Deep Reinforcement Learning

From Function Approximation to State-of-the-Art Continuous Control

Actor-Critic · DDPG · SAC & TD3 · Offline RL · Bridge to the Challenge

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Where We Are

Andrea's Lecture
Foundations, MDPs, TD

→

This Lecture
Deep RL, SAC, TD3

→

The Challenge
Hands-on coding

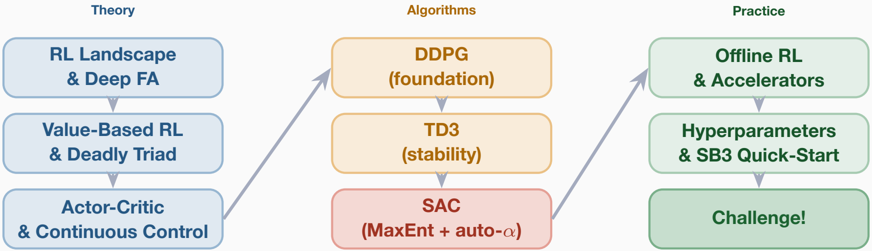
You already know: MDPs, value functions, Bellman equations, TD learning, Q-learning.

Now: How to scale RL to continuous actions and high-dimensional problems.

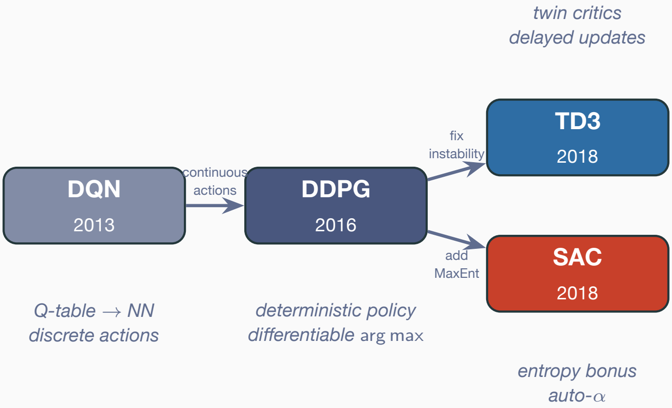
Goal: Understand SAC & TD3 well enough to use them in the challenge.

Outline

Goal: understand the path from Q-learning to TD3 & SAC, and deploy them on a real accelerator control problem.



Algorithm Family Tree: How We Got Here

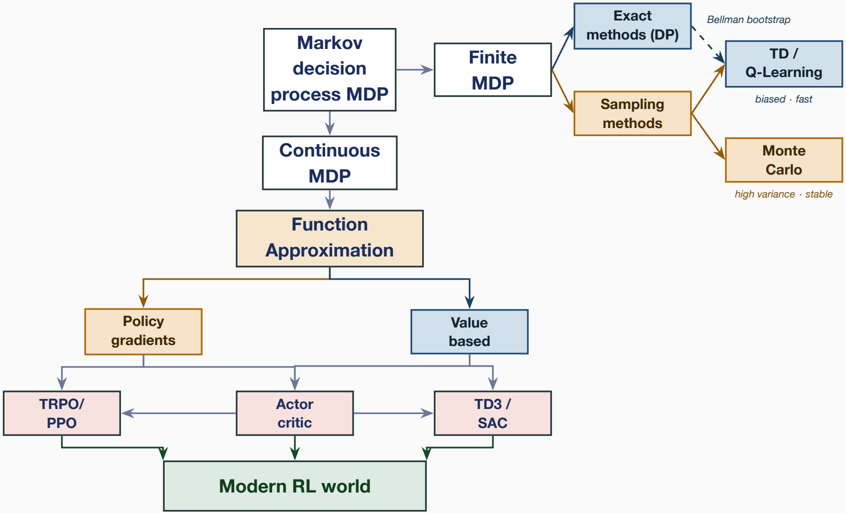


Today's journey: Each algorithm fixes a concrete flaw in its predecessor.

By the end you will understand every arrow above.

Overview: The RL Landscape

A Map of Reinforcement Learning Methods



Why Function Approximation (FA) in RL?

