ε-factorised form and numerical evaluation for elliptic Feynman integrals in diphoton production

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FACULTY OF SCIENCES

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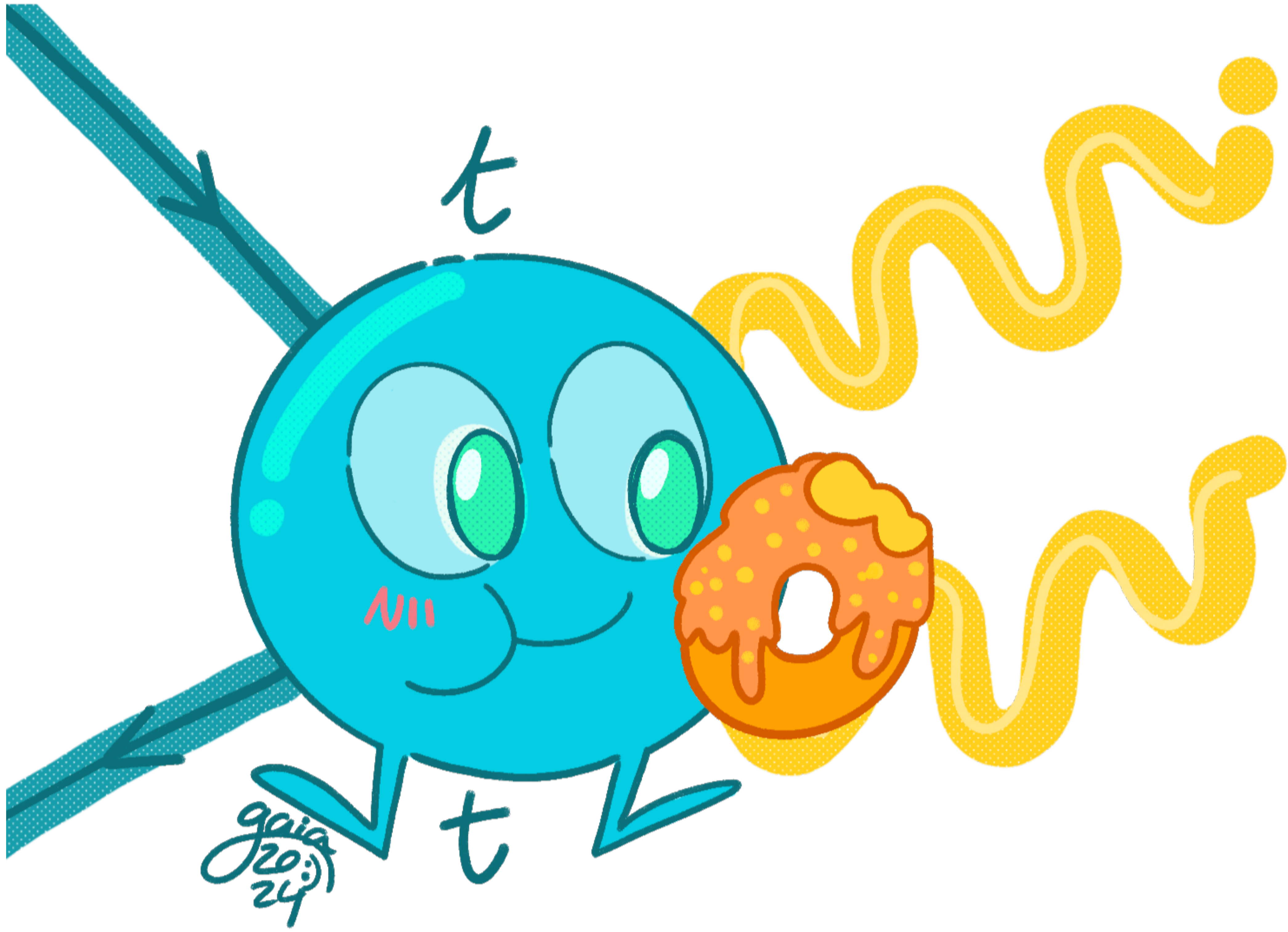
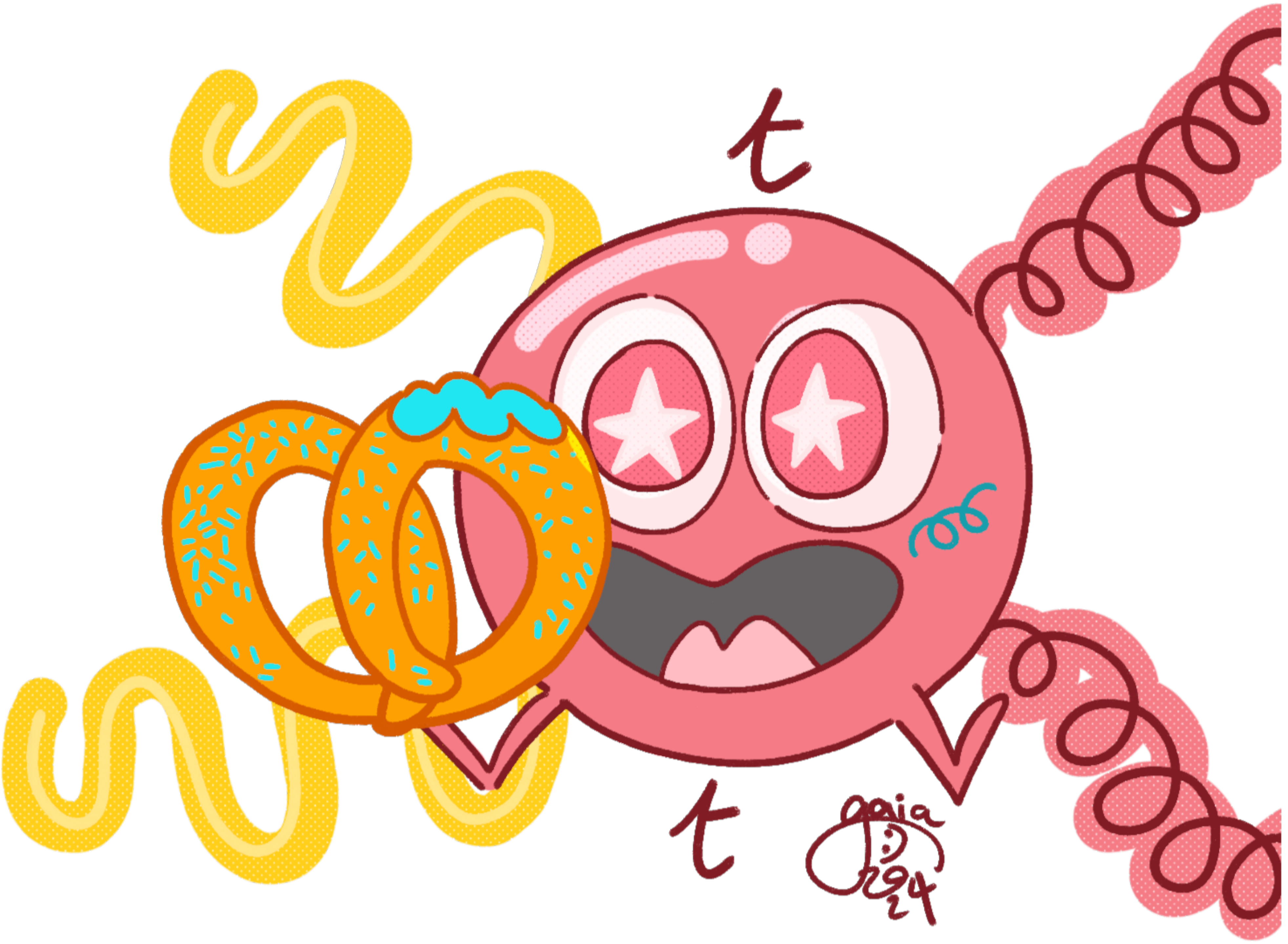
Massive corrections to diphoton production





