Error of the τ-lepton mass	0.009
Total	**0.101**

[M.Shapkin, Tau2014]

Tau lifetime systematics from detector misalignment

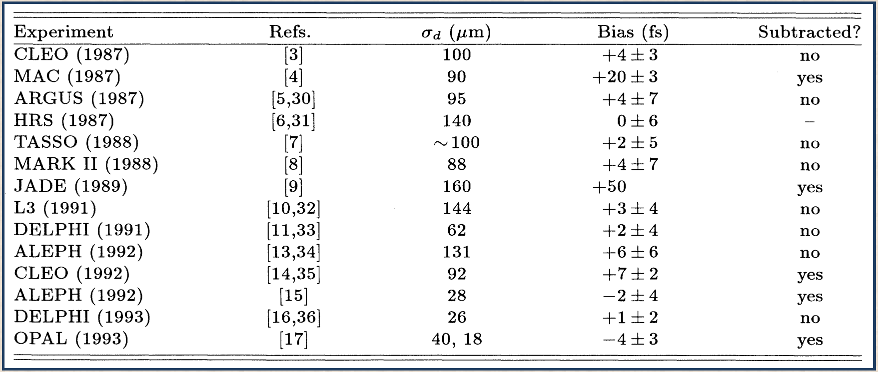
- crude simulation of ALEPH vertex detector with all wafers radially shifted out by 100 μm
- impact parameter offset $= 100\,\mu\text{m} \cdot \sin\alpha$
  - $\alpha =$ track angle w.r.t. normal of wafer
- decay length offset (measured using 3-tracks vertex)
  - negative when 3 tracks hit same wafer
  - positive when 3 tracks hit two wafers



- decay length offset ∼ derivative of impact parameter offset
  ⇒ decay length offset due to misalignment averages to zero over azymuth 2π
  - requires uniform acceptance (can be insured by event weighting)
  - approximately works over polar angle due to incomplete detector acceptance
- well confirmed with Monte Carlo full simulations
- [S.Wasserbaech, Nucl.Phys.Proc.Suppl. 76 (1999) 107]

Tau lifetime detector-related systematic biases

- reconstructed decay length bias $\Delta\lambda \sim \dfrac{\sigma^2_{d0,\phi0}}{\sqrt{\pi}\sigma_{\phi0}}$ for typical barrel-shaped vertex detectors
- often not subtracted in tau lifetime measurements before 1993, although estimated with simulations
- "...the average [tau lifetime] would be reduced by about 2 fs if all bias corrections were applied..."
- [S.Wasserbaech, PRD 48, 4216 (1993)]



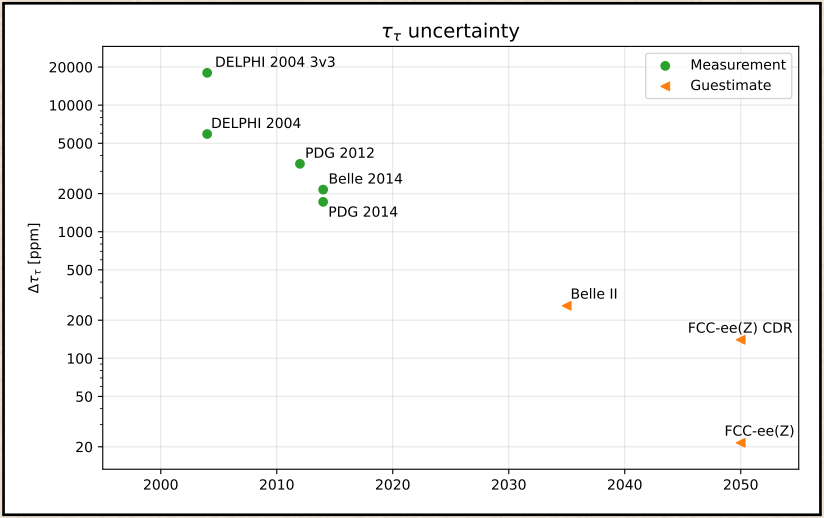
Experiment	Refs.	σ_d (μm)	Bias (fs)	Subtracted?
CLEO (1987)	[3]	100	$+4 \pm 3$	no
MAC (1987)	[4]	90	$+20 \pm 3$	yes
ARGUS (1987)	[5,30]	95	$+4 \pm 7$	no
HRS (1987)	[6,31]	140	0 ± 6	–
TASSO (1988)	[7]	~100	$+2 \pm 5$	no
MARK II (1988)	[8]	88	$+4 \pm 7$	no
JADE (1989)	[9]	160	$+50$	yes
L3 (1991)	[10,32]	144	$+3 \pm 4$	no
DELPHI (1991)	[11,33]	62	$+2 \pm 4$	no
ALEPH (1992)	[13,34]	131	$+6 \pm 6$	no
CLEO (1992)	[14,35]	92	$+7 \pm 2$	yes
ALEPH (1992)	[15]	28	-2 ± 4	yes
DELPHI (1993)	[16,36]	26	$+1 \pm 2$	no
OPAL (1993)	[17]	40, 18	-4 ± 3	yes

Tau Lifetime measurement prospects at FCC-ee(Z)

- extrapolate from DELPHI 2004 tau lifetime measurement using 3-3 prong topology tau pairs
- assume future tau mass uncertainty 10 ppm
- assume future 30× better simulation of radiation in $e^+e^- \to \tau^+\tau^-$