Tabular RL stores one value per state — fine for small problems, breaks for real ones.

The Curse of Dimensionality

- Chess: $\sim 10^{43}$ states
- Atari pixel obs: $256^{100\times200\times3}$ states
- Accelerator: continuous $s \in \mathbb{R}^n$

  $\Rightarrow$ infinitely many states
- A table simply cannot exist

FA: The Fix

- Represent $Q(s,a) \approx Q_{\mathbf{w}}(s,a)$ with a **compact model**
- **Generalise** across similar states automatically
- Same number of parameters $|\mathbf{w}|$ regardless of $|\mathcal{S}|$
- Neural networks $\Rightarrow$ **Deep RL**

$$\text{Tabular Q-table} \xrightarrow{\text{scale up}} Q_{\mathbf{w}}(s,a)\ \text{Neural Net} \xrightarrow{\text{Deep RL}} \textbf{DQN}$$

$|\mathcal{S}| \times |\mathcal{A}|$ entries — $|\mathbf{w}|$ parameters (fixed) — *first working idea* [Mnih et al., 2015]

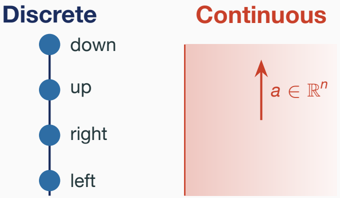
Why replacing the Q-table with a FA is not sufficient

DQN [Mnih et al., 2015] **works great for:**

- Discrete actions (Atari games, grid worlds)
- $\arg\max_a Q(s,a)$ is easy when $|\mathcal{A}|$ is finite

But in accelerator control:

- Actions are continuous (magnet currents, RF phases)
- $\arg\max_a Q(s,a)$ requires solving an optimization at every step
- We may want stochastic policies for exploration



Solution: Learn the policy directly — *Policy Gradient* methods, or scale Q-learning with function approximation.

FA in RL

Parametric (fixed-size $\mathbf{w}$)

- **Linear:** $\hat{V}(s) = \mathbf{w}^\top \phi(s)$ – converg. guarantees
- **Tile Coding / RBF** – hand-crafted features
- **MLP / CNN / RNN** – universal approx., GPU
- **Transformer** – sequences, multi-agent

Non-Parametric (grows with data)

- **Gaussian Processes** – uncertainty, poor scaling
- ***k*-Nearest Neighbours** – memory-based, no training
- **Decision Trees / Forests** – interpretable, discrete
- **Kernel Methods (SVM)** – $\mathcal{O}(n^2)$ cost

Why Parametric (Deep) Wins in RL

- Fixed memory; end-to-end gradient descent; scales with compute
- Cost: no convergence guarantee with bootstrapping + off-policy (*Deadly Triad*)

Why is this a probelm: The Bellman Operator and the Contraction Property

Define the Bellman optimality operator $\mathcal{T}^*$:

$$(\mathcal{T}^*Q)(s,a) = r(s,a) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q(s',a')$$

Contraction Mapping Theorem (Banach)

$\mathcal{T}^*$ is a γ-contraction in the ℓ_∞ norm:

$$\left\|\mathcal{T}^*Q_1 - \mathcal{T}^*Q_2\right\|_\infty \leq \gamma \left\|Q_1 - Q_2\right\|_\infty, \qquad \gamma \in (0,1)$$

Consequence: Repeated application converges to the *unique* fixed point Q^*:

$$Q^* = \mathcal{T}^*Q^* \quad \Longrightarrow \quad \text{Value iteration = guaranteed convergence!}$$

- Tabular: each Bellman update is **exact** $\rightarrow$ contraction is preserved
- Q-learning is a *stochastic* version of value iteration — still converges [Watkins & Dayan, 1992]
- Convergence rate: γ^k at step k — the lower γ, the faster

Function Approximation Breaks Convergence

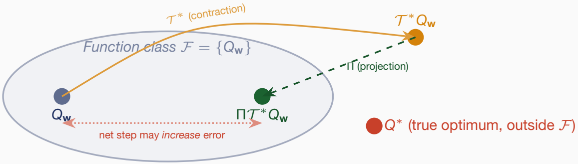
With FA we cannot store Q^* exactly. Each update becomes:

$$Q_{\mathbf{w}} \leftarrow \Pi \mathcal{T}^* Q_{\mathbf{w}}$$

where Π = projection onto the function class $\{Q_{\mathbf{w}} : \mathbf{w} \in \mathbb{R}^d\}$.