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In collaboration with **M. Becchetti, C. Nega, F. J. Wagner, L. Tancredi**

Outline of the talk

- Two-loop helicity amplitudes
  - Quark annihilation channel
  - Gluon fusion channel
- Differential equations
  - Elliptic Feynman integrals
  - ε-factorised basis
- Series expansions and numerical evaluation
  - Large-mass expansion
  - Expansion around threshold

Two-loop helicity amplitudes for diphoton production

We consider the amplitudes for diphoton production through a heavy-quark loop

At partonic level the scattering processes are :

$$q(p_1) + \overline{q}(p_2) \to \gamma(-p_3) + \gamma(-p_4)$$
$$g(p_1) + g(p_2) \to \gamma(-p_3) + \gamma(-p_4)$$

All the momenta are taken incoming and the external particles are on-shell: $p_i^2 = 0$

The kinematics is described by the usual Mandelstam invariants for $2 \to 2$ processes:

$$s = (p_1 + p_2)^2, \quad t = (p_1 + p_3)^2, \quad u = (p_2 + p_3)^2 \qquad\qquad s + t + u = 0$$

Physical scattering region:

$$s > 0$$
$$-s < t < 0$$

Relevant for numerical evaluation and phenomenology

Two-loop helicity amplitudes for diphoton production

- Quark annihilation channel: $$\mathscr{A}_{q\bar{q},\gamma\gamma}(s,t) = \delta^{kl}\left[\sum_{i=1}^{4} F_i(s,t)\bar{u}(p_2)\Gamma_i^{\mu\nu}u(p_1)\right]\epsilon_{3,\mu}(p_3)\epsilon_{4,\nu}(p_4)$$

- Gluon fusion channel: $$\mathscr{A}_{gg,\gamma\gamma}(s,t) = \delta^{a_1a_2}\left[\sum_{i=1}^{8} G_i(s,t)T_i^{\mu\nu\rho\sigma}u(p_1)\right]\epsilon_{1,\mu}(p_1)\epsilon_{2,\nu}(p_2)\epsilon_{3,\rho}(p_3)\epsilon_{4,\sigma}(p_4)$$

'tHV-scheme :

$$\Gamma_1^{\mu\nu} = \gamma^\mu p_2^\nu,\ \Gamma_2^{\mu\nu} = \gamma^\nu p_1^\mu,\ \Gamma_3^{\mu\nu} = p_{3,\rho}\gamma^\rho p_1^\mu p_2^\nu,\ \Gamma_4^{\mu\nu} = p_{3,\rho}g^{\mu\nu}$$

[F.Caola,A.Von Manteuffel,L.Tancredi]

$$T_1^{\mu\nu\rho\sigma} = p_3^\mu p_1^\nu p_1^\rho p_2^\sigma,\ T_2^{\mu\nu\rho\sigma} = p_3^\mu p_1^\nu g^{\rho\sigma},\ T^{\mu\nu\rho\sigma} = p_3^\mu p_1^\rho g^{\nu\sigma},\ T_4^{\mu\nu\rho\sigma} = p_3^\mu p_2^\sigma g^{\nu\rho},\ T_5^{\mu\nu\rho\sigma} = p_1^\nu p_1^\rho g^{\mu\sigma},$$

$$T^{\mu\nu\rho\sigma} = p_1^\nu p_2^\sigma g^{\mu\rho},\ T_7^{\mu\nu\rho\sigma} = p_1^\rho p_2^\sigma g^{\mu\nu},\ T_8^{\mu\nu\rho\sigma} = g^{\mu\nu}g^{\rho\sigma} + g^{\mu\sigma}g^{\nu\rho} + g^{\mu\rho}g^{\nu\sigma}$$

[T.Peraro,L.Tancredi]

[P.Bargiela,F.Caola,A.Von Manteuffel,L.Tancredi]

Two-loop helicity amplitudes for diphoton production

We can fix the helicities of the external states:

$$A_{qq}^{L++} = \frac{2[34]^2}{\langle 13\rangle[23]}\alpha(x,y)\,, \qquad A_{qq}^{L+-} = \frac{2\langle 24\rangle[13]}{\langle 23\rangle[24]}\beta(x,y)\,,$$

$$A_{qq}^{L-+} = \frac{2\langle 23\rangle[41]}{\langle 24\rangle[32]}\gamma(x,y)\,, \qquad A_{qq}^{L--} = \frac{2\langle 34\rangle^2}{\langle 31\rangle[23]}\delta(x,y)\,.$$

$$\lambda_a = \{L, R\} \qquad \lambda_i = \pm$$

$$A_{qq}^{R\lambda_3\lambda_4} = A_{qq}^{L\lambda_3^*\lambda_4^*}\left(\langle ij\rangle \leftrightarrow [ji]\right)$$

$$A_{gg}^{++++} = \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} f_{++++}(x,y), \qquad A_{gg}^{+++-} = \frac{\langle 42\rangle\langle 43\rangle[23]}{\langle 13\rangle\langle 21\rangle\langle 23\rangle} f_{+++-}(x,y),$$

$$A_{gg}^{-+++} = \frac{\langle 12\rangle\langle 14\rangle[24]}{\langle 34\rangle\langle 23\rangle\langle 24\rangle} f_{-+++}(x,y), \qquad A_{gg}^{--++} = \frac{\langle 12\rangle[34]}{[12]\langle 34\rangle} f_{--++}(x,y),$$

$$A_{gg}^{+-++} = \frac{\langle 21\rangle\langle 24\rangle[14]}{\langle 34\rangle\langle 13\rangle\langle 14\rangle} f_{+-++}(x,y), \qquad A_{gg}^{-+-+} = \frac{\langle 13\rangle[24]}{[13]\langle 24\rangle} f_{-+-+}(x,y),$$

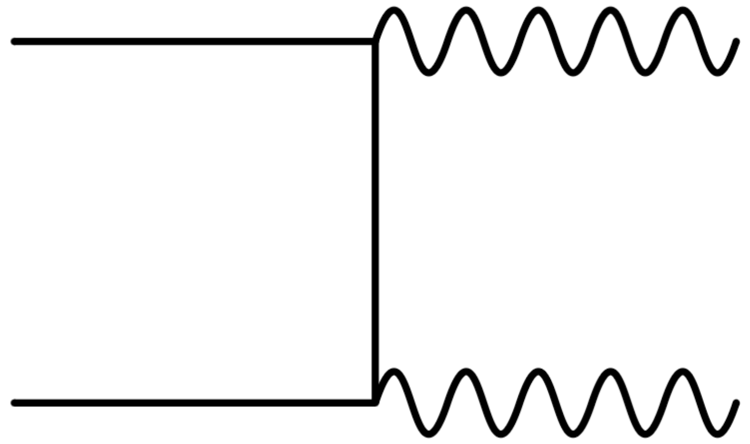
$$A_{gg}^{++-+} = \frac{\langle 32\rangle\langle 34\rangle[24]}{\langle 14\rangle\langle 21\rangle\langle 24\rangle} f_{++-+}(x,y), \qquad A_{gg}^{+--+} = \frac{\langle 23\rangle[14]}{[23]\langle 14\rangle} f_{+--+}(x,y)$$

$$A_{gg}^{\lambda_1\lambda_2\lambda_3\lambda_4} = A_{gg}^{\lambda_1^*\lambda_2^*\lambda_3^*\lambda_4^*}\left(\langle ij\rangle \leftrightarrow [ji]\right)$$