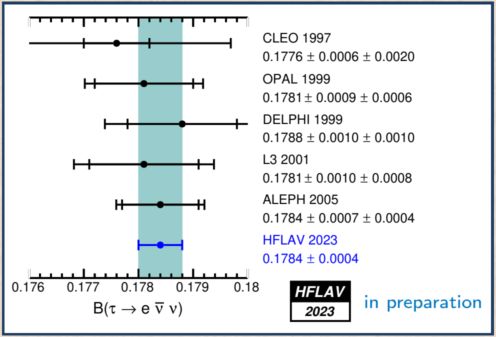
	DELPHI 2004 [fs]	DELPHI 2004 [ppm]	FCC-ee(Z) $6{\cdot}10^{12}$ Z [ppm]
statistical uncertainty	5.2	18000	15.0
luminosity-dependent systematics	1.3	4500	3.9
- background	0.2		
- reconstruction bias	0.8		
- vertex detector alignment	1.0		
luminosity-independent systematics			
- detector length scale	-	100	5.0
- average tau energy	-	-	1.0
- radiative energy loss	0.1	350	11.5
- tau mass	-	68	10.0
total systematics			15.9
total uncertainty			22.3

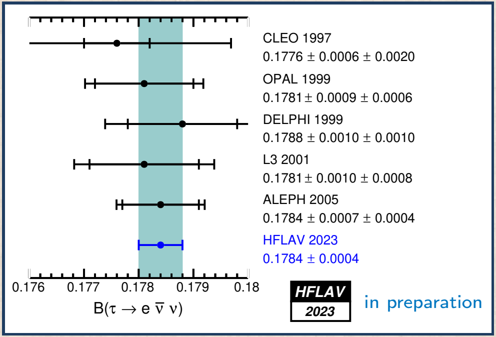
Tau Lifetime measurement prospects

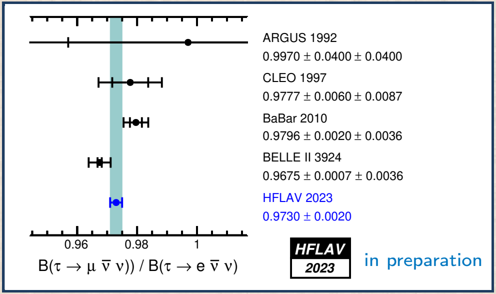


FCC-ee(Z) with $6{\cdot}10^{12}$ Z

Tau branching fractions measurements







Tau branching fractions measurements features

- branching fraction measurement: $\mathcal{B}(\tau \to X) = \dfrac{N(\tau \to X)}{N(\tau)}$

Z peak

- $\dfrac{N_{\text{obs}}(\tau \to X)/\epsilon(\tau \to X) - N_{\text{bkg}}(\tau \to X)}{N_{\text{obs}}(\tau)/\epsilon(\tau)}$
  - $N(\tau)$ from counting, not from luminosity
- $\epsilon(\tau \to X) \sim 100\%$
  small systematics from related MC modeling
- small non-tau N_{bkg}
  - easier to suppress hadrons, e^+e^-, $\mu^+\mu^-$
- $\Delta\mathcal{L} \sim 0.1\%$

$\Upsilon(4s)$ peak

- $\dfrac{N_{\text{obs}}(\tau \to X)/\epsilon(\tau \to X) - N_{\text{bkg}}(\tau \to X)}{2\sigma(e^+e^- \to \tau^+\tau^-)\mathcal{L}}$
  - $N(\tau)$ from from integrated luminosity
- $\epsilon(\tau \to X) \ll 100\%$
  large systematics from related MC modeling
- large non-tau N_{bkg}
  - more contamination of hadrons, e^+e^-, $\mu^+\mu^-$
- $\Delta\mathcal{L} \sim 1\%$

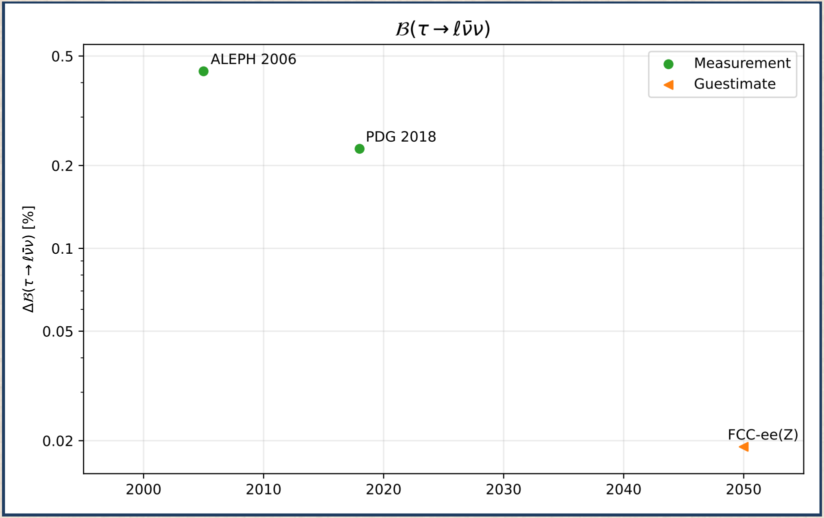
ALEPH at Z peak

- single analysis for all topological $\mathcal{B}$s
  (by n. of tracks and n. of π^0)
- non-tau bkg from cross-feed of other channels
  all channels and almost all bkg fit on data

Tau leptonic branching fractions measurement prospects at FCC-ee(Z)

