Light-quark EW contributions to $gg \to HH$

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[2503.16620]

From the Higgs boson to the Higgs sector

$$\frac{1}{2}\left(\partial^{\mu} H\right)\left(\partial_{\mu} H\right)+\frac{1}{2} m_H^2 H^2+\lambda_3 H^3+\lambda_4 H^4$$

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Trilinear Higgs coupling

- "Occam's razor" potential
- HL-LHC: $0.1 < \kappa_3 < 2.3$

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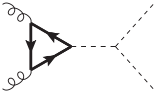

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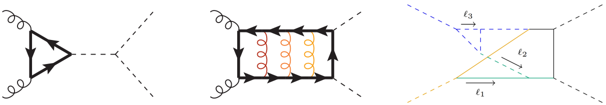

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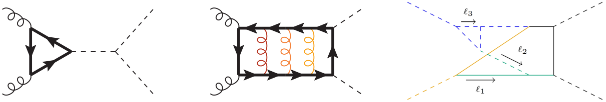

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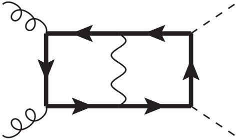
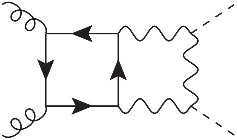
EW effects now relevant

Tackling EW

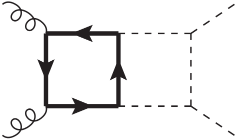
The EW family



Yukawa



Pure weak



Self-coupling

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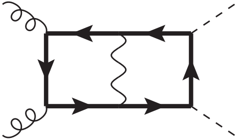
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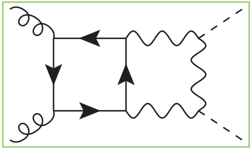
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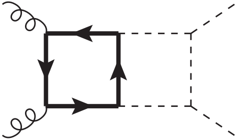
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- Self-contained: gauge invariant & finite
- λ_3 interference
- Light-quark contributions

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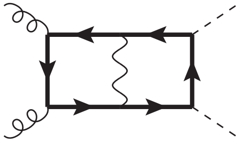

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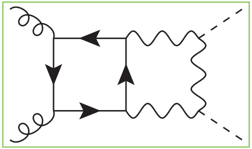

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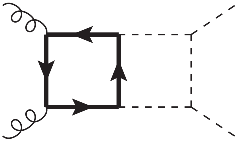

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  - No axial contributions
  - No elliptic curves, but algebraic dlogs
  - Weight drop in $\mathcal{M}^{++}$

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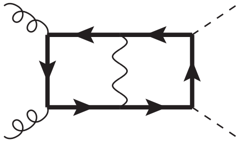

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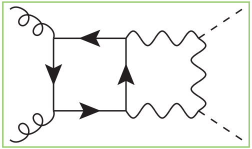

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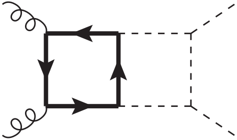

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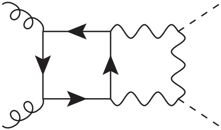
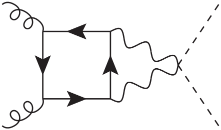
Analytic computation of light-quark pure weak corrections

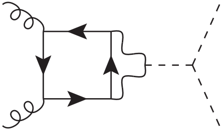
Light-quark pure weak corrections



VVV

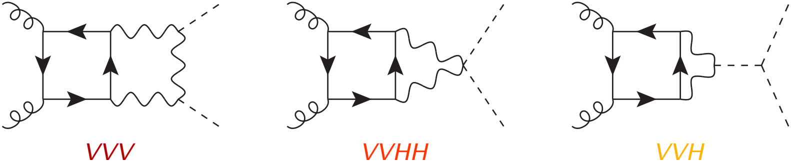


VVHH



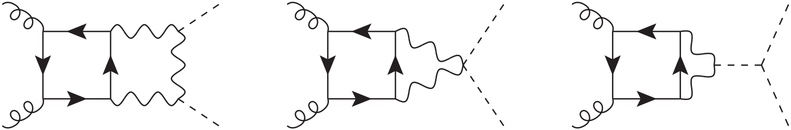
VVH

Light-quark pure weak corrections



$$\mathcal{M} \propto \begin{cases} \mathcal{F}^{++} = \mathcal{A}_1 + \mathcal{A}_3 + \frac{3m_H^2}{s - m_H^2}\mathcal{A}_3 \\ \mathcal{F}^{+-} = \mathcal{A}_2 \end{cases}$$

