Scattering Amplitudes in Liverpool

26-28 March 2025

Practicalities

- Funding
  - *School of physical sciences*
  - *Leverhulme grant LIP-2021-01*
  - *Local support by Math & Physics department*

Lauren Burton - Julie Clark - Joanna Seed

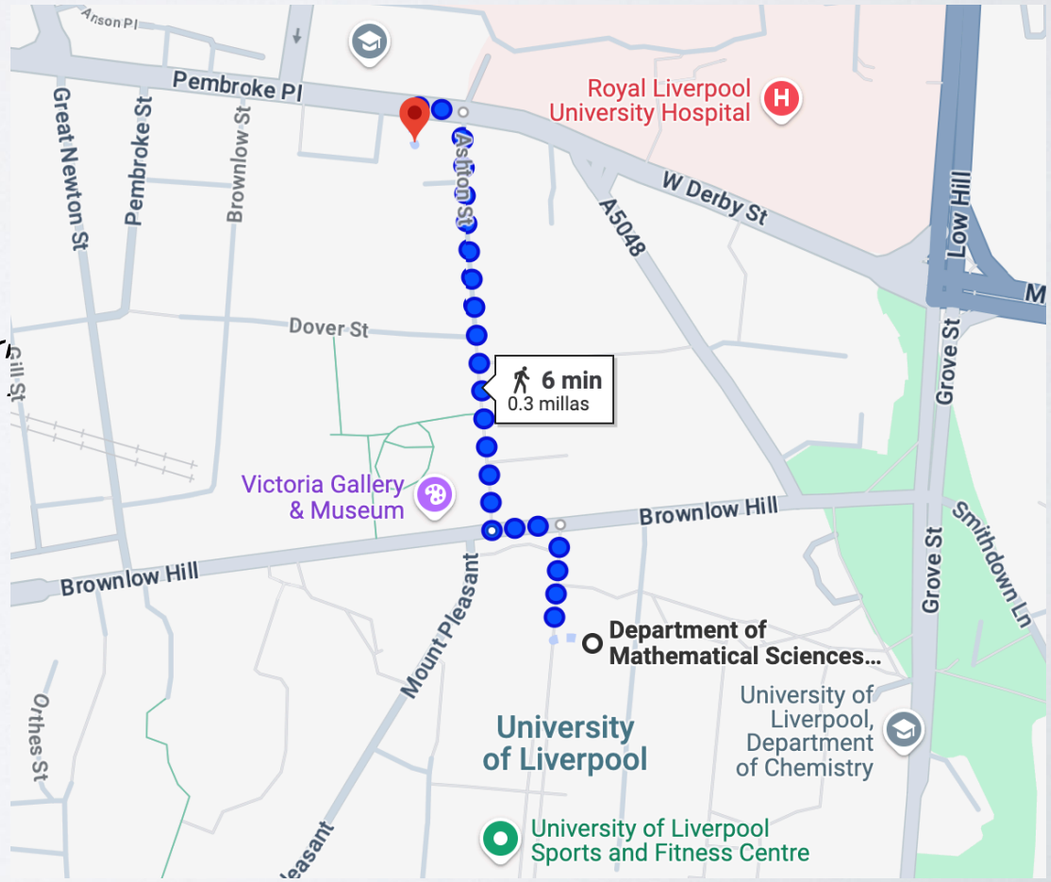
- Location
  - *Department of Mathematical Sciences (Today)*
  - *Cedar House (Thursday and Friday)*

Practicalities

- Funding
  - *School of physical sciences*
  - *Leverhulme grant LIP-2021-01*
  - *Local support by Math & Physics department*

Lauren Burton - Julie Clark - Joanna Seed

- Location
  - *Department of Mathematical Sciences (*
  - *Cedar House (Thursday and Friday)*



What is the workshop about?

Efficient evaluation of multi-loop scattering amplitudes in gauge theories
… but not a regular workshop

- *Motivate collaborative efforts*
- *State-of-the-art calculations*
- *From high-energy to low-energy physics*

An Analytic Computation of Three–Loop Five–Point Feynman Integrals

Yuanche Liu,[1,*] Antonela Matijašić,[2,†] Julian Miczajka,[3,‡] Yingxuan Xu,[4,§] Yongqun Xu,[5,¶] and Yang Zhang[5,6,**]

Feynman integral reduction: balanced reconstruction of sparse rational functions and implementation on supercomputers in a co-design approach

Alexander Smirnov[1] and Mao Zeng[2]

Analytic two-loop amplitudes for $q\bar{q} \to \gamma\gamma$ and $gg \to \gamma\gamma$ mediated by a heavy-quark loop

Matteo Becchetti,[a] Federico Coro,[b,c] Christoph Nega,[d] Lorenzo Tancredi,[d] Fabian J.