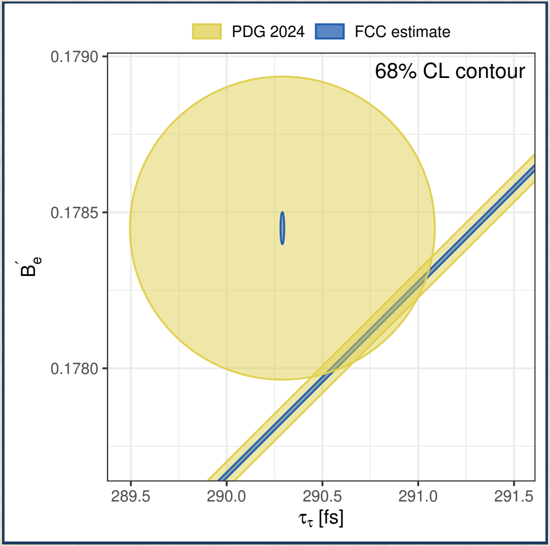
- ALEPH 2006 measurement precision: $4400\,\text{ppm} = [4000(\text{stat.}) \oplus 1900(\text{syst.})]\,\text{ppm}$ (average of the two similar electron and muon decays branching fractions)
  - complex simultaneous measurement of 12 tau branching fractions
  - many systematic uncertainties, no reliable extrapolations to FCC-ee statistics
  - several systematics related to photon and $\pi^0 \to \gamma\gamma$ reconstruction
- $\Delta^{\text{FCC}}_{\text{stat}}\mathcal{B}(\tau \to \ell\bar{\nu}\nu) = 4000\,\text{ppm} \cdot \sqrt{\frac{6.2{\cdot}10^6\ (\text{ALEPH Z bosons})}{6{\cdot}10^{12}}} = 4.1\,\text{ppm}\ (\text{FCC Z bosons})$
- $\Delta^{\text{FCC}}_{\text{syst}}\mathcal{B}(\tau \to \ell\bar{\nu}\nu) = \frac{1}{10} \cdot 1900\,\text{ppm} = 190\,\text{ppm}$ [assume also 100% correlated between $\mathcal{B}(\tau \to e\bar{\nu}\nu)$ and $\mathcal{B}(\tau \to \mu\bar{\nu}\nu)$]
- $\Delta^{\text{FCC}}\mathcal{B}(\tau \to \ell\bar{\nu}\nu) \simeq 190\,\text{ppm}$

Tau leptonic Branching fractions prospects at FCC-ee and other facilities



- FCC-ee(Z) with $6{\cdot}10^{12}$ Z
- Belle II is working on these measurements

Canonical tau lepton universality plot extrapolation at FCC-ee

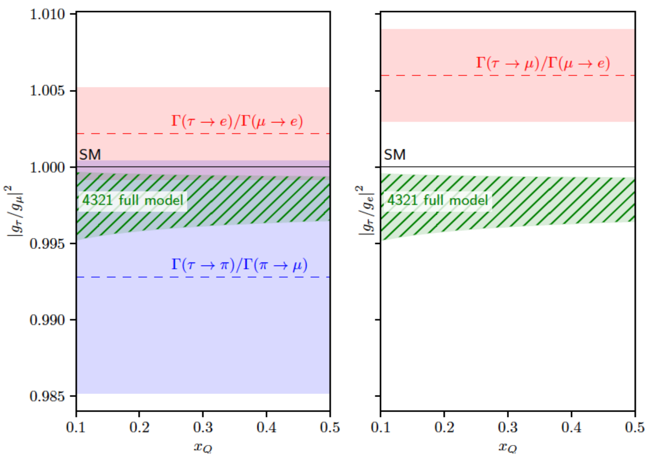


FCC-ee(Z) with $6{\cdot}10^{12}$ Z

Precise tau LFU measurements can confirm or exclude B anomalies models

- constraints on 4321 vector lepto-quark model for B anomalies
  [Allwicher, Isidori, Selimovic, 2021], [Allwicher, Isidori, Lizana, Selimovic, Stefanek, 2023]

today g_τ/g_μ, g_e/g_μ bounds



[G.Isidori, Jul 2023]

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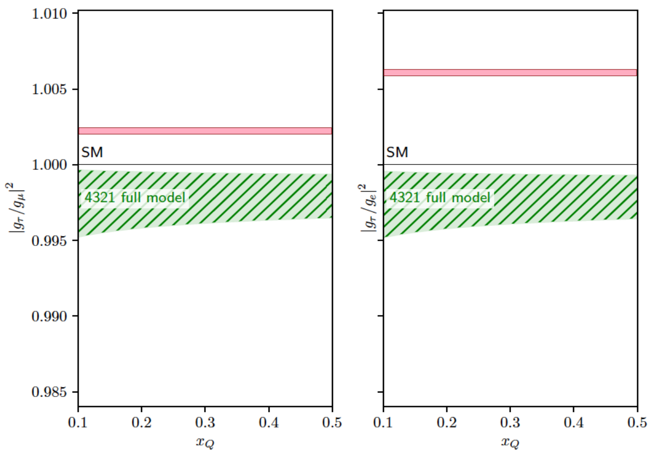
future FCC-ee g_τ/g_μ, g_e/g_μ may exclude model



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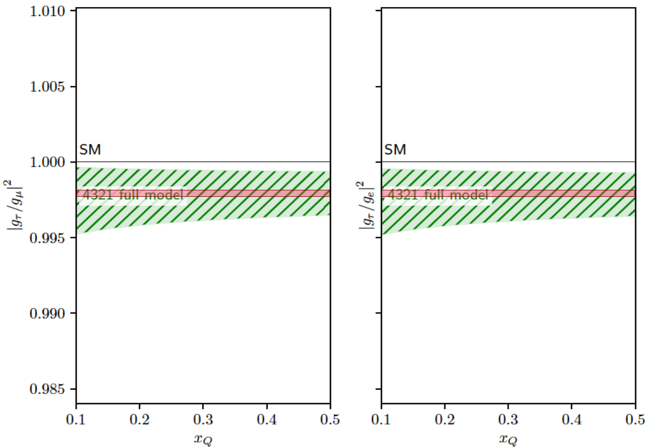


