Monte Carlo Techniques

-> Monte Carlo is a set of techniques for dealing with randomness

-> aim is to sample from a probability distribution that we can calculate (like a differential cross section)

-> Assumption : we have a uniform random number generator between 0 and 1

We have a function f(x) > 0

Method 1 : if we know the integral

If $f(x)$ can be indefinitely integrated, $F(x)$, and the inverse function found, $F^{-1}(x)$, then

$$\int_{x_{\text{min}}}^{x} f(x)\,\mathrm{d}x = R\int_{x_{\text{min}}}^{x_{\text{max}}} f(x)\,\mathrm{d}x$$

$$\implies x = F^{-1}(F(x_{\text{min}}) + R(F(x_{\text{max}}) - F(x_{\text{min}}))$$

Method 2 : not integrable by hand

Given two random numbers R_1 and R_2, if we know $f(x) \le f_{\text{max}}$ in the allowed x range:

1. select a uniform $x = x_{\text{min}} + R_1(x_{\text{max}} - x_{\text{min}})$
2. if $f(x)/f_{\text{max}} \le R_2$, reject this x and goto 1
3. else, retain the generated x value

Method 3 : better efficiency than method 2

Assume an overestimate function $g(x) \ge f(x)$ over the allowed x region, where $G(x)$ and its inverse are known.

1. select an x according to the distribution $g(x)$, using method 1, i.e. $x = G^{-1}(G(x_{\text{min}}) + R_1(G(x_{\text{max}}) - G(x_{\text{min}}))$
2. Hit and miss: if $f(x)/g(x) \le R_2$, reject this x and goto 1
3. else, retain the generated x value

Can also split g(x) up into different functions if f(x) is more complicated