Pilot-Wave Theory: A Plurality of Voices

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Looking at the history of Quantum Mechanics (QM), the pilot-wave programme deserves a notable mention for three main reasons:

- Well before the advent of a coherent axiomatization of QM, de Broglie proposed a radically new hypothesis concerning the nature of quantum objects and providing them with a consistent, non-classical dynamics governing their motion;

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- The pilot-wave approach provides solutions to the technical and conceptual conundra affecting QM (e.g. it explains contextuality and the q. measurement problem).

Outline

- Historical Background
- Bohm's Pilot-Wave Theory
- Bohmian Mechanics
- Outlook

Historical Background

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Streamlined history of Quantum Mechanics (1900–1935):

- 1900: Max Planck proposes the quantum hypothesis;
- 1900–1925: Old quantum theory (notable mention: Bohr atomic model, Bohr–Sommerfeld quantization conditions);
- 1925: Matrix mechanics (Born, Heisenberg, Jordan);
- 1925–1926: Wave mechanics (Schrödinger);
- 1926: Born statistical interpretation of the squared module of the wave function;
- 1927: Bohr formulation of the complementary principle & Heisenberg Uncertainty relations;
- 1927–1935: Criticisms of quantum mechanics;
- 1930–1932: Axiomatization of QM (consolidation of the 'Copenhagen spirit').

Historical Background

Although it is disputable whether a common metaphysical perspective existed among the fathers of QM, there is, however, a precise sense in which one may properly speak about a cohesive orthodox or ‘Copenhagen’ view of the theory:

O. Freire, *The Quantum Dissidents*, p. 81:

“in spite of the existence of important differences, both the intellectual backgrounds and the scientific views of people like Bohr, Pauli, Heisenberg, Born, and Jordan, who had been working together on the collective construction of quantum mechanics, had several points in common. All of them endorsed both indeterminism and the assumption of the corpuscular and discrete nature of atomic phenomena. They also firmly believed in the completeness of quantum theory. [...] [They] were attached to the revolutionary character of quantum mechanics, and were unsympathetic to any attempt to restore such classical ideals like causality and visualizability in microphysics.”

Historical Background

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- Measurement problem (1935).

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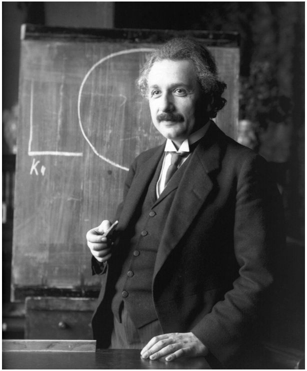
- Critique of orthodox formulation of quantum mechanics;



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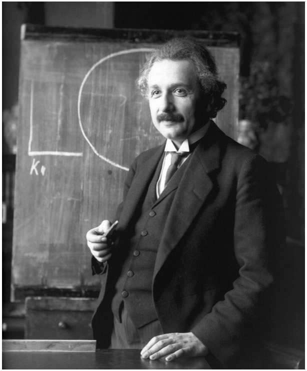

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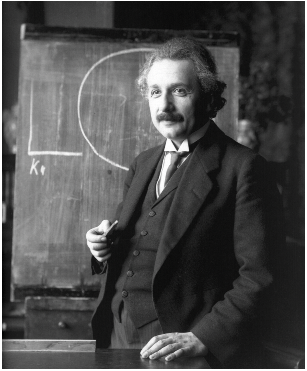

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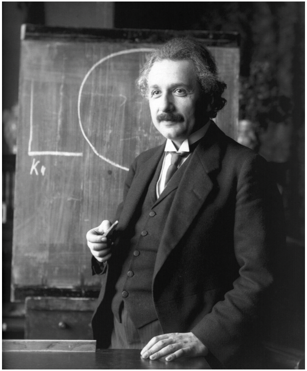

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- Generalization of Einstein's wave–particle description of light;
- Introduction of the particle-wave duality: *matter* shows a wave-like nature;
- Great explanatory power (interference of single photons, diffraction and interference of electrons, Bohr–Sommerfeld quantization conditions of atomic energy levels).

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- He formulated a dynamics for quantum particles where particles' velocities are determined by the associated wave;
- Particles have always determined positions (contrary to matrix- and wave-mechanics);
- PWT as part of a more general project called 'double solution program'.