LINE: Loop Integrals Numerical Evaluation

Renato Maria Prisco,[a] Jonathan Ronca,[b] Francesco Tramontano[a]

One-Loop QCD Corrections to $\bar{u}d \to t\bar{t}W$ at $\mathcal{O}(\varepsilon^2)$

Matteo Becchetti,[a] Maximilian Delto,[b,c] Sara Ditsch,[b,d] Philipp Alexander Kreer,[b] Mattia Pozzoli,[a] Lorenzo Tancredi[b]

Graded transcendental functions: an application to four-point amplitudes with one off-shell leg

Thomas Gehrmann,[a] Johannes Henn,[b] Petr Jakubčík,[a] Jungwon Lim,[b] Cesare Carlo Mella,[c] Nikolaos Syrrakos,[c] Lorenzo Tancredi,[c] and William J. Torres Bobadilla[d]

Two-loop light-quark Electroweak corrections to Higgs boson pair production in gluon fusion

Marco Bonetti,[a,b] Philipp Rendler,[b] and William J. Torres Bobadilla[c]

Feynman Integral Reductions by Intersection Theory with Orthogonal Bases and Closed Formulae

Giulio Crisanti,[a,b,c] Sid Smith[a,b,d]

Strategies of our group

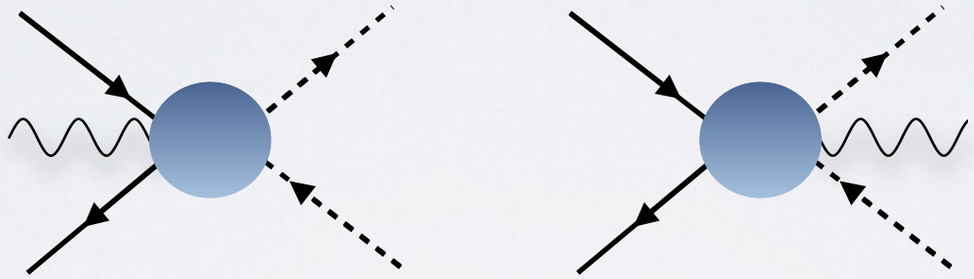
- Study analytic properties of Feynman integrals
  Grading of functions :: analytic cancellations at integral level

- Fast numerical evaluation
  Compute what we really want to evaluate

- Careful IBP generation
  Reconstruct amplitudes and/or ε-expansion

Low-energy physics

- Unravel physical properties of the muon $a_\mu = g - 2$
- Clean up tensions between experimental efforts through theoretical predictions
- Make use of $e^+e^- \to$ hadrons experiments to extract a_μ^{HVP}



$e^+e^- \to \pi^+\pi^-\gamma$

- Main focus on radiative return experiments
- Improve theoretical predictions
  —> From NLO to NNLO (and beyond?)

Towards $e^+e^- \to F^+F^-\gamma$ *@ NNLO*

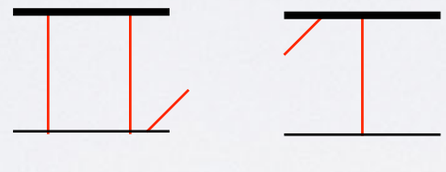
Anatomy @ LO

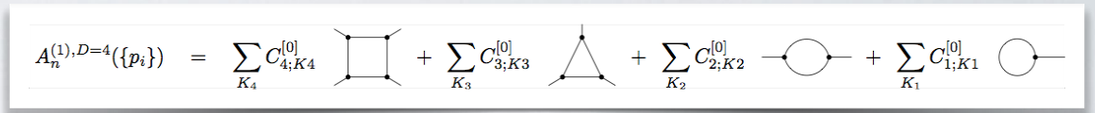
- Born matrix element
  tree-level & n-pt process

Anatomy @ NLO

- Real contribution tree-level (*n+1*)-particles

- Virtual Contribution one-loop (*n+1*)-particles





- ☑ Automated one-loop Feynman integral & phase-space evaluation
- ☑ IR subtraction schemes under control
- ☑ Efficient numerical evaluation (MC friendly)

Towards $e^+e^- \to F^+F^-\gamma$ *@ NNLO*

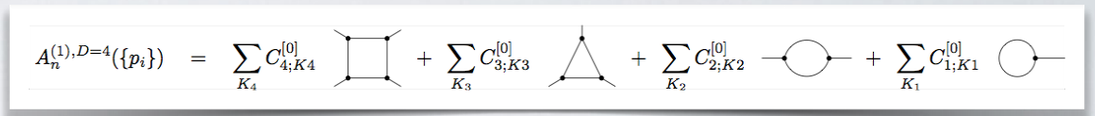
Anatomy @ LO

- Born matrix element
  tree-level & n-pt process

Anatomy @ NLO

- Real contribution tree-level (*n+1*)-particles

- Virtual Contribution one-loop (*n+1*)-particles

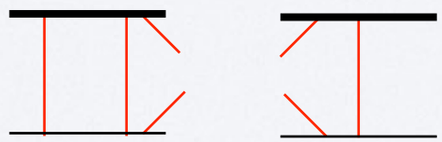


- ☑ Automated one-loop Feynman integral & phase-space evaluation
- ☑ IR subtraction schemes under control
- ☑ Efficient numerical evaluation (MC friendly)

Anatomy @ NNLO

- Real-Real contribution
  Tree-level (*n+2*)-particles

- Real-Virtual Contribution
  one-loop (*n+1*)-particles



- Virtual-Virtual Contribution
  two-loop *n*-particles

- ☐ Harder (but doable) phase-space integration
- ☐ Extend numerical evaluation of one-loop Feynman integrals
- ☐ Basis of two-loop Feynman integral not known

