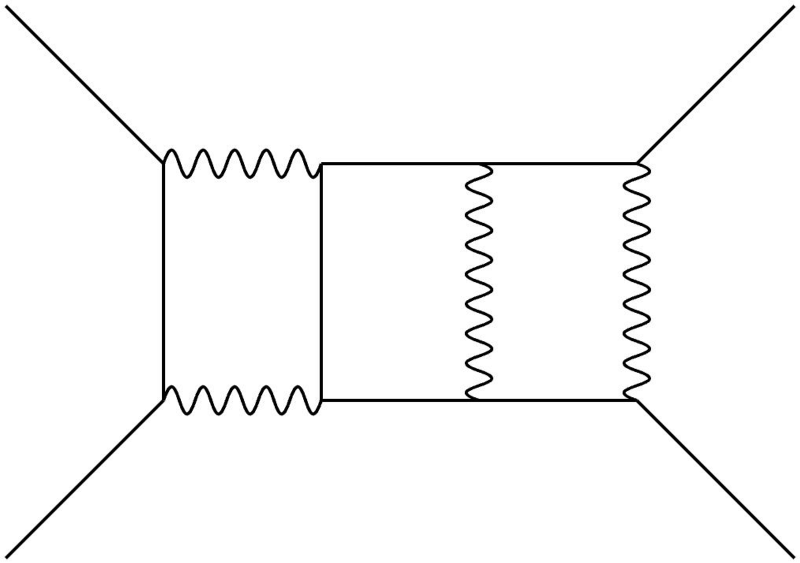
Overcoming the bottleneck

Integration-by-parts relations at Three-Loop



T. Dave

What are we calculating?

We are calculating the Helicity Amplitudes for the Three-Loop diagrams, in a massless model, that contribute to the following QED processes:

$$e^+e^- \to \mu^+\mu^- \,,$$
$$e^+\mu^- \to e^+\mu^- \,,$$
$$e^+e^- \to e^+e^- \,,$$
$$e^+e^- \to \gamma\gamma \,.$$

Helicity amplitudes in massless QED to higher orders in the dimensional regulator

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We analytically calculate one- and two-loop helicity amplitudes in massless QED, by adopting a four-dimensional tensor decomposition. We draw our attention to four-fermion and Compton scattering processes to higher orders in the dimensional regulator, as required for theoretical predictions at N^{3}LO. We organise loop amplitudes by proposing an efficient algorithm at integrand level to group Feynman graphs into integral families. We study the singular structure of these amplitudes and discuss the correspondence between QED and QCD processes. We present our results in terms of generalised polylogarithms up to transcendental weight six.

Two-Loop paper
(2411.07063)
Accepted to PRD!

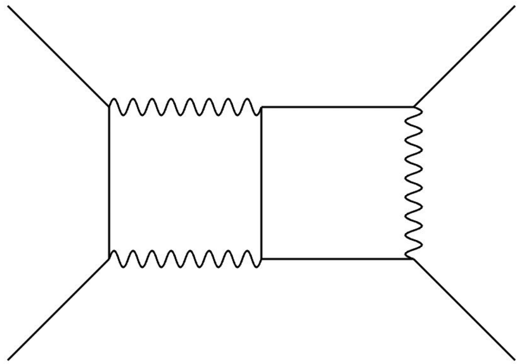
Why is Three-Loop so much harder?

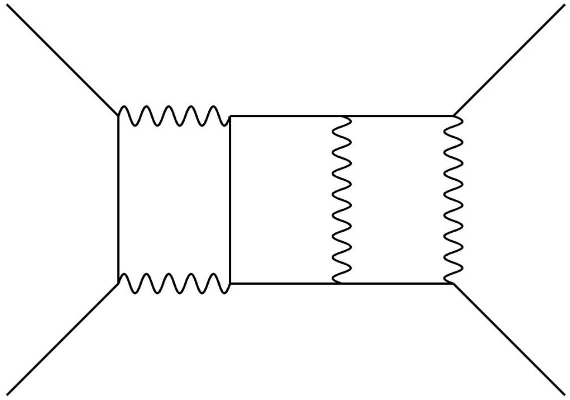
Increased rank

With increasing loop order, more virtual legs propagate through the loops.

