FERMILAB-SLIDES-25-0115-PPD

The Muon $g-2$ puzzle: theoretical and experimental status

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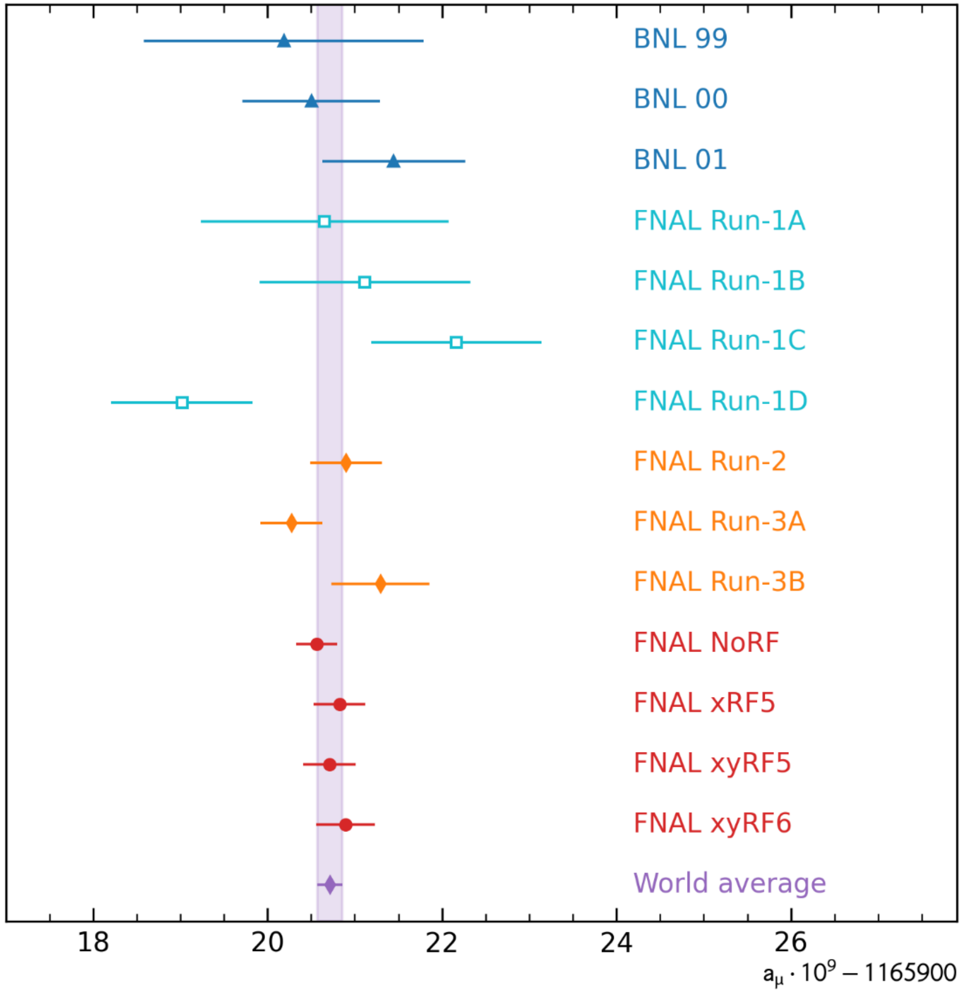
on behalf of the Muon g-2 collaboration

2025/09/23 | HEP Seminars | Liverpool

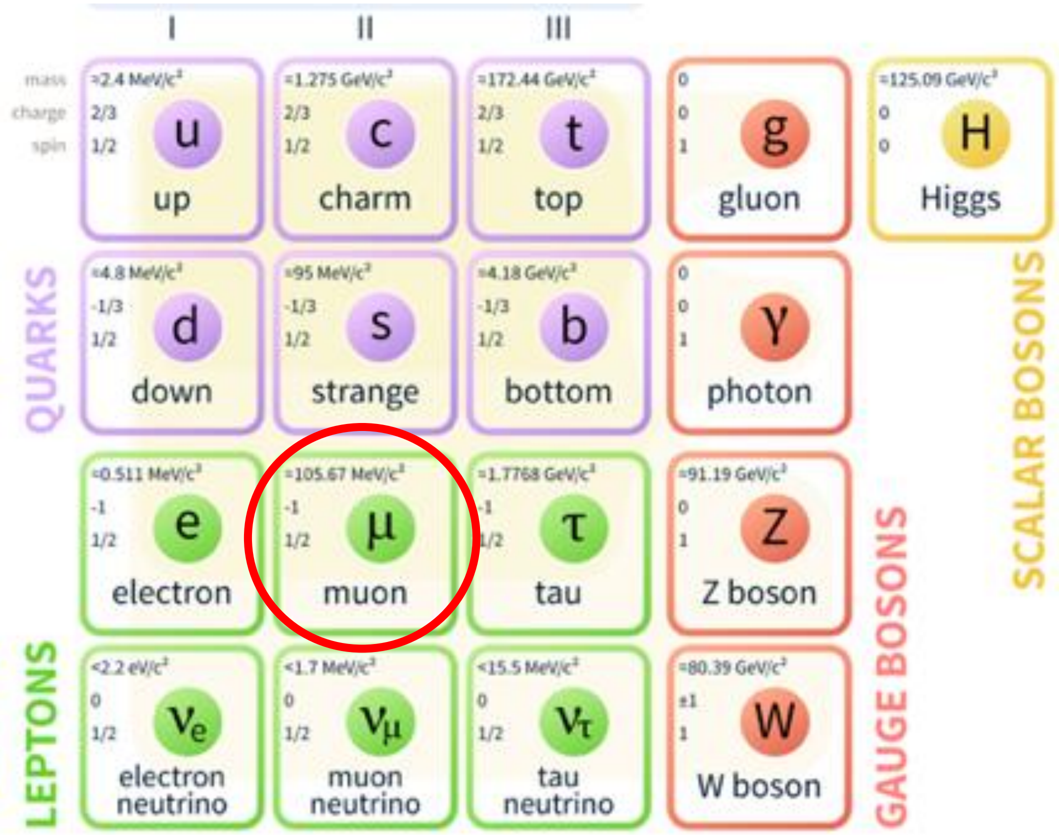
Outline

- Introduction: muons and $(g-2)_\mu$
- White Paper 2025 piece by piece
- Measurement principle at FNAL:
  - ω_a analysis
  - Magnetic field
  - Beam dynamics
- 2025 FNAL result: Run-4/5/6
- Future prospects



Introduction

What are «muons»?



Second generation charged lepton:

- We measure its anomalous magnetic moment as a stringent test of the Standard Model (SM) and a probe for New Physics
- 207 times more massive than electrons → more sensitive to Beyond SM
- We'll come back to its properties later

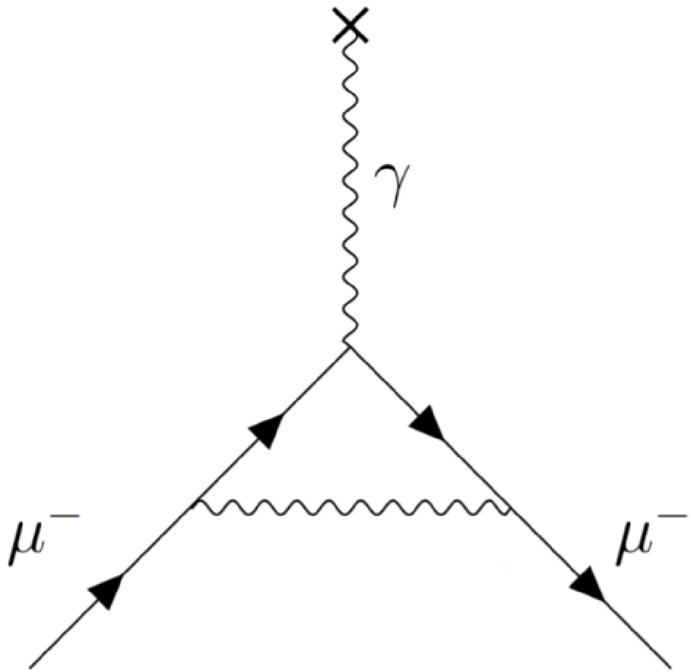
What is «$g-2$»?

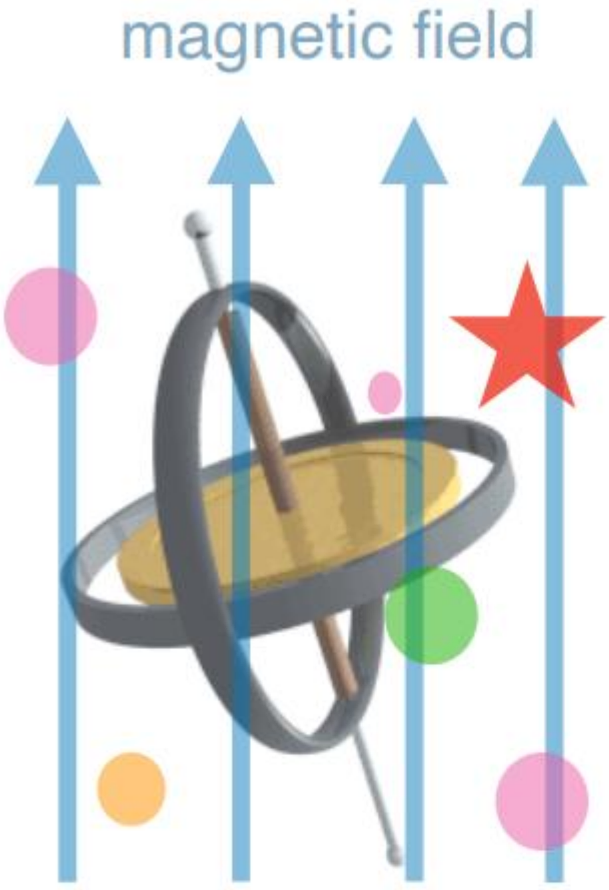
- Particle with spin in magnetic B-field: $\vec{\mu} \equiv g\frac{e}{2m}\vec{S} \rightarrow \tau = \vec{\mu} \times \vec{B}, \mathrm{U} = -\vec{\mu} \cdot \vec{B}$
- Dirac's prediction for spin-$\frac{1}{2}$ charged particles: $g = 2$
- Radiative corrections in Quantum Field Theories: $g \neq 2$
- Kusch and Foley's measurement, Schwinger's prediction: (1948, electron $g_e - 2$)

  $\frac{g_e-2}{2} \equiv a_e \approx 0.00116$

  1st order QED term: $\frac{\alpha}{2\pi}$

  universal to all leptons: e, μ, τ





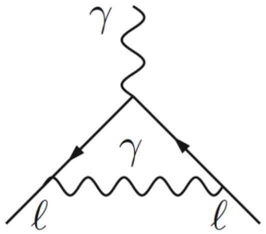
$g - 2 > 0$

SM prediction of $(g-2)_\mu$

$a_\mu \equiv \frac{g_\mu - 2}{2}$

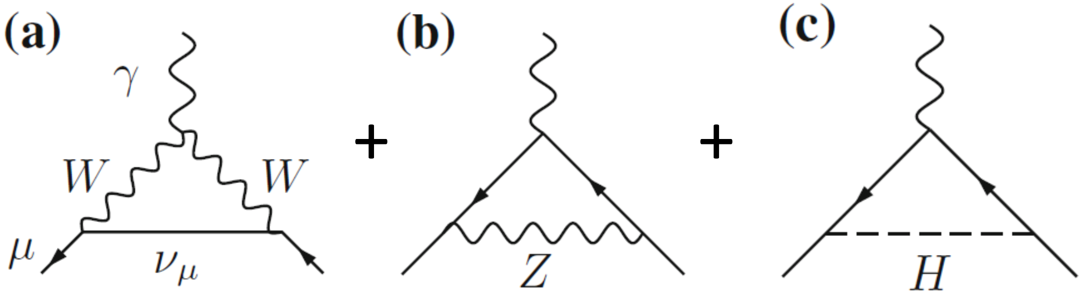
$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$$

$a_\mu^{QED} =$

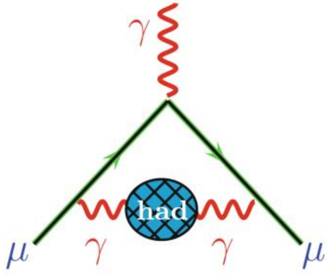


+ over 14’000 diagrams

$a_\mu^{EW} =$

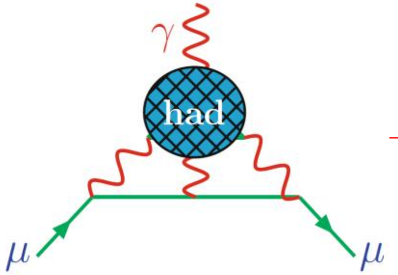


$a_\mu^{had} =$

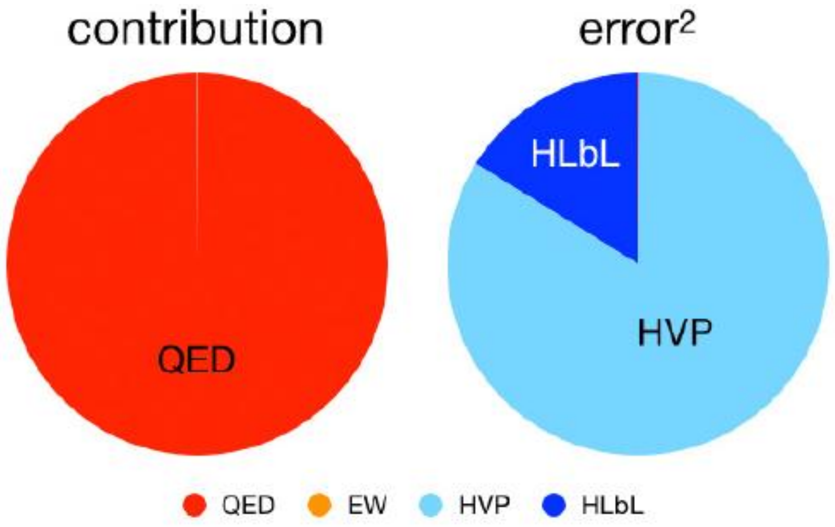


HVP

+



HLbL



a_μ^{had} cannot be evaluated perturbatively!

QCD dominates the uncertainty

Where can you read more about all of this?

(Suggestions)

Recent review: «Measured Lepton Magnetic Moments» by G. Venanzoni and G. Gabrielse, arXiv:2507.11268 [hep-ex], July 2025

Latest updates:

- Experiment: Run-4/5/6 result, Phys. Rev. Lett. **135**, 101802 (2025)

Liverpool has always been present in the history of $(g-2)_\mu$!

> BSM physics. Precision measurements of g_μ span decades of advances, beginning with early experiments at Columbia University Nevis Laboratory [7,8] and the University of Liverpool [9]. Direct measurement of a_μ started with the CERN-I [10], CERN-II [11], and CERN-III experiments [12], which the Brookhaven National Laboratory (BNL) E821 experiment further improved [13]. The E821 results

- Theory: White Paper 2025, Phys.Rept. 1143 (2025) 1-158

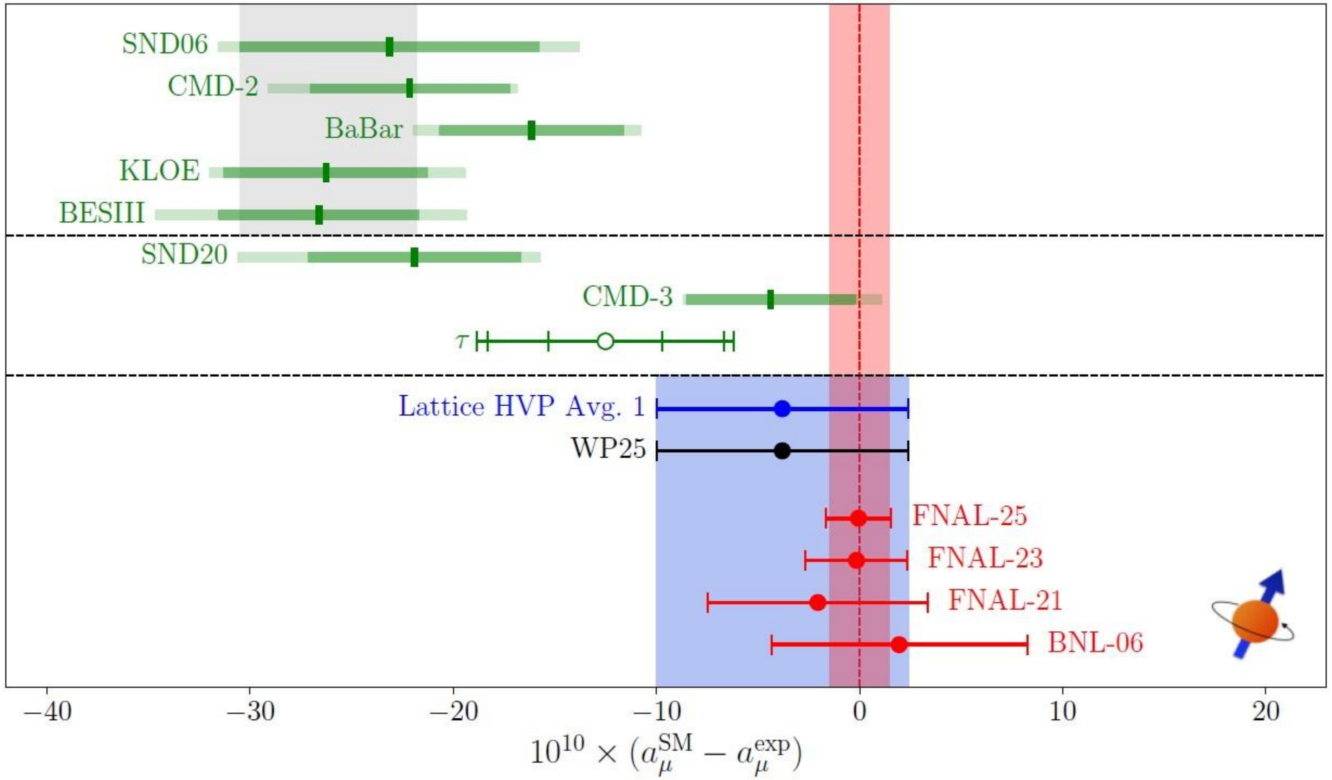
More in the next slides

A snapshot of the current scenario

Theory and Experiment

Muon $g-2$ Theory Initiative: 100+ theorists and experimentalists working together on the SM a_μ. Official website: https://muon-gm2-theory.illinois.edu/

Annual workshops: link to the 8th (latest) one, September 2025

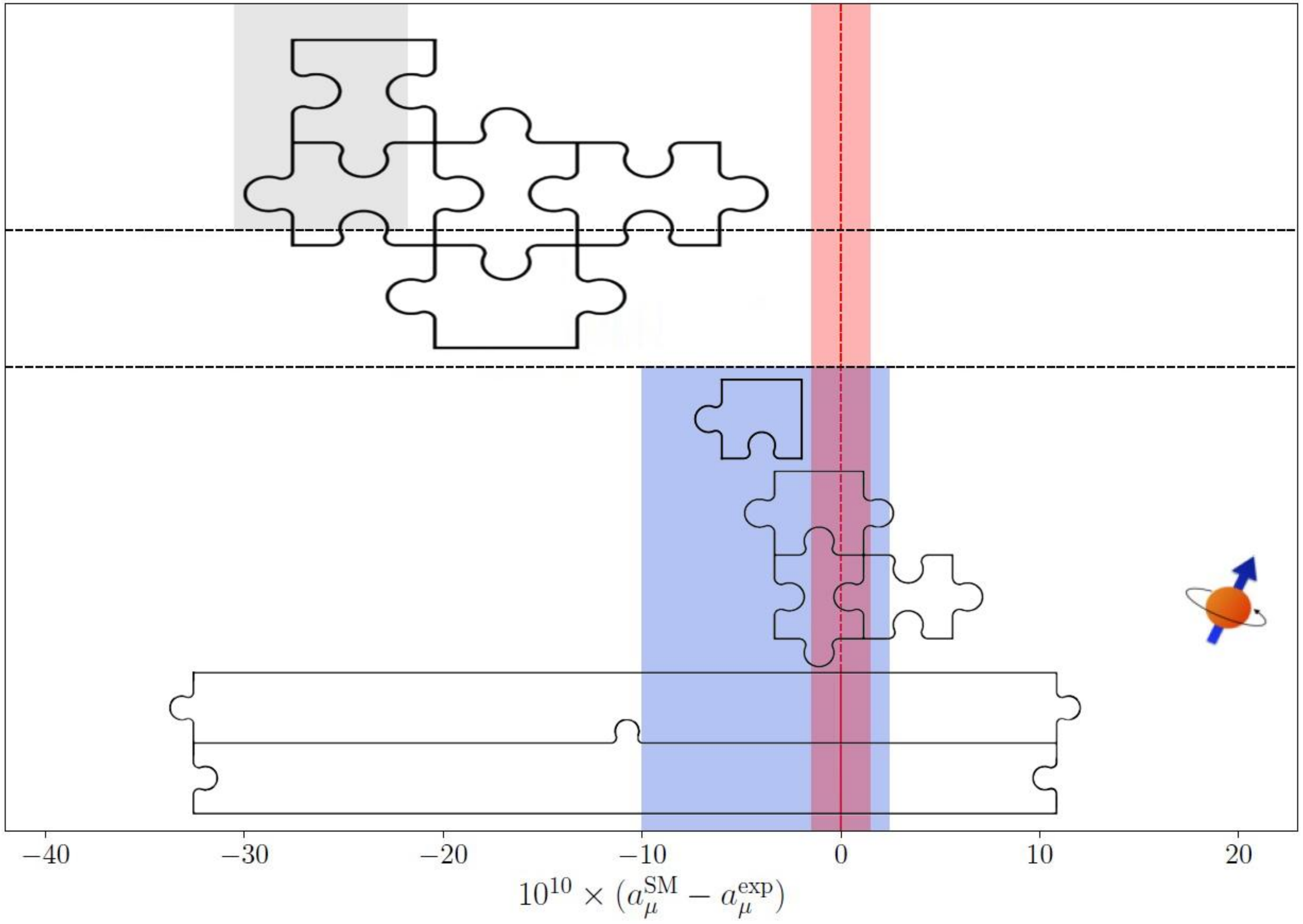


⟵ White Paper 2025

Confused?