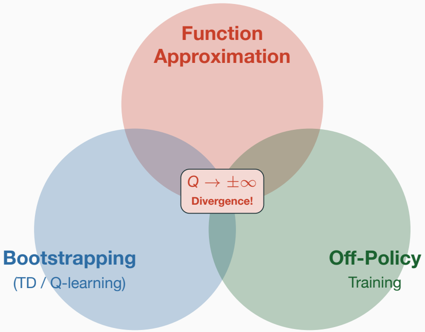
The Problem: $\Pi\mathcal{T}^*$ is NOT a contraction

- Projection can undo (or overshoot) the contraction step
- Errors may accumulate across iterations; no fixed-point theorem applies
- Divergence is possible even for simple linear FA *[Baird, 1995]*



The Deadly Triad

Sutton & Barto (2018) [Sutton & Barto, 2018]: instability or divergence arises when all three are combined.



Any two alone: manageable.
All three together: Q-values can diverge.

Practical mitigations:

- Target networks (freeze critic targets)
- Double Q (clipped Twin critics)
- Replay buffer (decorrelates data)
- Gradient clipping

These help in practice but provide *no* theoretical convergence guarantee.

Alternative: skip value estimation entirely — learn the policy directly.

No bootstrapping $\Rightarrow$ no deadly triad $\Rightarrow$ **Policy Gradient methods.**

Value-Based RL: Key Characteristics

Strengths

- **Solid theoretical basis:** unique Q^* via Bellman optimality
- **Tabular convergence:** Q-learning reaches Q^* [Watkins & Dayan, 1992]
- **Off-policy:** replay buffer reuses past experience
- **Fast credit assignment:** TD bootstrap, no full episodes needed
- **Explicit action values:** greedy policy readable from Q
- **Proven at scale:** DQN — superhuman on 57 Atari games [Mnih et al., 2015]

Limitations

- **Continuous actions:** $\arg\max_a Q(s, a)$ is an optimisation at every step
- **Deadly Triad:** FA + bootstrap + off-policy $\Rightarrow$ no convergence guarantee
- **Q-value divergence:** possible even for linear FA [Baird, 1995]
- **Heuristic fixes only:** target nets, double-Q, replay — no theoretical guarantee
- **Bootstrap bias:** max-operator overestimates Q
- **No native stochastic policy:** greedy $\Rightarrow$ poor exploration

Bottom line & bridge

Value-based RL is **sample-efficient and theoretically grounded**, but the Deadly Triad makes deep value-based learning fragile in practice. **Alternative:** learn $\pi_\theta(a|s)$ directly $\Rightarrow$ Policy Gradient methods — no bootstrapping, native continuous & stochastic actions.

The Actor: Parameterised Policy

The Actor: Parameterised Policy

The actor is a neural network that maps states directly to actions — replacing the intractable $\arg\max$ with a single forward pass.

Policy as a Neural Network

- **Input:** state s (e.g. BPM readings)
- **Output:** action a (e.g. corrector strengths)
- **Parameters:** θ trained by gradient ascent on $Q_\phi(s, a)$

Two flavours used in this lecture:

$$\mu_\theta(s) \quad \text{(deterministic — TD3)} \qquad \pi_\theta(a|s) = \mathcal{N}\big(\mu_\theta(s),\, \sigma_\theta(s)\big) \quad \text{(stochastic — SAC)}$$

The critic $Q_\phi(s, a)$ tells the actor *how good* each action is. Together they form the **Actor-Critic** architecture.

Why Actor-Critic?

Why Actor-Critic?