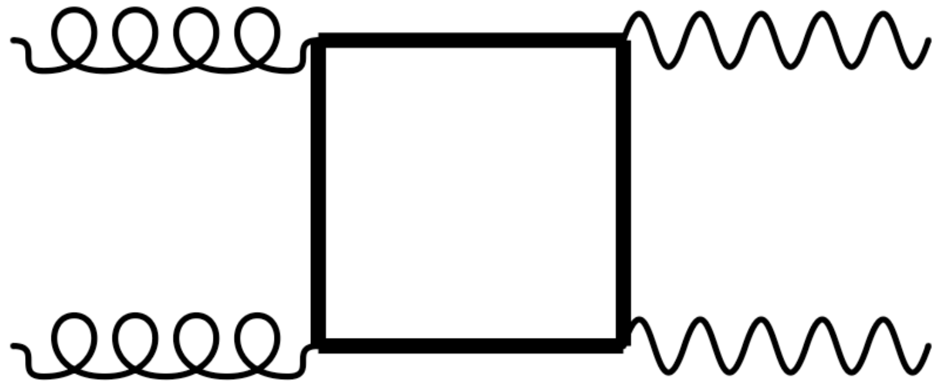
Two-loop helicity amplitudes for diphoton production

The spinor free helicity amplitudes can be expanded in the bare strong coupling α_s^b:

$$\Omega_{qq} = \delta_{kl}(4\pi\alpha_{em})\sum_{l=0}^{2}\left(\frac{\alpha_s^b}{2\pi}\right)^l \Omega_{qq}^{(l,b)}$$



$$\Omega_{gg} = \delta_{a_1a_2}(4\pi\alpha_{em})\sum_{l=1}^{2}\left(\frac{\alpha_s^b}{2\pi}\right)^l \Omega_{gg}^{(l,b)}$$



$$\Omega_{qq} = \{\alpha, \beta, \gamma, \delta\}$$

$$\Omega_{gg} = \{f_{++++}, \cdots, f_{+--+}\}$$

We can define the projector operators which act directly on the Amplitude represented in terms of Feynman diagrams:

$$\sum_{pol} P_{qq}^{(i)} A_{qq} = \delta^{kl}(4\pi\alpha_{em})e_q^2 F_i \qquad i = 1,\cdots,4$$

$$\sum_{pol} P_{gg}^{(j)} A_{gg} = \delta^{a_1a_2}(4\pi\alpha_{em}) G_j \qquad j = 1,\cdots,8$$

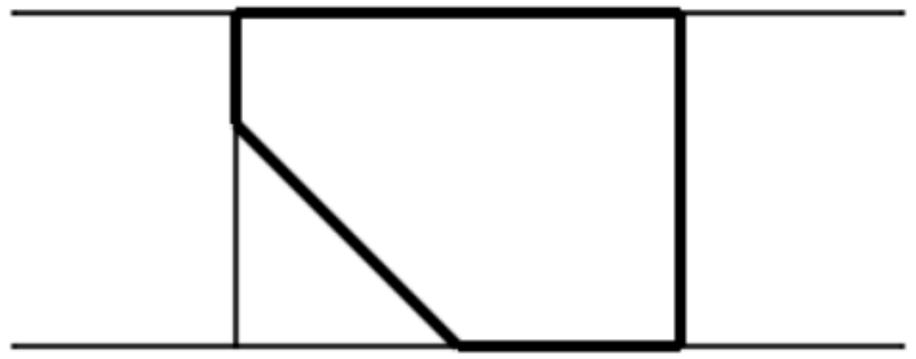
Scalar integral families

We have five scalar integral families (without counting the crossings)

- 3 planar topologies - PLA, PLB, PLC
- 2 non-planar topologies - NPA, NPB

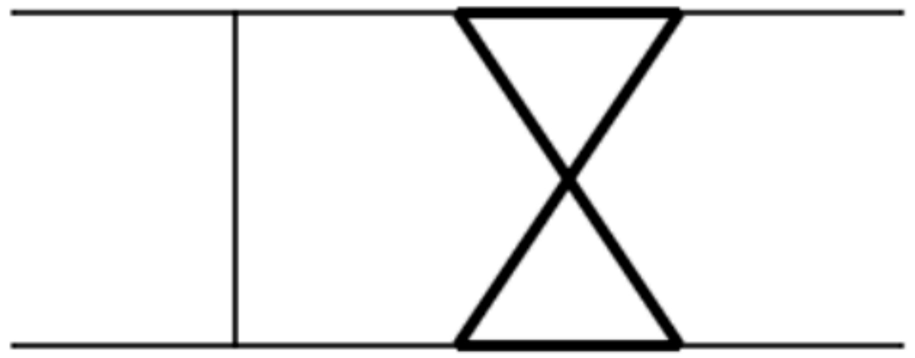
Relevant topologies for diphoton/dijet production

PLA



PLB

PLC



NPA

NPB

MIs for diphoton production

- gg : 138 diagrams ⟶ Form factors in terms of 32936 scalar integrals ⟶ (IBPs, Symmetries) 161 MIs

$$\text{PLA}_{\sigma_{12}} = \text{PLA}\Big|_{p_1\to p_2,\, p_2\to p_1}$$

$$\text{PLA}_{\sigma_{123}} = \text{PLA}\Big|_{p_1\to p_2,\, p_2\to p_3,\, p3\to p_1}$$

$$\text{PLA}_{\sigma_{124}} = \text{PLA}\Big|_{p_1\to p_2,\, p_2\to p_4,\, p_4\to p_1}$$

$$\text{PLA}_{\sigma_{1234}} = \text{PLA}\Big|_{p_1\to p_2,\, p_2\to p_3,\, p_3\to p_4,\, p_4\to p_1}$$

$$\text{PLA}_{\sigma_{1243}} = \text{PLA}\Big|_{p_1\to p_2,\, p_2\to p_4,\, p_4\to p_3,\, p_3\to p_1}$$

$$\text{PLC}_{\sigma_{12}} = \text{PLC}\Big|_{p_1\to p_2,\, p_2\to p_1}$$

$$\text{PLC}_{\sigma_{123}} = \text{PLC}\Big|_{p_1\to p_2,\, p_2\to p_3,\, p_3\to p_1}$$

$$\text{NPA}_{\sigma_{123}} = \text{NPA}\Big|_{p_1\to p_2,\, p_2\to p_3,\, p_3\to p_1}$$

$$\text{NPA}_{\sigma_{124}} = \text{NPA}\Big|_{p_1\to p_2,\, p_2\to p_4,\, p_4\to p_1}$$

$$\text{NPB}_{\sigma_{123}} = \text{NPB}\Big|_{p_1\to p_2,\, p_2\to p_3,\, p_3\to p_1}$$

$$\text{NPB}_{\sigma_{124}} = \text{NPB}\Big|_{p_1\to p_2,\, p_2\to p_4,\, p_4\to p_1}$$

