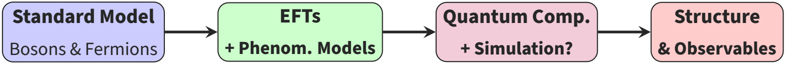
TOWARD HYBRID QUBIT–OSCILLATOR SIMULATION OF NUCLEAR EFFECTIVE FIELD THEORIES

Sam Godwood

HEP Annual Meeting – 22/05/25

STANDARD MODEL → PREDICTIVE PHYSICS



- **Target observables**
  - Scattering cross sections e.g. single pion production
- **Direct QCD?** Not (yet) practical
  - Lattice QCD real-time dynamics → sign problems
  - Quantum algorithms $\gtrsim 10^{50}$ gates for simple quark transport[1]
- **Nuclear Effective Field Theory (EFT)**
  - **Most general** Lagrangian consistent with QCD symmetries & *approx.* chiral symmetry
  - Include all terms up to a given order in energy expansion: Pionless, one-pion exchange, dynamical pions

[1] arXiv:2107.12769

QUANTUM SIMULATION OF EFT DYNAMICS

- Key subroutine is **real-time evolution** of $H_{\mathrm{EFT}} \rightarrow e^{-iH_{\mathrm{EFT}}t}$

- (bosonic) pion fields drive the resource cost for a ‘minimal’ $10 \times 10 \times 10$ lattice[1]

Model	Circuit Depth	Qubits
Pionless	5.8×10^{8}	6,000
Dynamical Pions	1.6×10^{44}	138,000

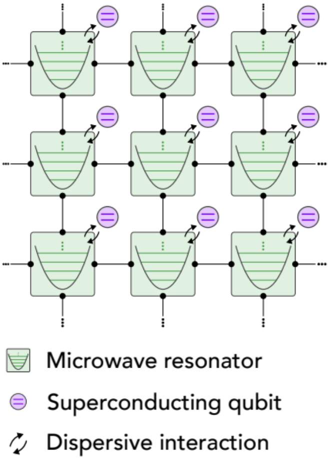
- Can native bosonic hardware reduce this overhead?

[1] arXiv:2312.05344

HYBRID QUBIT–OSCILLATOR ARCHITECTURES

- **Circuit QED[1,2,3] (this work)**: cavity modes; qubits = transmons
- **Trapped ions**: phonon modes; qubits = ion-spin levels
- **Neutral atoms**: motional modes; qubits = atomic internal states

- Universal control via natively available operations
- Assume arbitrary Fock-level access & full qubit–oscillator connectivity



[1] Nat. Phys. **16**, 247–256 (2020).
[2] arXiv:2005.12667
[3] arXiv:2407.10381

MINIMAL EXAMPLE: BOSONIC DISPLACEMENT OPERATOR

- **Displacement operator:**

$$\hat{D}(\alpha) = e^{\alpha \hat{b}^\dagger - \alpha^* \hat{b}}, \quad \alpha \in \mathbb{C}$$

  displaces bosonic mode in phase space

- **Native in cQED:** Realized directly by a resonant microwave drive

- **Representing bosonic modes with qubits:** $q = \lceil \log_2(n_{\max} + 1) \rceil$ qubits, cutoff $n_{\max}$

Method	**Gates**	**Qubits**
Pauli decomposition	$q\,2^{q-1}$	q
Square-root method	$\frac{11}{2}q^2 + 23q - 28$	$3q + 2$

⇒ Native displacement is orders-of-magnitude cheaper

TOWARD A DYNAMICAL PION EFT

$$H_{D\pi} = \underbrace{H_{\text{free}}}_{\text{free nucleon}} + \underbrace{H_C + H_{C_{I2}}}_{\text{NN contact}} + \underbrace{H_\pi}_{\text{free pion}} + \underbrace{H_{\pi N}}_{\text{pion–nucleon}}$$

- Consider minimal, non-trivial 1D interaction term:

$$H_{\text{int}}^{(1\text{D})} = \frac{g_A}{2f_\pi} \sum_x \left[\hat{n}_{00}(x) - \hat{n}_{01}(x) - \hat{n}_{10}(x) + \hat{n}_{11}(x) \right] \partial_1 \pi(x) \,.$$

$\Downarrow$ *Mapping to spin–boson DOF*

- Terms $\propto Z\,(b^\dagger + b)$
- Native time evolution:

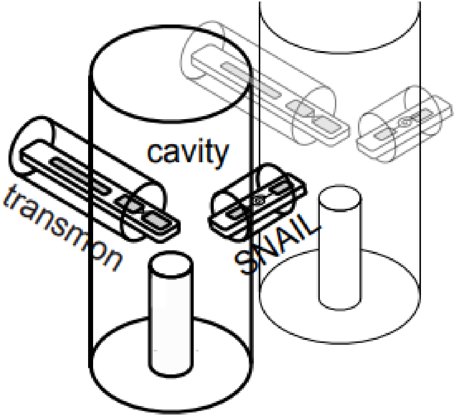
$$\underbrace{e^{-i Z \phi\,(b^\dagger + b)} = e^{-i \frac{\pi}{2} Z n}\, e^{i\phi\,(b^\dagger + b)}\, e^{i \frac{\pi}{2} Z n}}_{\text{CD}(\phi) \;=\; \text{C}\Pi\, D(i\phi)\, \text{C}\Pi^\dagger}$$

OUTLOOK

- **Short-term goal: Quantify savings**
  - Real-time evolution of hybrid vs. qubit-only encoding
- **Next step: Observables costs**
  - Identify target observables (e.g. linear-response via two-point functions)
  - Estimate & compare overhead for extracting
- **Bosonic-mode protection**
  - QEC schemes available
  - Fault-tolerance remains challenging
- **Regime of advantage?**
  - NISQ vs. Early fault-tolerance vs. Full fault-tolerance

BACKUP

SUPERCONDUCTING QUBITS & MICROWAVE CAVITIES



Comprehensive reviews:

- Blais et al., Nat. Phys. **16**, 247–256 (2020)
- Blais et al., Rev. Mod. Phys. **93**, 025005 (2021)
- Crane et al., arXiv:2409.03747
- Liu et al., arXiv:2407.10381

REAL TIME EVOLUTION ON A QUANTUM COMPUTER

- **Trotter–Suzuki Approximation** We split the full Hamiltonian into simple pieces and apply each term in short time slices:

$$e^{-iHt} \approx \left(\prod_{j=1}^{\Gamma} e^{-iH_j\,\delta t}\right)^{t/\delta t}$$

(Error falls as you shrink the time step or increase repetition)

- **Jordan–Wigner transform:**

$$c_j^\dagger \to \left(\prod_{m<j} Z_m\right)\frac{X_j - iY_j}{2},$$

maps fermionic modes → qubit Pauli strings, preserving $\{c_i, c_j^\dagger\} = \delta_{ij}$