Let's take it step by step…

The «Muon $g - 2$ puzzle»



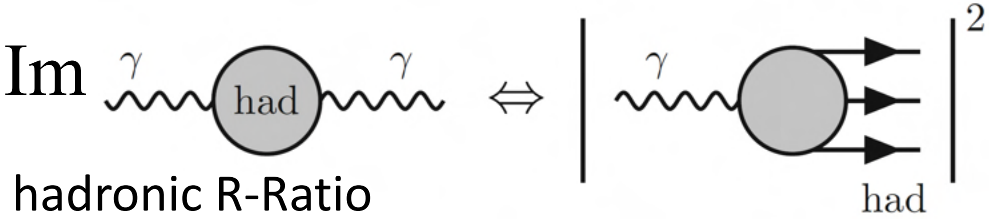
In this presentation:

What makes it a «puzzle»? ✓

Why is it like this? ✗

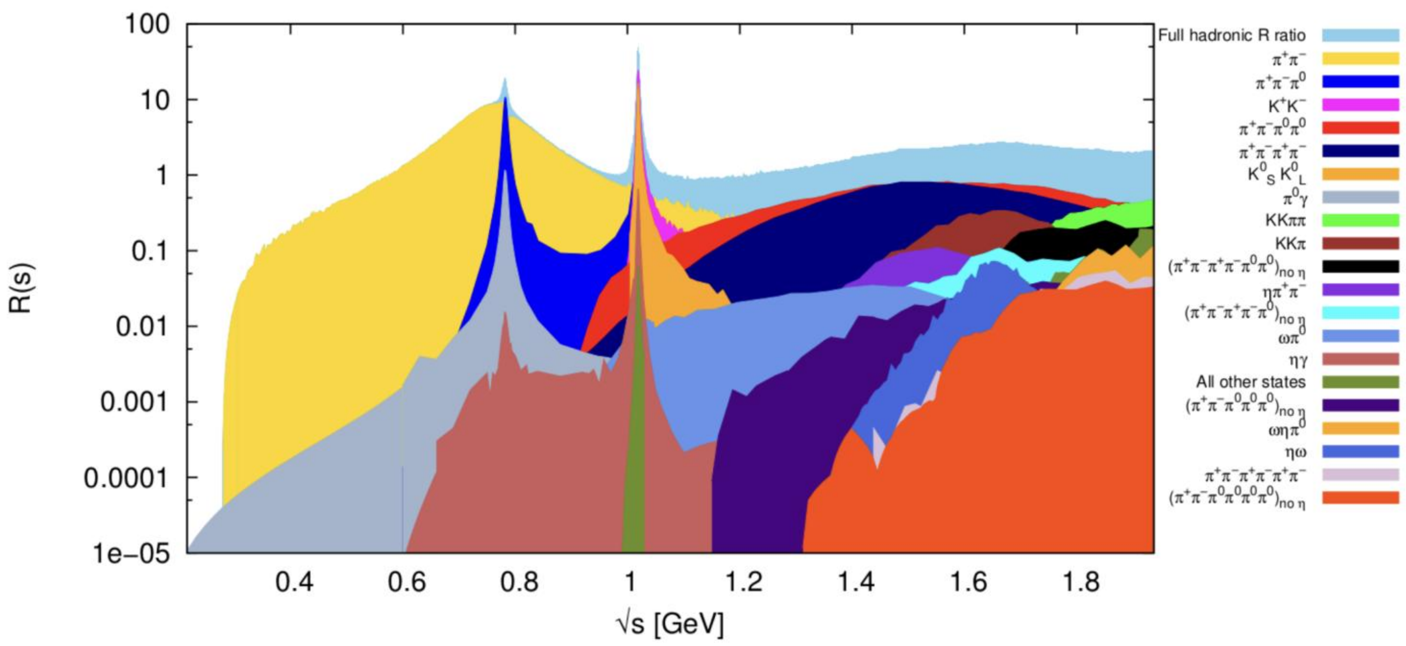
(we don't know…)

Dispersive method (e^+e^-) for HVP-LO

- Optical theorem: 

$$a_\mu^{HVP-LO} = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

- $R(s)$ is data-driven hadronic R-Ratio
- 20+ years of e^+e^- experiments: situation is, unfortunately, already unclear



Phys. Rev. D 97.11 (2018)

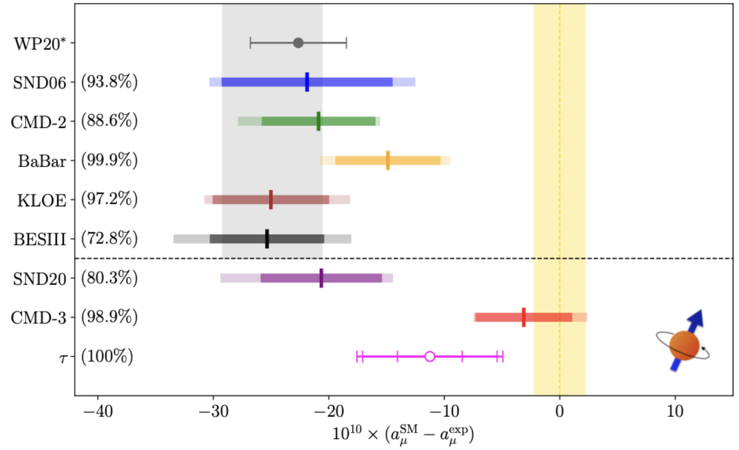


Figure 27: Summary of current data-driven evaluations of HVP, propagated to a_μ^{SM} (the yellow band indicates a_μ^{exp}, the gray band the WP20 SM prediction based on the e^+e^- data sets above the dashed line and the remainder from WP20, in particular, the WP20 HLbL value; the data point labeled WP20* indicates the shift upon using WP25 input for the other contributions besides LO HVP). The τ point corresponds to WP25 in Fig. 13, with the third, outmost error including the additional uncertainties beyond the 2π channel (the remainder of HVP is taken from WP20, the other contributions from WP25). The other points use input from the various $e^+e^- \to \pi^+\pi^-$ experiments according to Fig. 26 (again with HVP remainder from WP20 and the other contributions from WP25), where for each experiment the central values are obtained as simple average of the three combination methods, the inner ranges as simple average of the uncertainties obtained in each method, and the outer ranges reflect the maximal range covered by all methods (the percentages indicate how much of the 2π contribution to the HVP integral is covered by each measurement). We emphasize that these ranges are merely meant to illustrate the current spread, they cannot be interpreted as uncertainties with a proper statistical meaning. The numerical values follow from Tables 1 and 5.

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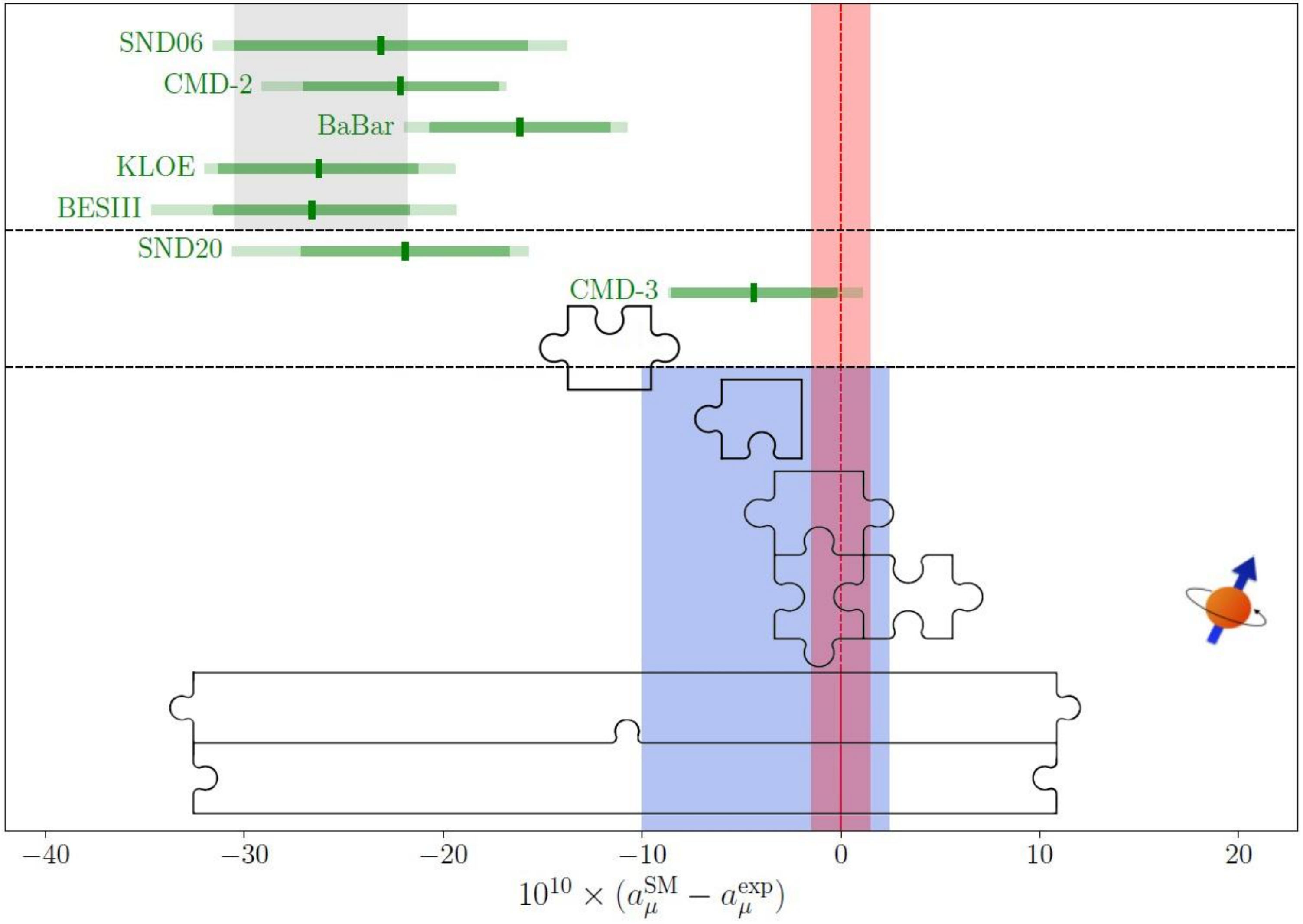
The «Muon $g-2$ puzzle»

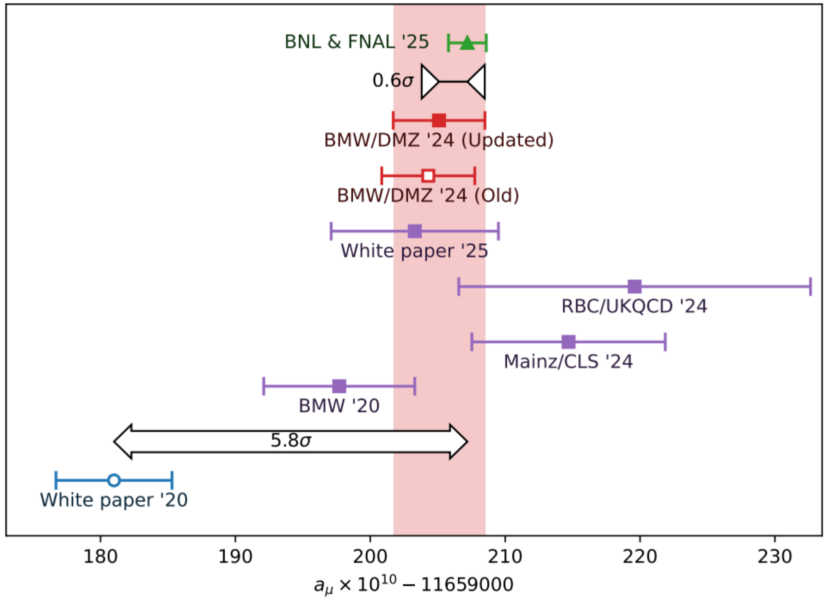


We have included:

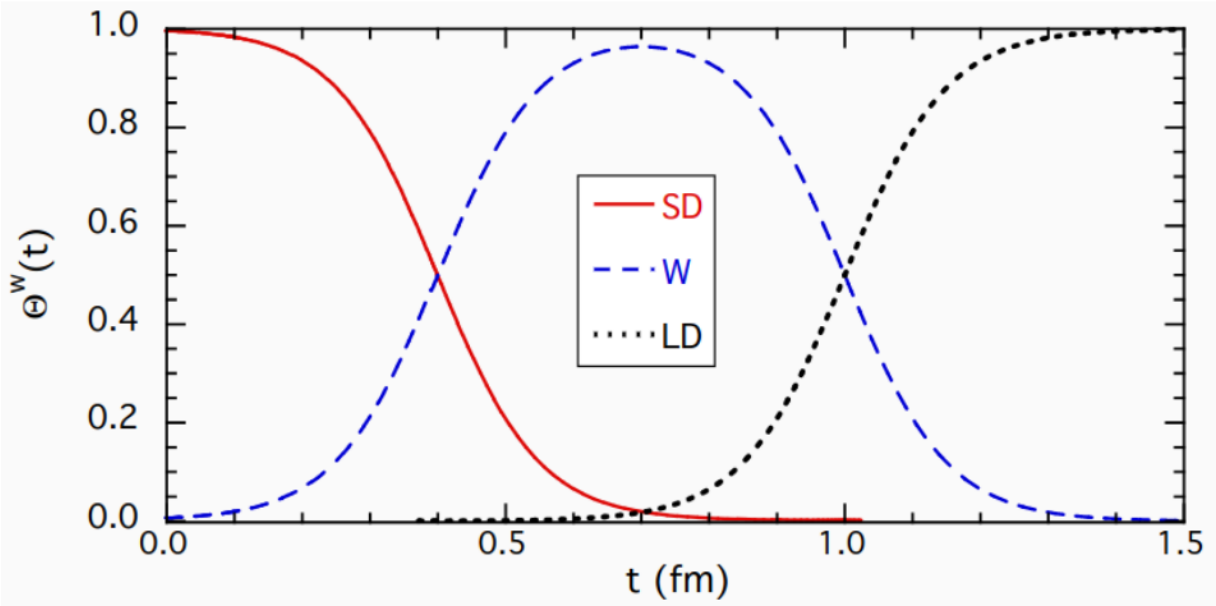
- HVP dispersive

Lattice QCD method for HVP-LO

- Ab-initio calculation of HVP from first principles, approximation of discrete space-time
- After WP2020: BMW collaboration published first sub-percent uncertainty

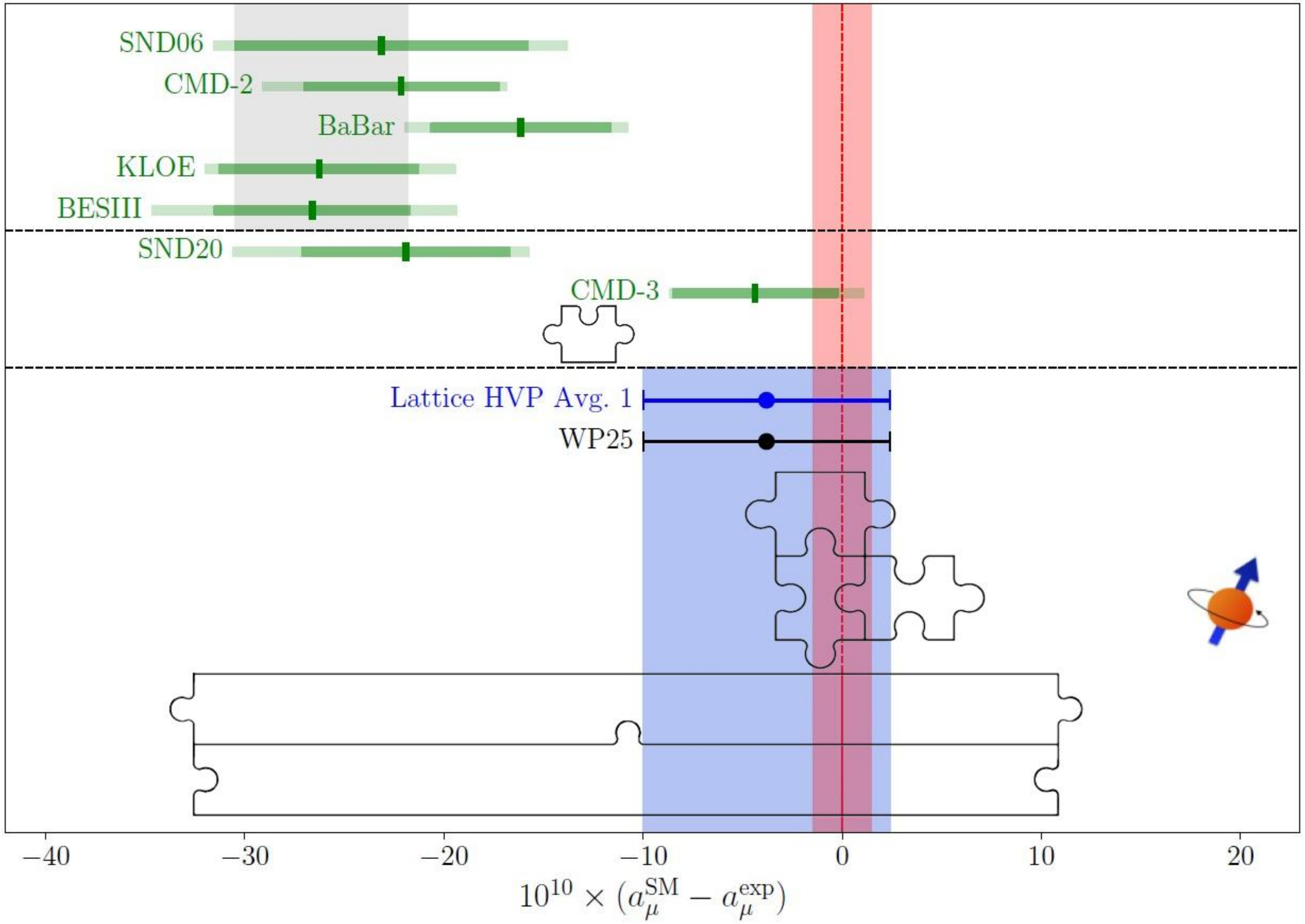


Finn M. Stokes, «Status of HVP from the BMW/DMZ collaboration», 10/09/25
Link to slides at the 8th T.I. workshop



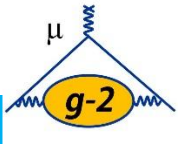
Window observables: excellent agreement in the intermediate window

The «Muon $g-2$ puzzle»

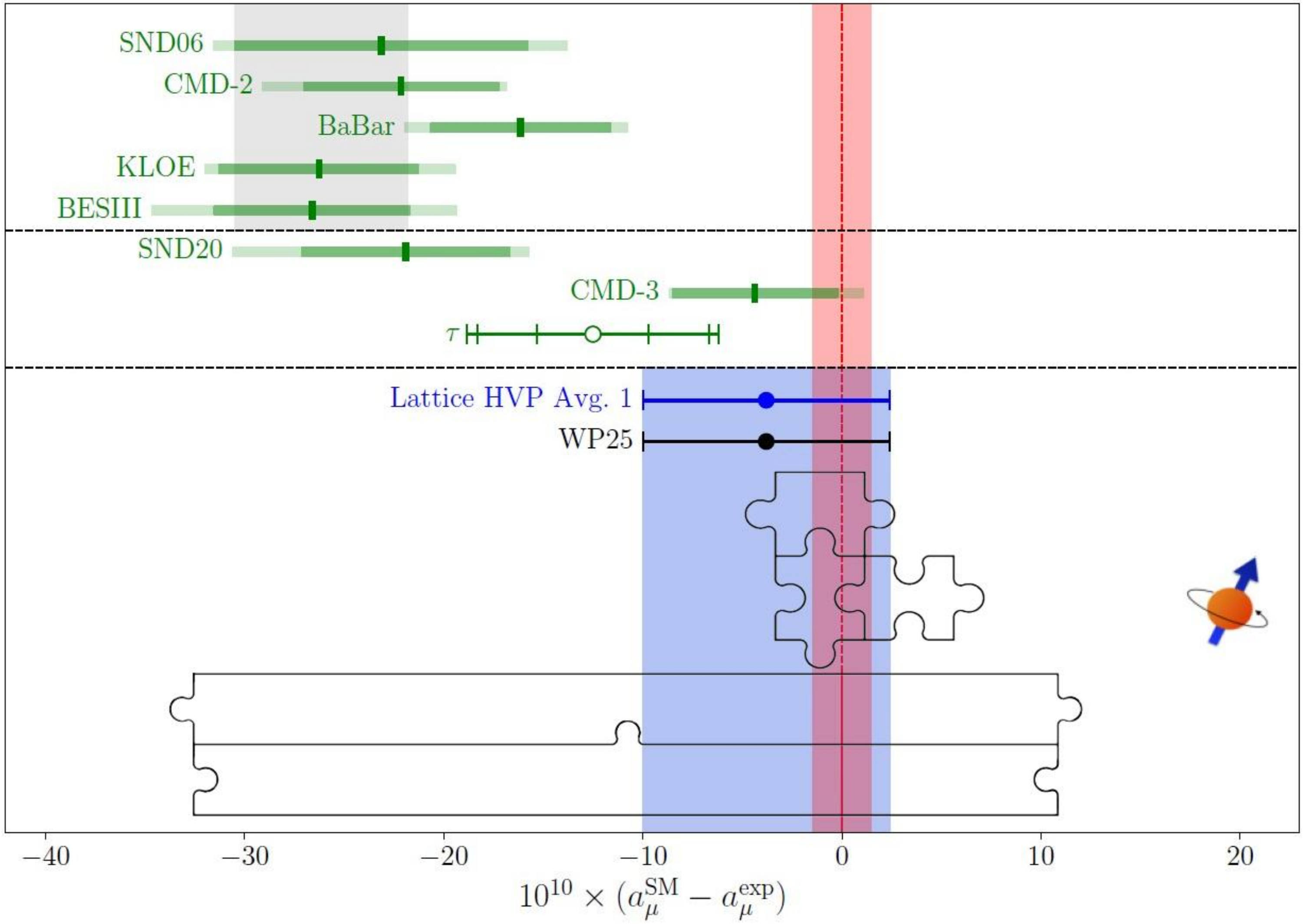


We have included:

- HVP dispersive
- HVP lattice



The «Muon $g-2$ puzzle»



We have included:

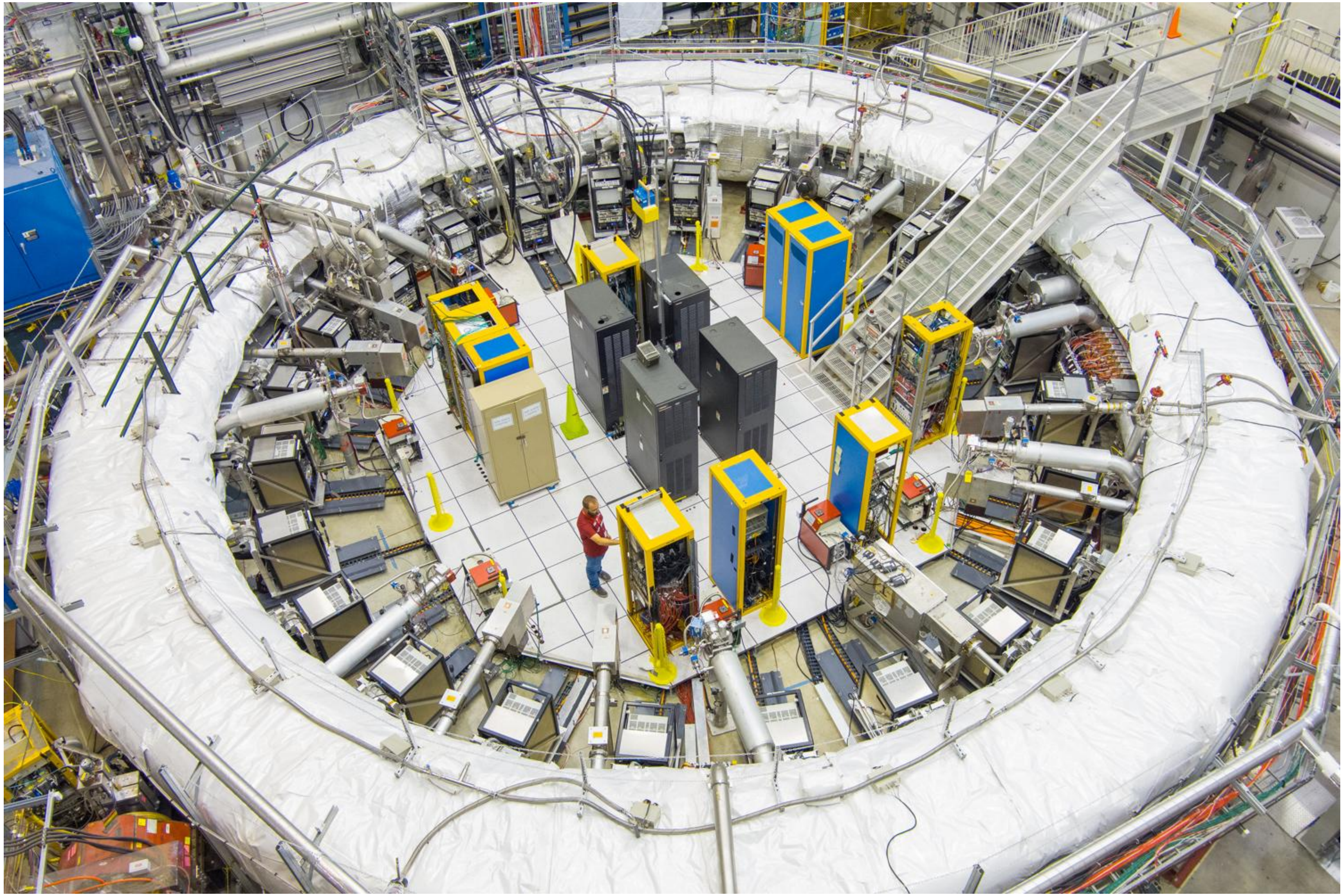
- HVP dispersive
- HVP lattice
- HVP from τ data

… Let's talk about the experiment(s), from now on

The «Muon g-2 Experiment» at Fermilab

Fermilab Muon Campus

The Muon $g - 2$ ring



The collaboration

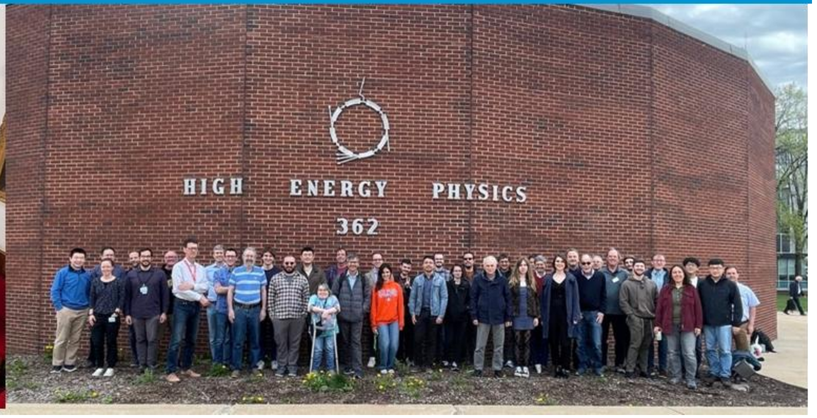
Collaboration meeting at Fermilab, March 2017

Collaboration meeting at Elba, Italy, May 2019

Online Collaboration Meeting
during Covid-19 period
April 2021

Collaboration meeting at Liverpool, UK, July 2023

Collaboration meeting at Argonne National Laboratory, April 2024



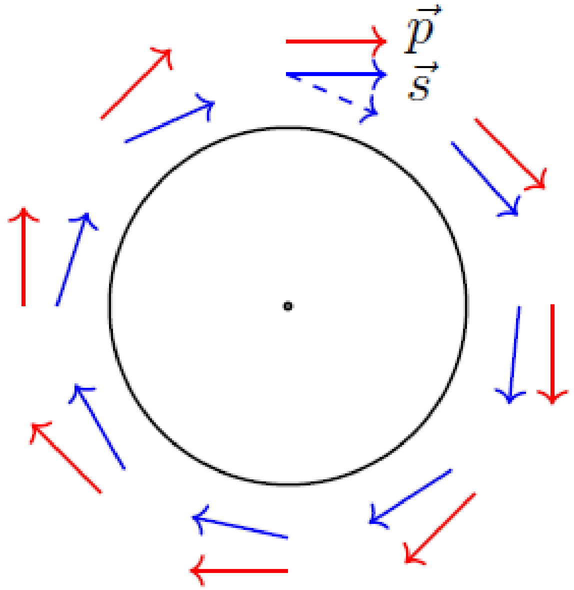
Anomalous spin precession in a B-field

$$\left.\begin{matrix} g-2 \neq 0 \\ a_\mu \neq 0 \end{matrix}\right\} \rightarrow \text{spin precesses with anomalous frequency } \vec{\omega}_a = \vec{\omega}_{\text{spin}} - \vec{\omega}_{\text{c}}$$

$$\vec{\omega}_a = -\frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E} - a_\mu \frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta}\right]$$

$\gamma = 29.3 \rightarrow p = 3.094$ GeV/c
"magic momentum"

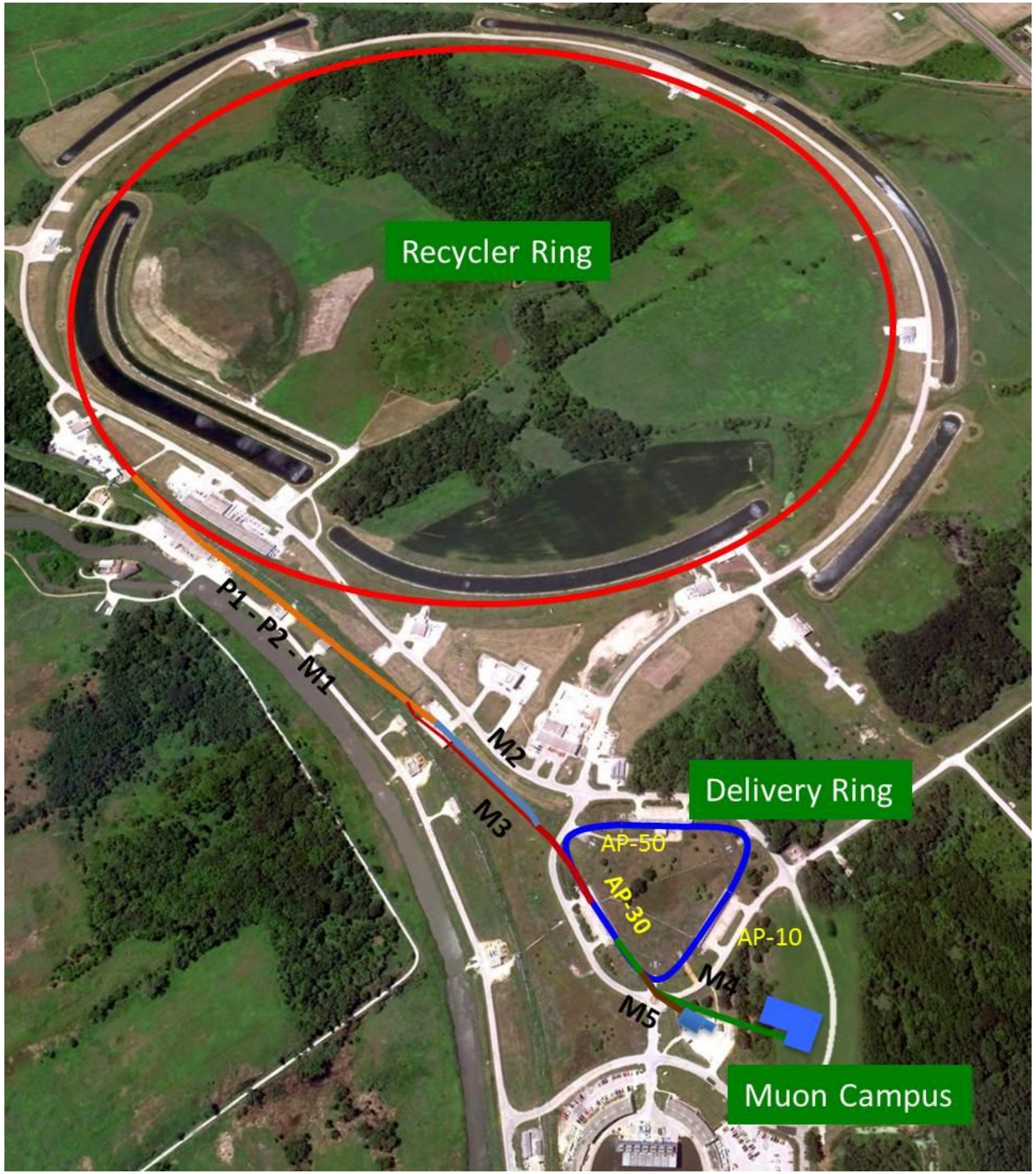
$\vec{\beta}\cdot\vec{B} = 0$



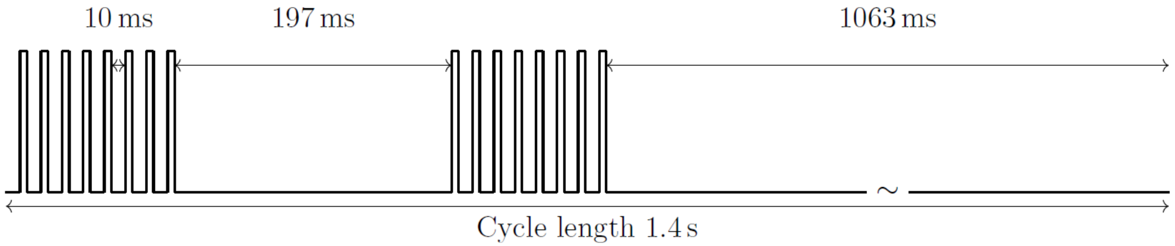
$\omega_c \sim 42.1$ rad/μs

$\omega_a \sim 1.439$ rad/μs ~
~ 12.4° per turn

FNAL accelerator complex

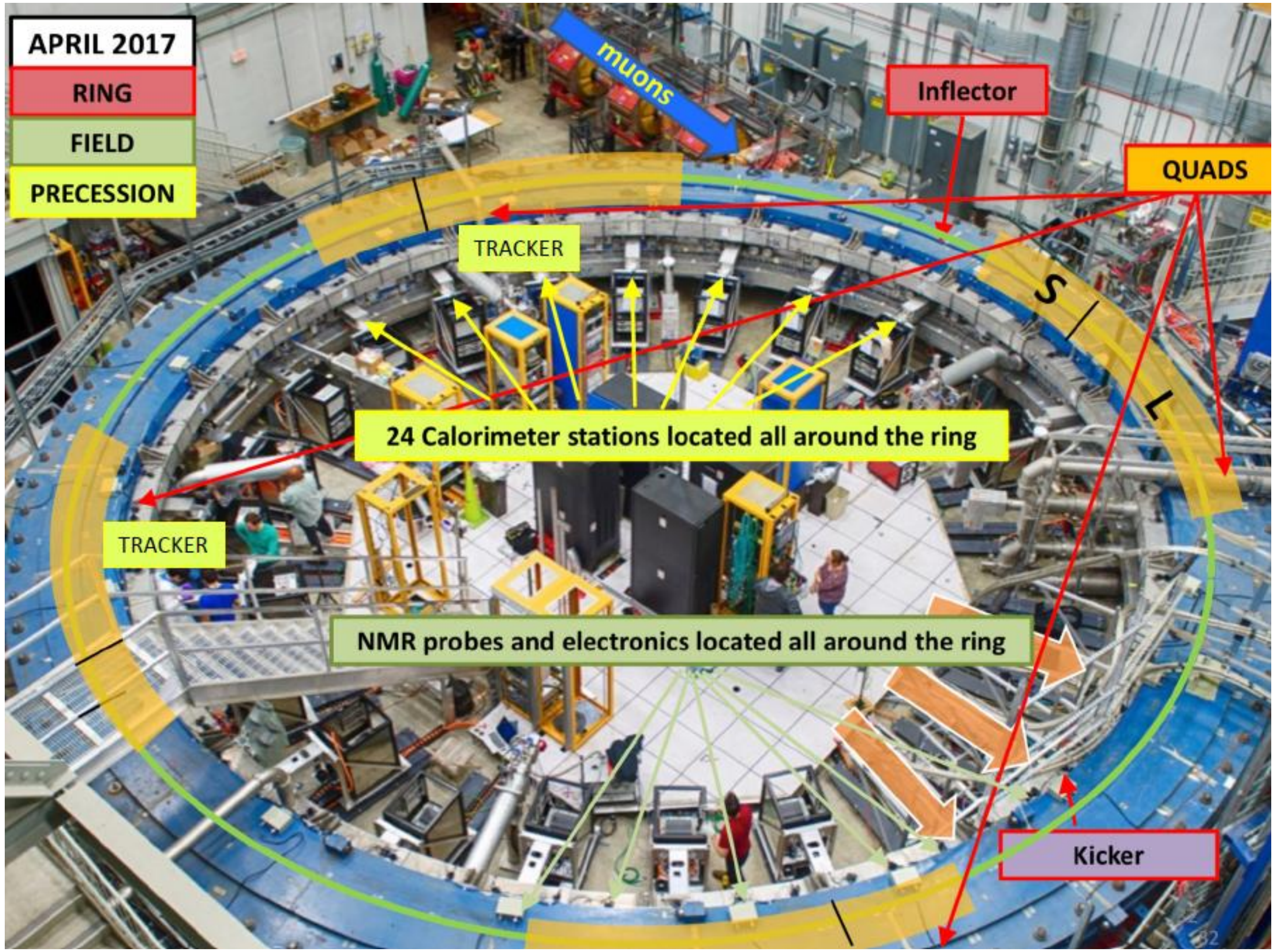


- Bunches of 4×10^{12} protons @ 8 GeV
- Boosted and delivered via the recycler ring every 1.4 s

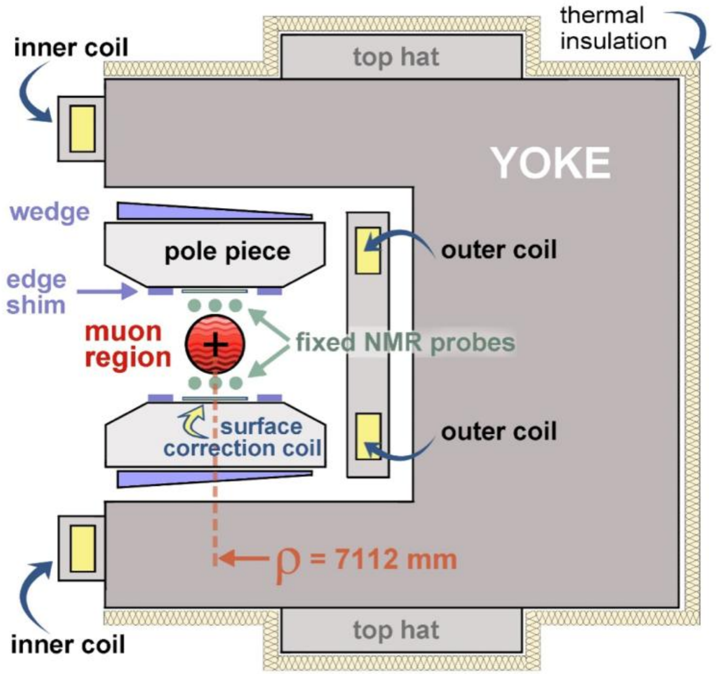


- Collide against fixed target and generate pions
- Pions decay into muons along ~ 2 km line in Delivery Ring
- Muons are injected into the 7 m radius $g-2$ storage ring

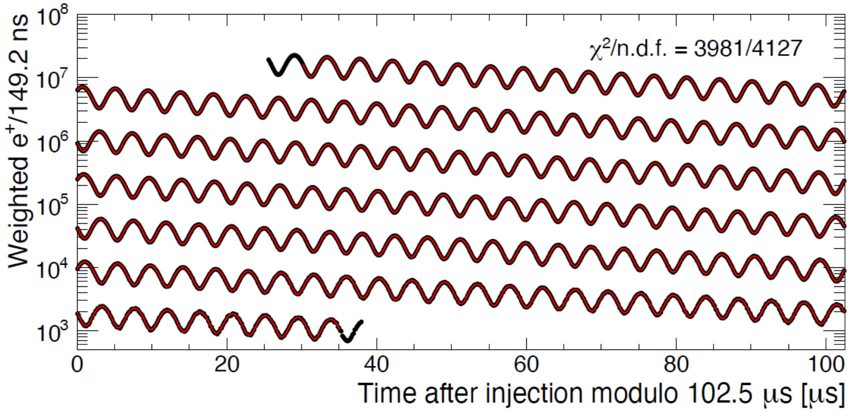
Subsystems of the experiment

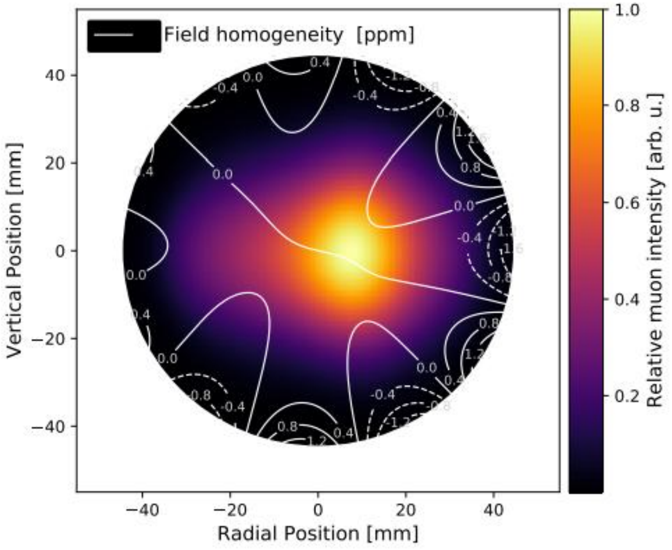


- 3.1 GeV polarized, positively charged muons stored for 700 μs
- Superconductive storage ring: 1.45 T B-field



Master formula for a_μ





$$a_\mu = \frac{\omega_a}{\tilde{\omega}'_p} \times \boxed{\frac{\mu'_p}{\mu_B} \frac{m_\mu}{m_e}}$$

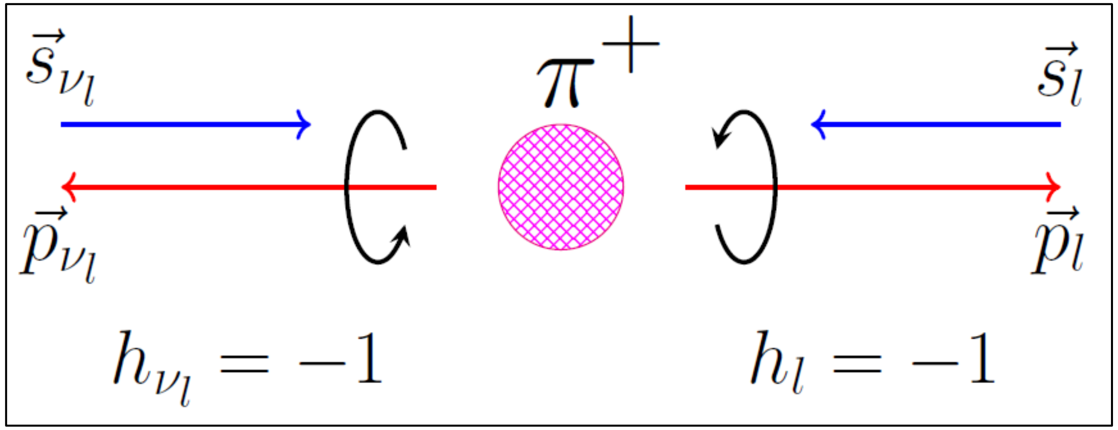
External factors, known to 25 ppb

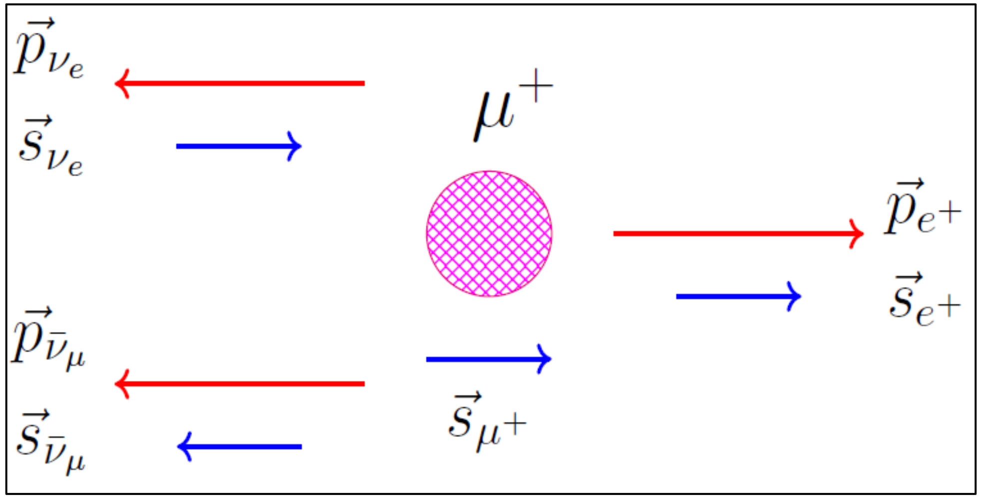
$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{\omega_a^m}{\tilde{\omega}'^m_p} \times$$ corrections for effects that…

m=Measured values

- Make spin precess slower (E-field, vertical motion)
- Make phase change within 700 μs
- Are synchronous to the muon injection so induce transient magnetic fields

Back to the properties of a muon!