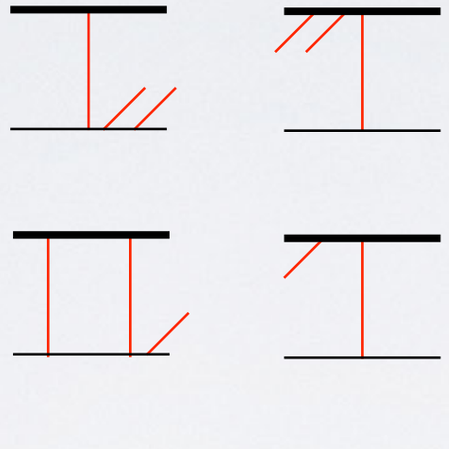
Towards $e^+e^- \to F^+F^-\gamma$ *@ NNLO*

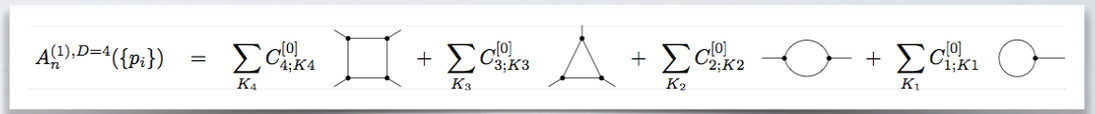
Anatomy @ LO

- Born matrix element
  tree-level & n-pt process

Anatomy @ NLO

- Real contribution tree-level (*n+1*)-particles
- Virtual Contribution one-loop (*n+1*)-particles

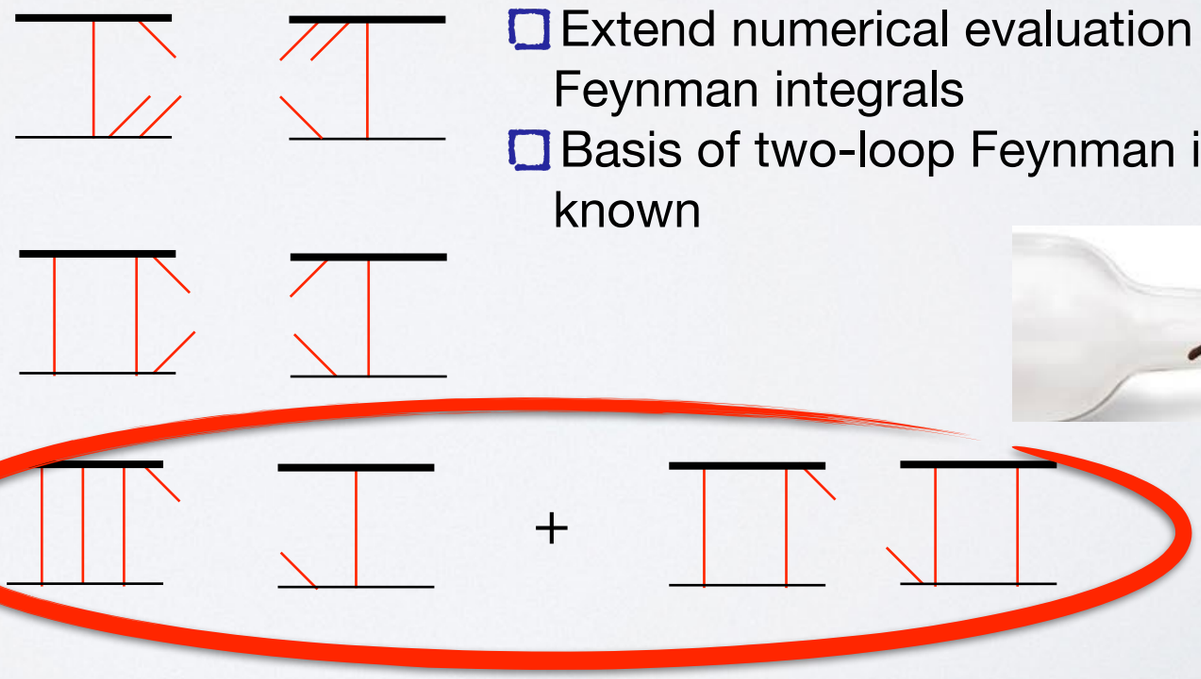




- ☑ Automated one-loop Feynman integral & phase-space evaluation
- ☑ IR subtraction schemes under control
- ☑ Efficient numerical evaluation (MC friendly)

Anatomy @ NNLO

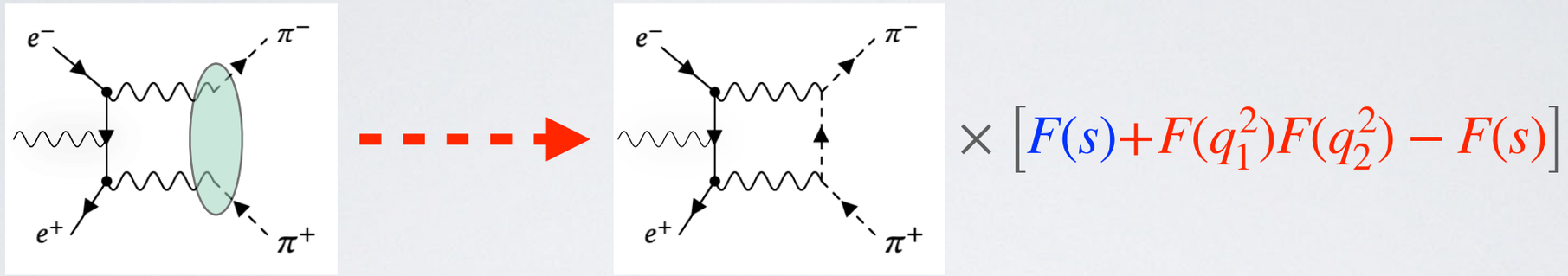
- Real-Real contribution
  Tree-level (*n+2*)-particles
- Real-Virtual Contribution
  one-loop (*n+1*)-particles
- Virtual-Virtual Contribution
  two-loop *n*-particles