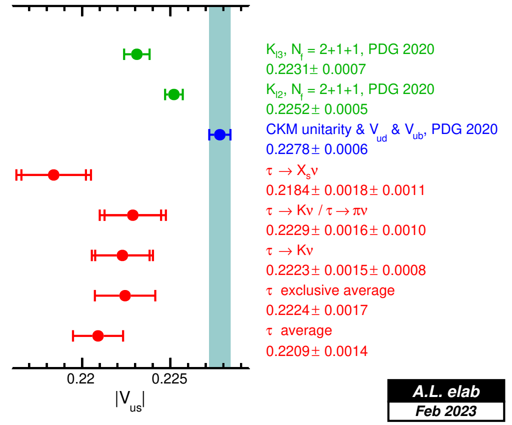
[G.Isidori, Jul 2023]

$|V_{us}|$ from tau measurements

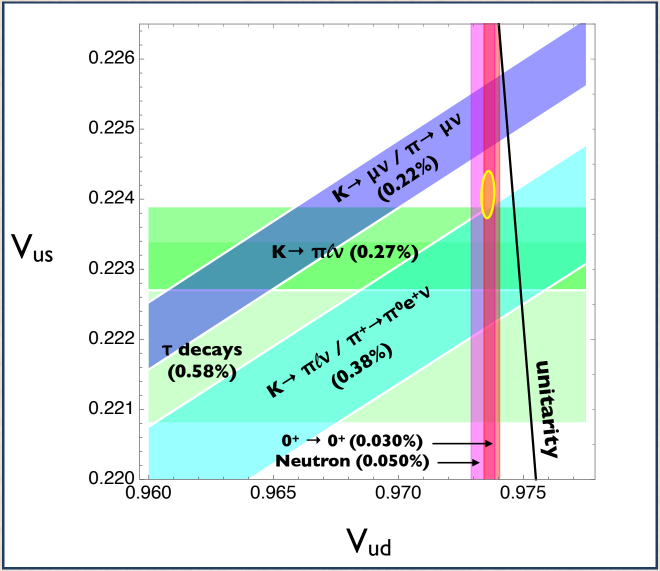
Cabibbo angle anomaly

- 2018: CMK 1st row unitarity OK
- 2019: $\Delta_{\text{CKM}} > 3\sigma$ unitarity violation
  - dispersive calculation of Δ_R^V inner or universal electroweak radiative corrections (RC) to super-allowed nuclear beta decays
    Seng, Gorchtein & Ramsey-Musolf, Phys. Rev. D 100, 013001 (2019)
    - $\sim 2\times$ more precise
    - significant shift
- 2020: revision & inflation of $|V_{ud}|$ th. syst. unc.
- 2020-2023: PDG reviews quote $\Delta_{\text{CKM}} \sim 3\sigma$
- 2023: short review [Cirigliano *et al.*, 2023]
- 2024: PDG review quotes $\Delta_{\text{CKM}} \sim 2\sigma$

$|V_{us}|$ after [Seng *et al.* 2019]



Cabibbo angle anomaly



[Brynman *et al.*, Ann.Rev.Nucl.Part.Sci. 72 (2022) 69-91]

$|V_{us}|$ determinations from kaons

- $\Gamma(K \to \pi\ell\bar{\nu}_\ell[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2 S_{\text{EW}}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1 + \delta_{\text{EM}}^{K\ell} + \delta_{\text{SU(2)}}^{K\pi}\right)^2$ $\boxed{K_{\ell 3}}$
  - C_K Clebsch-Gordan coefficients, $= 1$ for K^0, $= 1/2$ for K^-
  - $I_K^\ell =$ phase-space form factor integral
- $\frac{\Gamma(K^- \to \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{m_K (1 - m_\ell^2/m_K^2)^2}{m_\pi (1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{\text{EM}})$ $\boxed{K_{\ell 2}}$

$|V_{us}|$ calculation with “inclusive” tau branching fractions

$|V_{us}|_{\tau\text{-OPE}}$ from $\mathcal{B}(\tau \to X_s \nu)$ and OPE

- $|V_{us}|_{\tau\text{-OPE}} = \sqrt{R_s / \left[\frac{R_{ud}}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}} \right]}$

$\boxed{\tau\text{-OPE}}$

  - $R_s = \mathcal{B}(\tau \to X_s \nu)/\mathcal{B}(\tau \to e\bar{\nu}\nu)$, $\quad R_{ud} = \mathcal{B}(\tau \to X_{ud} \nu)/\mathcal{B}(\tau \to e\bar{\nu}\nu)$
  - $\delta R_{\tau,\text{SU3 breaking}}$ computed with Operator Product Expansion (OPE) perturbative QCD, $+$ m_s W.A.
  - E. Gamiz *et al.*, JHEP 01 (2003) 060, PRL 94 (2005) 011803, Nucl. Phys .Proc. Suppl. 169 (2007) 85, PoS KAON (2008) 008
  - no strong isospin breaking correction

$|V_{us}|_{\tau\text{-latt}}$ from $\mathcal{B}(\tau \to X_s \nu)$ and lattice QCD

- $|V_{us}|_{\tau\text{-latt}} = \sqrt{\left(\frac{|V_{us}|^2}{R_s}\right)_{\text{latt-incl}} \cdot R_s}$

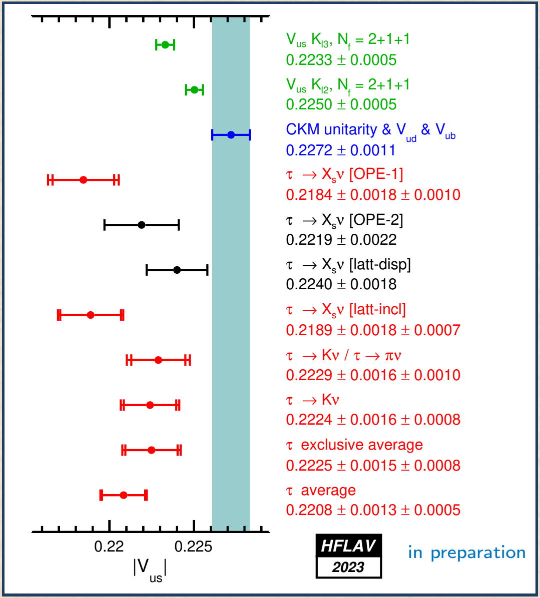
$\boxed{\tau\text{-latt}}$

  - $(|V_{us}|^2 / R_s)$ computed with lattice QCD
  - Extended Twisted Mass collaboration, Phys. Rev. D 104 7, (2021) 074520
  - no strong isospin breaking correction