Problem	Solution
$\arg\max_a Q(s, a)$ is **intractable** for continuous actions $a \in \mathbb{R}^n$	**Actor** $\mu_\theta(s)$: neural net that outputs a directly — no argmax
μ_θ needs a gradient signal: *"which way should the action go?"*	**Critic** $Q_\phi(s, a)$: differentiable $\Rightarrow \nabla_a Q_\phi$ backpropagates into μ_θ

Actor-Critic $=$ differentiable argmax trained by a differentiable value function.

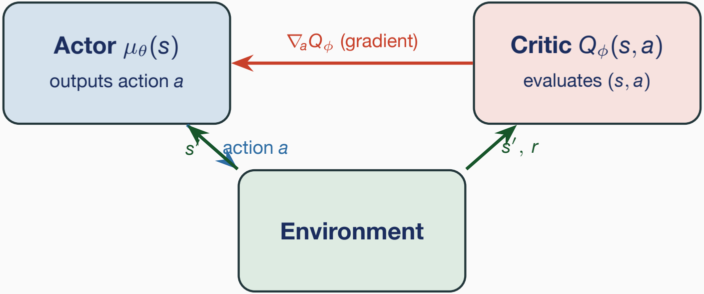
$\longrightarrow$ This is exactly what DDPG, TD3, and SAC do.

Actor-Critic & Off-Policy Learning

The Actor-Critic Architecture — Best of Both Worlds

Key insight: the critic $Q_\phi(s, a)$ is differentiable w.r.t. a. Its gradient trains the actor directly — no Monte Carlo rollouts needed:

$$\nabla_\theta J \approx \mathbb{E}\Big[\nabla_a Q_\phi(s, a)\big|_{a=\mu_\theta(s)} \cdot \nabla_\theta \mu_\theta(s) \Big]$$



- **Actor** (μ_θ): selects actions — the differentiable argmax
- **Critic** (Q_ϕ): evaluates (s, a), backpropagates $\nabla_a Q_\phi$ into actor

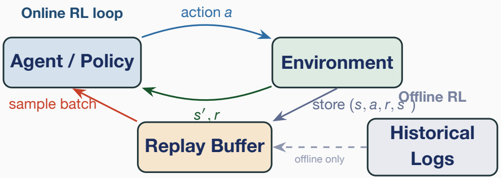
On-Policy vs. Off-Policy

On-Policy [Schulman et al., 2017] (e.g. PPO)

- Learns from data collected by *current* policy
- Data used once, then discarded
- Stable but sample inefficient
- Needs millions of environment steps

Off-Policy (e.g. SAC, TD3)

- Learns from *any* past data (replay buffer)
- Data reused many times
- Much more sample efficient
- Can learn from demonstrations



Key enabler: The replay buffer stores transitions and allows reuse.

In **offline RL** (coming soon) the buffer is filled from logs — no live environment needed.

DDPG — The Foundation

DDPG: Deep Deterministic Policy Gradient

DDPG [Lillicrap et al., 2016] = DQN ideas + Actor-Critic for continuous actions.

Solving the $\arg\max$ problem for continuous actions

DQN needs $\arg\max_a Q(s,a)$ — intractable when $a \in \mathbb{R}^n$.
DDPG learns a deterministic policy $\mu_\theta(s)$ so that:

$$\max_a Q(s,a) \;\approx\; Q(s,\, \mu_\theta(s))$$

The policy becomes the *differentiable* argmax.

Key ingredients:

1. **Deterministic policy:** $a = \mu_\theta(s)$
2. **Critic:** $Q_\phi(s,a)$ trained with Bellman error
3. **Replay buffer:** off-policy data reuse
4. **Target networks:** slow-moving copies

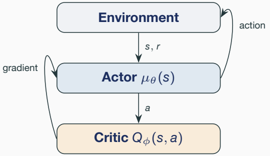
$$\phi' \leftarrow \tau\,\phi + (1-\tau)\,\phi'$$

5. **Exploration:** Gaussian noise on actions

$$a = \mu_\theta(s) + \epsilon, \quad \epsilon \sim \mathcal{N}(0,\sigma)$$

The Key Insight

The policy *becomes* a differentiable $\arg\max$:
a beautiful idea all modern algorithms inherit.



But DDPG Has Problems...