Light-quark pure weak corrections



VVV VVHH VVH

$$(\bar{q}qV)^2 \quad \Rightarrow \quad (g_V^2 + g_A^2)\,\mathrm{Tr}[\ldots] + g_V g_A\,\mathrm{Tr}[\gamma_5 \ldots]$$

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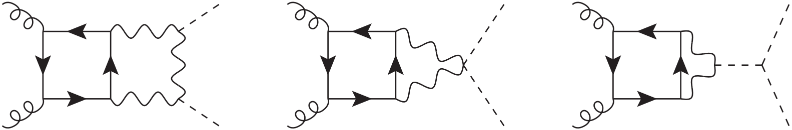

VVV *VVHH* *VVH*

$$\sum_{u,d,s,c,b} (\bar{q}qV)^2 \quad \Rightarrow \quad (g_V^2 + g_A^2)\,\mathrm{Tr}[\ldots] + \cancel{g_V g_A \mathrm{Tr}[\gamma_5 \ldots]} \quad \Rightarrow \quad \text{no } \gamma_5$$

$$|V_{ij}|^2 \qquad \frac{1}{\cos^4\theta_W}\left(\frac{5}{4} - \frac{7}{3}\sin^2\theta_W + \frac{22}{9}\sin^4\theta_W\right)$$

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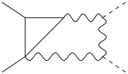
Building the $\mathcal{A}$'s

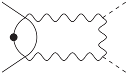
Amplitude to integrals

- qgraf: diagrams (Feynman & unitary)
- FORM: color and Dirac, γ_5, projectors
- Reduze & kira: full symbolic reduction to MIs

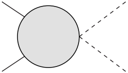
Square roots



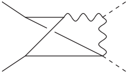
$\Rightarrow r_2, r_6$



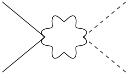
$\Rightarrow r_3, r_8$



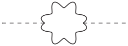
$\Rightarrow r_1$



$\Rightarrow r_7$



$\Rightarrow r_4$



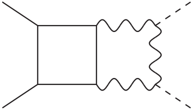
$\Rightarrow r_5$

- No rationalization
- No linear reducibility

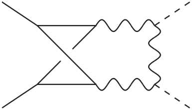
DEs in ϵ-factorized form

Differential equations for canonical MIs

74 Master Integrals



45 MIs



43 MIs

Differential equations for canonical MIs

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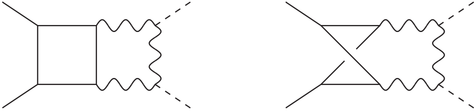

- LiteRed + FiniteFlow + DlogBasis + Magnus series

Canonical form

$$\partial_x \mathrm{J} = \epsilon A_x \mathrm{J} \qquad x = s, t, u, m_V^2$$

Differential equations for canonical MIs

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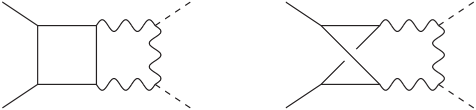

45 MIs   43 MIs

- LiteRed + FiniteFlow + DlogBasis + Magnus series
- Landau singularities, Effortless, direct integration

Canonical form

$$\mathrm{d}\,\mathrm{J} = \epsilon \left[\sum_{i=0}^{77} \mathbb{A}_i \,\mathrm{dlog}\, \alpha_i(x) \right] \mathrm{J}$$

P	$\frac{P - r_i}{P + r_i}$	$\frac{P - r_i r_j}{P + r_i r_j}$	$\frac{P - Q r_i}{P + Q r_i}$
even (30)	—— odd $(19+6)$ ——		mixed (22)