$|V_{us}|$ calculation with "exclusive" tau branching fractions

- $\Gamma(\tau \to \bar{K}\nu_\tau[\gamma]) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 \, S_{\text{EW}}^\tau \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\tau \left(1 + \delta_{\text{EM}}^{K\tau} + \delta_{\text{SU(2)}}^{K\pi}\right)^2$ $\boxed{\tau \to K\pi}$
  - C_K Clebsch-Gordan coefficients, $= 1$ for K^0, $= 1/2$ for K^-
  - $I_K^\ell =$ phase-space form factor integral
  - Antonelli *et al.*, JHEP 1310 (2013) 070 (2013)
  - (not updated for HFLAV 2023 report in preparation)
- $\frac{\mathcal{B}(\tau^- \to \mathcal{B}(\tau^- \to K^-\nu_\tau))}{\mathcal{B}(\tau^- \to \mathcal{B}(\tau^- \to \pi^-\nu_\tau))} = \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{|V_{us}|^2_{\tau K/\pi}}{|V_{ud}|^2} \frac{\left(m_\tau^2 - m_K^2\right)^2}{\left(m_\tau^2 - m_\pi^2\right)^2} (1 + \delta R_{\tau K/\tau\pi})$ $\boxed{\tau \to K/\tau \to \pi}$
- $\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{1}{16\pi} \left(\frac{G_F}{\hbar^3 c^3}\right)^2 |V_{us}|^2_{\tau K} f_{K\pm}^2 \frac{\tau_\tau}{\hbar} m_\tau^3 c^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}^{m_\tau} (1 + \delta R_{\tau K})$ $\boxed{\tau \to K}$

$|V_{us}|$ from tau measurements [HFLAV 2023 report, in preparation]



Tau Lepton Flavour Violation Searches

- Standard Model accidental symmetry conserves L_e, L_μ, L_τ
- νSM (including neutrino mixing) predicts extremely suppressed charged Lepton Flavour Violation, $\propto m^2_{\nu_i}/m^2_W$, e.g., $\mathcal{B}(\mu \to e\gamma) \sim 10^{-54}$
  $\Rightarrow$ can do clean searches for New Physics
- LFV processes
  - muon LFV decays
  - tau LFV decays
  - hadrons, W/Z, H decays to leptons

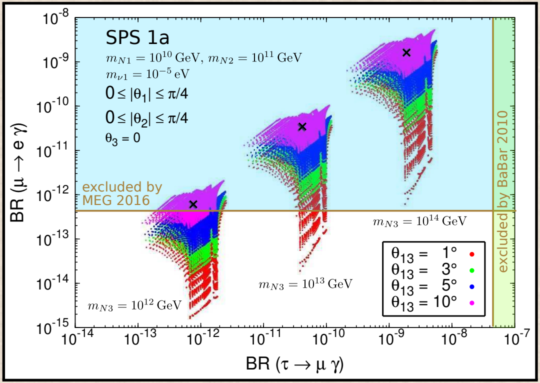
Experimental searches for LFV processes

Process	**Current bound on BR**	**Future Sensitivity**
$\mu \to e\gamma$	$< 4.2 \times 10^{-13}$ MEG	10^{-14} MEGII
$\mu \to \bar{e}ee$	$< 1.0 \times 10^{-12}$ SINDRUM	10^{-16} Mu3e
$\mu A \to eA$	$< 7 \times 10^{-13}$ SINDRUMII	$10^{-16} \to 10^{-18}$ COMET, Mu2e
$\tau \to l\gamma$	$< 3.3 \times 10^{-8}$	$3 \times 10^{-9}(e), 10^{-9}(\mu)$
$\tau \to e\bar{e}e$	$< 2.7 \times 10^{-8}$	5×10^{-9}
$\tau \to \mu\bar{\mu}\mu$	$< 2.1 \times 10^{-8}$	4×10^{-9}
$\tau \to \mu\bar{e}e, e\bar{\mu}\mu$	$< 1.8, 2.7 \times 10^{-8}$ Belle	$3, 5 \times 10^{-9}$ BelleII
...	...	...
$\tau \to l\pi^0$	$< 8.0 \times 10^{-8}$	4×10^{-9}
$\tau \to l\eta$	$< 6.5 \times 10^{-8}$	7×10^{-9}
$\tau \to l\rho$	$< 1.2 \times 10^{-8}$ Belle	10^{-9} BelleII
$K^0 \to \mu^\pm e^\mp$	$< 4.7 \times 10^{-12}$	
$B_d^0 \to \tau^\pm\mu^\mp$	$< 1.2 \times 10^{-5}$ LHCb	$\sim 10^{-6}$?
...	...	...
$h \to e^\pm\mu^\mp$	$< 6.1 \times 10^{-5}$ Atlas	2.1×10^{-5}
$h \to e^\pm\tau^\mp$	$< 2.2 \times 10^{-3}$ CMS	2.4×10^{-4}
$h \to \tau^\pm\mu^\mp$	$< 1.5 \times 10^{-3}$ CMS	2.3×10^{-4} ILC
$Z \to e^\pm\mu^\mp$	$< 7.5 \times 10^{-7}$ Atlas	
$Z \to l^\pm\tau^\mp$	$< 10^{-7}$ Atlas	[M.Ardu, CLFV 2023]