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- de Broglie abandoned his project in 1930;
- There are various conflicting reconstructions explaining de Broglie's abandonment of his pilot-wave approach, we opt for a moderate view (dB rejoined the PW program in the 1950s due to Bohm's work).

Bohm's Pilot-Wave Theory

Bohm's 1952 Papers on *Physical Review*

PHYSICAL REVIEW VOLUME 85, NUMBER 2 JANUARY 15, 1952

A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. I

DAVID BOHM*
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received July 5, 1951)

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- Discussion about possible extensions of the theory.

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Outline of Bohm's theory (re-interpretation of the Schrödinger equation):

- $\psi(x,t) = R(x,t)e^{iS(x,t)/\hbar}$, where R, S are the real-valued amplitude and phase. Posing $P(x) = R^2(x)$, where $P(x)$ represents the probability density, one obtains the following equations for R and S:

$$\frac{\partial P}{\partial t} + \nabla \cdot \left(P \frac{\nabla S}{m} \right) = 0,$$

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- The former is the quantum continuity equation for the probability density, the latter is the *quantum Hamilton–Jacobi* equation describing the motion of a particle (or a configuration of particles) subjected to the influence of both a classical and a new *quantum* potential.

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- It contains information about the whole experimental set-up.

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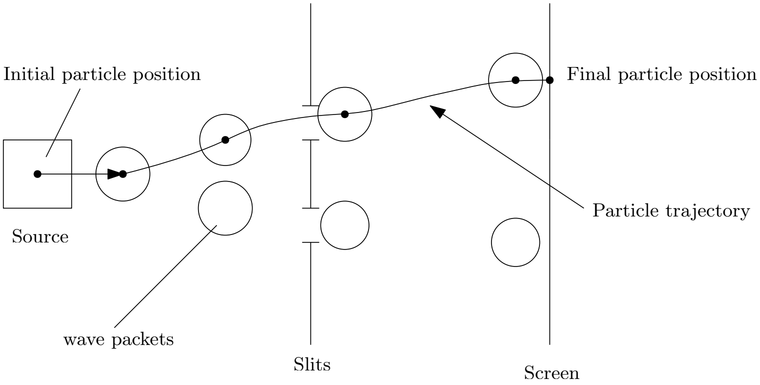
- The chosen branch of the wave function then guides the particle configuration towards the actualization of a definite state of the macroscopic device, i.e., the measurement result.

Examples of Bohmian Trajectories



Idealized representation of the double-slit experiment. Individual particles go through one slit, ψ passes through both creating the interference pattern.

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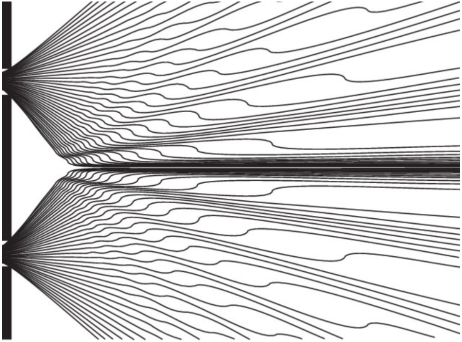

Picture taken from C. Philippidis, C. Dewdney and B.J. Hiley, (1979), Quantum interference and the quantum potential, Il Nuovo Cimento, **52***: 15–28.*

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- Randomness arises in Bohm's theory from the ignorance about the exact particles' position. **Epistemic** interpretation of quantum probability.

Some metaphysical remarks:

Following Falkenburg's analysis of the particles concept, it is possible to define a classical particle an object with the following properties:

- carriers of mass m and charge q;
- *independent* from each other;
- *localized* in space;
- spatio-temporally *individuated*;
- following continuous *trajectories*.

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Some metaphysical remarks:

Quantum particles (Fermions) are instead defined by the following attributes:

- carriers of mass m, charge q and spin s;
- *independent* from each other;
- *localized* only by particle detectors;
- only *probabilistically determinate* by the Schrödinger equation;
- in states that superpose and interfere;
- not spatio-temporally *individuated*, only distinguished by their quantum numbers;
- unsharp in momentum and position due to the *Heisenberg uncertainty relations*.

Some metaphysical remarks:

In Bohm's theory particles instantiate the following properties:

- carriers of mass m, charge q and spin s;
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These objects share the features with classical particles, and avoid obscure properties characterizing quantum particles.