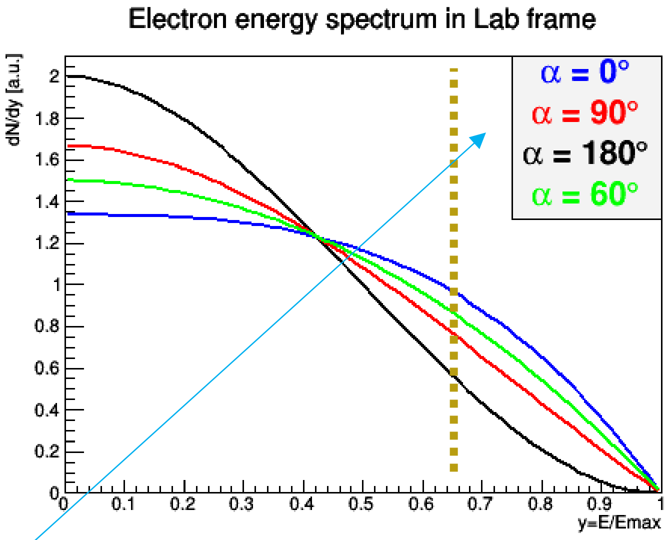


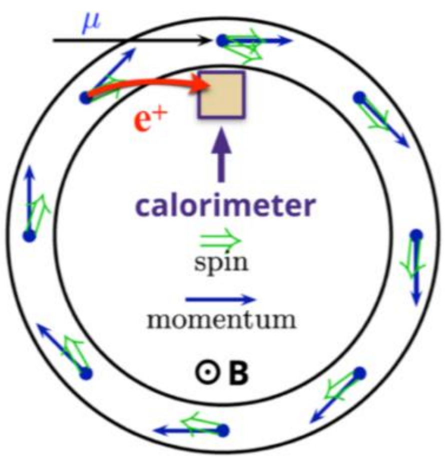


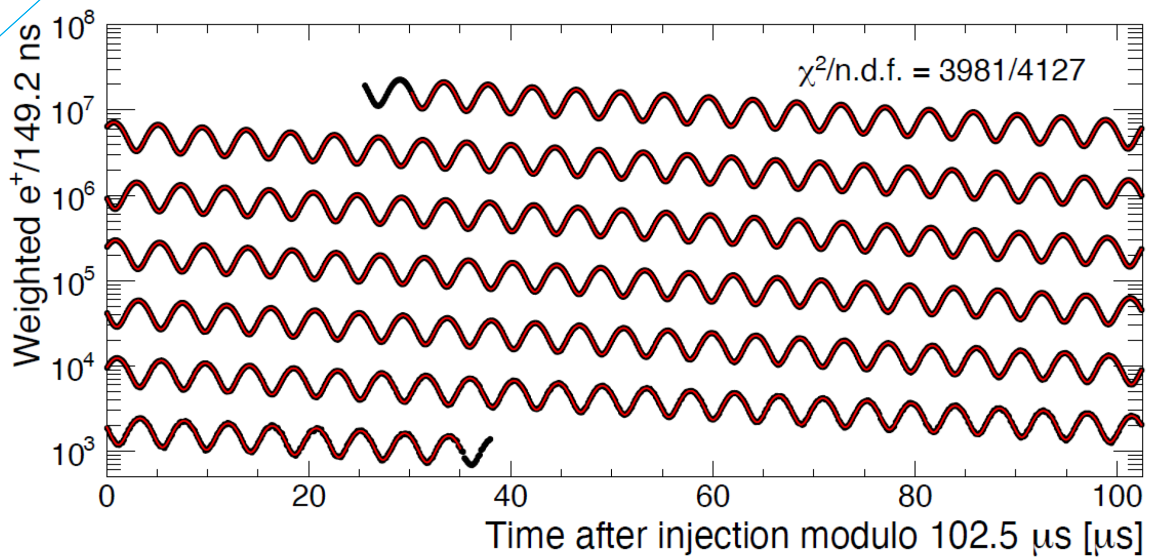
1. Polarized muon beam can be obtained by pion decay: $\pi^+ \to \mu^+ \nu_\mu$
2. Muons are unstable and almost always decay like this: $\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$
   No hadronic channels because they are lighter than a pion
3. Lifetime of 2.2 μs at rest: feasible to make and store
4. Parity violation: high-energy e^+ are preferentially emitted along μ^+ spin

Principle of ω_a measurement

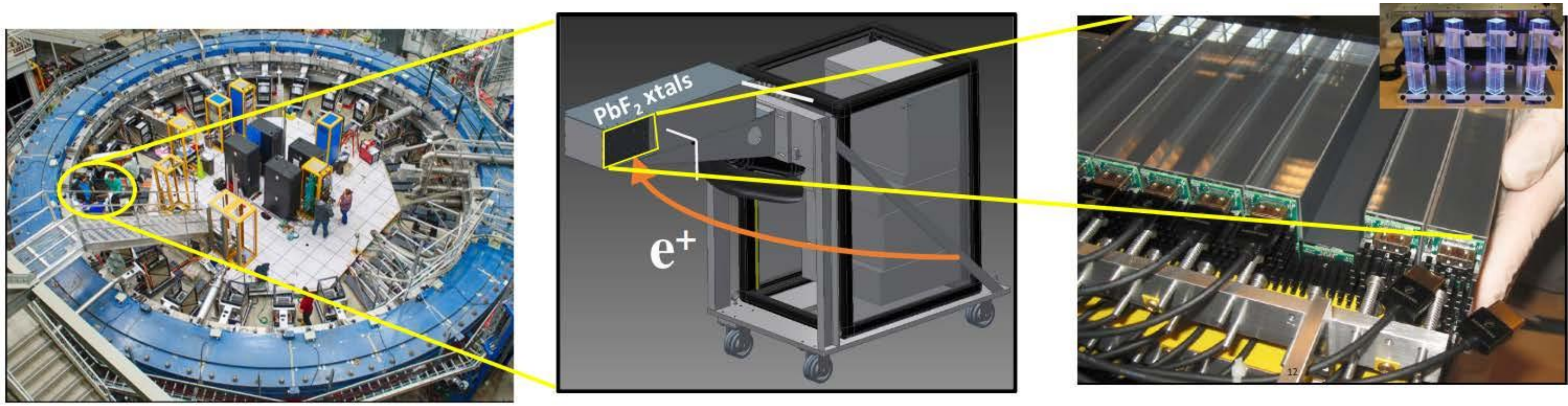
1. Weak decays violate parity:
   - polarized muon beam
   - preferred high-energy e^+ direction
2. Correlation in the lab frame between e^+ energy spectrum and ω_a phase (α)
3. «Wiggle plot»: count high-energy e^+ over time, for about 700 μs (muon lifetime is ~ 64 μs in the lab)







Detectors

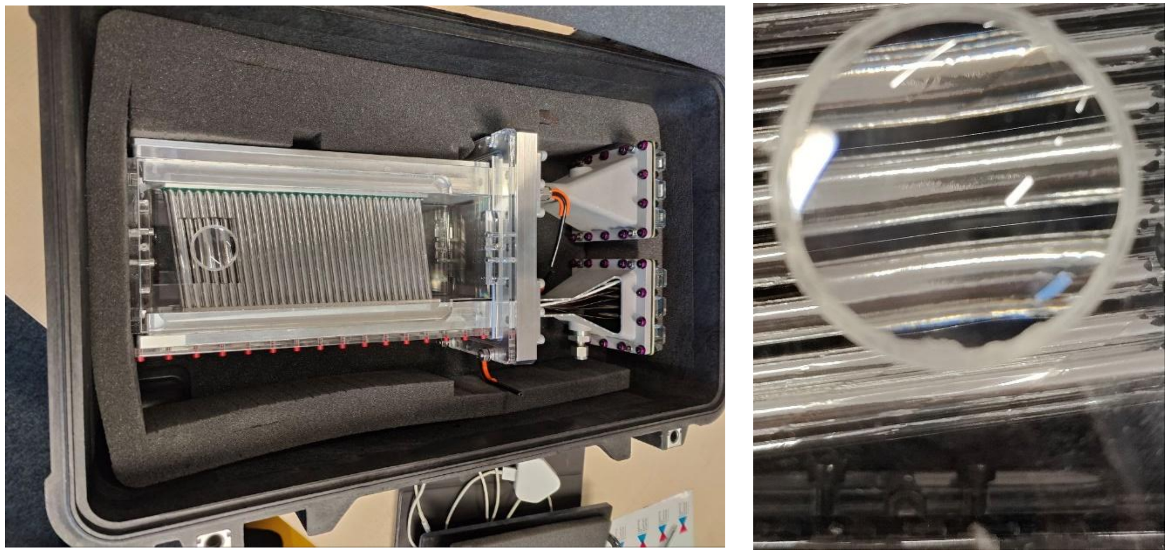


24 e.m. calorimeters

- Measure (E,t) of e^+
- Each made of 6×9 PbF_2 crystals, $15X_0$, read out by large-area SiPMs
- e^+ generate electromagnetic shower, SiPMs detect Cherenkov light ($n = 1.8$)

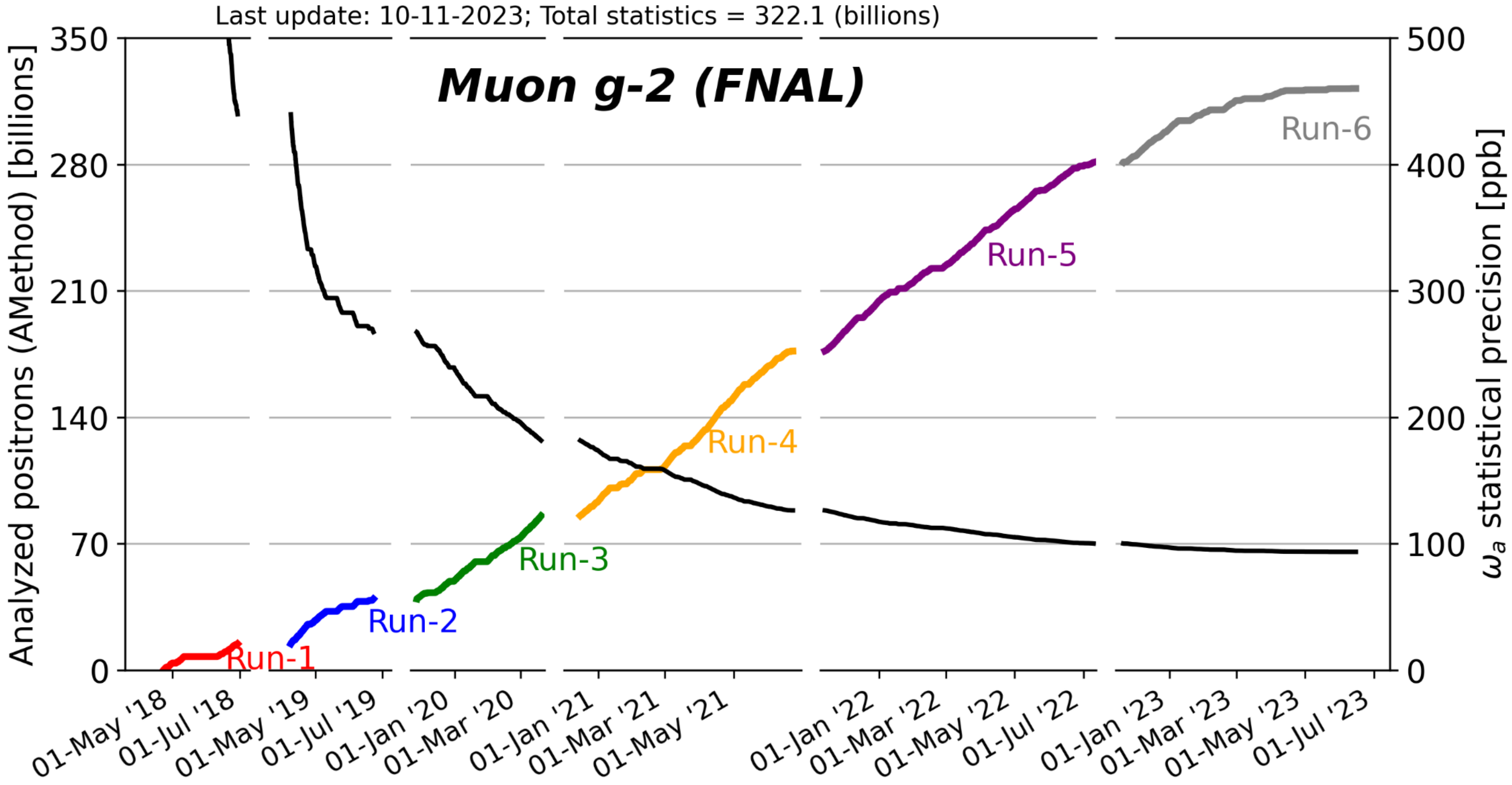
2 straw tube trackers

- Each has 8 modules and 32 planes
- 50:50 Argon:Ethane at 1 atm pressure
- Extrapolate decay vertex location to measure beam distribution



Made in Liverpool! We show prototypes at outreach events :D

Statistics: total number of analyzed e^+

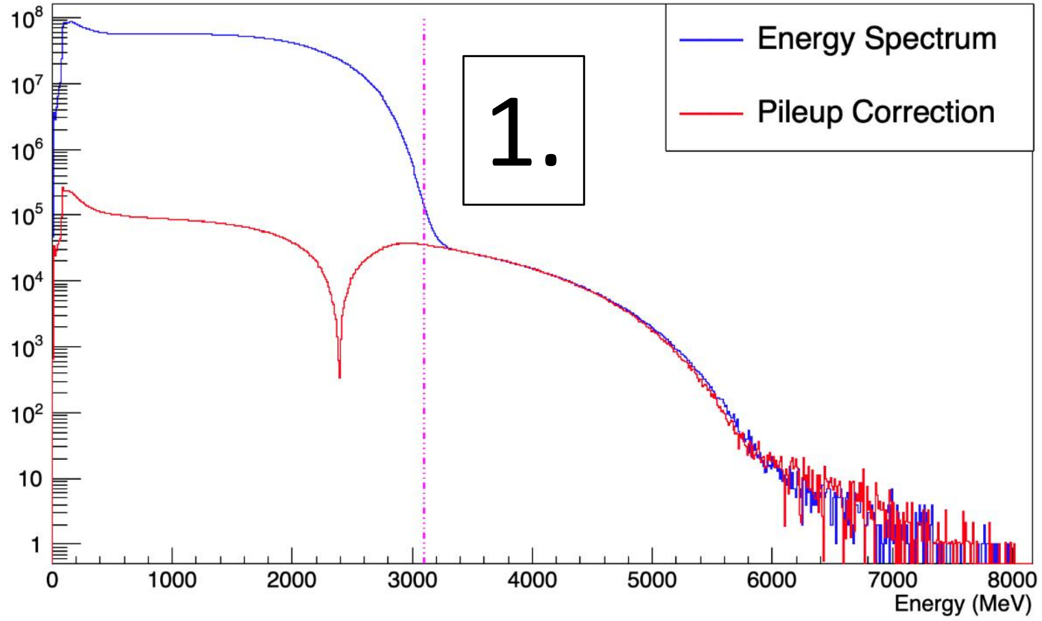


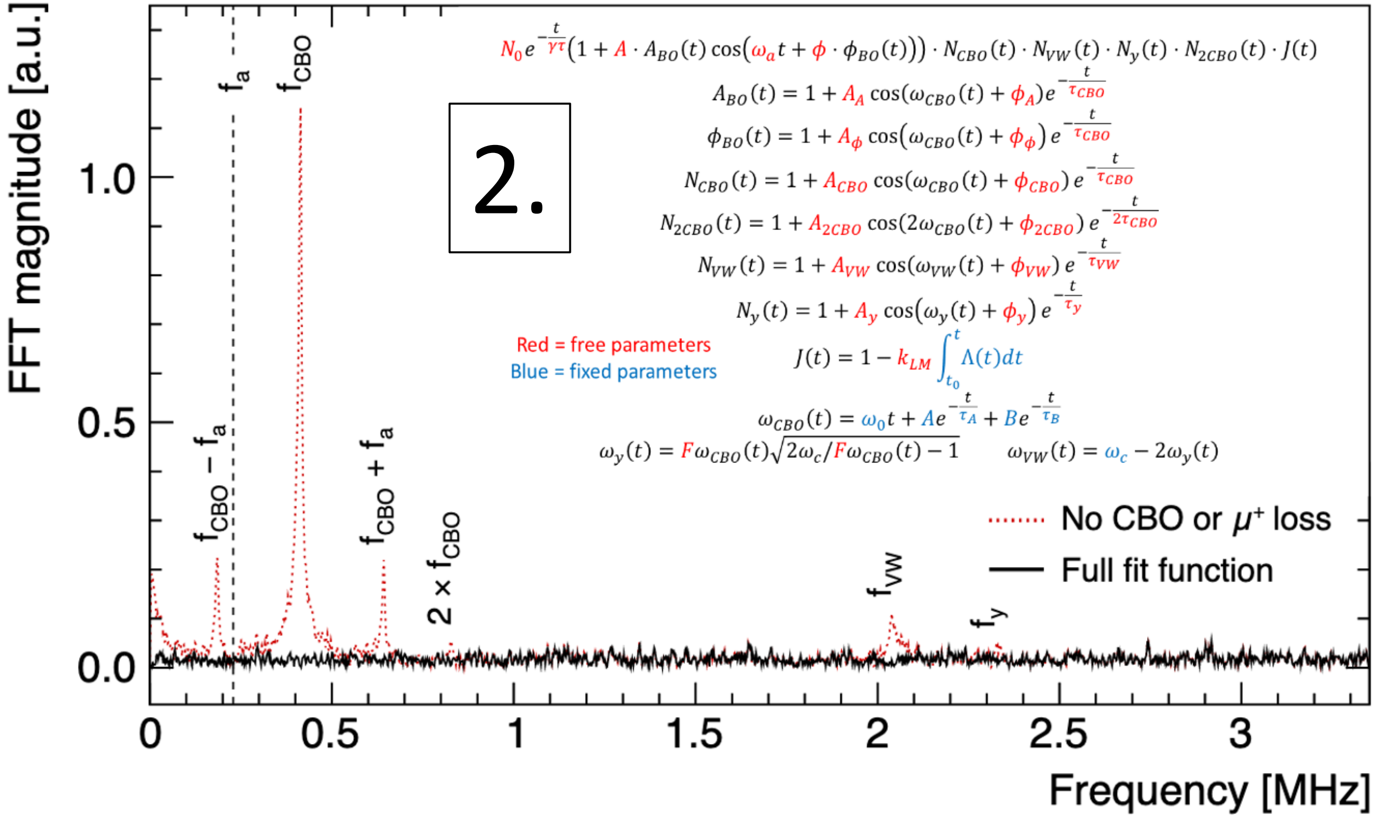
≳21 times the previous experiment at BNL

FNAL goal from TDR: 100 ppb → surpassed!