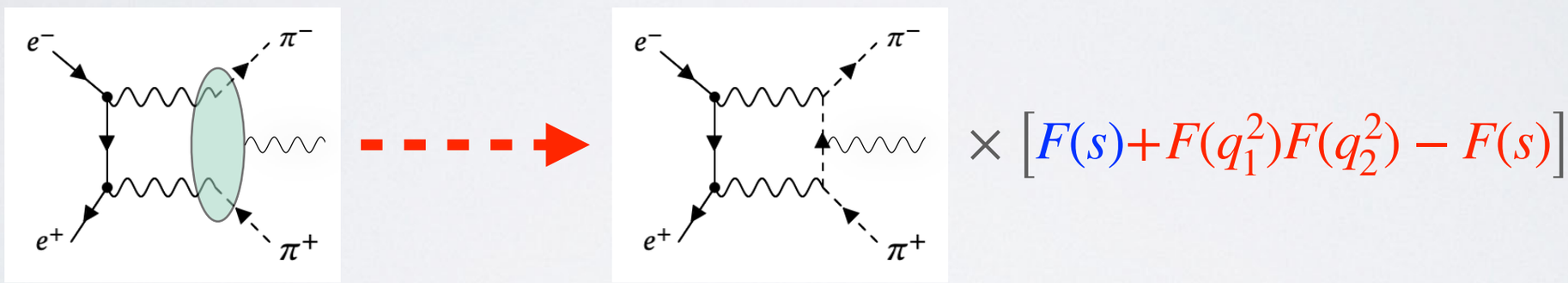
- ☐ Harder (but doable) phase-space integration
- ☐ Extend numerical evaluation of one-loop Feynman integrals
- ☐ Basis of two-loop Feynman integral not known

$$e^+e^- \to \pi^+\pi^-(\gamma)$$

- Initial & Final State radiation @ NLO



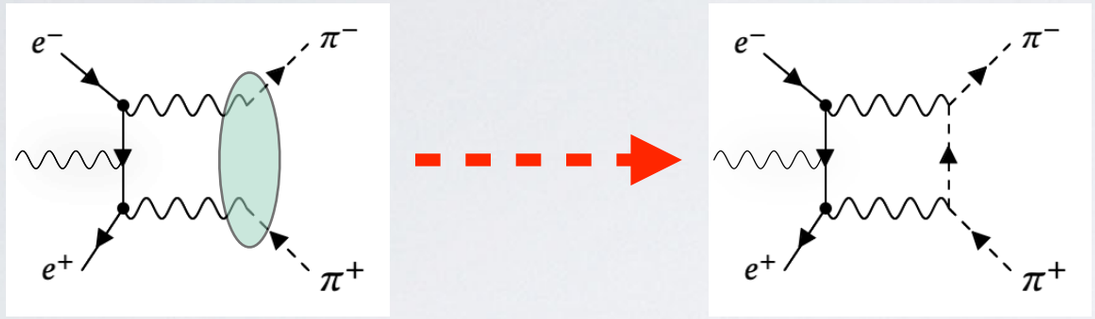
$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$



$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

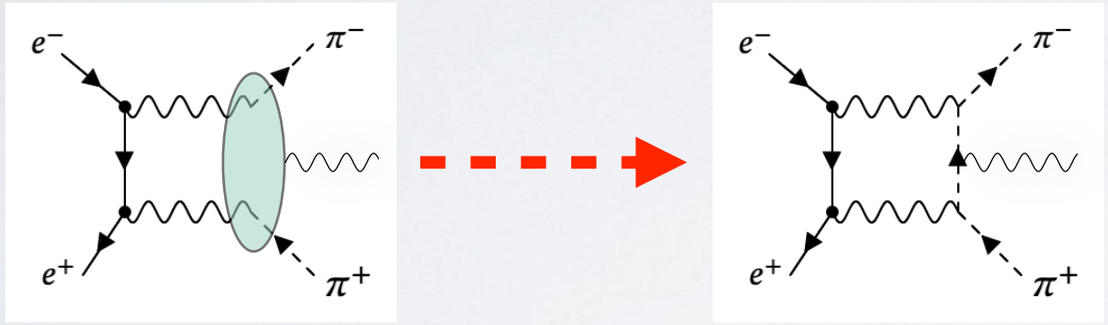
$$e^+e^- \to \pi^+\pi^-(\gamma)$$

Initial & Final State radiation @ NLO



$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

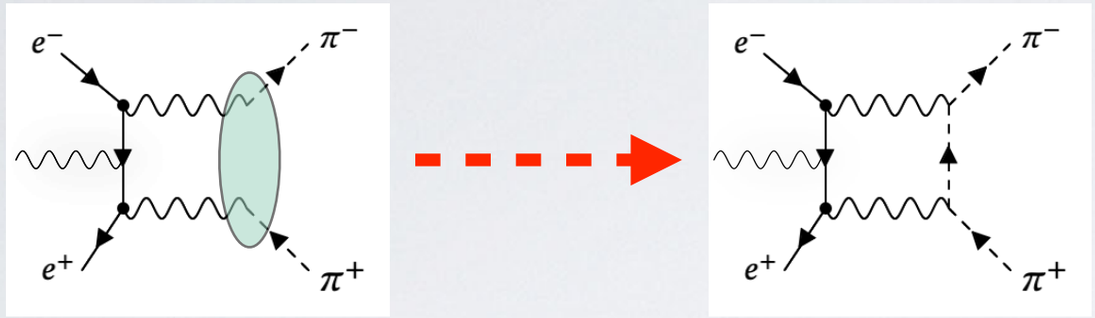
- Semi-analytic computation
- In-house implementation of FSR
- To be included in Phokhara-X