PLB does not contribute to the amplitude

- $q\bar{q}$: 14 diagrams [M.Becchetti,R.Bonciani,L.Cieri,F.Coro,F.Ripani]

Total of 165 MIs for diphoton production

MIs - Diphoton/Dijet production

220 MIs — Dijet production in gluon fusion

↓

165 MIs — Diphoton production

- Same elliptic sectors
- More crossing of the transcendental functions and periods

↑

- Same elliptic sectors
- Same transcendental functions

↑

DEs in ϵ-factorised form for all the system, including the NPA family. This introduces new transcendental functions, in particular periods of the elliptic curve

Differential equations

Let's denote by $\underline{I}$ the set of all the MIs: $\underline{I} = \{I_1, \cdots, I_{165}\}$ $\quad I_i = I_i(x, y; \epsilon) \quad i = 1, \cdots, 165$

The amplitude depend on two dimensionless ratios: $x = -\frac{t}{s}, \quad y = \frac{m^2}{s}, \quad 0 \leq x \leq 1$

We can use IBPs to obtain a closed system of linear DEs: $d\underline{I}(x, y; \epsilon) = \mathbb{A}(x, y; \epsilon)\underline{I}(x, y; \epsilon)$

The choice of the basis of MIs is not unique $\underline{I}(x, y; \epsilon) = \mathbb{B}(x, y; \epsilon)\underline{f}(x, y; \epsilon) \longrightarrow d\underline{f}(x, y; \epsilon) = \tilde{\mathbb{A}}(x, y; \epsilon)\underline{f}(x, y; \epsilon)$

It is convenient to derive an ε-factorised basis: $d\underline{f}(x, y; \epsilon) = \epsilon\tilde{\mathbb{A}}(x, y)\underline{f}(x, y; \epsilon)$ [J.M.Henn]

Formal solution: $\underline{f}(x, y) = \mathbb{P}\left[exp\left(\epsilon\int_\gamma \tilde{\mathbb{A}}(x, y)\right)\right]\underline{f}(x_0, y_0)$

ε-factorised form beyond MPLs

Procedure to obtain an ε-factorised form for any Feynman integral family [L.Görges,C.Nega,L.Tancredi,F.J.Wagner]

Choice of the initial basis $\underline{I}$ → Related to the underlying geometry associated to the integral family

$$\partial_{m^2} \underline{I} = \left(\mathbb{A}_0 + \mathcal{O}(\epsilon)\right) \underline{I} \xrightarrow{\underline{J} = W^{-1}\underline{I}} \partial_{m^2} \underline{J} = \mathcal{O}(\epsilon) \underline{J}$$

Solution at $\epsilon = 0$ for the DEs required as first step

$$W = \begin{pmatrix} \varpi_0 & \varpi_1 \\ \partial_{m^2}\varpi_0 & \partial_{m^2}\varpi_1 \end{pmatrix} \qquad \varpi_0, \varpi_1 \sim K$$

Complete elliptic integral of the first kind

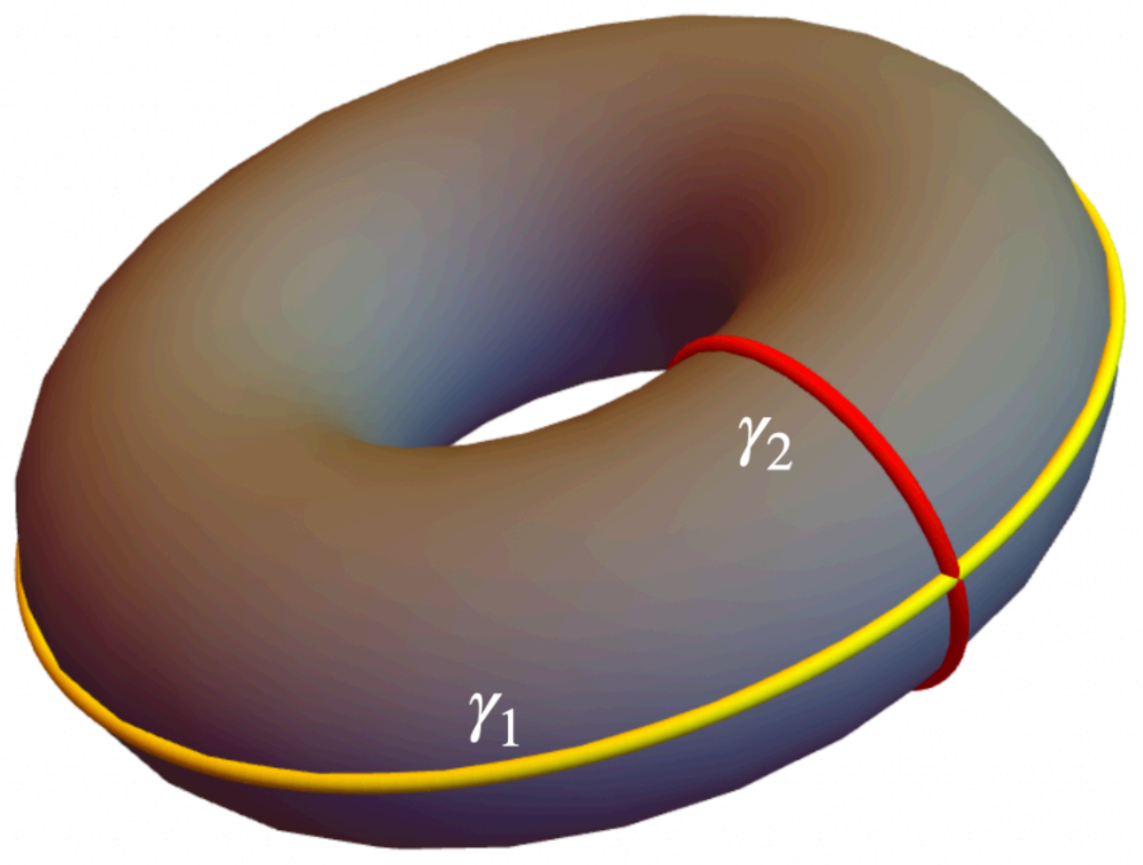
$$W = W_{ss} \cdot W_u$$

Rotate to unipotent basis

$$\underline{f} = \mathrm{T}_d \cdot \mathrm{T}_\epsilon \cdot W_{ss}^{-1} \underline{I} \longrightarrow d\underline{f} = \epsilon \, d\tilde{\mathbb{A}}(s, m^2) \underline{f}$$

ε-factorised form

Periods - local solutions around MUM points



Given an elliptic curve defined by the algebraic equation : $Y^2 = P_4(X)$

$$\partial_{m^2} W = \mathbb{A}_0 W \longrightarrow \mathscr{L}^{(2)}_{m^2} \varpi_i = 0 \qquad \mathscr{L}^{(2)}_{m^2} = \sum_{j=0}^{2} a_j(m^2) \partial^j_{m^2}$$

$$\varpi_0(s, m^2) = \int_{\mathscr{C}_1} \frac{dX}{\sqrt{P_4(X)}} \qquad \varpi_1(s, m^2) = \int_{\mathscr{C}_2} \frac{dX}{\sqrt{P_4(X)}}$$

Solutions of the associated Picard-Fuchs operator (PF)

