Reduction of Feynamn Integrals by IBP (Integration By Parts) method

- Main idea: use an algeric technique to reduce complicated amplitudes involving Feynman diagrams to simplest ones
- Usually loop diagrams can be expressed as integrals. We will use the following constraint:

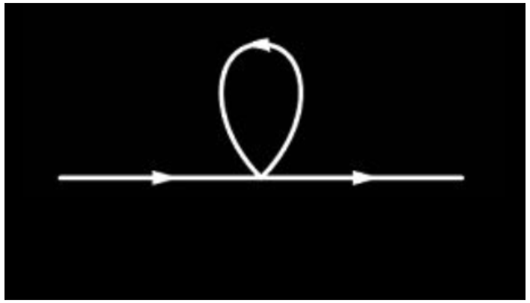
$$0 = \int d^D k \, \frac{\partial}{\partial k^\mu} \left(v^\mu \, f(k, p) \right)$$

D corresponds to the number of dimension of particles (4 -2ε)

f($^{\mu}$, p^{μ}) is the integrand corresponding to the amplitude;

v^{μ}=k^{μ}, p^{μ} are the momenta involved

Example: Massive tadpole



$$\mathcal{I}(a) = \int d^D k \frac{1}{(k^2 - m^2)^a}$$

Using the relation before (believe, Tom did the math for me):

$$\left(\frac{D - 2a}{2m^2 a}\right) \mathcal{I}(a) = \mathcal{I}(a+1)$$

We can use this relation in a recurrive way to express I(n) as function of I(1)

Example: Compute: $I(5) - m^2 I(2)$

We will use the previous relation:

$$\left(\frac{D-2a}{2m^2a}\right)\mathcal{I}(a) = \mathcal{I}(a+1)$$

Result:

$$I(5) - m^2 I(2) = \left(\frac{(D-8)(D-6)(D-4)(D-2)}{384m^8} + \frac{D-2}{2}\right)\mathcal{I}(1)$$