$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

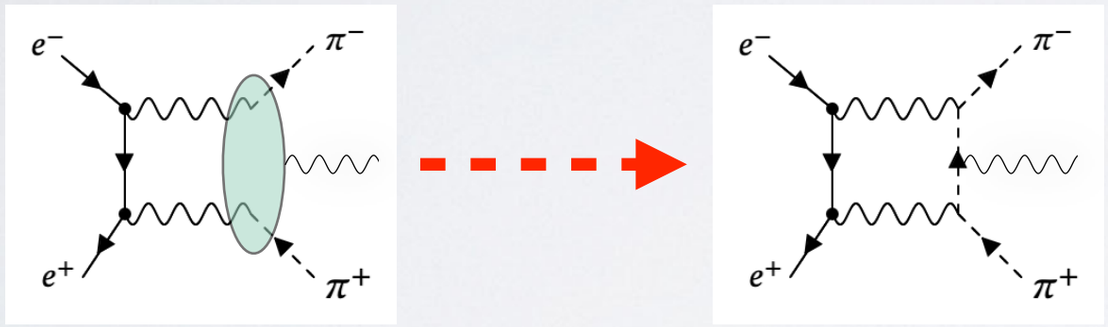
$$e^+e^- \to \pi^+\pi^-(\gamma)$$

- Initial & Final State radiation @ NLO



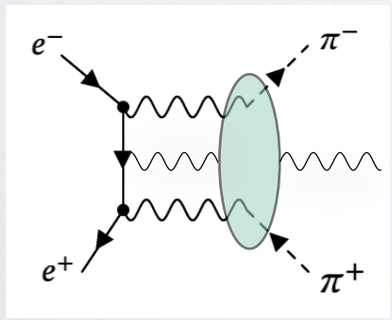
$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

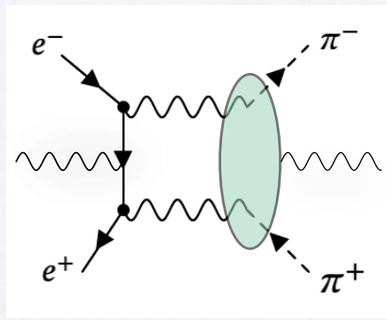
- Semi-analytic computation
- In-house implementation of FSR
- To be included in Phokhara-X

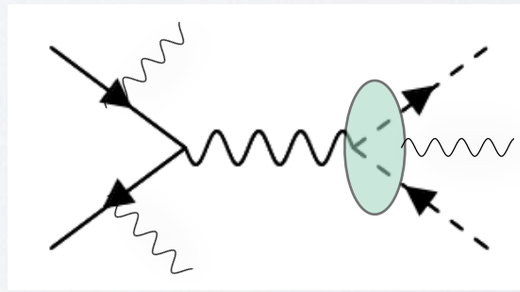


$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

- Same treatment at NNLO (VV, VR, RR) ?