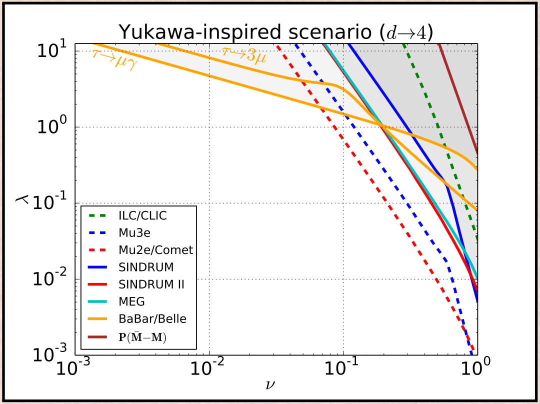
- muon searches most sensitive
- tau searches probe many operators
  can distinguish models
- *W*,*Z*,*H*, hadrons decays complementary

Tau LFV searches probe & constrain New Physics models

MSSM Seesaw
Antusch, Arganda, Herrero, Teixeira 2006



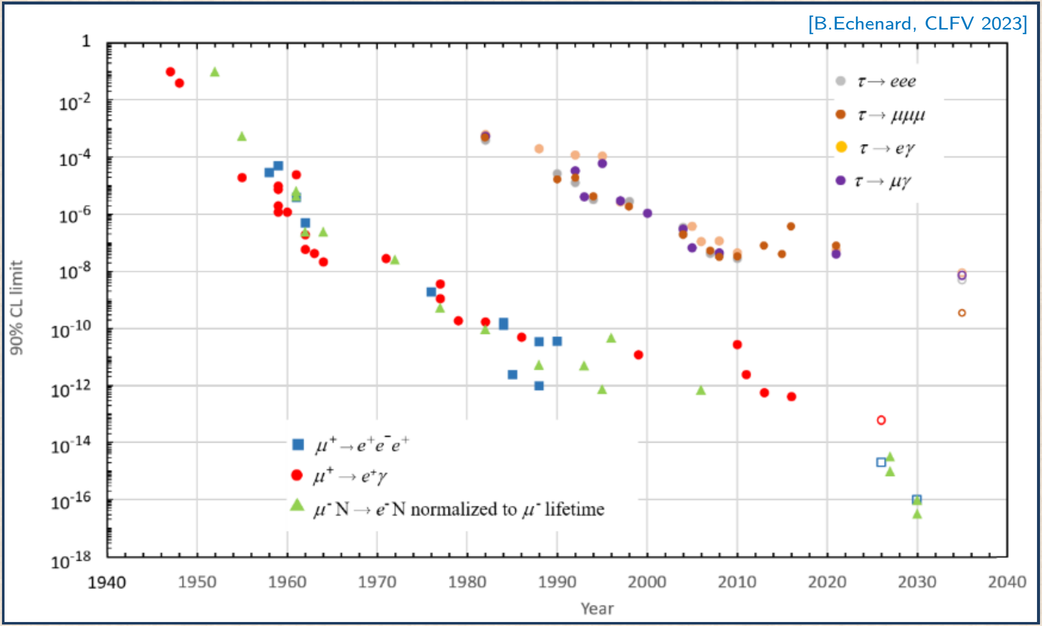
doubly charged scalar
Crivellin, Ghezzi, Panizzi, Pruna, Signer 2019



- typical "dipole-dominance" models
  $\mathcal{B}(\tau \to \mu\gamma)$ 10-10000 larger than $\mathcal{B}(\mu \to e\gamma)$
  but μ searches sensitivity $>$ 10000 larger than τ
  $\Rightarrow$ experimental muon LFV searches more effective

- tau LFV upper limits may be most constraining
  - for specific New Physics models
  - and / or for specific parameters space regions

LFV searches status and prospects



Searches for LFV in tau decays

- present upper limits mainly from CLEO, *BABAR*, Belle
- LHCb, CMS and ATLAS performed searches on $\tau^- \to \mu^-\mu^+\mu^-$, $\tau^- \to \bar{p}\mu^+\mu^-$
- Belle II will significantly improve B-factories limits
- possible future players FCC-ee(Z), CEPC, STCF

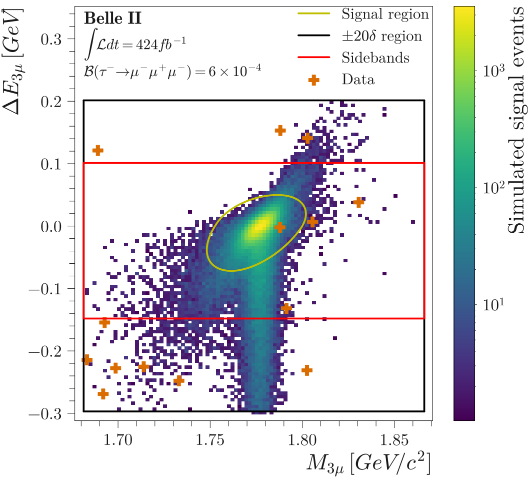
new Belle II limit on $\tau^- \to \mu^-\mu^+\mu^-$

- $\mathcal{B}(\tau^- \to \mu^-\mu^+\mu^-) < 1.9{\cdot}10^{-8}$ JHEP 09 (2024) 062, 424 fb^{-1}
- $>2\times$ larger selection efficiency of previous Belle analysis!
- previous $\mathcal{B}(\tau^- \to \mu^-\mu^+\mu^-) < 2.1{\cdot}10^{-8}$ 90% CL, Phys.Lett.B687 (2010) 139-143, 782 fb^{-1}