ω_a analysis in a nutshell

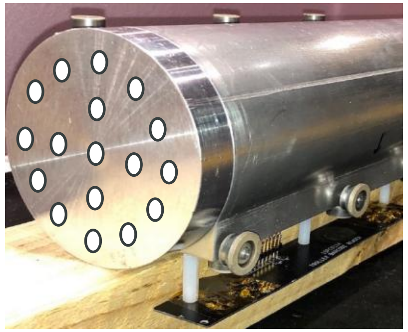
1. Count positrons and subtract pileup



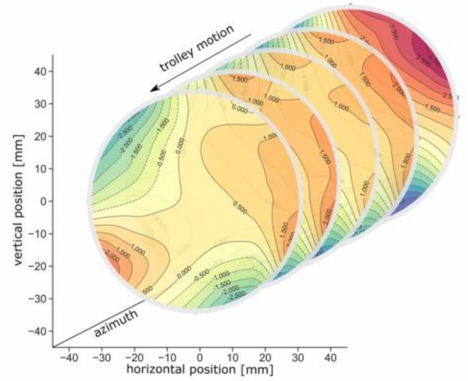


2. Fit wiggle plots with many parameters to account for beam dynamics effects

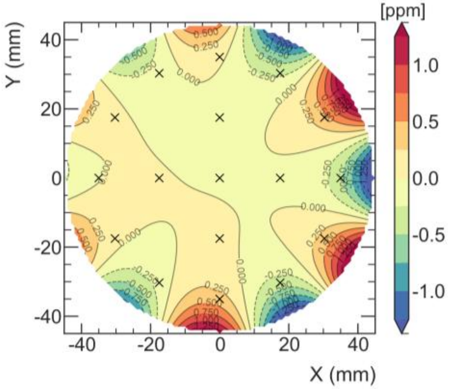
ω_p (field) analysis in a nutshell



17 petroleum jelly NMR probes

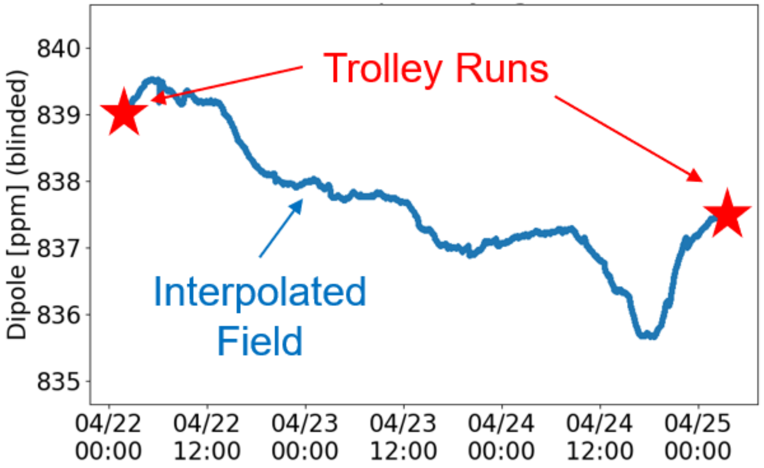


2D field maps (~9000 points)



Azimuthally-Averaged **Variation < 1 ppm**

Nuclear Magnetic Resonance (NMR) probes: placed on trolley for special runs, every 2 or 3 days between muon fills



378 fixed NMR probes monitor field during muon storage at 72 azimuthal locations

Calibration with respect to **shielded protons in a spherical sample**

Beam dynamics and field transients - briefly

$$\frac{\omega_a}{\tilde{\omega}'_p} = \frac{\omega_a^m\left(1 + C_e + C_p + C_{pa} + C_{dd} + C_{ml}\right)}{\langle \omega'_p \times M \rangle \left(1 + B_k + B_q\right)}$$

Quantity	Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)		114
ω_a^m (systematic)		30
C_e	347	27
C_p	175	9
C_{pa}	−33	15
C_{dd}	26	27
C_{ml}	0	2
$\langle \omega'_p \times M \rangle$ (mapping, tracking)		34
$\langle \omega'_p \times M \rangle$ (calibration)		34
B_k	−37	22
B_q	−21	20
μ'_p/μ_B		4
m_μ/m_e		22
Total systematic for $\mathcal{R}'_\mu$		76
Total for a_μ	572	139

C_e: $\vec{E}$ field, muon momentum distribution

C_p: muon vertical betatron amplitude

C_{pa}: time-dependent phase and acceptance

C_{dd}: spread of muon lifetimes in the beam

C_{ml}: momentum distribution of lost muons

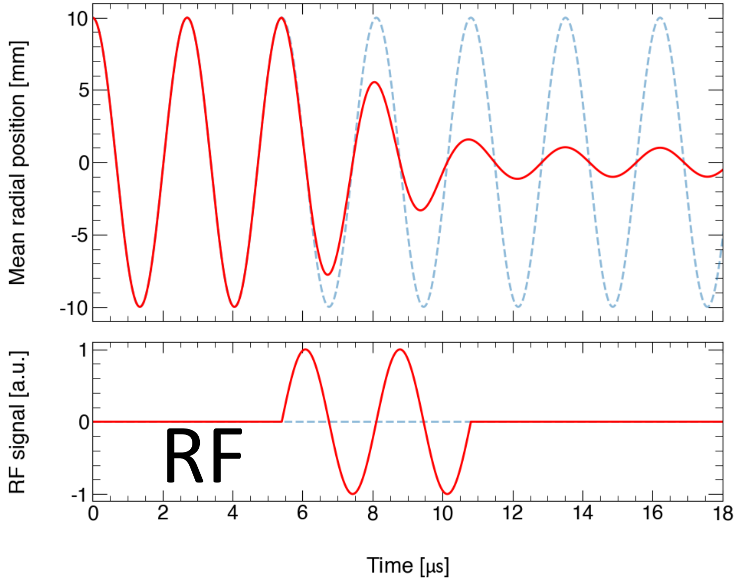
B_k: eddy currents in the kicker

B_q: pulsing of electrostatic quadrupoles

Latest result from Fermilab

Run-4/5/6 improvements

- Improved running conditions: Quad RF reduced muon oscillations; MiniSciFi detector for systematic runs
- More statistics and better understanding of systematics: surpassed Technical Design Report (TDR) goal

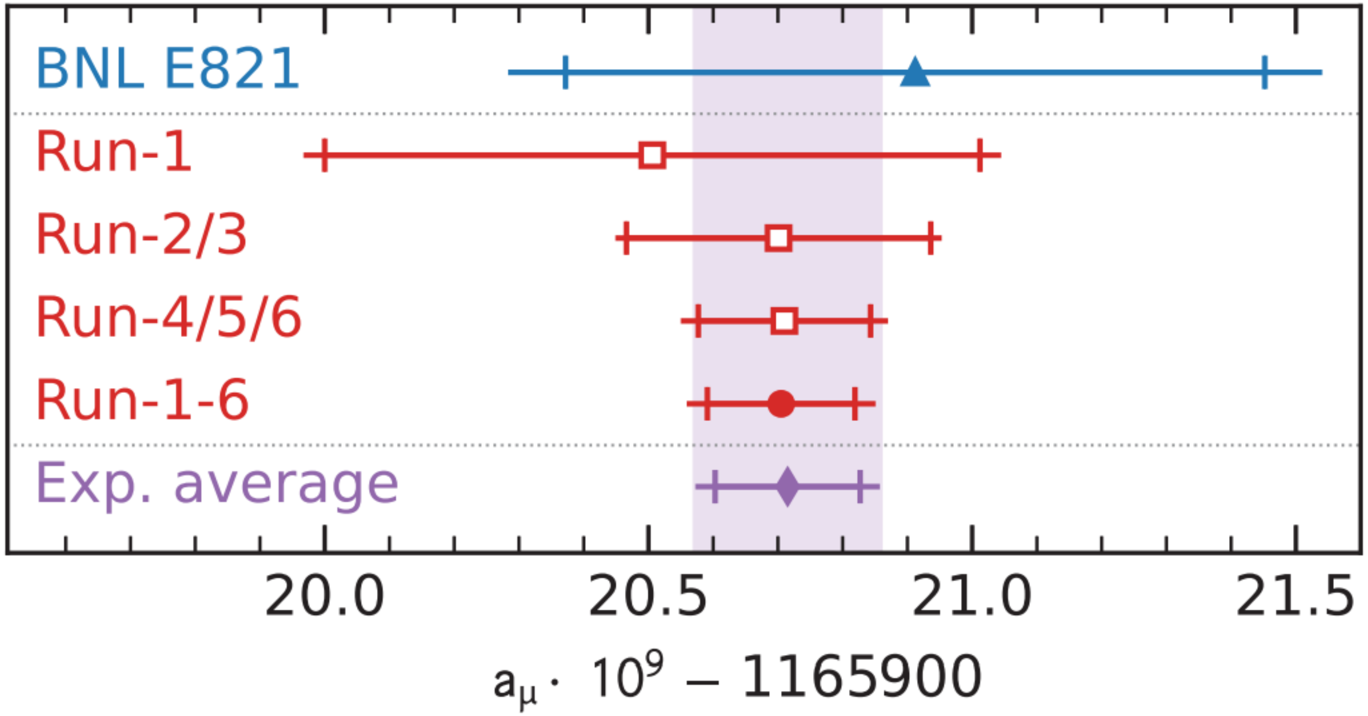


$\frac{\omega_a}{\tilde{\omega}'_p}$	Stat. Uncertainty (ppb)	Syst. Uncertainty (ppb)	Total Uncertainty (ppb)
Run-1	434	159*	462
Run-2/3	201	78*	216
Run-4/5/6	114	76	137
Run-1-6	**98**	**78**	**125**
	TDR goal 100 ppb ✓	TDR goal: 100 ppb ✓	TDR goal: 140 ppb ✓

Run-4/5/6 result: new World Average

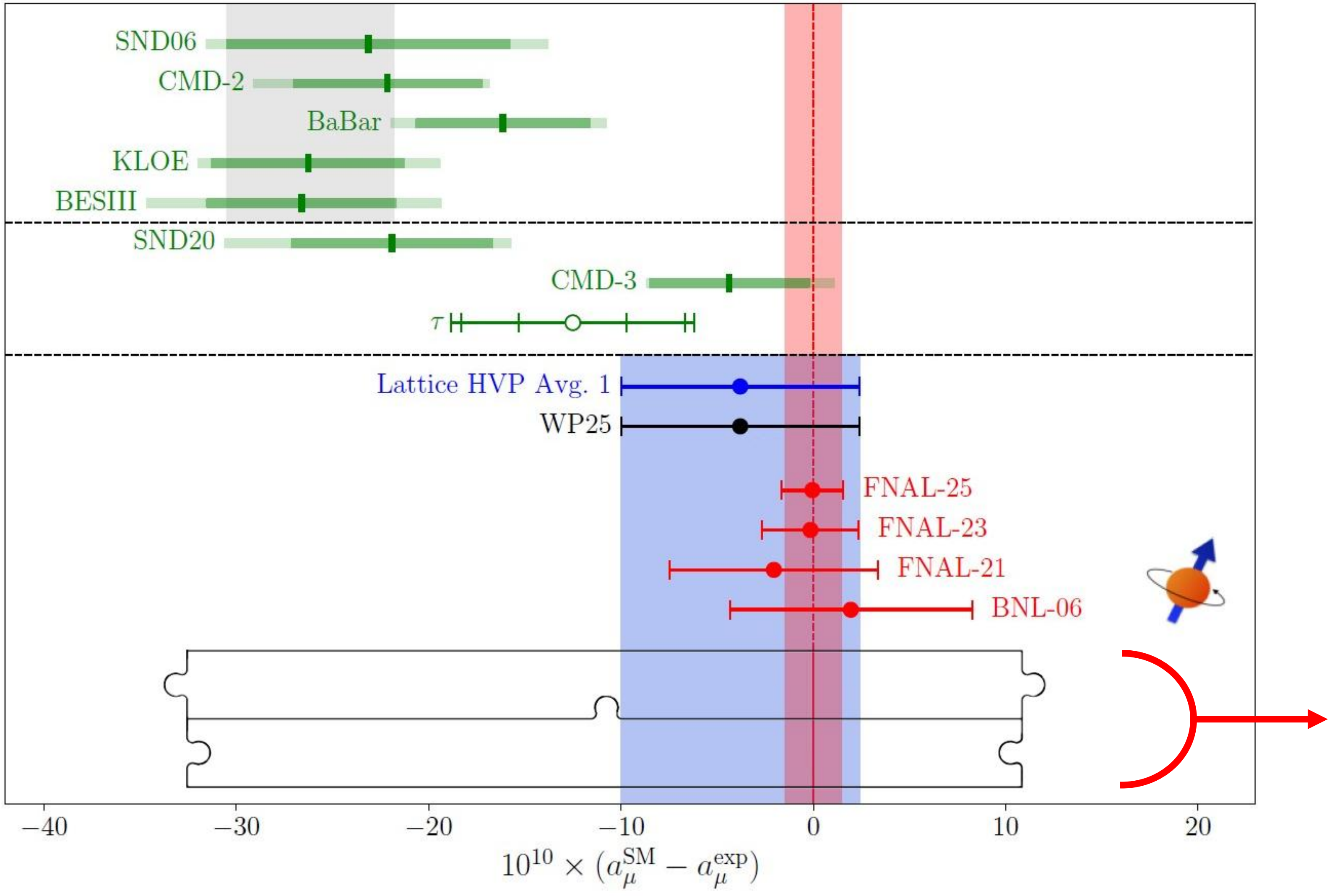
Most precise determination of a_μ for years to come

Stringent test of the SM and benchmark for BSM models



$$a_\mu^{exp}(\mathrm{W.A.}) = 116592071 5(145) \times 10^{-12}\ (124\ \mathrm{ppb})$$

The «Muon $g-2$ puzzle»



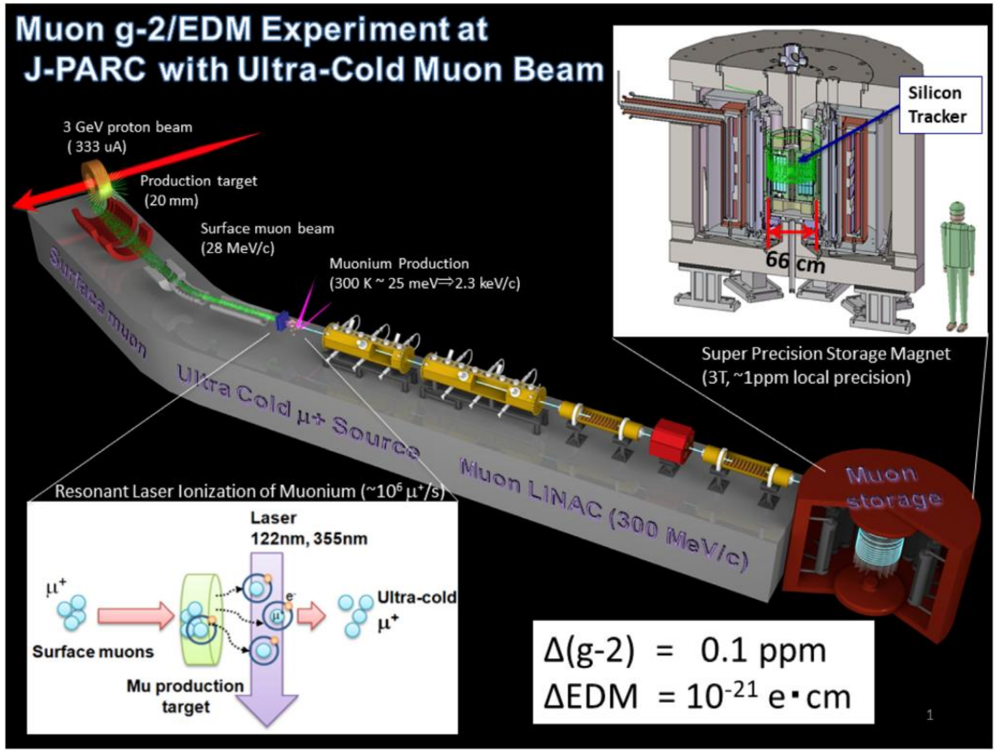
We have included:

- HVP dispersive
- HVP lattice
- HVP from τ data
- Experiments at BNL and FNAL

Who are these?

Future prospects

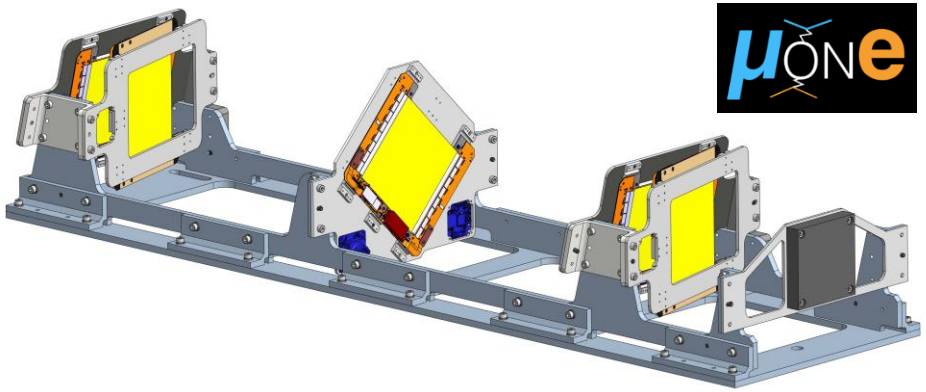
Future experiments: JPARC and MUonE

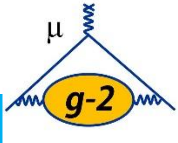


J-PARC Muon g-2/EDM (new technology, different systematics from Fermilab)

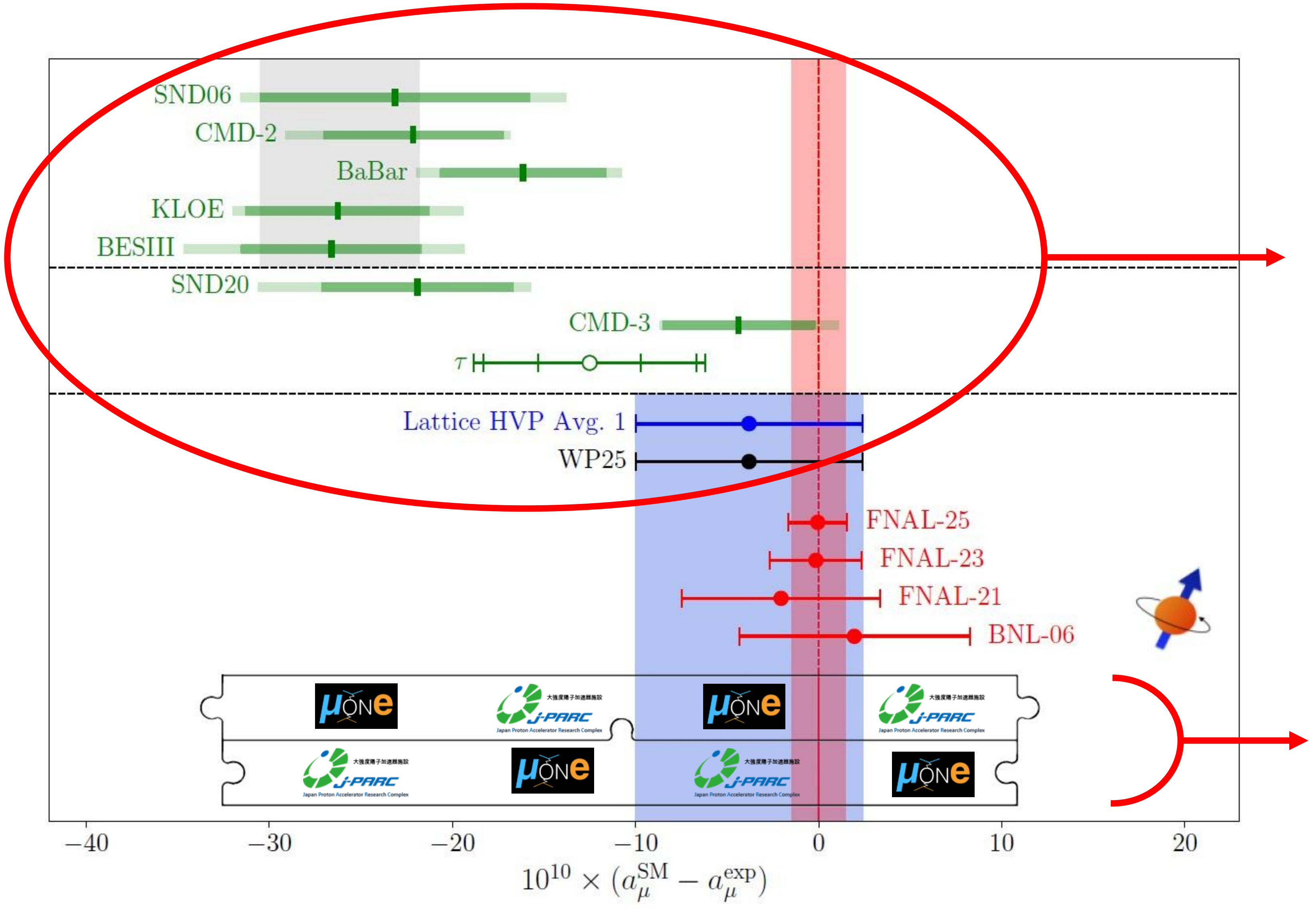
MUonE:

- Extract leading-order HVP from hadronic running of α_{QED} in the space-like region
- Measure directly muon-electron elastic scattering differential cross section shape
- Goal of 0.3% uncertainty statistical, similar systematic; comparable with data-driven HVP





The future «Muon $g-2$ puzzle»



Hopefully will be resolved!!

More points (some already at latest T.I. workshop), improved MC generators, …

More independent measurements

Summary and conclusions

- New a_μ experimental world average has **unprecedented precision of 124 ppb**
  FNAL collaboration now focusing on Beyond SM searches (EDM, CPTLIV, DM)
- Theoretical prediction for a_μ will hopefully consolidate in the coming years, with new, competitive, independent inputs (HVP dispersive, lattice, tau; MUonE, J-PARC g-2/EDM)
- Topics of related HEP seminars at Liverpool:
  - D. Tarazona, 23/03/2022: Muon $g-2$ beam dynamics
  - G. Venanzoni, 29/11/2023, Inaugural Lecture
  - R. Pilato, 28/06/2024: The MUonE experiment
  - B. Quinn, 05/07/2024: CPTLIV searches in Muon $g-2$
  - T. Mibe, 03/10/2024: g-2/EDM at J-PARC
  - E. Bottalico, 08/11/2024: Muon $g-2$ at Fermilab

  Future HEP seminar by Dominika Vasilkova on muonEDM

THANK YOU FOR YOUR ATTENTION!

lorenzo.cotrozzi@liverpool.ac.uk

July 2023 collaboration meeting @ Liverpool, UK

Extra: MUonE and J-PARC (taken from latest T.I.)