- **Q-function overestimates values** — max-operator bias compounds over training
- **Brittle to hyperparameters** — learning rate, noise σ, τ all require tuning
- **Exploration noise is manual** — $\mathcal{N}(0, \sigma)$ schedule must be set by hand
- **Often unstable in practice** — training can collapse without warning

TD3 and SAC keep the idea — fix the problems.

TD3 — Twin Delayed DDPG

TD3: Three Tricks to Fix DDPG

TD3 [Fujimoto et al., 2018] — three targeted fixes:

1. **Twin Q-Networks** (Clipped Double Q-Learning)
   - Train *two* critics Q_{ϕ_1}, Q_{ϕ_2}
   - Use the minimum for the target: $y = r + \gamma \min_i Q_{\phi'_i}(s', \tilde{a}')$
   - Prevents overestimation bias
2. **Delayed Policy Updates**
   - Update actor every d critic updates (typically $d = 2$)
   - Lets the critic stabilise before the actor chases it
3. **Target Policy Smoothing**
   - Add clipped noise to target actions:

$$\tilde{a}' = \mu_{\theta'}(s') + \text{clip}(\epsilon, -c, c), \quad \epsilon \sim \mathcal{N}(0, \sigma)$$

   - Regularises the critic, prevents sharp Q-peaks

TD3: Algorithm Overview

1. Sample batch (s, a, r, s') from replay buffer

2. Compute target: $y = r + \gamma \min_{i=1,2} Q_{\phi_i'}(s',\ \mu_{\theta'}(s') + \text{clip}(\epsilon))$

3. Update both critics: minimise $(Q_{\phi_i}(s, a) - y)^2$

4. **Every** *d* **steps:** update actor via $\nabla_\theta Q_{\phi_1}(s, \mu_\theta(s))$

5. Soft-update target networks: $\phi' \leftarrow \tau\phi + (1-\tau)\phi'$

Deterministic policy $\rightarrow$ exploration via explicit noise added during data collection.

SAC — Soft Actor-Critic

Control as Inference: The Probabilistic Foundation of SAC

Optimality Variable $\mathcal{O}_t \in \{0, 1\}$

$$p(\mathcal{O}_t{=}1 \mid s_t, a_t) \;\propto\; \exp\big(r(s_t, a_t)\big)$$

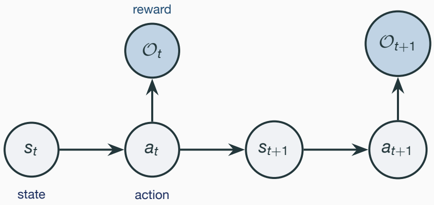
Goal: compute the posterior policy

$$\pi^*(a_t \mid s_t) = p(a_t \mid s_t,\, \mathcal{O}_{t:\infty}{=}1)$$

Variational Objective — minimise KL divergence

$$\max_{\pi} \;\mathbb{E}[\textstyle\sum_t r_t] + \alpha\, \mathbb{E}[\textstyle\sum_t \mathcal{H}(\pi(\cdot|s_t))]$$

Entropy term $\mathcal{H}$ emerges naturally from the KL, with temperature α.



Shaded nodes observed as optimal.

Solving this posterior *exactly* yields the MaxEnt RL objective.

SAC: The Maximum Entropy Framework

SAC [Haarnoja et al., 2018] — key insight: maximise reward AND entropy.

$$\boxed{J(\theta) = \sum_t \mathbb{E}\Big[r_t + \alpha\,\mathcal{H}\big(\pi_\theta(\cdot|s_t)\big)\Big]}$$

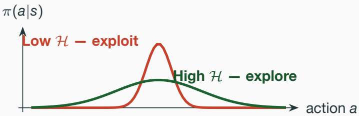
where $\mathcal{H}(\pi) = -\mathbb{E}[\log \pi(a|s)]$ is the entropy of the policy.

Why entropy matters:

- Automatic exploration — no manual noise
- More robust policies
- Better multi-modal behaviour
- Prevents premature convergence

α — the temperature:

- Controls exploration–exploitation trade-off
- α large $\rightarrow$ more exploration
- α small $\rightarrow$ more exploitation
- SAC auto-tunes α!