Chen Iterated Integrals

Canonical MIs

$$\mathsf{J} = \sum_k \epsilon^k \left[\mathsf{J}_0^{(k)} + \mathbb{A} \int_\gamma \mathrm{dlog}\, \alpha \, \mathsf{J}^{(k-1)} \right]$$

Chen Iterated Integrals

Canonical MIs

$$\mathrm{J} = \sum_k \epsilon^k \left[\sum_l \mathbb{A}_{i_1} \dots \mathbb{A}_{i_l} \mathrm{J}_0^{(k-l)} \, [\alpha_{i_1}, \dots, \alpha_{i_l}]_0 \right]$$

Chen Iterated Integrals

- Shuffle algebra $\Rightarrow$ Symbol map
- Integration path γ: straight line from 0 to x
- Starting point singularity: $[x, \dots, x]_0 := \frac{1}{n!} \log^n x$

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Numerical evaluation via DiffExp

- Differentiation w.r.t line parameter

$$\mathrm{d} \left[\alpha_{i_1}, \dots, \alpha_{i_{n-1}}, \alpha_{i_n}\right]_0 = \left[\alpha_{i_1}, \dots, \alpha_{i_{n-1}}\right]_0 \, \mathrm{d}\, \alpha_{i_n}$$

- Boundary values at 0

$$\left[\alpha_{i_1}, \dots, \alpha_{i_{n-1}}, \alpha_{i_n}\right]_0 (x = 0) = 0$$

Large-mass Expansion

Integration constants matched to

Large-mass Limit

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- $s, t, u, m_H^2 \ll m_V^2$

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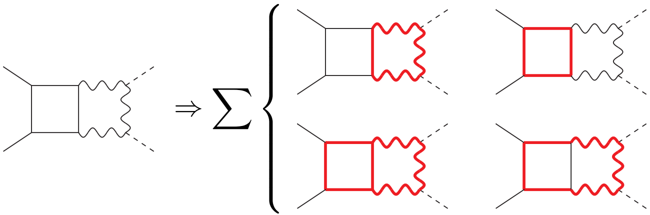

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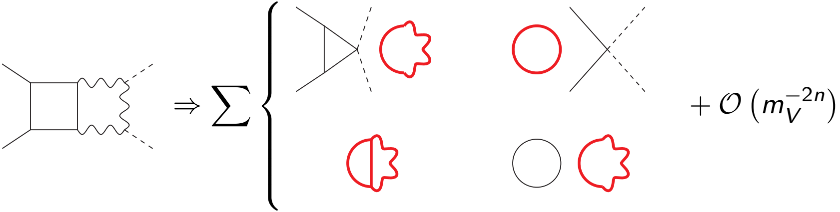

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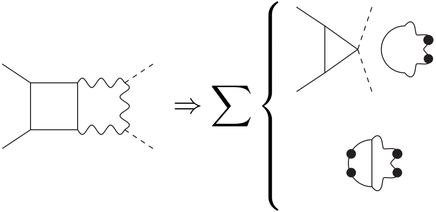

$$+\,\mathcal{O}\left(m_V^{-2n}\right)$$

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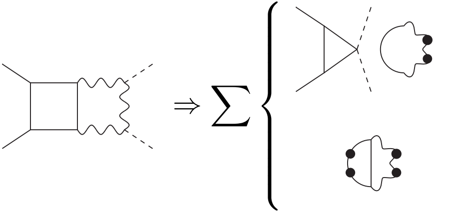

$$\Rightarrow \sum \left\{ \cdots \right. + \mathcal{O}\left(m_V^{-2n}\right)$$

- Few non-zero sectors
- Tadpoles & 1-loops: analytic result
- LME for W almost always zero
- Direct match to $x = 0$ in CIIs

Improving the basis

Weighted rotations

- CIIs are a vector space

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- J: overabundant or scattered letters

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Weighted rotations

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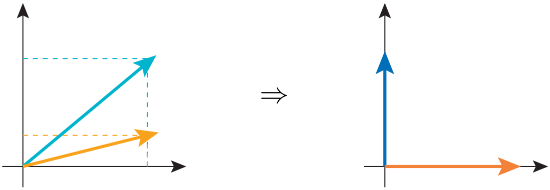
$$RJ = W$$