Belle II 2024 search for $\tau^- \to \mu^-\mu^+\mu^-$

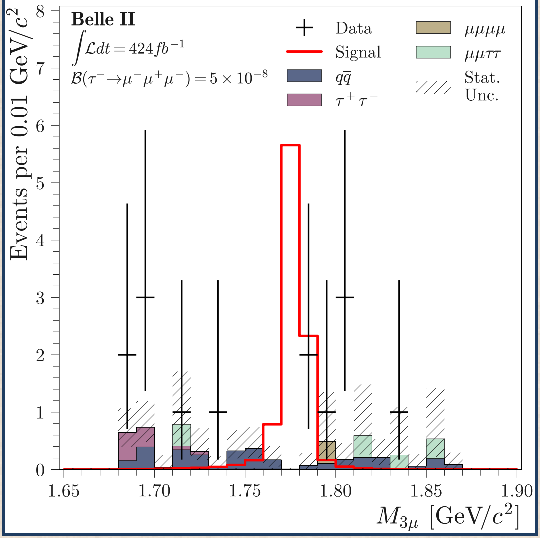
- select $\tau^- \to \mu^-\mu^+\mu^-$ candidates
- compute $M_{3\mu}$, $\Delta E_{3\mu} = E_{3\mu} - E_{\text{CM}}/2$
- estimate expected background events in signal region with simulation calibrated on sidebands
- search for excess over background
- compute upper limnit if no evidence of signal