Additional fermionic propagators introduce Lorentz indices, thus increasing the **rank** of integrals that appear in our calculation.

As rank increases, so does the complexity of the IBP relations.

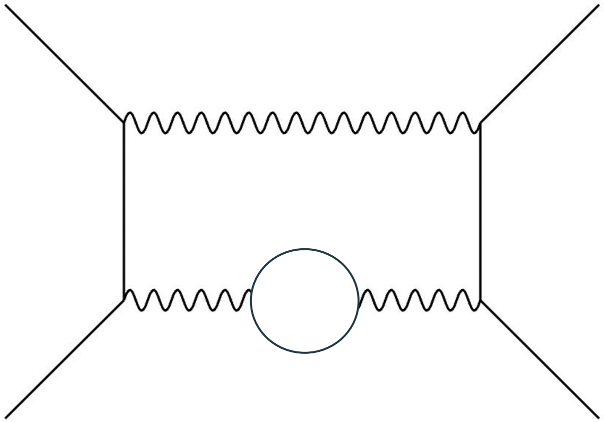


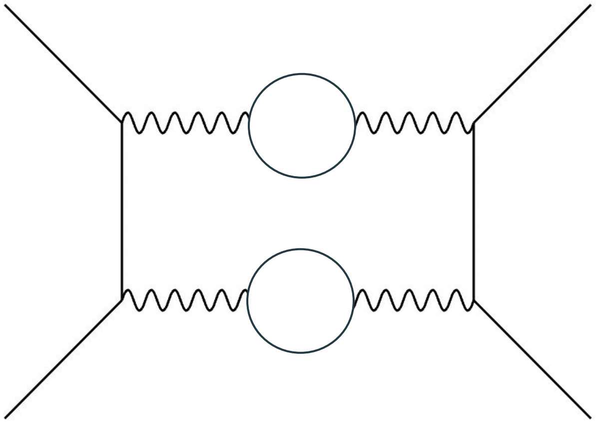


Why is Three-Loop so much harder?

Dotted Propagators

“Dotted” propagators appear when two internal legs carry the same momentum.

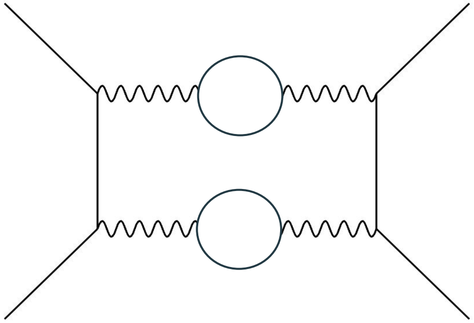




Why is Three-Loop so much harder?

Dotted Propagators

This is problematic as it increases the power of a propagator in the scattering amplitude of “Dotted” diagrams.



$$\int \prod_{j=1}^{L} d^d \ell_j \frac{\partial}{\partial \ell_f^\mu} \left(\frac{q_l^\mu}{P_1^{a_1} \ldots P_N^{a_N}} \right) = 0,$$

Taken from (**1705.05610)**

$$\frac{\text{Numerator}}{q2^2\ q3^2\ q1^2 .(\ q1 - p2\ )^{22} .(\ -p2 + q1 + q3\ )^2 .(\ -p2 - p3 + q1\ )^2 .(\ -p2 - p3 - p4 + q1\ )^{22} .(\ -p2 - p3 - p4 + q1 - q2\ )^2}$$

Why is Three-Loop so much harder?

Due to the increasing number of loops, many integrals have to be considered. High rank integrals have to be broken down into many lower rank integrals.

This makes the system of equations generating for IBP reduction extremely large, which is very computationally expensive.

Strategies to overcome this bottleneck

- We have grouped diagrams into families that obey the same IBPs.
- We have collected these families into three groups; One Planar family and two Non-Planar families.
- Within each of these groups we have determined which families contain diagrams that have no dotted propagators, one dotted propagator or two dotted propagators.
- We then apply an in-house IBP code that we can tweak to suit our needs. We start with simple families then will progress to harder families.

When performing initial reductions this has proved to be quicker than black box packages that are more general.