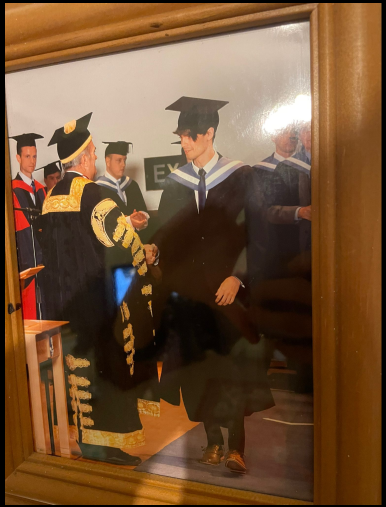
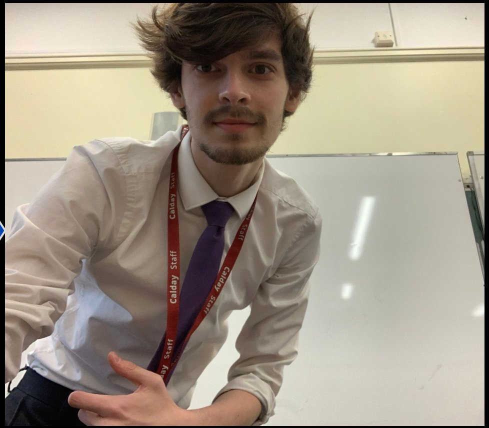
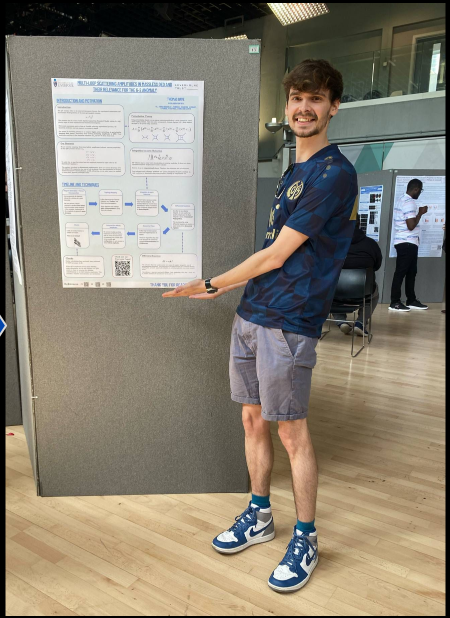
Amplitudes made (somewhat) simple: Tensor Decomposition

Tom Dave

My journey to the PhD







Amplitudes made (somewhat) simple: Tensor Decomposition

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What is Tensor Decomposition?

Feynman amplitudes are complicated expressions made up of many **tensor structures** including **spinors**, the metric tensor, gamma matrices etc.

Many of these structures depend on each other once Dirac algebra is performed.

Tensor Decomposition allows us to extract the coefficients of a Feynman amplitude that are proportional to a minimal basis of independent tensor structures that comprise the whole amplitude.

We name these coefficients **Form Factors**.

$$\mathcal{A} \;\longrightarrow\; \{\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n\}$$

How does tensor decomposition work?

1. Consider a minimal set of tensor structures $\mathcal{T}_i$ that will appear in the amplitudes of a process.

2. Form a Gram Matrix from this basis.
$$c_{ij} = T_i T_j^\dagger$$

3. Find the inverse of this matrix, then multiply by the conjugate tensors. This gives us a **Projector**.
$$P_i = (c_{ij}^{-1}) T_j^\dagger$$

4. Apply this Projector to Amplitudes to find **Form Factors**.
$$P_i \mathcal{A} = \mathcal{F}_i$$

These form factors form gauge invariant groups, so all intermediate steps can be performed on them before reconstructing the amplitude.

How does tensor decomposition work?

How can we reconstruct the amplitudes?

We can find the sum of the Form Factors
multiplied by their corresponding tensor structures.

$$\mathcal{A} = \mathcal{F}_i T_i$$

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$$\downarrow$$

$$(c_{ij}^{-1}) T_j^\dagger \mathcal{A} = \mathcal{F}_i$$

$$\rightarrow$$

$$(c_{ij}^{-1}) T_i T_j^\dagger \mathcal{A} = \mathcal{F}_i T_i$$

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$$P_i \mathcal{A} = \mathcal{F}_i$$

$$(c_{ij}^{-1}) T_j^{\dagger} \mathcal{A} = \mathcal{F}_i$$

$$(c_{ij}^{-1}) T_i T_j^{\dagger} \mathcal{A} = \mathcal{F}_i T_i$$

$$(c_{ij}^{-1})(c_{ij}) \mathcal{A} = \mathcal{F}_i T_i$$

What can we gain from Tensor Decomposition?

From Form Factors we can gain two things:

- Helicity (polarised) amplitudes.
  These are not the ultimate goal of our calculations but an added bonus.

- Interference (amplitude squared).
  This is what we would like, as it is what would would like to calculate in MC Generators.

$$\mathcal{A}_{(l_1)}\mathcal{A}^*_{(l_2)}$$

Why use tensor decomposition?

Classic Approach

1. Generate Amplitudes. $\mathcal{A}$ — Analytic
2. Multiply amplitudes by conjugate amplitudes. $\mathcal{A}_{(l_1)}\mathcal{A}^*_{(l_2)}$ — Analytic
3. Perform Dirac Algebra. — Analytic
4. Substitute in phase space point. — Numeric

Why use tensor decomposition?

Tensor Decomposition Approach

1. Generate Amplitudes. $\mathcal{A}$ — Analytic
2. Calculate Form Factors $\mathcal{F}_i$ — Analytic or Numeric
3. Multiply amplitudes by conjugate amplitudes. — Analytic or Numeric

$$\mathcal{A}_{(l_1)}\mathcal{A}^*_{(l_2)} = \mathcal{F}_{(l_1),i}\mathcal{F}^*_{(l_2),j}T_iT_j^\dagger = \mathcal{F}_{(l_1),i}\mathcal{F}^*_{(l_2),j}(c_{ij})$$

4. Substitute in phase space point. — Numeric

What do Monte-Carlo tools do

Need to know amplitude for the process and be able to compute it fast

$$\sigma_{LO} = \int d(\mathrm{PS})_2 \, |\mathcal{M}_{LO}|^2$$

Numerical integration

Phase-space algorithm

Beyond LO, need to cancel IR singularities with a scheme, but principle is the same

NLO is automated for most processes (sometimes NNLO)

What about effects that are important but arise at NN…NLO?

What calculations am I currently completing with tensor decomposition?

- 3L fully massless QED processes.

$$e^+e^- \to \mu^+\mu^- \qquad e^+e^- \to e^+e^-$$
$$e^+\mu^- \to e^+\mu^- \qquad e^+e^- \to \gamma\gamma\,,$$

- Fully mass dependent $e^+e^- \to \mu^+\mu^-$
- Fully mass dependent $e^+e^- \to \pi^+\pi^-$
- Fully mass dependent $e^+e^- \to \pi^+\pi^-\gamma$

Conclusion

Tensor decomposition is an effective method for calculating the interference of scattering amplitudes at higher perturbative orders. This is as we can remove the complexity of spinors at an early stage, then perform remaining steps numerically.

With this method we aim to compute the NLO contribution to $e^+e^- \to \pi^+\pi^-\gamma$ to higher orders in the dimensional regulator. This is a crucial step to finding the finite contribution for this process at NNLO.

This method is also applicable to many other processes that we aim to benefit from additionally.

Thank you for listening and feel free to ask any questions!