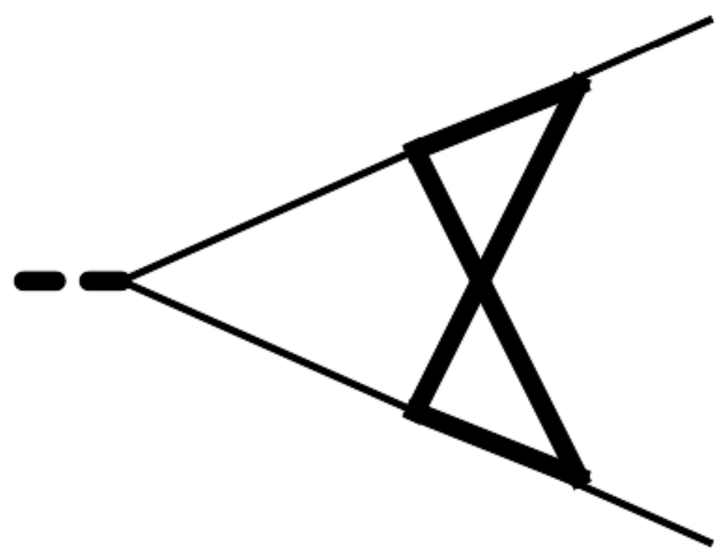
For an elliptic curve, each regular singular point is a **MUM point** $\longrightarrow$ Local solution fo the PF operator

Frobenius basis :

$\varpi_0^{[z]}(s, m^2)$ holomorphic solution

$\varpi_1^{[z]}(s, m^2)$ contains a single power of a logarithm

Elliptic sectors NPA - $I_{\text{NPA}}(1,1,1,0,0,1,1,1,0)$



2 MIs

Choice of the initial basis:

- Abel differential of the first kind
- Derivative of the first one wrt internal mass

Maximal cut in loop-by-loop Baikov representation (d=4):

$$\text{MC}\left(I_{\text{NPA}}(1,1,1,0,0,1,1,1,0)\right) \sim \frac{1}{s}\int \frac{dz_9}{\sqrt{(m^2-z_9)(s+m^2-z_9)(m^2(m^2-3s)-(2m^2+s)z_9+z_9^2)}}$$

Abel differential of the first kind

[J.Broedel,C.Duhr,F.Dulat,B.Penante,L.Tancredi]

Choice of the initial basis:

$$I_1 = s\epsilon^4\, I_{\text{NPA}}(1,1,1,0,0,1,1,1,0)$$
$$I_2 = \partial_{m^2} I_1$$

$$\partial_{m^2}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{4(1+2\epsilon)(1+4\epsilon)}{m^2(s+16m^2)} & -\frac{s+32m^2+\epsilon(2s+48m^2)}{m^2(s+16m^2)} \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} + \text{inhomogeneous part}$$

Elliptic sectors NPA - $I_{\textbf{NPA}}(1,1,1,0,0,1,1,1,0)$

We start with the homogeneous part at $\epsilon = 0$: $\partial_{m^2}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\frac{4}{m^2(s+16m^2)} & -\frac{s+32m^2}{m^2(s+16m^2)} \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$

Homogeneous solution at $\epsilon = 0$ $\longrightarrow$ $\partial_{m^2} W = \begin{pmatrix} 0 & 1 \\ -\frac{4}{m^2(s+16m^2)} & -\frac{s+32m^2}{m^2(s+16m^2)} \end{pmatrix} W$

$W = W_u \cdot W_{ss}$

- ❖ $W_u = \begin{pmatrix} 1 & \frac{\varpi_1}{\varpi_0} \\ 0 & 1 \end{pmatrix}$ $\longrightarrow$ $\partial_\tau W_u = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} W_u$
- ❖ $W_{ss} = \begin{pmatrix} \varpi_0 & 0 \\ \partial_{m^2}\varpi_0 & \frac{1}{sm^2(s+16m^2)\varpi_0} \end{pmatrix}$ $\longleftarrow$ $\varpi_0\left(\partial_{m^2}\varpi_1\right) - \varpi_1\left(\partial_{m^2}\varpi_0\right) = \left[sm^2(s+16m^2)\right]^{-1}$ Legendre relation

Elliptic sectors NPA - $I_{\text{NPA}}(1,1,1,0,0,1,1,1,0)$

Change to a unipotent basis: $\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = W_{ss}^{-1} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \longrightarrow \partial_{m^2} \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{m^2 s(s+16m^2)\varpi_0^2} \\ -\epsilon\left(24s\varpi_0^2 + 2s(s+24m^2)\varpi_0(\partial_{m^2}\varpi_0)\right) & -\frac{2\epsilon(s+24m^2)}{m^2(s+16m^2)} \end{pmatrix} \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$

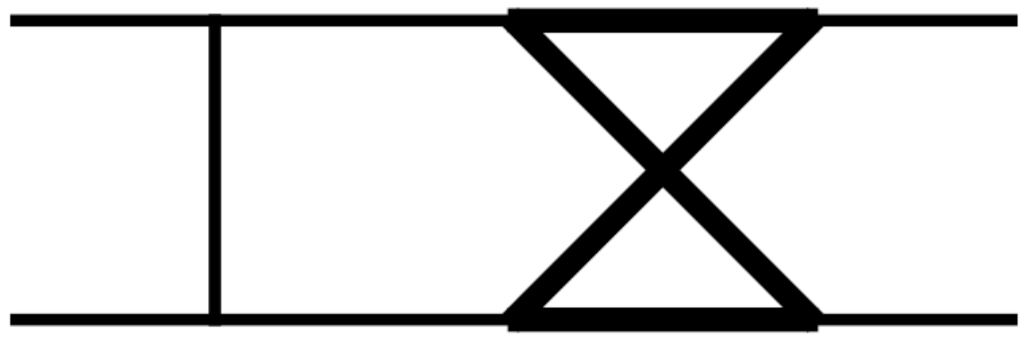
Rescale the second integral: $\begin{pmatrix} \tilde{K}_1 \\ \tilde{K}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\epsilon} \end{pmatrix} \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$

Still a part not proportional to ϵ $\quad -\left(24s\varpi_0^2 + 2s(s+24m^2)\varpi_0(\partial_{m^2}\varpi_0)\right) - 32\epsilon s\varpi_0^2 \longrightarrow$ We need to integrate out a total derivative

$$\begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ s(s+24m^2)\varpi_0^2 & 1 \end{pmatrix} \begin{pmatrix} \tilde{K}_1 \\ \tilde{K}_2 \end{pmatrix} \longrightarrow$$

$$\partial_{m^2} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} = \epsilon \begin{pmatrix} -\frac{(s+24m^2)}{m^2(s+16m^2)} & \frac{1}{m^2 s(s+16m^2)\varpi_0^2} \\ \frac{s(s+8m^2)\varpi_0^2}{m^2(s+16m^2)} & -\frac{(s+24m^2)}{m^2(s+16m^2)} \end{pmatrix} \begin{pmatrix} J_1 \\ J_2 \end{pmatrix} + \text{inhomogeneous part (modified)}$$

Elliptic sectors NPA - $I_{\textbf{NPA}}(1,1,1,1,0,1,1,1,0)$



$$\text{MC}\left[I_{\text{NPA}}(1,1,1,1,0,1,1,1,0)\right] \propto \frac{1}{s}\int \frac{dz_5 dz_9}{P_{2,3}(z_5, z_9)}$$

$P_{2,3}(z_5, z_9)$ is quadratic in z_5 and cubic in z_9

Two residues in z_5 ⟶

$$\text{MC}\left[I_{\text{NPA}}(1,1,1,1,0,1,1,1,0)\right] \propto \frac{1}{s}\int \frac{dz_9}{(m_t^2 - t - z_9)\sqrt{P_4(z_9)}}$$