SAC: Automatic Temperature Tuning

The killer feature: SAC automatically adjusts α to maintain a target entropy.

$$\alpha^* = \arg\min_{\alpha} \mathbb{E}_{a\sim\pi}\left[-\alpha \log \pi(a|s) - \alpha \bar{\mathcal{H}}\right]$$

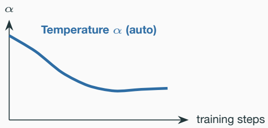
where $\bar{\mathcal{H}}$ is the **target entropy** (typically $= -\dim(\mathcal{A})$).

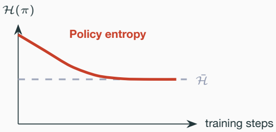
Early training:

- Policy is uncertain $\rightarrow$ entropy is high
- α decreases to match target
- Agent naturally explores

Late training:

- Policy converges $\rightarrow$ entropy drops
- α increases to prevent collapse
- Maintains healthy exploration





$\rightarrow$ Always use automatic α. One less hyperparameter to tune!

SAC: Algorithm Overview

1. Sample batch (s, a, r, s') from replay buffer
2. Sample $\tilde{a}' \sim \pi_\theta(\cdot|s')$, compute $\log \pi$
3. Target: $y = r + \gamma\big(\min_i Q_{\phi_i'}(s', \tilde{a}') - \alpha \log \pi(\tilde{a}'|s')\big)$
4. Update critics: minimise $(Q_{\phi_i}(s, a) - y)^2$
5. Update actor: $\nabla_\theta \mathbb{E}[\alpha \log \pi_\theta(a|s) - Q_{\phi_1}(s, a)]$
6. Update α: adjust temperature toward target entropy
7. Soft-update targets: $\phi' \leftarrow \tau\phi + (1-\tau)\phi'$

SAC vs. TD3 — When to Use What

SAC vs. TD3: Head-to-Head

Property	**TD3**	**SAC**
Policy	Deterministic	Stochastic
Exploration	Gaussian noise (manual)	Entropy (automatic)
Q-networks	Twin (min)	Twin (min)
Temperature α	—	Auto-tuned
Policy updates	Delayed ($d = 2$)	Every step
Target smoothing	Yes (noise)	Yes (entropy)
Hyperparameter sensitivity	Medium	Low
Sample efficiency	High	High
Robustness	Good	Better

Rule of thumb: Start with **SAC**. Try TD3 if you need simpler debugging or lower compute.

SAC vs. TD3: Schematic Performance Across Seeds

Limitation: Both still require online interaction — impossible when beam time is scarce. $\Rightarrow$ Next section: learn from historical data, without touching the machine.

SAC's **narrower band** means more reliable results across random seeds.

Both converge to similar final performance — SAC simply gets there more consistently.

Offline Reinforcement Learning

Offline RL: Learning Without Interaction

Problem: Online RL needs many environment interactions — expensive & risky!

Historical Data
Logs, simulations

$\xrightarrow{\text{batch}}$

Train Policy
Offline RL

$\xrightarrow{\text{evaluate}}$

Deploy
On machine

No new interactions!

Key Challenge: Distributional Shift

The policy may choose actions never seen in the dataset $\rightarrow$ Q-values become unreliable.

Solutions: Conservative Q-Learning (CQL), IQL, or world-model approaches (our work).

Offline RL: Why It Matters for Accelerators

- Accelerators generate **terabytes** of logged operations data
- Beam time is precious — cannot afford millions of online RL steps
- Safety-critical: must avoid catastrophic actions during learning

Our approach (Koopman-Stabilised World Models):

1. Archive data from simulations or operations
2. Learn a stable surrogate model (Koopman operator)
3. Train RL policies **entirely offline** on the surrogate
4. Use **epistemic uncertainty** to detect out-of-distribution states

$\rightarrow$ Policies match online PPO performance **without any machine interaction.**

Tomorrow morning (Tue 09:15): Keynote by Samuele Tosatto

"Accelerating RL with Off-Policy Data" — going deeper on exactly these challenges!