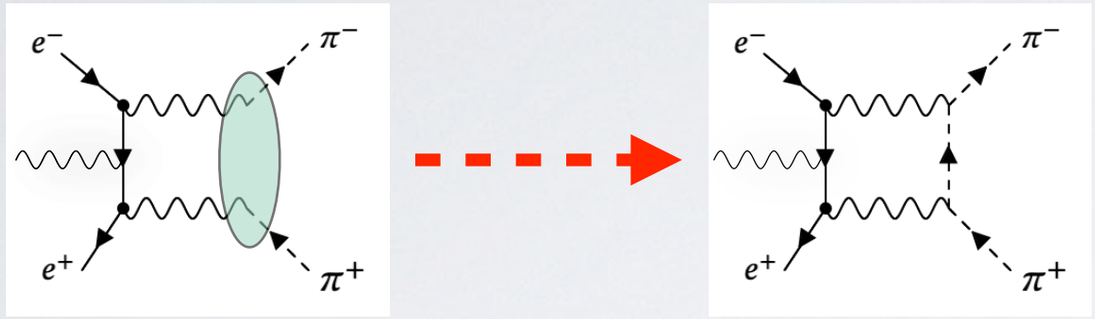






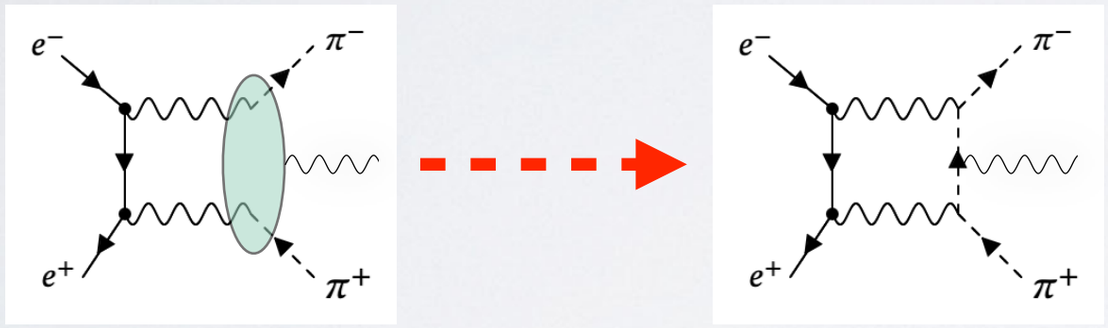
$$e^+e^- \to \pi^+\pi^-(\gamma)$$

- Initial & Final State radiation @ NLO



$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

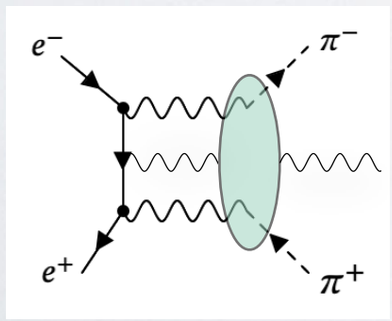
- Semi-analytic computation
- In-house implementation of FSR
- To be included in Phokhara-X

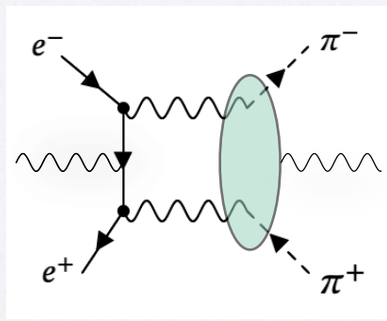


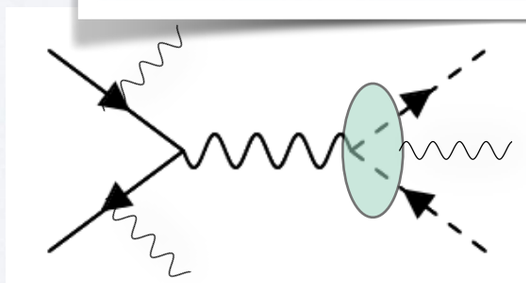
$$\times \left[F(s)+F(q_1^2)F(q_2^2) - F(s)\right]$$

- Same treatment at NNLO (VV, VR, RR) ?

- Fully numerical computation
- Use Collier
- Focus on RR & RV (at the moment)



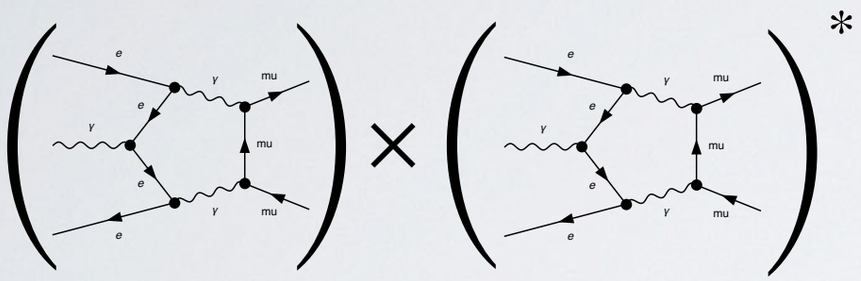




Radiative return processes @ NNLO

- Ultimate goal :: efficient evaluation of $e^+e^- \to \mu^+\mu^-\gamma$ and $e^+e^- \to \pi^+\pi^-\gamma$

Virtual-Virtual Contribution



Extend evaluation of one-loop Feynman integrals

Compute two-loop Feynman integrals

- Strategy :: chop problem into smaller pieces

  - ☆Divide the amplitude in terms of gauge invariant pieces
  - ☆Take into account the formal properties of Feynman integrals
  - ☆Make use of simple variables for their calculation
  - ☆Profit from relations at integrand and integral level
  - ☆Get experimental insights

Radiative return processes @ NNLO

Amplitude generation

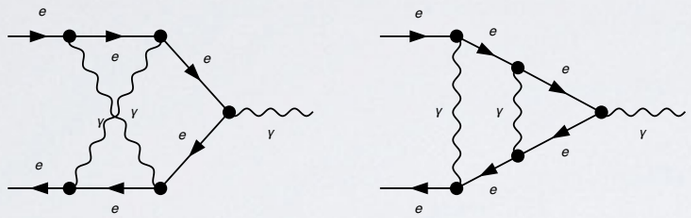
Algebraic decomposition

Loop integral evaluation

MC input

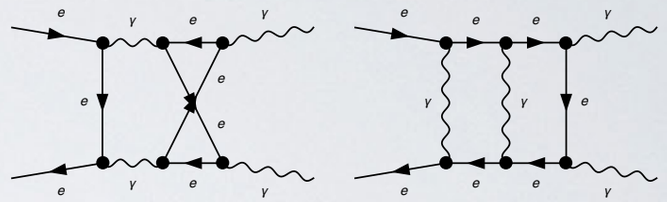
- Two-loop gauge invariant pieces

$f^+f^- \to \gamma^* \to F^+F^- + \gamma$



Easy (m_f^2, s)

$f^+f^- \to \gamma\,\gamma^* \to F^+F^-$



Normal (s, t, m_e^2, q^2)

- Preliminary results for planar contributions w/o closed fermion loops.