Bohm's 1952 Papers on *Physical Review*

Bohm's proposes to interpret ψ as a physical field. Analogy with the electromagnetic field:

- "This field exerts a force on the particle in a way that is analogous to, but not identical with, the way in which an electromagnetic field exerts a force on a charge" (Bohm (1952), p. 170);
- The electromagnetic field obeys Maxwell's equation, while ψ obeys the Schrödinger dynamics;
- "In both cases, a complete specification of the fields at a given instant over every point in space determines the values of the fields for all times" (*ibid.*);
- "In both cases, once we know the field functions, we can calculate force on a particle, so that, if we also know the initial position and momentum of the particle, we can calculate its entire trajectory" (*ibid.*).

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- Multi-category ontology, contrary to de Broglie's double solution program (wave monistic);
- Particle dynamics, for Q is absent in de Broglie's original pilot-wave theory.

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- Bohm did not aim at restoring a particle ontology, in the Appendix A to his 1952 papers he provided a guidance equation for field coordinates describing the electromagnetic field;
- Bohm was against reductionism and mechanistic philosophy (cf. his book *Causality and Chance in Modern Physics* 1957, where he postulated a form of metaphysical infinitism, i.e. the infinite richness of nature and the impossibility to reduce it to a fixed set of laws and objects).

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- 1957: *Causality and Chance in Modern Physics*, Bohm's most important philosophical publication in the 1950s;
- In the late 1950s Bohm abandoned his pilot-wave approach due to the negative (and biased) reception of his proposal.

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- Mathematical structures accounting for the motion of the PO.

Primitive Ontology

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- Explanatory function of the PO: every physical phenomena/process is explained in terms of the PO (reductionism);
- Primitive variables *are not deduced* from other more fundamental notions;
- Primitive variables as “building blocks”: their histories provide a fundamental picture of the world.

Primitive Ontology

According to DGZ the PO *do not* exhaust the ontology of a physical theory.

The PO divides the structure of a given theory:

1. Primitive variables in 3-dimensional space;
2. Mathematical structures implementing their motion (e.g. wave function in Bohmian Mechanics (BM)).

DGZ consider the ontology of a theory composed by *every* object appearing within its vocabulary; therefore, even mathematical structures are part of the ontology of the theory.

However, only the subset of the PO refers to material entities.

Bohmian Mechanics

Physical systems are described by the couple (Q, ψ) where:

- $Q_t = (Q_1(t), \ldots, Q_n(t)) \in \mathbb{R}^3$
- $\psi \in \mathbb{R}^{3N}$

Dynamical Equations of Bohmian Mechanics:

$$\text{SE:} \quad i\hbar\frac{\partial\psi}{\partial_t} = \hat{H}\psi_t = -\sum_{i=1}^{N}\frac{\hbar}{2m_k}\nabla_k^2\psi + V\psi;$$

$$\text{GE:} \quad \frac{dQ_k}{dt} = v_k^{\psi}(Q_1, \ldots, Q_N) = \frac{\hbar}{m_k}\text{Im}\frac{\psi^*\nabla_k\psi}{\psi^*\psi}(Q_1, \ldots, Q_N).$$

Equivariance

If $\rho(Q,t_0) = |\psi(Q,t_0)|^2$, then $\rho(Q,t) = |\psi(Q,t)|^2$ for all times $t \in \mathbb{R}$

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- Projection Postulate not needed (cf. next section).

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- DGZ are realist about laws of nature, thus, the wave function still belong to the general ontology of BM.

A Primer on Bohmian Measurement Theory (Dürr & Teufel, 2009)

- Consider the wave function $\psi = c_1\psi_1 + c_2\psi_2$, where ψ_1, ψ_2 correspond to the possible eigenstates of a two-valued operator O, with eigenvalues "left" (L) and "right" (R);

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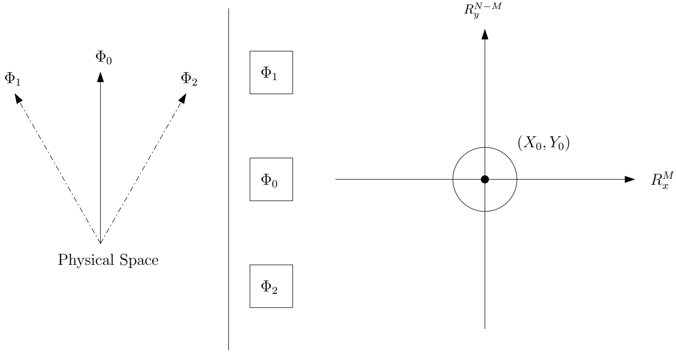
- Although system and apparatus are initially independent, we obtain a macroscopic superposition due to the linear Schrödinger dynamics:

$$\sum_{i=1,2} c_i\psi_i\Phi_0 \longrightarrow \sum_{i=1,2} c_i\psi_i\Phi_i.$$

Bohmian Mechanics



Left: *Schematic representation of the pointer pointing in the neutral direction in physical space before the measurement* (solid line). *Dashed lines represent physically possible but not actualized measurement outcomes.* Right; *The relative support of the pointer's wave function and particle configuration describing the experimental situation before the measurement's performance.*

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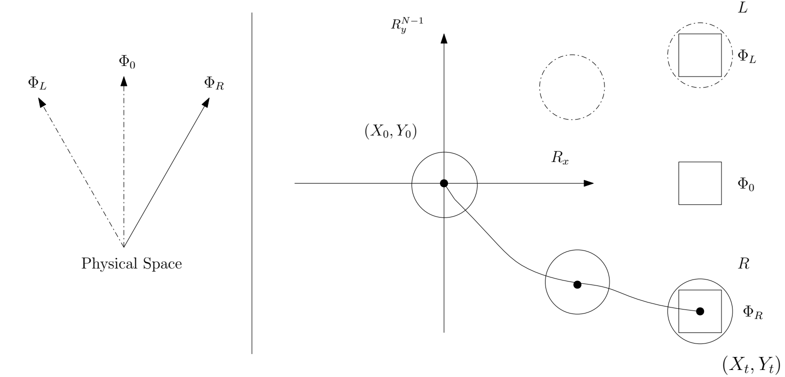
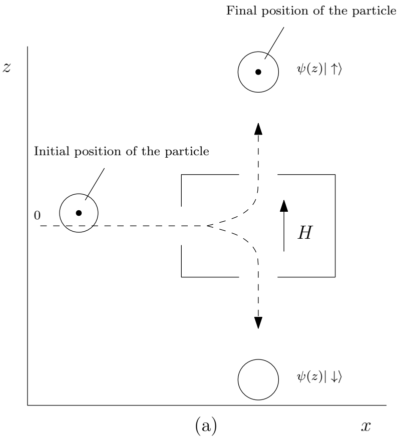
- As a consequence of SE, the system's wave function entangles with the apparatus' wave function, and two disjoint wave packets are formed—each one corresponding to a possible measurement outcome;
- The configuration (X_0, Y_0), which is always well-defined and never in superposition, enters in only one wave packet. The chosen branch of ψ guides the particle configuration towards the actualization of a definite state of the macroscopic device (X_t, Y_t) at a later time t.

Bohmian Mechanics



Left: *Schematic representation of the pointer pointing in the right direction in physical space* (solid line). *The relative support of the pointer's wave function and particle configuration describing the experimental result are shown on the right. FAPP the empty branch of the wave function can be neglected.*

Bohmian Mechanics



Schematic representation of the contextual nature of spin in BM.

Some Consequences of the Bohmian Measurement Theory

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- Therefore, spin is not real;
- Eliminativist approach to reductionism.

Outlook

Open questions and future research directions:

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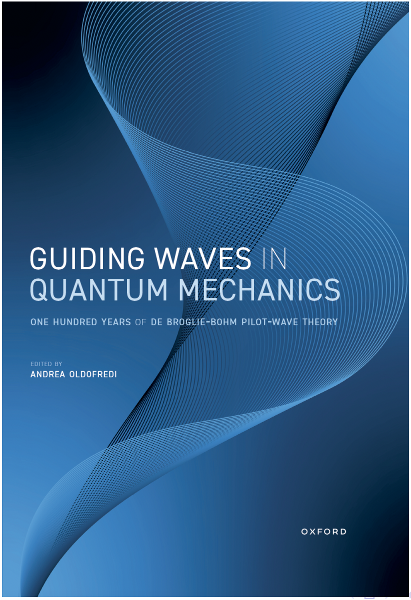
- On the philosophical side, understand the nature of the wave function, quantum observables and the very notion of particle;
- On the historical side, rediscover and reassess Bohm's works, changing the current (and wrong) ideas concerning his philosophical perspective, often times promoted by contemporary Bohmians.

Self-Promotion – *Guiding Waves in Quantum Mechanics*



Thank You!

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