Practical Guide: Hyperparameters & Tips

Hyperparameter Cheat Sheet

Hyperparameter	SAC	TD3	Advice
Learning rate (actor & critic)	3e-4	3e-4	Start here (Adam)
Replay buffer size	1e6	1e6	Bigger is better
Batch size	256	256	128–512, rarely critical
τ (soft update)	0.005	0.005	Don't touch
γ (discount)	0.99	0.99	Lower for short horizons
α (entropy)	auto	—	Always use auto
Policy delay	1	2	TD3-specific
Exploration noise σ	—	0.1	TD3 only
Hidden layers	[256, 256]	[256, 256]	Good default

Start with defaults. Change one thing at a time.

Top Practical Tips

1. **Normalise observations** — the single biggest improvement
   - Use running mean/std or fixed normalisation from your data
2. **Reward shaping matters** — scale rewards to roughly $[-1, 1]$
   - SAC's entropy interacts with reward scale
3. **Action scaling** — ensure actions map to $[-1, 1]$ internally
   - Rescale to physical units outside the algorithm
4. **Monitor Q-values** — if they diverge, something is wrong
   - Common sign of reward scaling or learning rate issues
5. **Seed averaging** — always run 3–5 seeds, report mean $\pm$ std
   - RL results are notoriously noisy across seeds

Quick Start: SAC & TD3 with Stable-Baselines3

SAC (recommended default)

```
import gymnasium as gym
from stable_baselines3 import SAC

env = gym.make("Pendulum-v1")

model = SAC(
   "MlpPolicy", env,
   learning_rate=3e-4,
   buffer_size=1_000_000,
   batch_size=256,
   tau=0.005,
   gamma=0.99,
   ent_coef="auto", # auto!
   verbose=1,
)
model.learn(100_000)
```

TD3 (if you prefer deterministic)

```
import gymnasium as gym
from stable_baselines3 import TD3

env = gym.make("Pendulum-v1")

model = TD3(
   "MlpPolicy", env,
   learning_rate=3e-4,
   buffer_size=1_000_000,
   batch_size=256,
   tau=0.005,
   gamma=0.99,
   policy_delay=2, # every 2 steps
   verbose=1,
)
model.learn(100_000)
```

Evaluate: `action, _ = model.predict(obs, deterministic=True)` — same API for both!

Summary & Bridge to the Challenge

What You've Learned

Foundations

- **Why FA?** Tables break for continuous/high-dim spaces
- **Bellman + FA** = no convergence guarantee (Deadly Triad)
- **Policy Gradients:** learn π_θ directly via $\nabla_\theta \log \pi \cdot G_t$
- **Actor-Critic:** advantage $A(s, a)$ cuts variance; off-policy replay cuts cost

State-of-the-Art Algorithms

- **DDPG:** deterministic policy = differentiable $\arg\max$
- **TD3:** twin critics + delayed updates + target smoothing
- **SAC:** MaxEnt + auto-α = most robust default
- **Offline RL:** learn from logs, no machine interaction

Decision rule for the challenge

Start with SAC · normalise observations · scale rewards to $[-1, 1]$ · run 3 seeds

Good Luck with the Challenge!

Questions?

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Next up: Challenge Group Assignment

Backup Slides

Not part of the main talk — available if questions arise

Deep RL Since 2018: What Has Changed?

Sample Efficiency & Stability

- **REDQ** (2021): ensemble of *N* critics
  Massively improves sample efficiency; random subset for updates
- **CrossQ** (2024): batch-norm trick
  SAC-level performance with far fewer gradient steps
- **TQC** (2021): truncated quantile critics
  Distributional RL; better uncertainty; top SB3 benchmark

Model-Based RL (MBRL)

- **DreamerV3** (2023): world model in latent space
  One algorithm, no tuning, across domains (Atari, DMC, Minecraft)
- **TD-MPC2** (2024): MPC + implicit world model
  State-of-the-art on continuous control, scales to 1B params