$f^+f^- \to F^+F^-\,\gamma$

Hard $(s_{12}, s_{23}, s_{34}, s_{45}, s_{51}, m_f^2, m_F^2)$

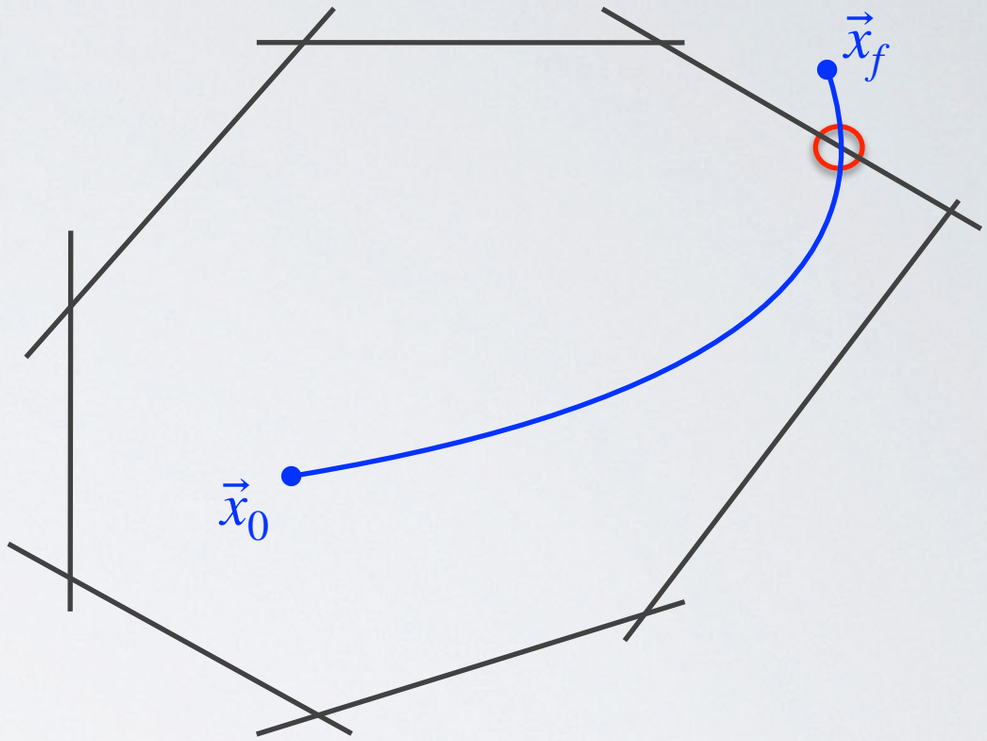
- Very very hard!

Radiative return processes @ NNLO

- Evaluation of Feynman integrals by the method of differential equations

$$\partial_x \vec{I}(\vec{x};\epsilon) = A_x(\vec{x};\epsilon)\,\vec{I}(\vec{x};\epsilon)$$

$\vec{x}$ (kinematic invariants)



- ☑ When possible find a canonical basis $\vec{J} = R\vec{I}$ **[Henn 2013]**
- ☑ Solve DEQ along the path **[Moriello 2019]**
- ☑ Get boundary constants $\vec{J}_0$ analytically or numerically **[AMFlow 2017]**
- ☑ Account for analytic continuations when crossing regions

- *Currently working on C++ implementation*
  Input :: DEQ+ boundary constants

What is this workshop about?

* *Wednesday afternoon*

Analytic tools

MATH-103, Department of Mathematical Sciences 14:30 - 16:00

Tea & coffee

Department of Mathematical Sciences 16:00 - 16:30

Numerical tools

MATH-103, Deparment of Mathematical Sciences 16:30 - 18:00

* *Thursday "early" morning*

Light talks

Forshall Room (Room 427), Cedar House 09:00 - 10:30

* *Thursday "late" morning*

Moderated discussion

Forshall Room (Room 427), Cedar House 11:00 - 12:45

What is this workshop about?

* *Wednesday afternoon*

From Impossible to Doable: Complete Function Space for Two-Loop Six-Point Scattering Amplitudes — *Antonela Matijašić*
MATH-103, Department of Mathematical Sciences — 14:30 - 15:00

Progress on two-loop integrals for top-pair production plus a W boson — *Mattia Pozzoli*
MATH-103, Department of Mathematical Sciences — 15:00 - 15:30

ε-factorised form and numerical evaluation for elliptic Feynman integrals in diphoton production — *Federico Coro*
MATH-103, Department of Mathematical Sciences — 15:30 - 16:00

Tea & coffee
Department of Mathematical Sciences — 16:00 - 16:30

Efficient supercomputer-scale IBP reduction for Feynman integrals — *Mao Zeng*
MATH-103, Deparment of Mathematical Sciences — 16:30 - 17:00

Loop Integral Numerical Evaluation with LINE — *Jonathan Ronca*
MATH-103, Deparment of Mathematical Sciences — 17:00 - 17:30

Fast evaluation of Feynman integrals for MC generators — *Pau Petit Rosàs*
MATH-103, Deparment of Mathematical Sciences — 17:30 - 18:00

* *Thursday "early" morning*

Modern techniques for Integration-by-parts reduction — *Sid Smith*
Forshall Room (Room 427), Cedar House — 09:00 - 09:25

Progress on three-loop four-point integrals with one massive leg — *Jungwon Lim*
Forshall Room (Room 427), Cedar House — 09:30 - 09:55