$M_{3\mu}$ vs. $\Delta E_{3\mu}$

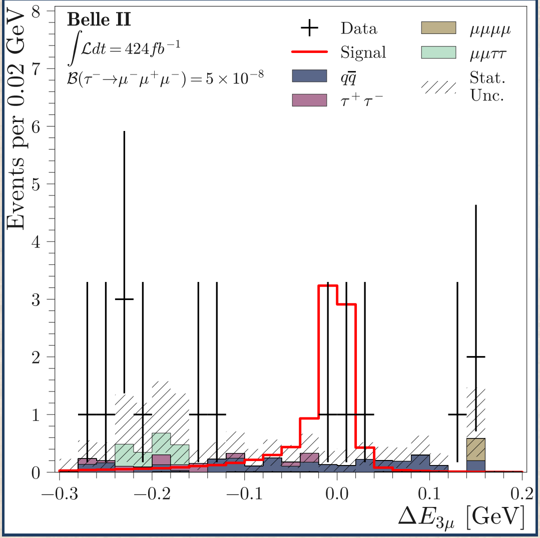


Belle II 2024 search for $\tau^- \to \mu^-\mu^+\mu^-$

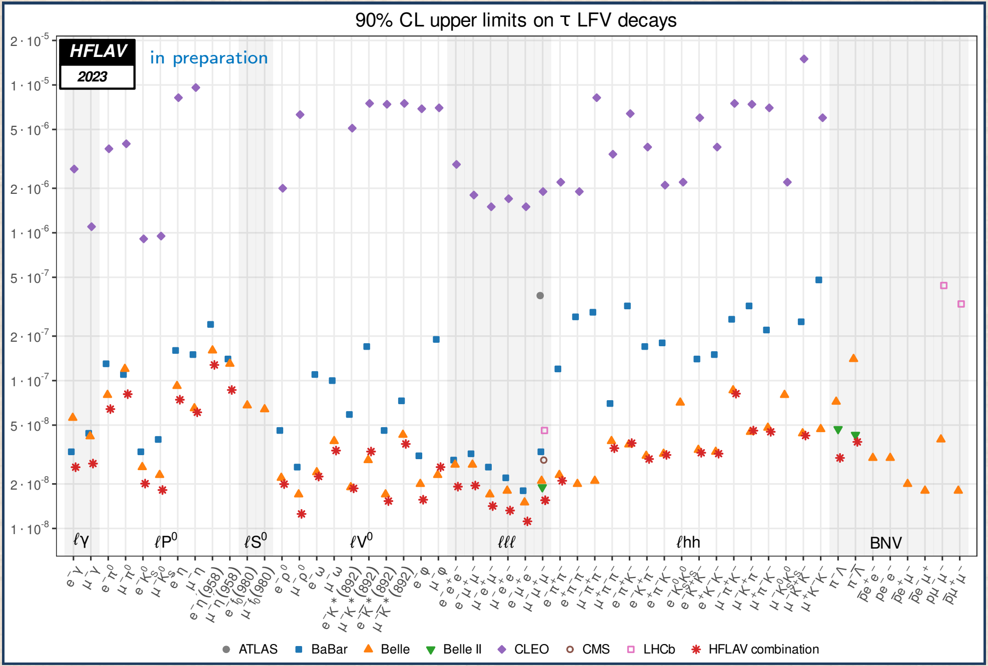
$M_{3\mu}$ projection



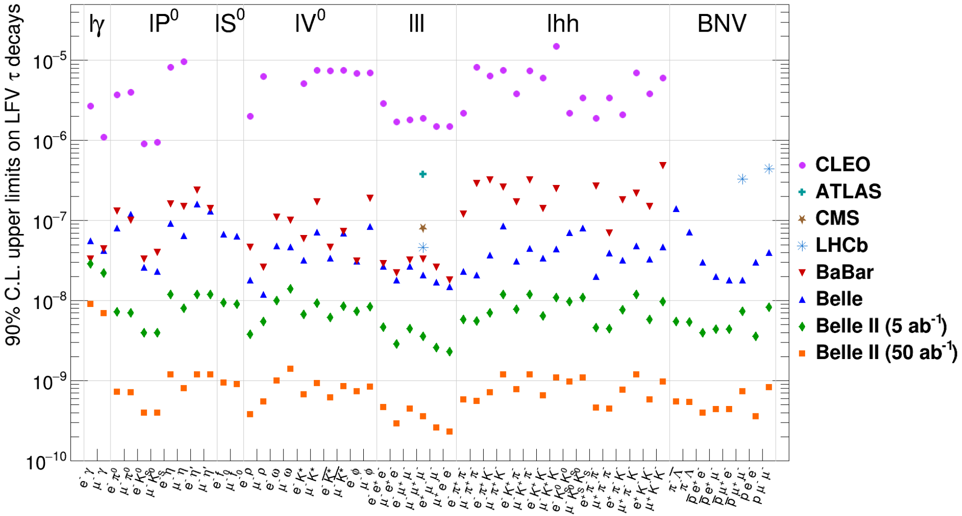
$\Delta E_{3\mu}$ projection



Tau LFV decay upper limits

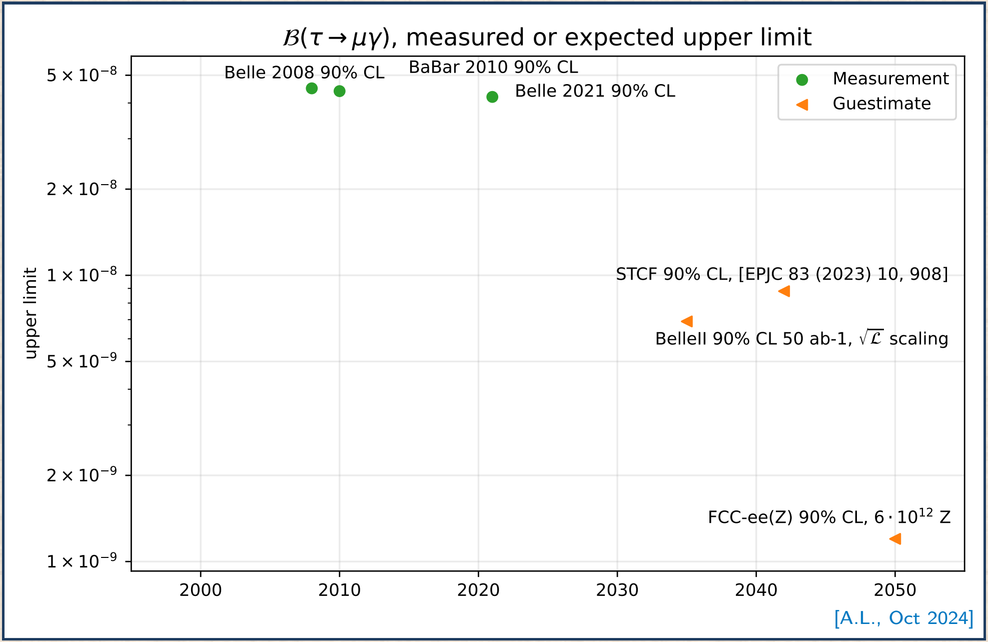


Tau LFV decay upper limits, Belle II expected

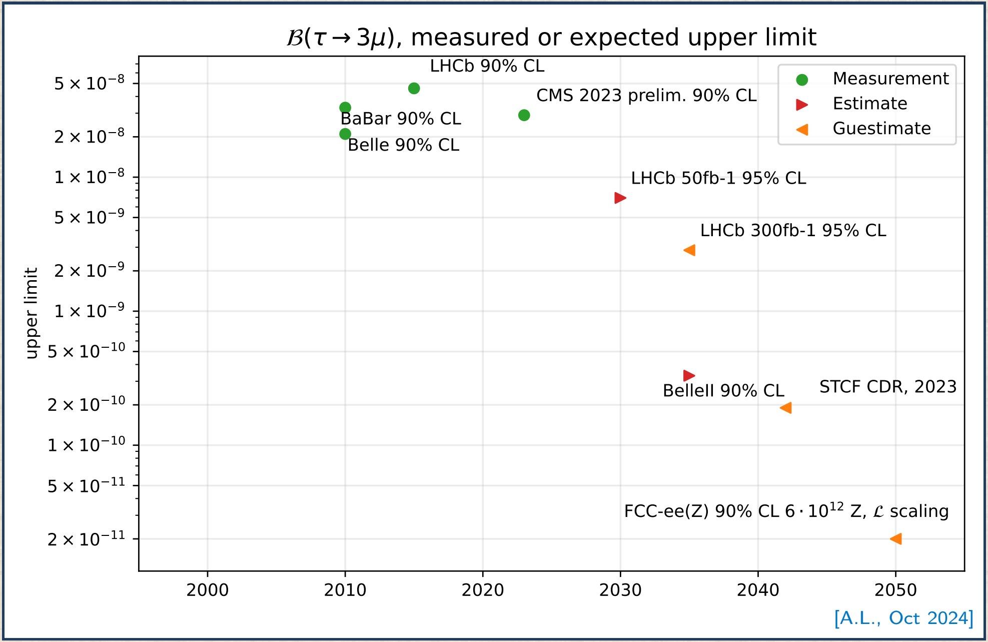


[Snowmass 2021 White Paper: Charged lepton flavor violation in the tau sector]

LFV search for $\tau \to \mu\gamma$



LFV search for $\tau \to 3\mu$



Conclusion

- reported selection of most interesting & actual tau physics measurements
- some interesting measurements left out to respect time constraints
  - $\alpha_s(m_\tau)$ & tau spectral functions
  - $(g{-}2_\tau)$, EDM$_\tau$
  - Michel parameters

End

Backup slides

μ/e universality from $\mathcal{B}(\pi \to e\nu(\gamma))/\mathcal{B}(\pi \to \mu\nu(\gamma))$

- $R^{\pi}_{e/\mu,\ \text{SM}} = 1.23524(15)\cdot 10^{-4}$
- $R^{\pi}_{e/\mu,\ \text{PIENU(2015)}} = 1.2344(23)\text{stat}(19)\text{syst}\cdot 10^{-4}$ PRL 115 (2015) 071801
- $R^{\pi}_{e/\mu,\ \text{EXP}} = 1.2327(23)\cdot 10^{-4}$ PDG 2024
- on-going PEN experiment at PSI also aims to measure $R^{\pi}_{e/\mu}$

- future: PIONEER (PSI) uncertainty goal equal to SM prediction precision