We have an extra residue in $z_9 = t - m_t^2$

$$\text{Res}_{z_9 = t - m^2}\left[\frac{1}{s}\frac{1}{(m^2 - t - z_9)\sqrt{P_4(z_9)}}\right] = \frac{1}{s\sqrt{P_4(m^2 - t)}}$$

Perfect candidate for the Abel differential of the third kind

We expect as initial basis:

- Abel differential of the first kind
- Derivative of the first one wrt internal mass
- Abel differential of the third kind
- Abel differential of the third kind

Elliptic sectors NPA - $I_{\textsf{NPA}}(1,1,1,1,0,1,1,1,0)$

MC $\left(I_{\text{NPA}}(1,1,1,1,0,1,1,1,a)\right) \propto \frac{1}{s}\int dz_9 \frac{z_9^{-a}}{(m_t^2 - t - z_9)\sqrt{P_4(z_9)}}$

Choice of the initial basis:

$$I_1 = s\,\epsilon^4\left((m_t^2 - t)I_{NPA}(1,1,1,1,1,0,1,1,1,0) - I_{NPA}(1,1,1,1,1,0,1,1,1,-1)\right)$$

$$I_2 = \epsilon^4 s\,\partial_{m^2}\left((m^2 - t)I_{\textsf{NPA}}(1,1,1,1,0,1,1,1,0) - I_{\textsf{NPA}}(1,1,1,1,0,1,1,1,-1)\right)$$

$$I_3 = \epsilon^4 s\left((m^2 - t)I_{\textsf{NPA}}(1,1,1,1,0,1,1,1,-1) - I_{\textsf{NPA}}(1,1,1,1,0,1,1,1,-2)\right)$$

$$I_4 = \epsilon^4 s\sqrt{P_4(m^2 - t)}I_{\textsf{NPA}}(1,1,1,1,0,1,1,1,0)$$

The procedure introduces a new transcendental function: $G(s,t,m_t^2) = \int^{m_t^2} dx \frac{s(s+2t)\sqrt{P_4(x-t)}}{(t(s+t)-4sx)^2}\varpi_0(s,x)$

Helicity Amplitudes and Iterated Integrals

We obtain a fully ε-factorised basis :

$$d\underline{f} = \epsilon \left[\sum_{i=1}^{74} G_i \omega_i \right] \underline{f}$$

- ❖ 12 letters are purely rational
- ❖ 45 letters are algebraic
- ❖ 17 contains kernels of elliptic functions

17 contains kernels of elliptic functions → 4 are modular letters and 13 mix the periods of the elliptic curve

We substitute the MIs as iterated integrals in the amplitudes :

- ❖ All the iterated integrals with elliptic kernels identically cancel in the poles of the bare amplitudes
- ❖ A large number of algebraic letters drop

→ Analytic expression for the poles in terms of weight 3 iterated integrals, depending only on the one-loop letters

Helicity Amplitudes - Renormalisation

Mixed renormalisation scheme:

- $\overline{MS}$ scheme for massless quarks
- On-shell scheme for the heavy quark

Bare helicity amplitudes

- UV and IR poles for gg-channel and UV poles for $q\bar{q}$-channel
- The poles are polylogarithmic

Renormalization

Strong coupling

Mass (gg-channel)

Quark/gluon wave function

The poles in the bare amplitudes are not elliptic (rewriting the amplitudes in terms of iterated integrals)

$$\Omega_{qq}^{(2,fin)} = \Omega_{qq}^{(2,UV)}$$

$$\Omega_{gg}^{(2,fin)} = \Omega_{gg}^{(2,UV)} - I_{gg}^{(1)}\Omega_{gg}^{(1,UV)} \qquad \Omega_{gg}^{(1,fin)} = \Omega_{gg}^{(1,UV)}$$

$$I_{gg}^{(1)} = -\frac{e^{\gamma_E\epsilon}}{\Gamma(1-\epsilon)}\left(\frac{C_A}{\epsilon^2}+\frac{\beta_0}{\epsilon}\right)\left(\frac{-s-i0^+}{\mu_R}\right)^{-\epsilon}$$

[S.Catani]

Helicity Amplitudes - Finite Remainders

Finite remainders in terms of 154 MIs $\longrightarrow$ Extra relations between MIs

Analytical solutions for the helicity amplitudes in terms of iterated integrals

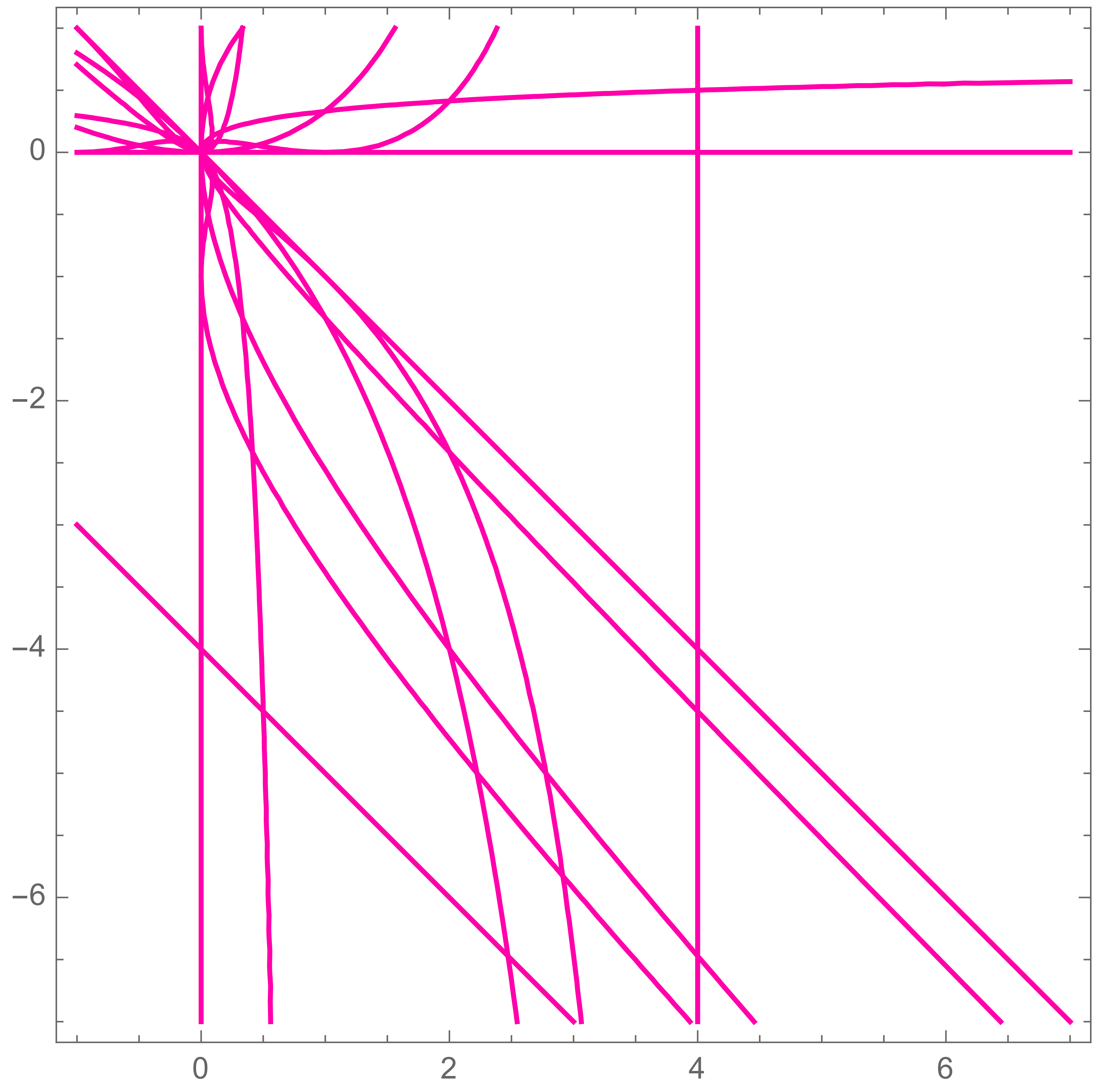
- $gg \to \gamma\gamma$:
  - All the algebraic letters appear in the finite remainder
  - 2 rational letters drop
  - 6 of the elliptic letters drop $\longrightarrow$ All the letters with ϖ_0^2 at the numerator drop (or mix of ϖ_0 and G)
    - 1 modular letter
    - 5 elliptic letters
- $q\bar{q} \to \gamma\gamma$:
  - Only 4 rational and 26 algebraic letters contribute to the finite remainders
  - Another elliptic letter drop