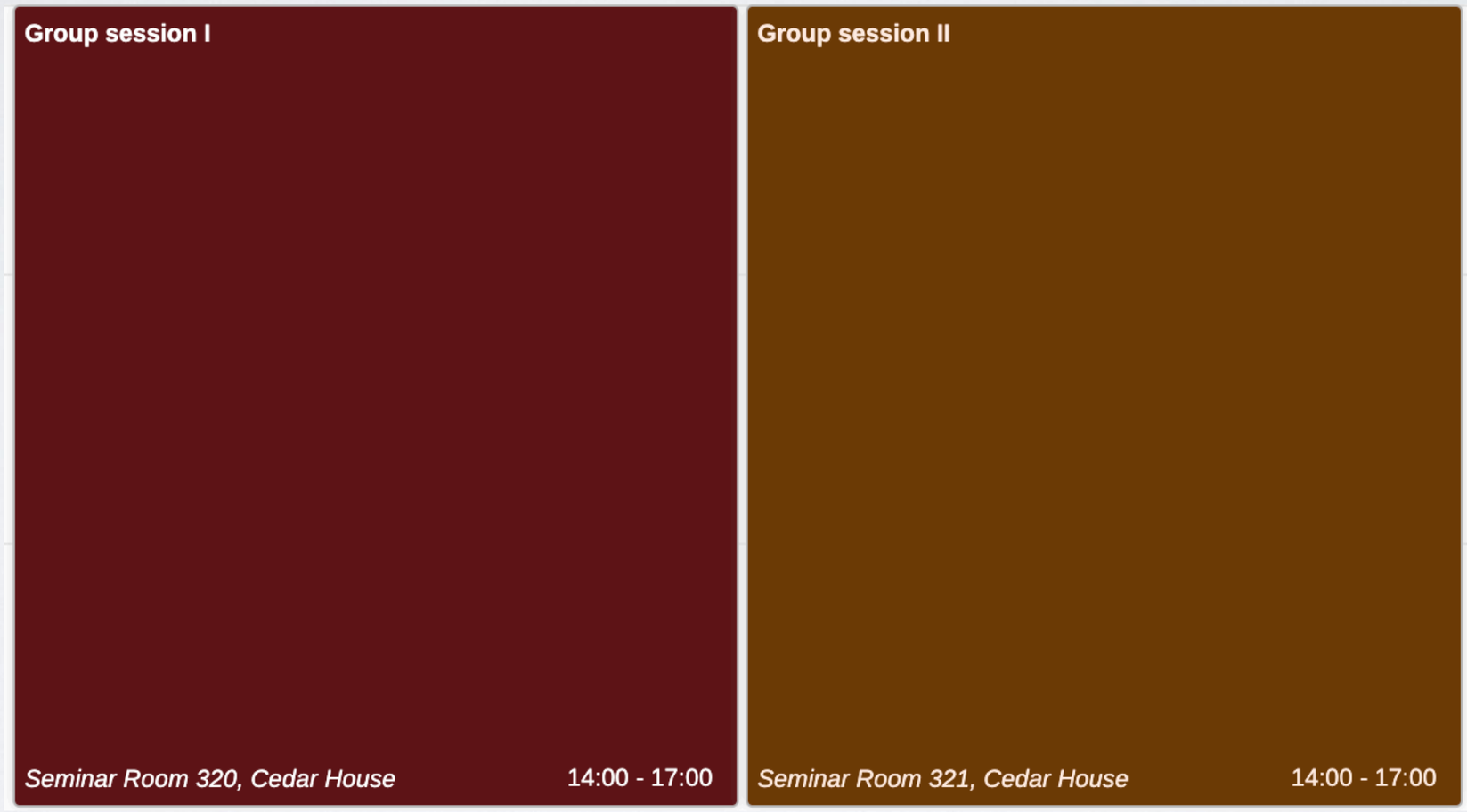
Electroweak corrections to Higgs boson pair production in gluon fusion — *Marco Bonetti*
Forshall Room (Room 427), Cedar House — 10:00 - 10:25

* *Thursday "late" morning*

Moderated discussion
Forshall Room (Room 427), Cedar House — 11:00 - 12:45

What is this workshop about?

* *Thursday afternoon :: core of the workshop*



What is this workshop about?

* *Friday morning*

Moderated discussion

Forshall Room (Room 427), Cedar House 09:00 - 10:30

Tea & coffee

Department of Mathematical Sciences 10:30 - 11:00

Final Remarks

Forshall Room (Room 427), Cedar House 11:00 - 13:00

- Discussion on automated tools (Line, Fuel, …)
- New integration methods for Feynman integrals
- Look into $e^+e^- \to \mu^+\mu^-\gamma$
- …

Das Ende ist der Anfang

- Let's profit from being in the same place these days
- Remember: not a standard workshop
- Promote collaborations as much as we can
- Let's boost new understanding in the evaluation of Feynman integrals.

Das Ende ist der Anfang

- Let's profit from being in the same place these days
- Remember: not a standard workshop
- Promote collaborations as much as we can
- Let's boost new understanding in the evaluation of Feynman integrals.

Social activities

- Lunches @ 1:00 PM
  Thursday: Victoria Gallery & Museum

- Conference dinner:
  The Philharmonic Dining Rooms

- Friday: Bertie and Bella's (Vine court)

- Networking activity:
  The Blackburne Arms Gastro Pub