Offline & Foundation RL

- **CQL** (2020): conservative Q-learning
  Penalises OOD actions; seminal offline RL paper
- **IQL** (2021): implicit Q-learning
  Avoids OOD queries entirely; simpler & often better
- **Decision Transformer** (2021): RL as sequence modelling
  GPT-style; conditions on return-to-go tokens
- **RLHF** (2022+): RL from human feedback
  Powers ChatGPT, Gemini; reward model from preferences

The Big Shift

- **LLM + RL:** agents that plan with language (ReAct, GRPO)
- **Diffusion policies:** multi-modal, contact-rich control

Algorithms Worth Exploring Next

If you liked SAC / TD3…

1. **TQC** — drop-in upgrade, often outperforms SAC
   `pip install sb3-contrib`
2. **CrossQ** — same quality, $5\times$ faster training
   Batch norm in critic; minimal code change
3. **REDQ** — N-ensemble critics for hard exploration
   Tune N and UTD ratio
4. **SAC-N** — push ensemble to $N{=}500$ critics
   Offline RL *without* explicit conservatism

For Accelerator / Safety Problems

1. **MBPO / DreamerV3** — model-based; few real steps
2. **TD-MPC2** — MPC + learned model; interpretable
3. **IQL / CQL** — offline RL from logs
4. **BEAR / MOPO** — offline RL with uncertainty

If you want to go deeper…

1. **PPO + RND** — curiosity-driven on-policy exploration
2. **TRPO** — trust-region; theoretically grounded PPO ancestor
3. **MPO** (DeepMind) — maximum a-posteriori policy optimisation
4. **Diffusion Policies** — multi-modal; robotics SOTA
5. **Decision Transformer** — offline RL without Bellman

Starting Points

- **Stable-Baselines3** + `sb3-contrib` — most algorithms above
- **CleanRL** — single-file, readable implementations
- **CORL** — offline RL benchmark suite
- **Gymnasium / Shimmy** — environment wrappers

References

Core papers:

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Further reading:

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- Chen et al. (2021). **REDQ.** *ICLR*.
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- Ouyang et al. (2022). **InstructGPT/RLHF.** *NeurIPS*.
- Hafner et al. (2023). **DreamerV3.** *arXiv:2301.04104*.
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Backup: Policy Gradient Theorem

Objective: Maximise expected return

$$J(\theta) = \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \gamma^t r_t\right]$$

The Policy Gradient Theorem *[Sutton et al., 1999]*:

$$\boxed{\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim \pi_\theta}\left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t)\, G_t\right]}$$

where $G_t = \sum_{k=t}^{T} \gamma^{k-t} r_k$ is the return-to-go.

Intuition:

- Actions that led to high return $\rightarrow$ increase probability
- Actions that led to low return $\rightarrow$ decrease probability

Backup: REINFORCE and Its Problems

REINFORCE [Williams, 1992]: simplest policy gradient algorithm.

1. Collect a full episode using π_θ
2. Compute returns G_t
3. Update: $\theta \leftarrow \theta + \alpha \sum_t \nabla_\theta \log \pi_\theta(a_t|s_t)\, G_t$

The Variance Problem

- MC returns G_t are noisy — high variance
- Needs many episodes for stable gradients
- **On-policy:** data discarded after 1 use

Fix: replace G_t with critic estimate $Q_\phi(s, a) \rightarrow$ **Actor-Critic**.

Backup: SAC — Stochastic Policy with Squashing

Policy outputs (reparameterisation trick):

$$\mu_\theta(s),\ \sigma_\theta(s) \xrightarrow{\epsilon \sim \mathcal{N}(0,I)} z = \mu_\theta(s) + \sigma_\theta(s) \odot \epsilon \xrightarrow{\tanh} a = \tanh(z)$$

- Reparameterisation: gradients flow through $\mu_\theta, \sigma_\theta$ (not through sample)
- The tanh **squashes** actions to $[-1, 1]$ (bounded action space)
- Requires a log-probability correction (change of variables):

$$\log \pi(a|s) = \log \mathcal{N}(z|\mu, \sigma) - \textstyle\sum_i \log\left(1 - \tanh^2(z_i)\right)$$

Key Advantage over TD3

- Exploration is **built into the policy** via entropy — no manual noise schedule
- Stochastic $\rightarrow$ can represent multi-modal action distributions