Simplifications at integral and amplitude level

Series expansion & numerical evaluation

We want to efficiently evaluate the helicity amplitudes numerically

- Usual approaches → Numerical evaluation on the fly producing series expansions in different phase-space points
- Our strategy → Series expansion representation for the whole amplitude



Small number of expansions

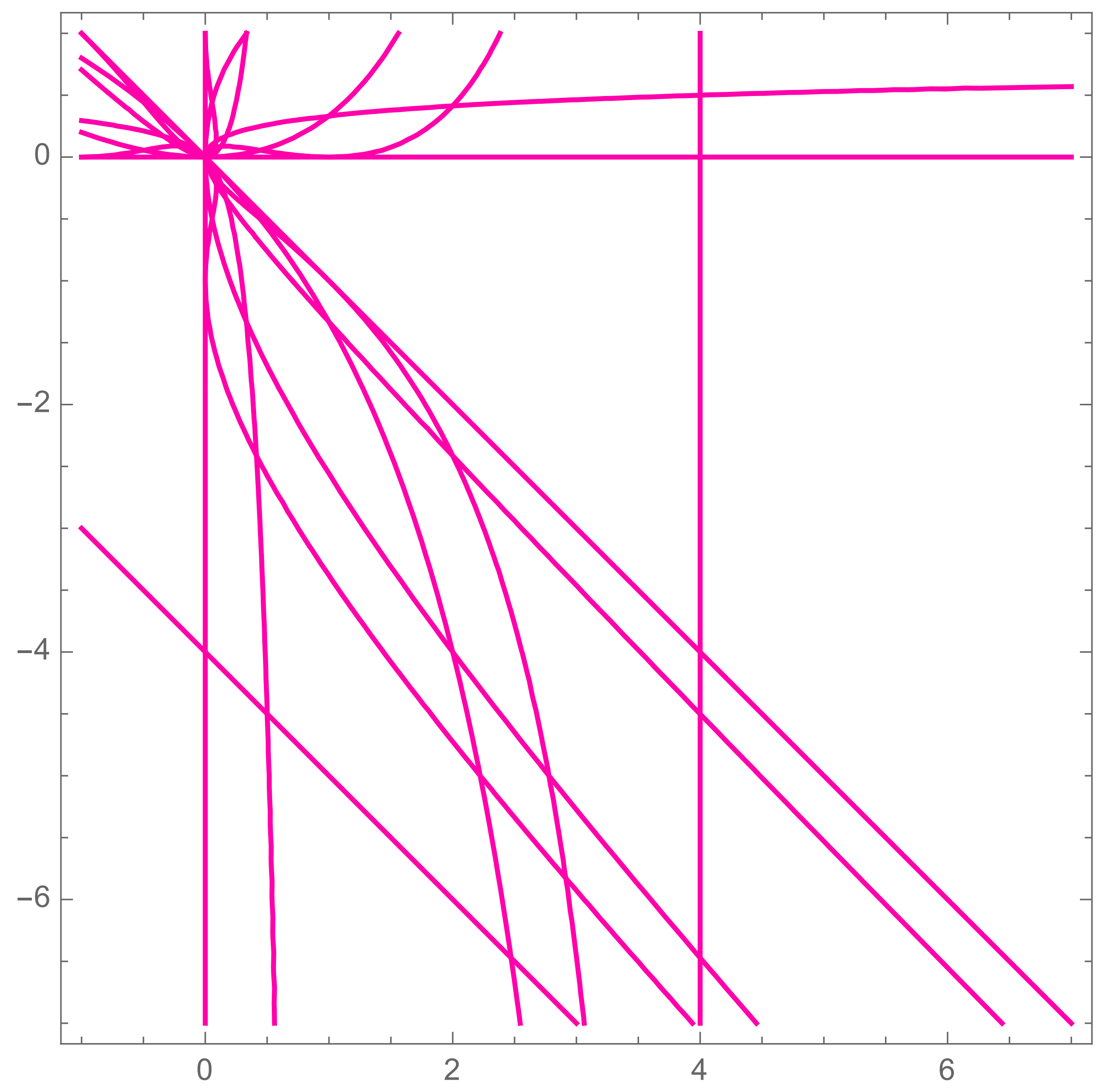
Large mass expansion | Around threshold | Small mass expansion

To consistently expand around a regular singular point the DEs:

- Normal crossing divisors
- Blow-up

Large mass expansion

The point m^2 corresponds to $(s, t) = (0,0)$



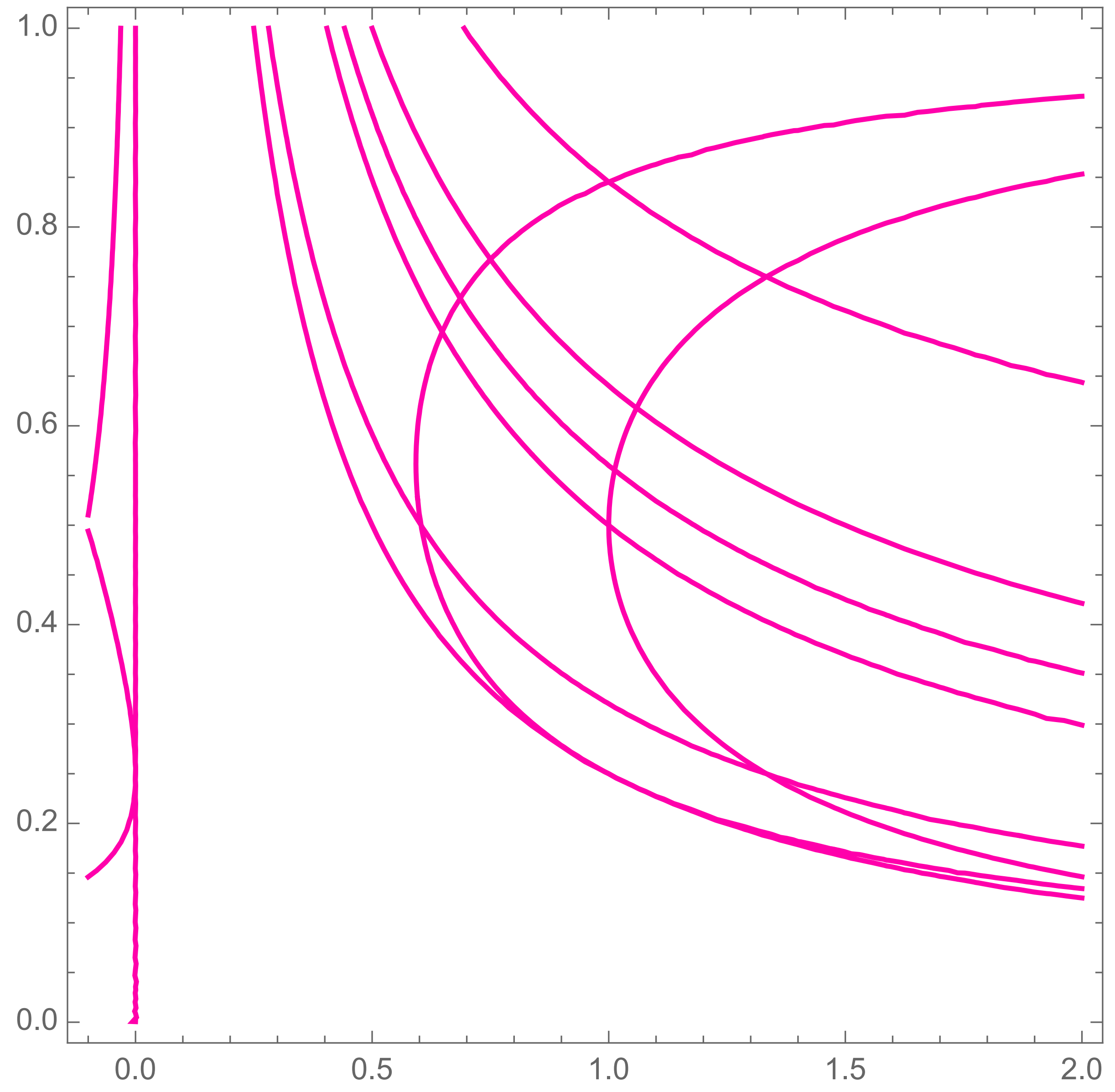
A lot of singular lines around that point cross tangentially to each other

Blow-up:

$$x_1 = -\frac{tm^2}{s^2}$$

$$x_2 = \frac{s}{4m^2}$$

Good set of variables for the double expansion



Singular lines are disentangled in the expansion point

Denominators $\sim \dfrac{1}{s+t}$ ⟶ Problem with the expansion order

Expansion at $s = 4m^2$

$s = 4m^2$ is a regular point on the elliptic curve

The elliptic curve does not degerate at this point

$\varpi_0(s, m^2)$ does not depends on t, but $G(s, t, m^2)$ ⟶ The expansion of the amplitudes is non-trivial

Double series expansion in $(s, t) = (4m^2, 0)$ to avoid the integration problem

We don't need a blow-up here: only two singular lines cross this point

Expansion variables:

$$y_1 = 4 - \frac{s}{m^2}$$

$$y_2 = -\frac{t}{m^2}$$

BC trivial in (0,0) ⟶ $\gamma(\lambda) = (4m^2\lambda, 0), \ \lambda \in [0,1]$ To transport them using the DEs in the new point

Numerical Evaluation

Large mass expansion: $(0,0)$

Symmetry under the exchange of the photons : $p_3 \leftrightarrow p_4 \quad (t \leftrightarrow u)$

Expansion around threshold: $(4m^2,0)$

$(s,u) = (4m^2,0) \longrightarrow (s,t) = (4m^2,-4m^2)$ Same for the large mass expansion

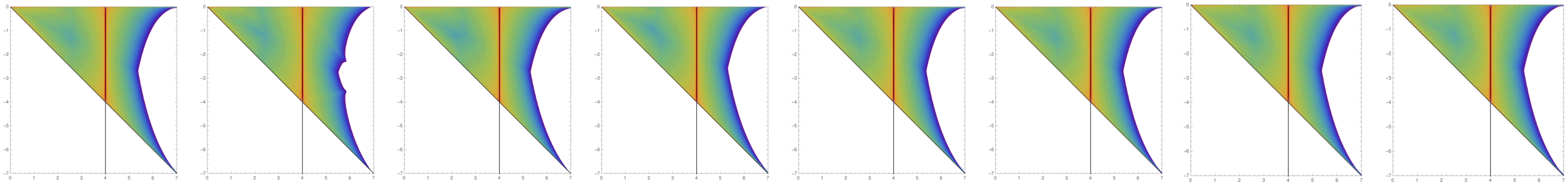
We have the expansion around these four points to cover large part of the phase-space

Evaluation time: $0.02s - 0.07s$ On a single core For the evaluation of a helicity coefficient!

s/m^2	$13/10$	$23/10$	$28/9$	$11/3$	$22/5$	$51/10$
t/m^2	-3/5	-1	-1/10	-5/2	-3/5	-11/10
α_h	3.0×10^{-9}	4.1×10^{-7}	2.3×10^{-17}	1.0×10^{-21}	5.2×10^{-21}	3.5×10^{-8}
β_h	8.4×10^{-12}	7.7×10^{-9}	2.1×10^{-18}	6.7×10^{-23}	7.0×10^{-22}	1.5×10^{-9}
γ_h	5.7×10^{-11}	2.9×10^{-8}	3.3×10^{-19}	6.8×10^{-21}	3.3×10^{-22}	2.5×10^{-9}
δ_h	3.0×10^{-9}	4.1×10^{-7}	2.3×10^{-17}	1.0×10^{-21}	5.2×10^{-21}	3.5×10^{-8}

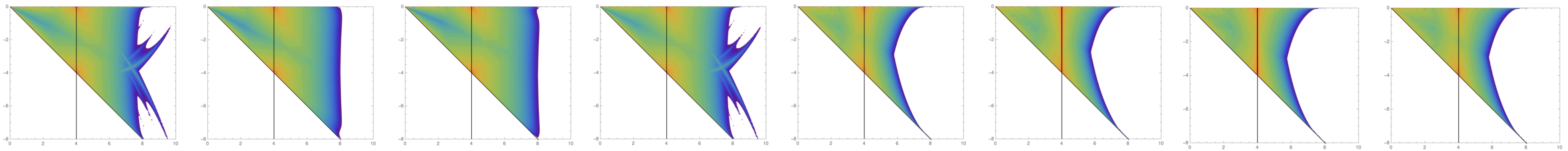
Numerical Evaluation

$gg \to \gamma\gamma$

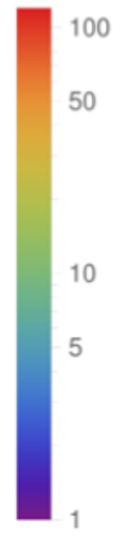


(a) $f^{(2,b)}_{++++}$ (b) $f^{(2,b)}_{--++}$ (c) $f^{(2,b)}_{-+-+}$ (d) $f^{(2,b)}_{+--+}$ (e) $f^{(2,b)}_{-+++}$ (f) $f^{(2,b)}_{+-++}$ (g) $f^{(2,b)}_{++-+}$ (h) $f^{(2,b)}_{+++-}$

$q\bar{q} \to \gamma\gamma$



(a) α_l (b) β_l (c) γ_l (d) δ_l (e) α_h (f) β_h (g) γ_h (h) δ_h



Number of digits of precision

Thank you for your attention!