From R. Pilato

The MUonE experiment

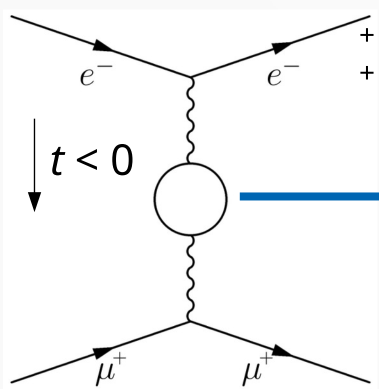
Phys. Lett. B 746 (2015), 325
Eur. Phys. J. C 77.3 (2017), 139
Letter of Intent CERN-SPSC-2019-026
Proposal for Phase 1 of the MUonE experiment

New independent evaluation of $a_\mu^{\mathrm{HVP,LO}}$, based on the measurement of $\Delta\alpha_{\mathrm{had}}(t)$ in the space-like region

$$a_\mu^{HLO} = \frac{\alpha_0}{\pi} \int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)] \qquad t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

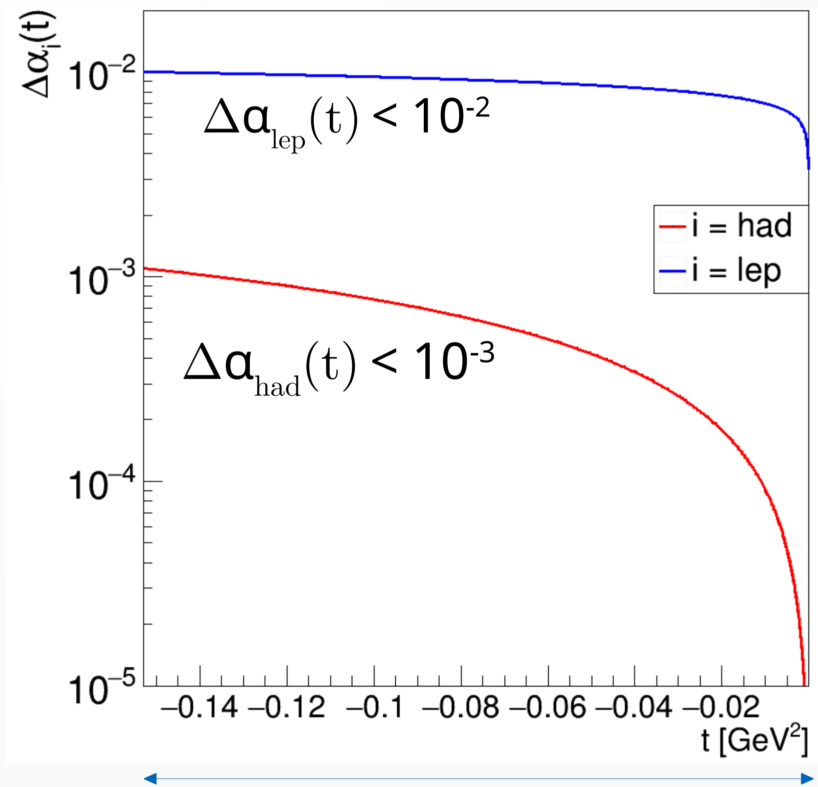
Extract $\Delta\alpha_{\mathrm{had}}(t)$ from the *shape*

of ***μe → μe*** differential cross section



+ higher orders
+ radiative corrections

$$\Delta\alpha(t) = \Delta\alpha_{\mathrm{lep}}(t) + \Delta\alpha_{\mathrm{had}}(t) + \Delta\alpha_{\mathrm{top}}(t)$$



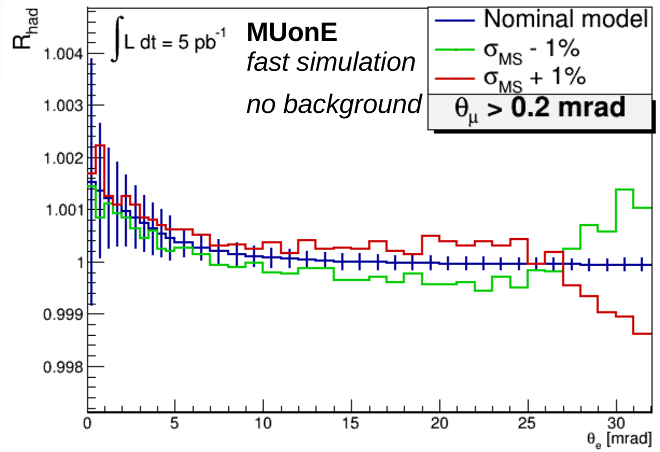
MUonE kinematic range

The MUonE experiment

From R. Pilato

MUonE final goal:

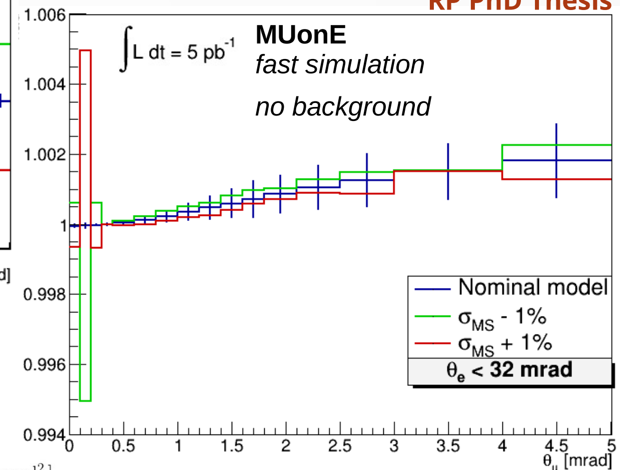
- ~3 years post LS3 (>2030)
- 40 stations
- $a_{\mu}^{\mathrm{HVP,LO}} < 0.5\%$



multiple scattering ±1% systematic effect

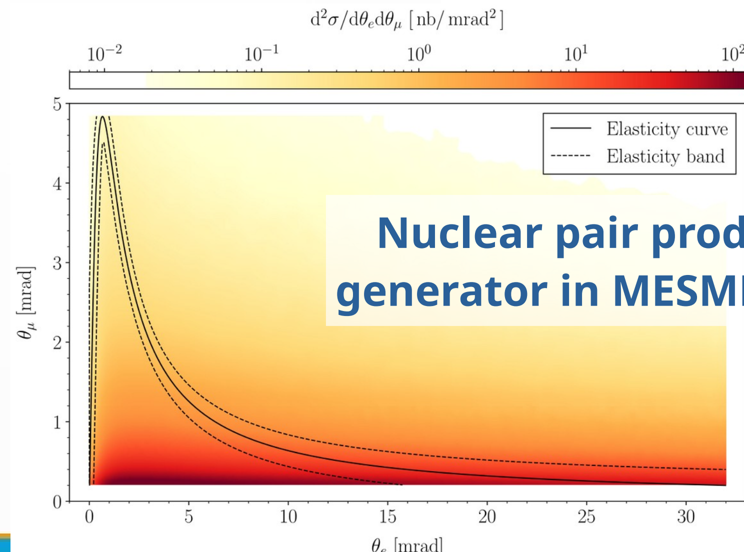
Phys. Scr. 97 (2022) 054007

RP PhD Thesis



Systematic error goal: 10 ppm

- 10 µm longitudinal alignment
- Beam energy measured to few MeV
- Multiple scattering 1%
- Angular intrinsic resolution
- Uniform detector response over full angular range
- Need of dedicated MC generators: signal (>NNLO), main backgrounds



Phys. Lett. B 854 (2024)

Nuclear pair production event generator in MESMER (also signal)

MESMER

McMule

J-PARC muon g-2/EDM experiment

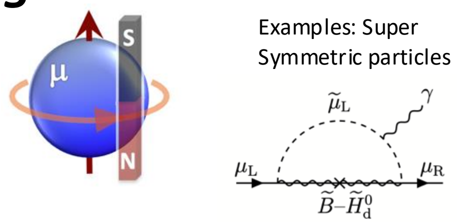
From T. Mibe

Studies on physics beyond the standard model in quantum loops

***g*-2**

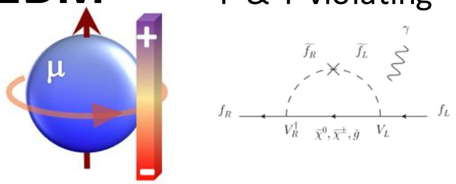
C, P, T conserved

Examples: Super Symmetric particles

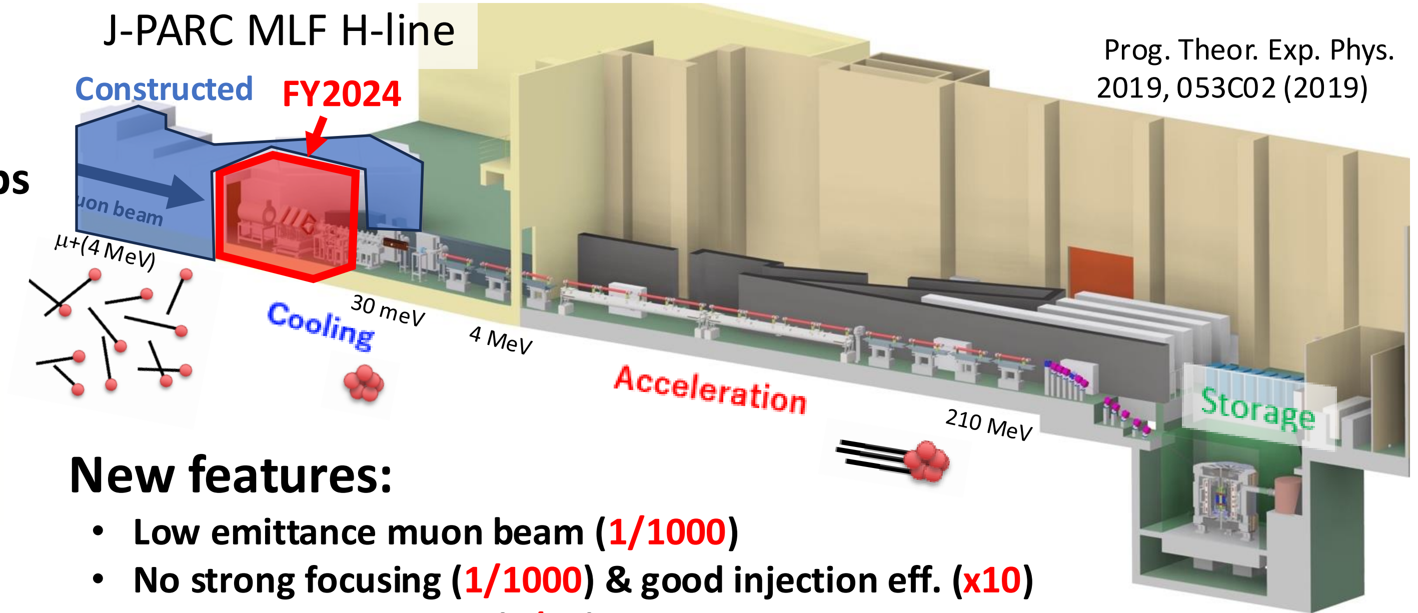


EDM

P & T-violating



J-PARC MLF H-line



Prog. Theor. Exp. Phys. 2019, 053C02 (2019)

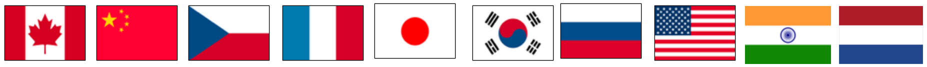
New features:

- Low emittance muon beam (1/1000)
- No strong focusing (1/1000) & good injection eff. (x10)
- Compact storage ring (1/20)

The only experiment to test FNAL/BNL *g*-2 results.

***g*-2 : 450 ppb**

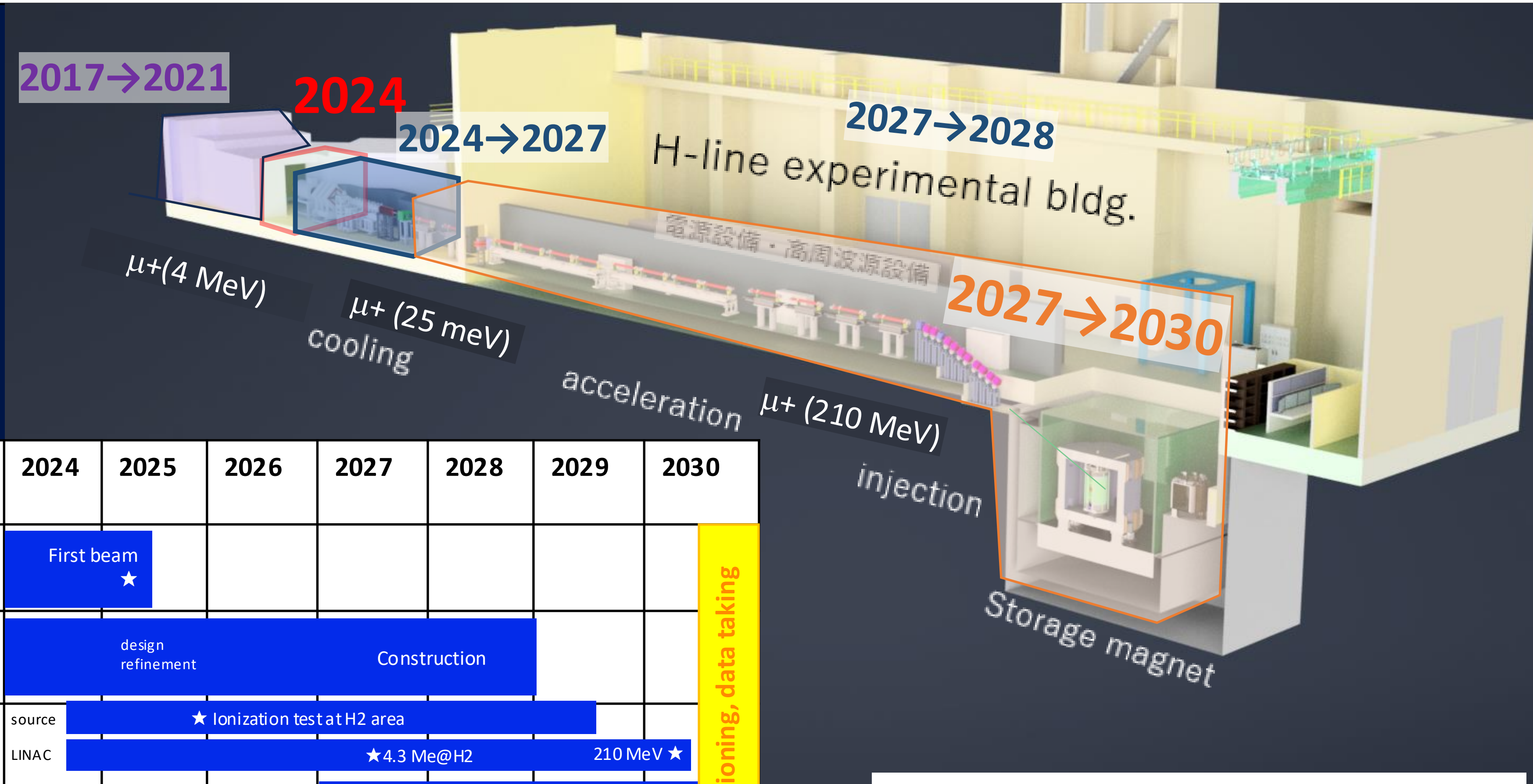
EDM : 1.5 E-19 ecm



Welcome U. Liverpool group in Dec, 2024 !!

Muon g-2/EDM : intended schedule

From T. Mibe



		2024	2025	2026	2027	2028	2029	2030
Beamline		First beam ★						Commissioning, data taking
Bldg. & facility			design refinement		Construction			
Source, LINAC, storage	source			★ Ionization test at H2 area				
	LINAC				★4.3 Me@H2		210 MeV ★	
	Storage				procurement		Installation ★	
Detector			positron tracker magnetic field monitors					Installation ★

Strong supports from the community is necessary to complete remaining construction.

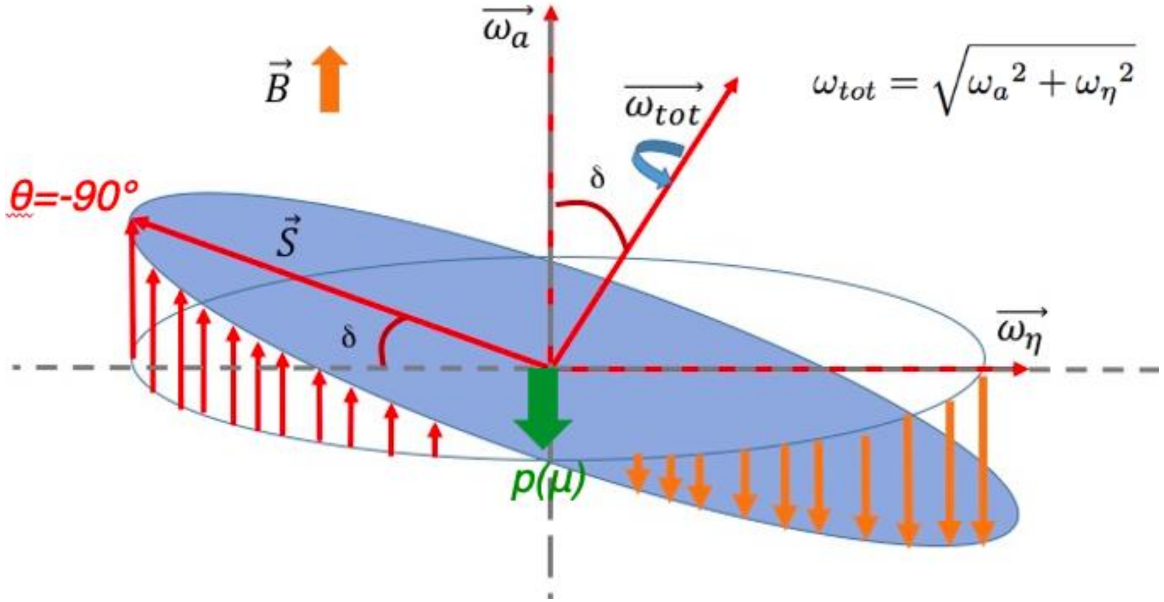
History

- 2009 proposal
- 2015 TDR
- 2016 IPNS focused review
- 2016 SAC (priority #3)
- 2019 KEK-IPNS stage-2, KEK-IMSS stage-2
- 2024 MEXT funding (partial construction)
- 2025 MEXT funding (partial construction)

Extra: BSM searches at FNAL Muon $g-2$

Muon Electric Dipole Moment (EDM)

- Muon EDM causes tilt in precession plane
- Asymmetry in vertical decay angle of positrons
- Vertical angle measured by tracking detectors
- Momentum binned analysis for maximum sensitivity



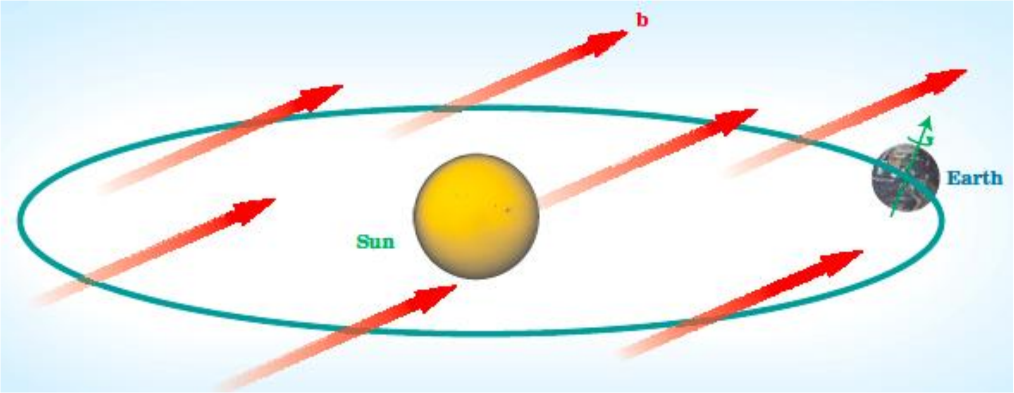
- Run 1 analysis still blinded. Assuming zero signal expecting limit of:

  $|d_\mu| < 2.0 \times 10^{-19}$ e.cm (95% C.L.)

- Still statistically limited in tracker analysis
- Factor of **~10 improvement** for statistics accumulated so far, with tracking improvements

CPT and Lorentz Violations

Lorentz Violation –
existence of a preferred direction



- Uniform background vector, *b*
- What could it come from?
  Spontaneous Symmetry Breaking,
  - **SM**: In EWSB, scalar field gets non-zero vacuum expectation value, filling vacuum with *Lorentz Symmetric quantities*
  - **SME**: Can have Lorentz SB, where vector field gets non-zero vev, filling vacuum with *4-dimensionally oriented quantities* → preferred direction in space → LV!
  - Possibilities: string theory, loop-quantum gravity, etc.

CPT Violation

- LV *allows* but does not *require* CPTV, because CPT Theorem no longer holds (but CPTV does require LV)
- SME Lagrangian coefficients provide a quantitative way to measure CPT/LV.
- CPT/LV signatures: Sidereal (or annual) variation in $\boldsymbol{\omega_a}$.

Ultralight Dark Matter

Muon $g-2$ has a competitive sensitivity to the **ultralight (thus bosonic and wave-like field) muonic DM**. It is the first direct DM search with muons in a storage ring.

- Scalar field (Yukawa coupling, y) $\phi = \phi_0 \cos(m_\phi t)$
  - It induces oscillating m_μ.

$$\mathcal{L} \supset -y\phi\bar{\mu}\mu - y'\phi^2\bar{\mu}\mu \quad \Rightarrow \quad m_\mu \to m_\mu + y\phi + y'\phi^2$$

  - It leads ω_a to oscillate: $\omega_a \to \omega_a(1 + A_\phi \cos m_\phi t)$

- Pseudoscalar axion-like field $a = a_0 \cos(m_a t)$
  - EDM coupling induces oscillating EDM (d_μ).