- W: orthogonality in ϵ and minimal set of CIIs weight-by-weight



Improving the basis

Weighted rotations

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Weighted functions

- Amplitude: linear combination of independent transcendental functions

$$\mathrm{W} = \sum \epsilon^k w^{(k)}$$

Improving the basis

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Weighted functions

- Amplitude: linear combination of independent transcendental functions

$$\mathrm{W} = \sum \epsilon^k w^{(k)}$$

- DEs for $w^{(k)}$: essential transcendental blocks
  - ϵ-independent
  - BCs from LME weight by weight

The amplitude

Amplitude to ϵ^2 & weight 6

The amplitude

- W basis

Amplitude to ϵ^2 & weight 6

	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$
Poles	ϵ^0	ϵ^0	ϵ^0
MIs	$38, 32, 2$	$67, 5, 0$	$7, 2, 0$

The amplitude

Amplitude to ϵ^2 & weight 6

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- w functions

	$\mathcal{A}_1$	$\mathcal{A}_2$	$\mathcal{A}_3$
Letters	66	72	4
Maximum weight	$4, 5, 6$	$4, 5, 6$	$3, 4, 5$

The amplitude

Amplitude to ϵ^2 & weight 6

- W basis

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Poles	ϵ^0	ϵ^0	ϵ^0
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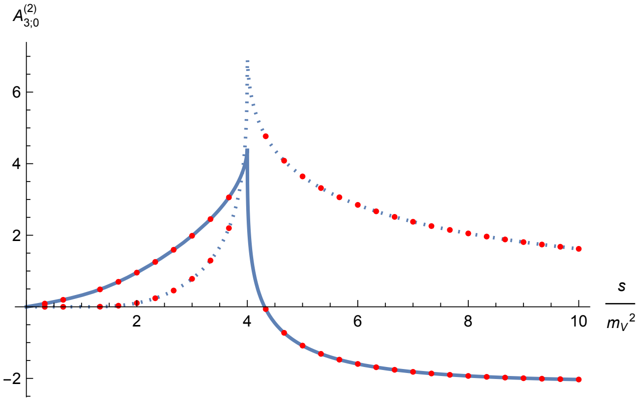
Letters	66	72	4
Maximum weight	$4, 5, 6$	$4, 5, 6$	$3, 4, 5$

- Hierarchy of complexity
  - 3-point amplitude: $++$ only, weight drop
  - $++$: 4-point, -2 MIs, $t \leftrightarrow u$ symmetric
  - $+-$: 4-point, -2 MIs, no symmetry

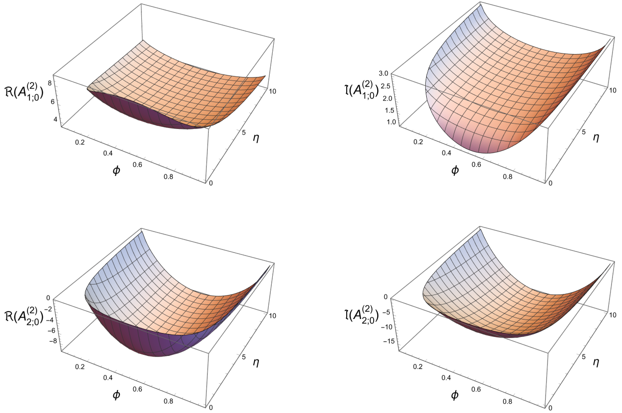
Triangles: $\mathcal{A}_3$

- Only r_4 $\Rightarrow$ log, Li, GPL
- Thresholds: $s = m_V^2$ & $s = 4m_V^2$, both below production



Boxes: $\mathcal{A}_1$ & $\mathcal{A}_2$

- Smooth: thresholds below production
- Double evolution: LME → Base point → Physical region



$$\eta = \frac{s}{4m_H^2} - 1 \qquad \phi = \frac{m_H^2 - t}{s}$$

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Future Projects

- Implementation of the amplitude in `ggHH` for `Powheg-Box`
- $q\bar{q} \to HH$
- Mixed QCD-EW corrections

Thank you for your attention