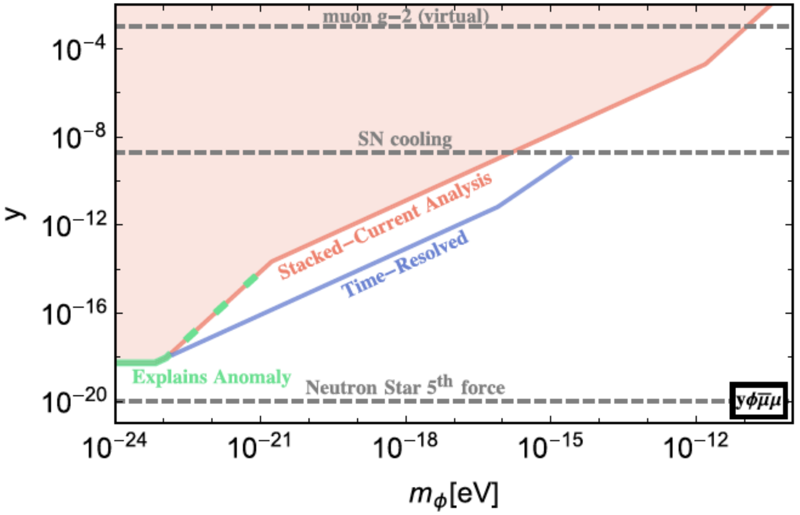
$$\mathcal{L} \supset -ig_{\rm EDM} a\bar{\mu}\sigma^{\lambda\nu}\gamma_5\mu F_{\lambda\nu} \;\Rightarrow\; d_\mu \to d_\mu + g_{\rm EDM} a$$

  - Gradient coupling induces oscillating spin along the axis of the muon's motion.

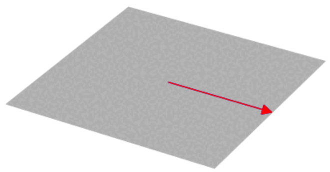
$$\mathcal{L} \supset g_{a\mu}\partial_\lambda a\bar{\mu}\gamma^\lambda\gamma_5\mu \;\Rightarrow\; \mathcal{H} \supset g_{a\mu}\nabla a \cdot \mathbf{S}$$

  - Both lead to oscillating $\delta\omega_a$ components perpendicular to ω_a.

Janish & Ramani, PRD 102, 115018 (2020)



Spin precession



No DM

Gradient coupling (10% of ω_a)

Extra: beam dynamics and field transient corrections

E-field & Pitch Corrections:

- Non-simplified spin-motion is described by BMT equation:

Jackson Eq. (11.171)

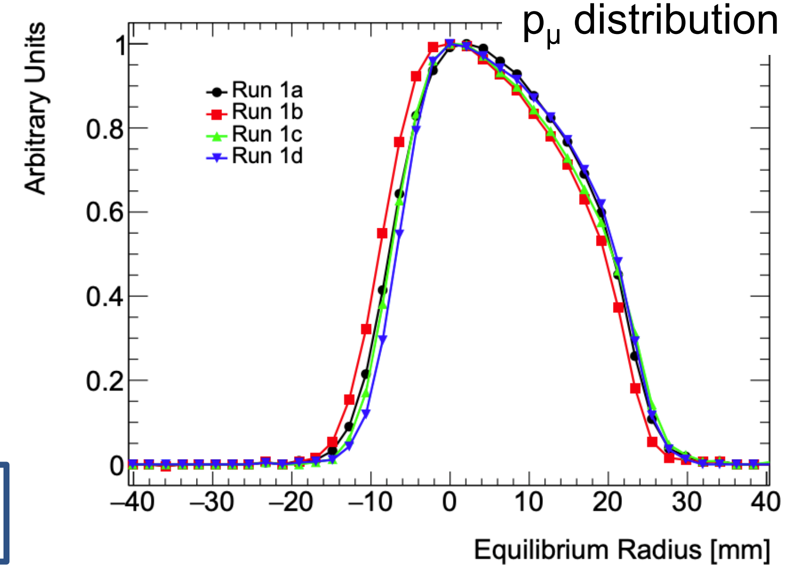
$$\frac{d(\hat{\beta}\cdot\vec{S})}{dt} = -\frac{q}{m}\vec{S}_T \cdot \left[a_\mu \hat{\beta}\times\vec{B} + \beta\left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{E}}{c}\right]$$

$p_\mu = 3.094$ GeV

$\omega_a \propto a_\mu B$

- Muons travel in **E-field** from focusing quadrupoles: experience a **motional magnetic field** in their rest frame
- Term vanishes at “magic” momentum (p_μ = 3.094 GeV)
- But not all muons are at p_{magic}
- C_E comes from p_μ distribution measured using timing data from calorimeters

p_μ distribution

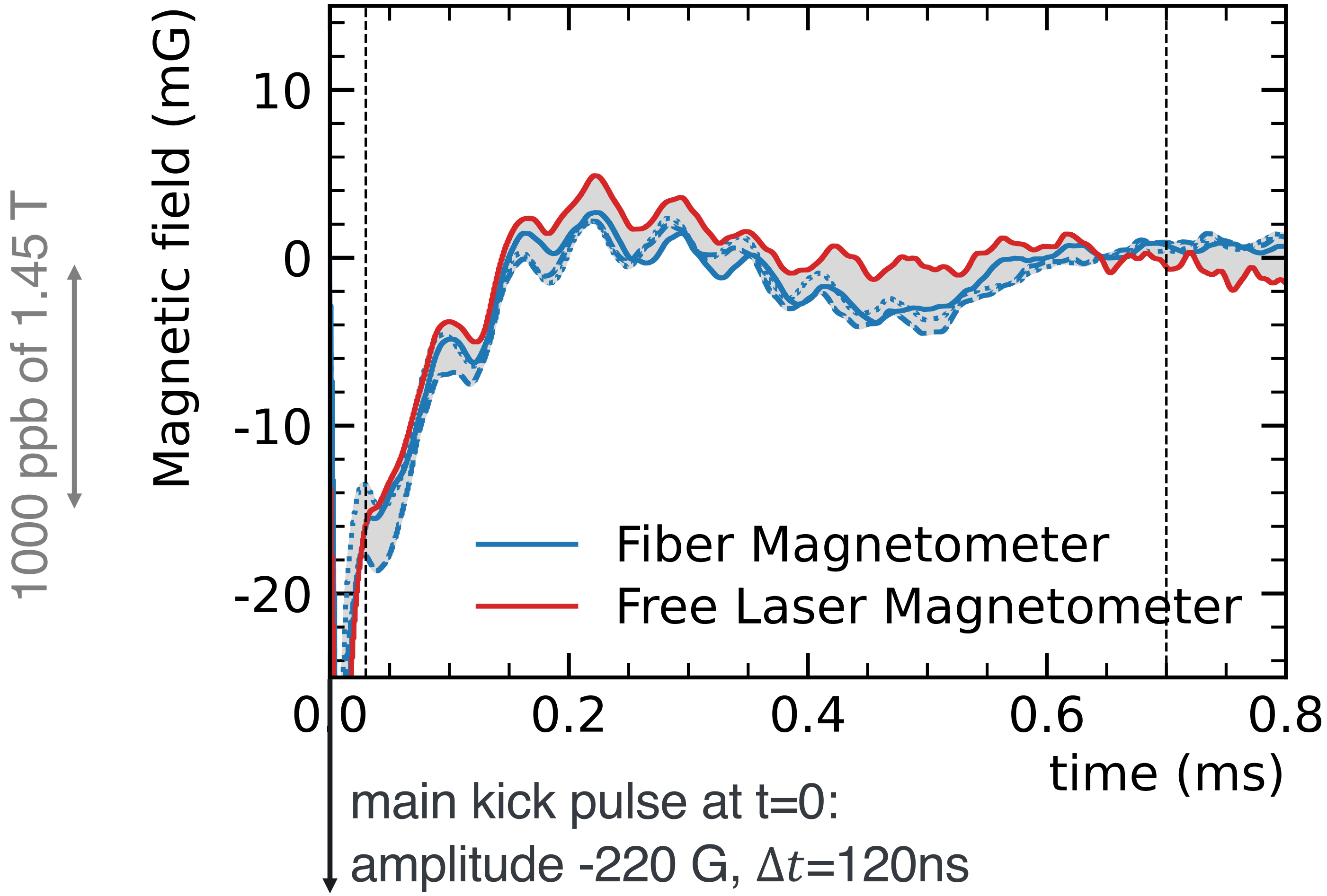


C_E = 489 ± 53 ppb

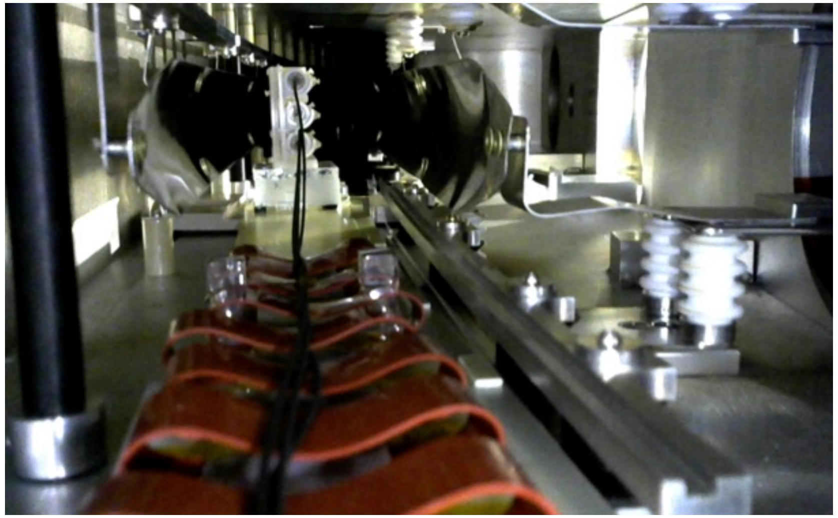
Magnetic Transients: Kicker

Kick causes **eddy currents** → **transient** magnetic field

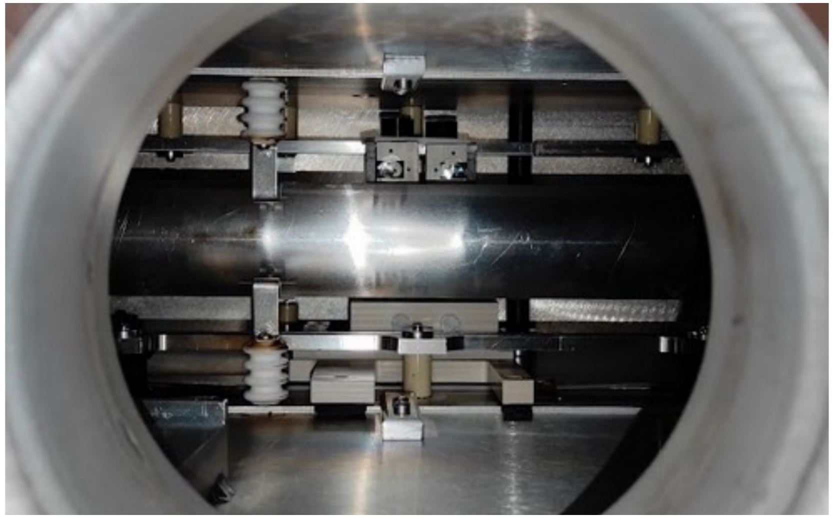
Measured **newly** with **two different magnetometers** both based on Faraday effect in TGG crystals



main kick pulse at t=0:
amplitude -220 G, Δt=120ns



Fiber magnetometer

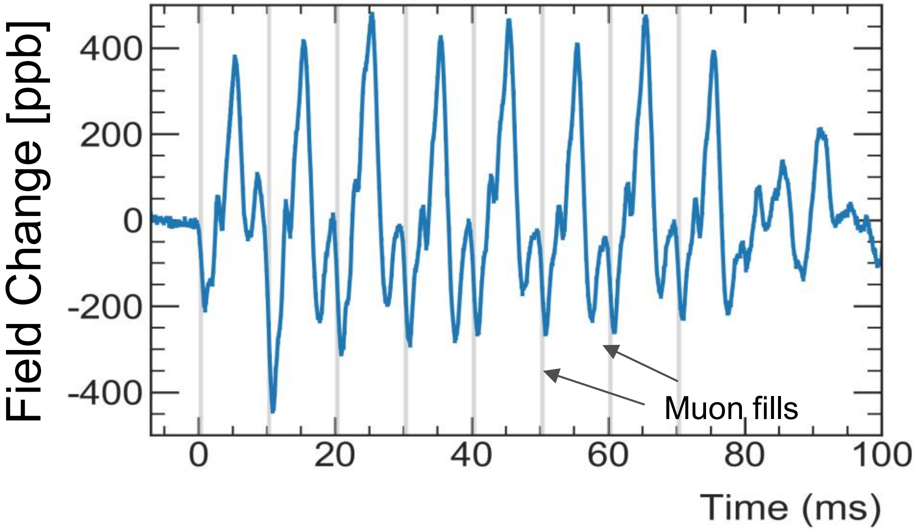


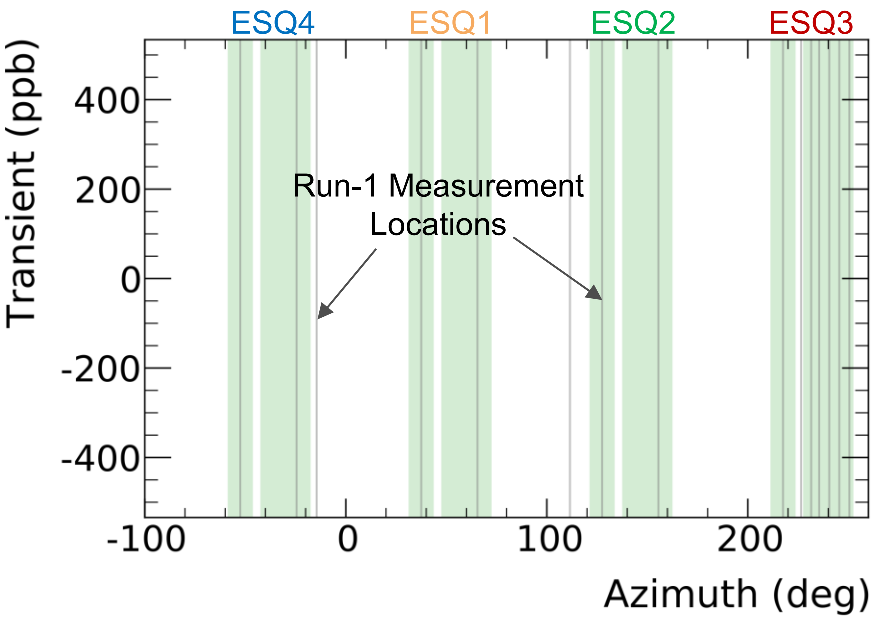
Free laser magnetometer

FERMI NATIONAL ACCELERATOR LABORATORY

Improved Measurements: Quad Transient Field

- Pulsing **quads vibrate ⇒ oscillating magnetic fields**
- Measured with a **new NMR probe** housed in insulator

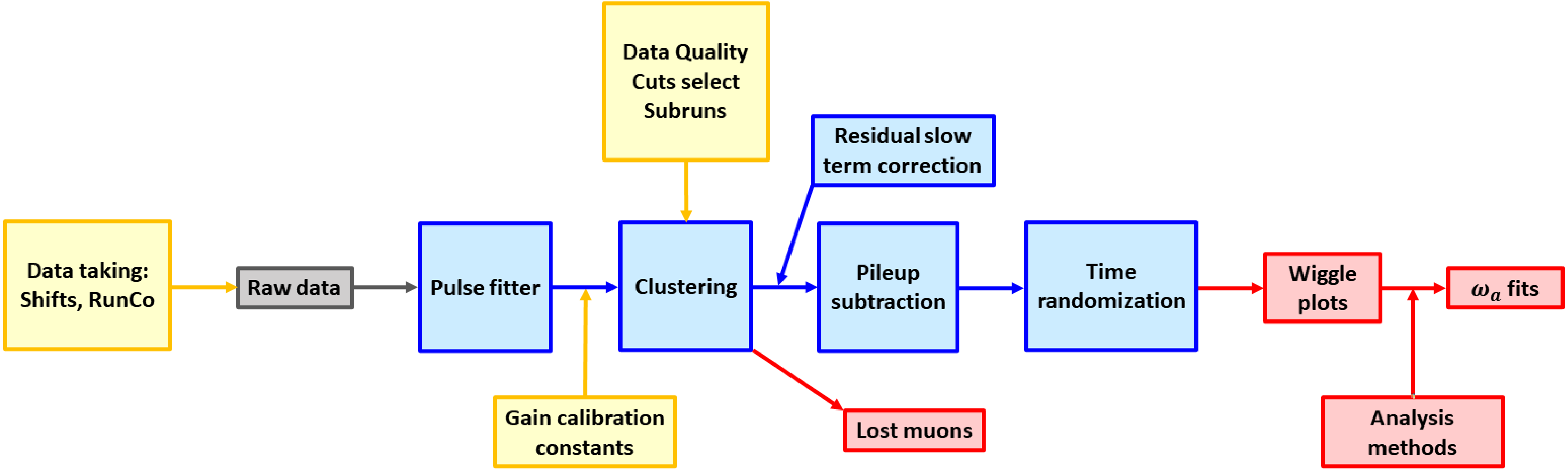


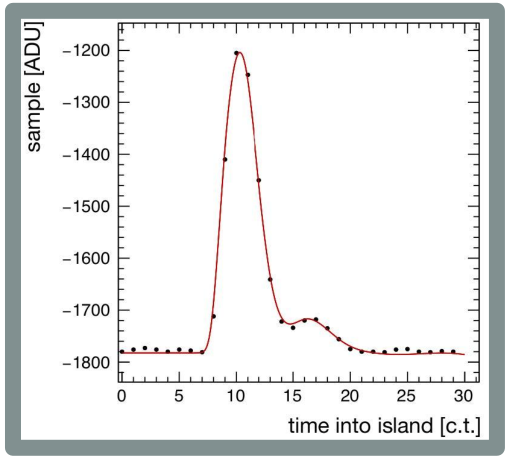


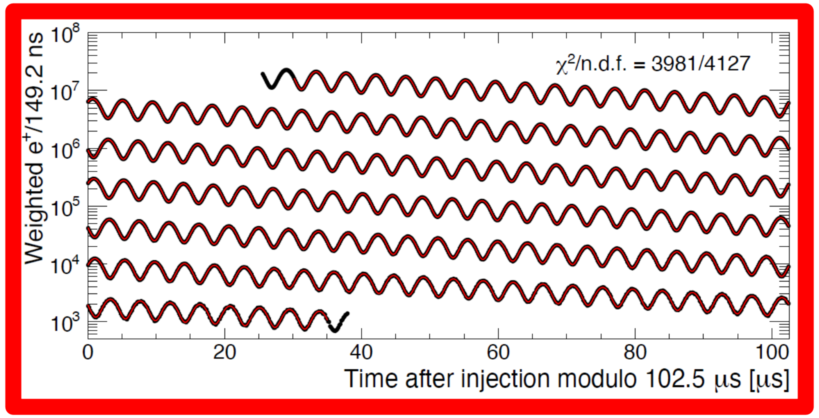
- For Run-1 analysis, we had **limited measurement positions**
- Largest Run-1 systematic: **92 ppb**

Extra: ω_a backup

ω_a analysis flowchart

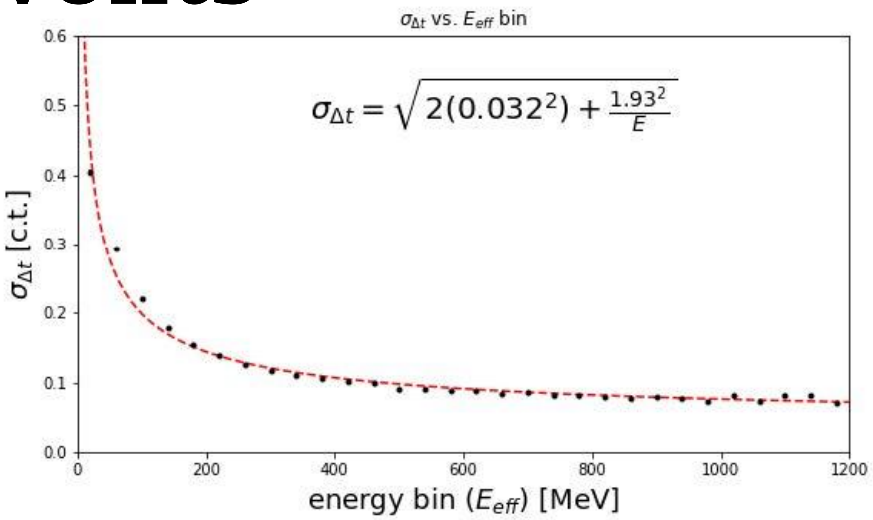


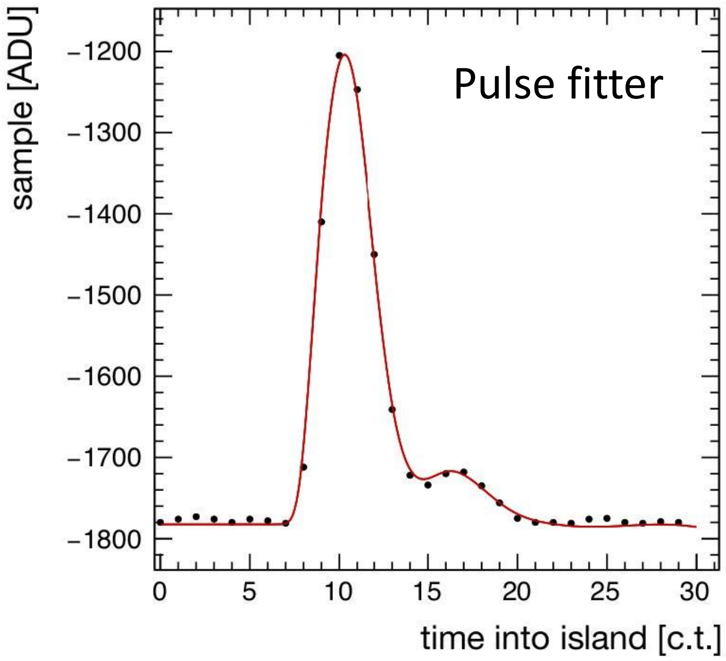


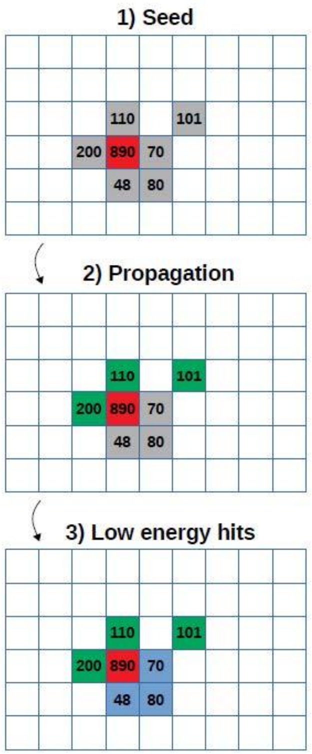


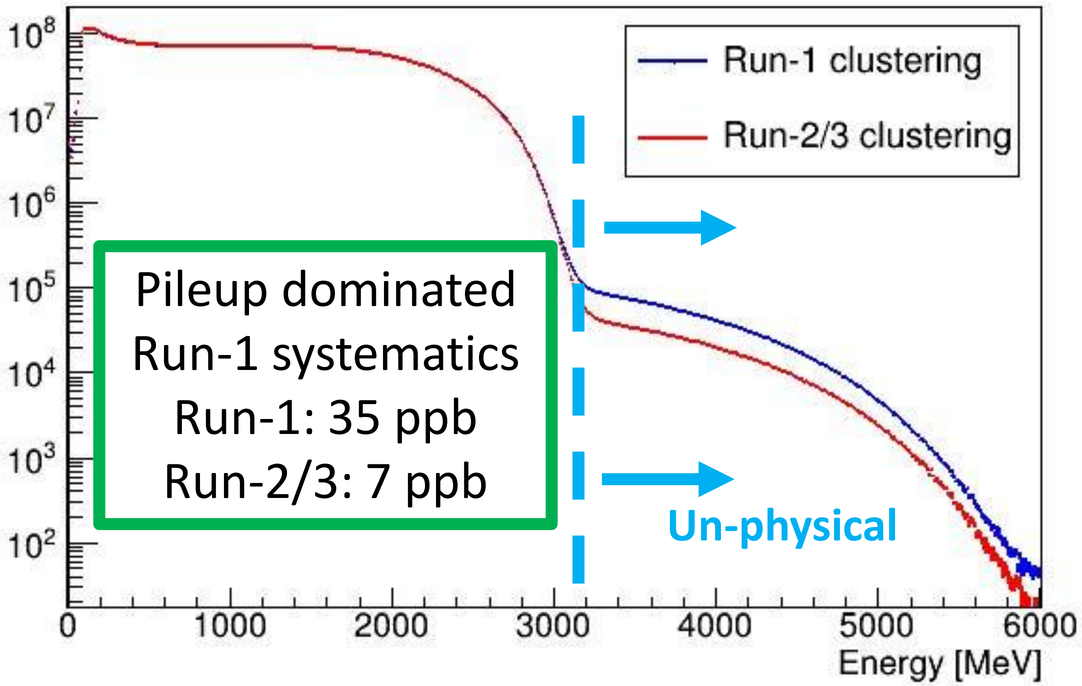
Reconstruct e^+ events

- Pulse fitter identifies traces on crystals
- Clustering algorithms reconstruct total time and energy of positron hit
- Better algorithms reduced pileup after Run-1

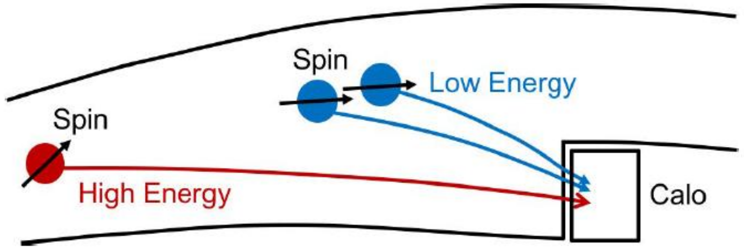


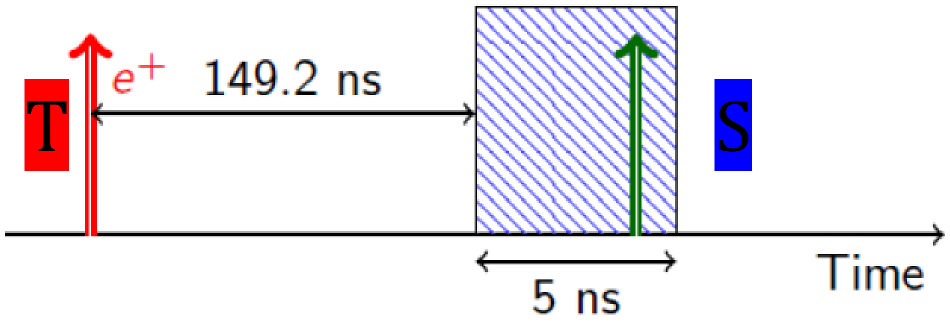




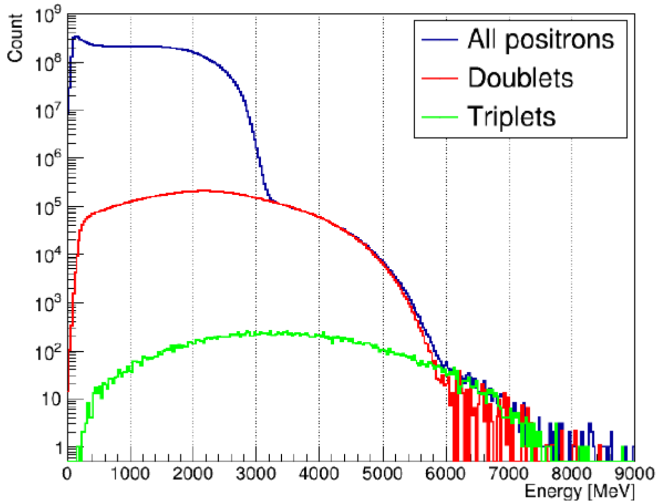


Improved pileup correction since Run-1
(example for «local» method)





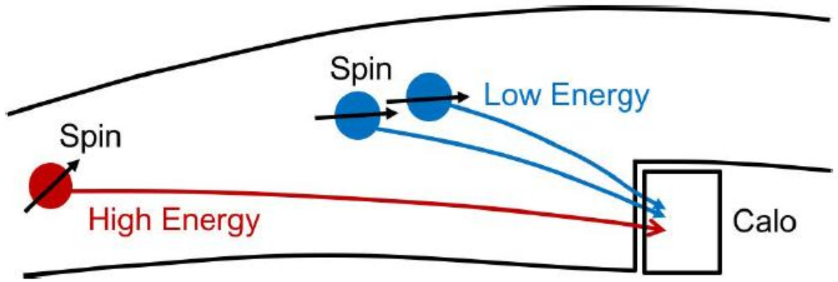
For each T (Trigger) cluster that we find:
- Search for coincidence e^+ in S (Shadow) window, after 149.2 ns
- Superimpose the two clusters and pass to reconstruction algorithm

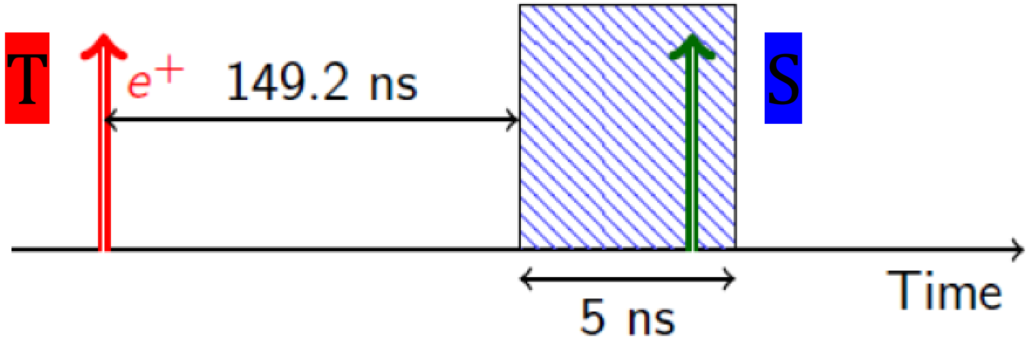


If not resolved: merge them and build pileup

After Run-1: also searched for triplets (2 shadows)

Example of new method to subtract pileup





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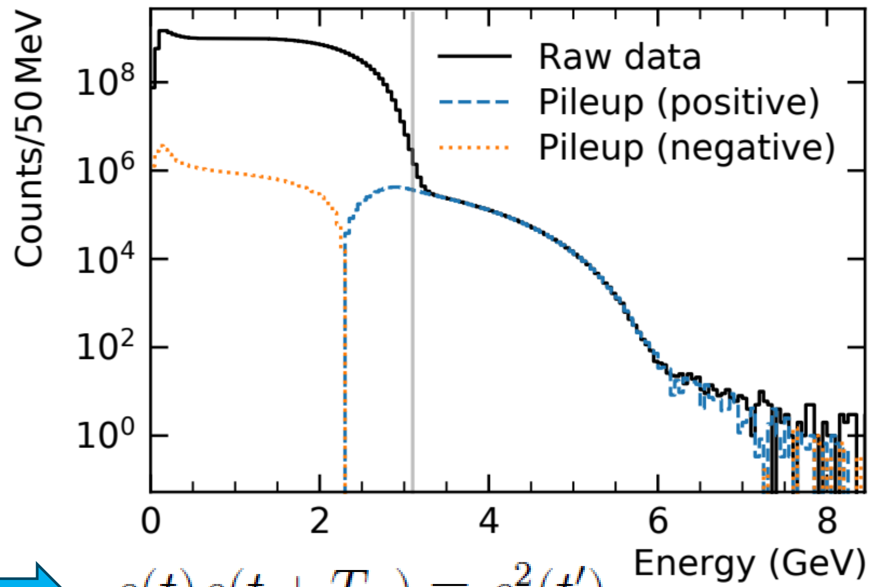


$$E_2 = (E_T + E_{S_1}) \qquad t_2 = \frac{(t_T + T_G/2)E_T + (t_{S_1} - T_G/2)E_{S_1}}{E_T + E_{S_1}} \quad \Rightarrow \quad \rho(t)\rho(t+T_G) \equiv \rho^2(t')$$

Finally: subtract merged event and add single events

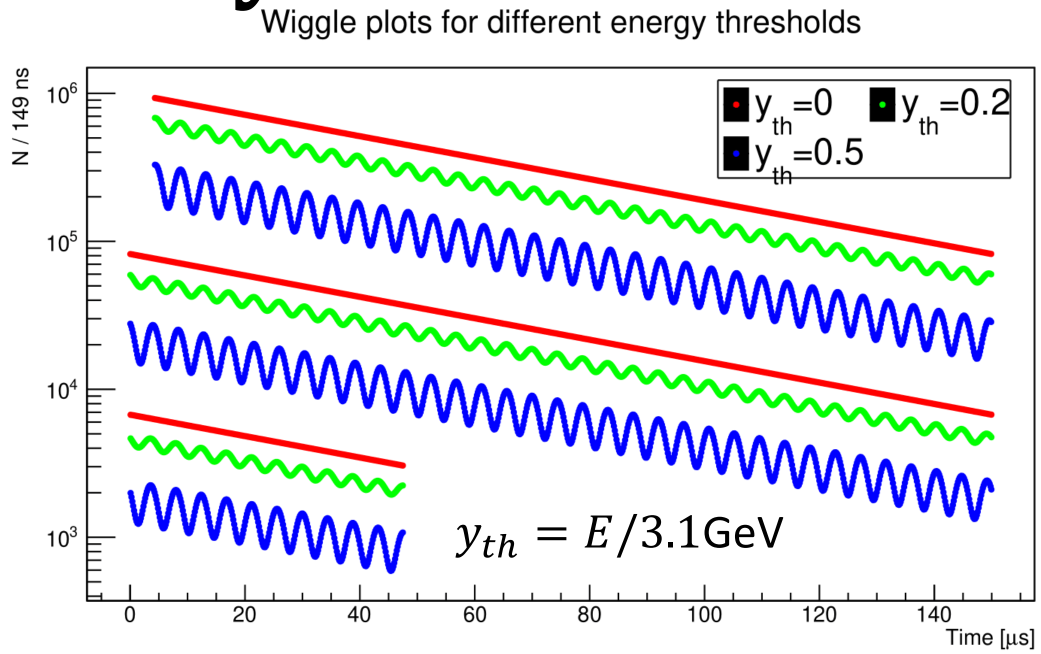
Methods for ω_a analysis

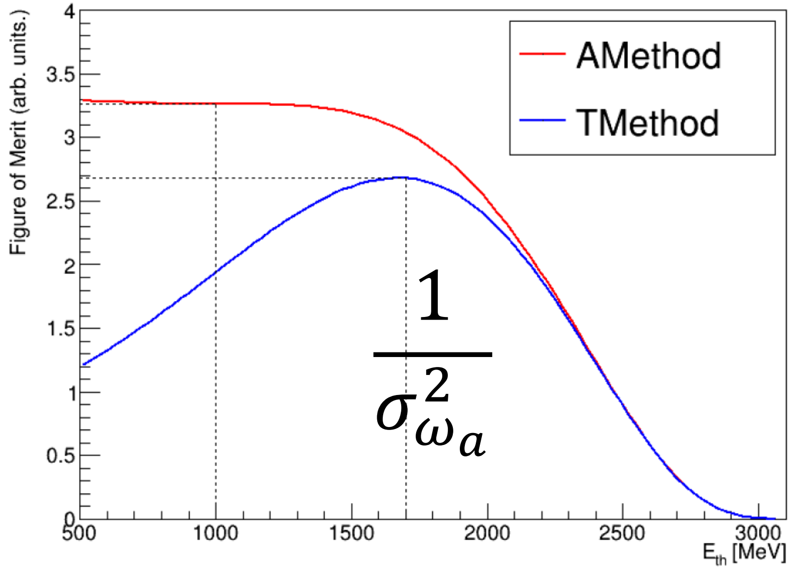
T-Method:

- Greater threshold: wider ω_a oscillations
- Lower threshold: more positrons
- Compromise: $\sim$ 1.7 GeV

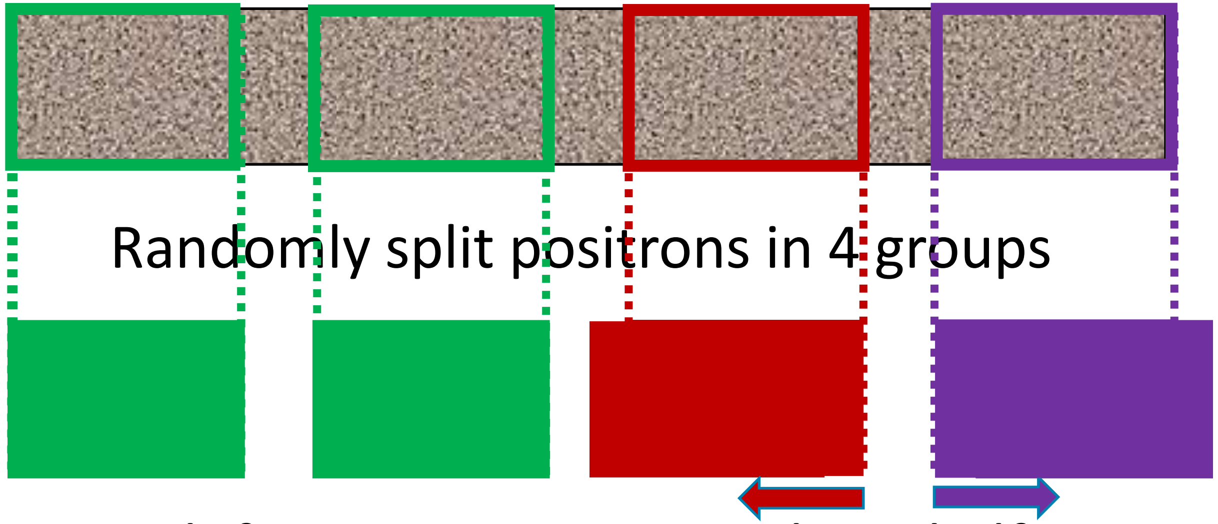
A-Method:

- Extract asymmetry (oscillation amplitude) as function of positron energy → $A(E)$
- Weight each positron event with $A(E)$
- σ_{ω_a}(A-Method)$\sim$ 90% σ_{ω_a}(T-Method)





Ratio method wiggle plots

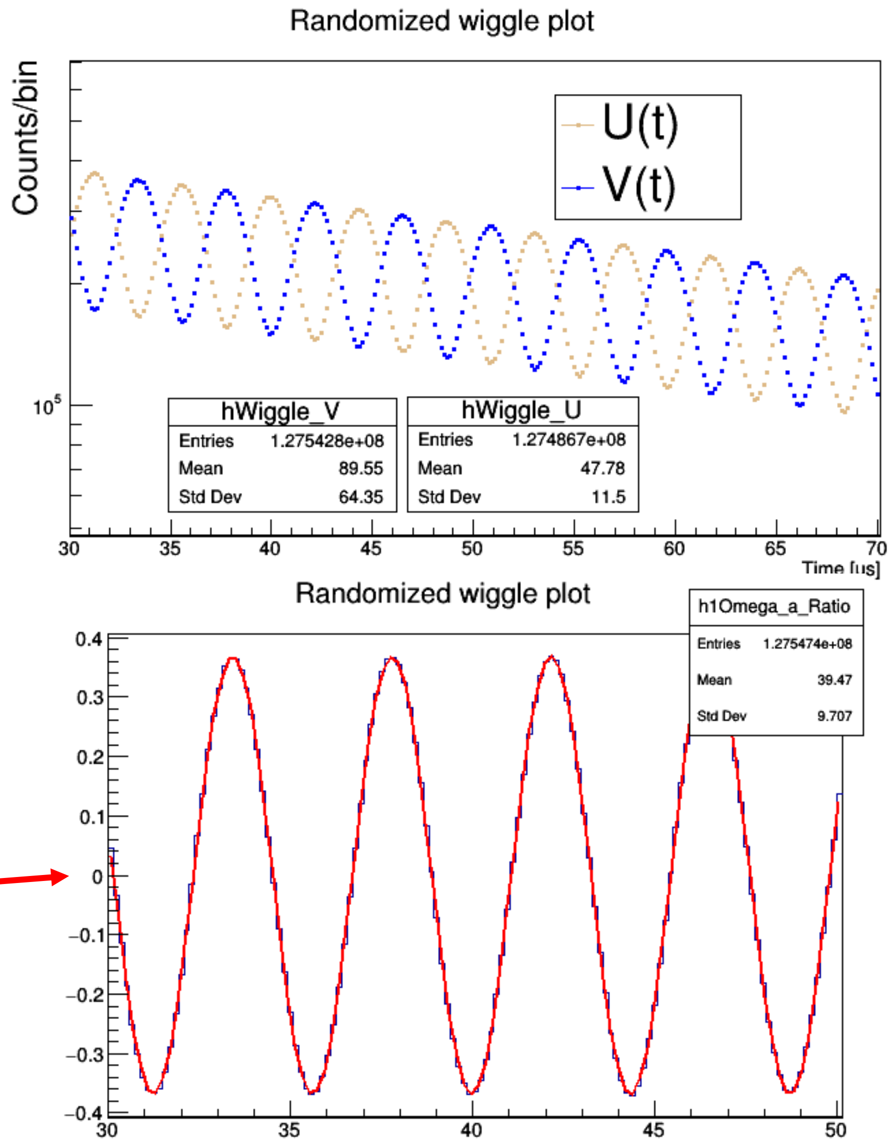


Randomly split positrons in 4 groups

Shift two groups in time by $\pm$ half precession period $T_a/2$. Recombine:

$\boldsymbol{V(t)}$	$\boldsymbol{U(t)}$

$$R(t) = [V(t) - U(t)]/[V(t) + U(t)]$$



It gets rid of muon lifetime and normalization N_0 in fit function. Any «slow» effect is highly reduced!

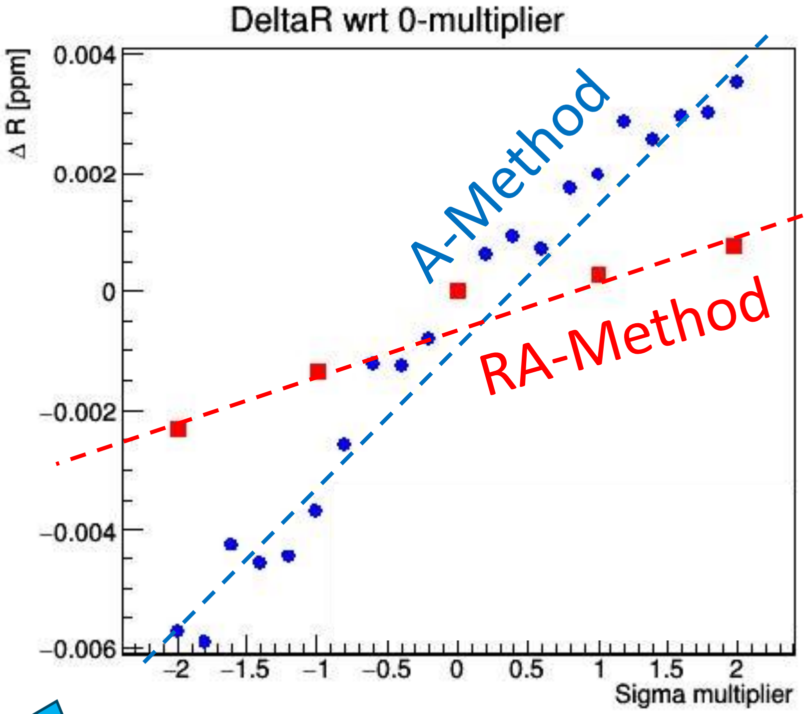
“Ratio” A-Method, new in Run-2/3

- Weight each positron with asymmetry function (like A-Method)

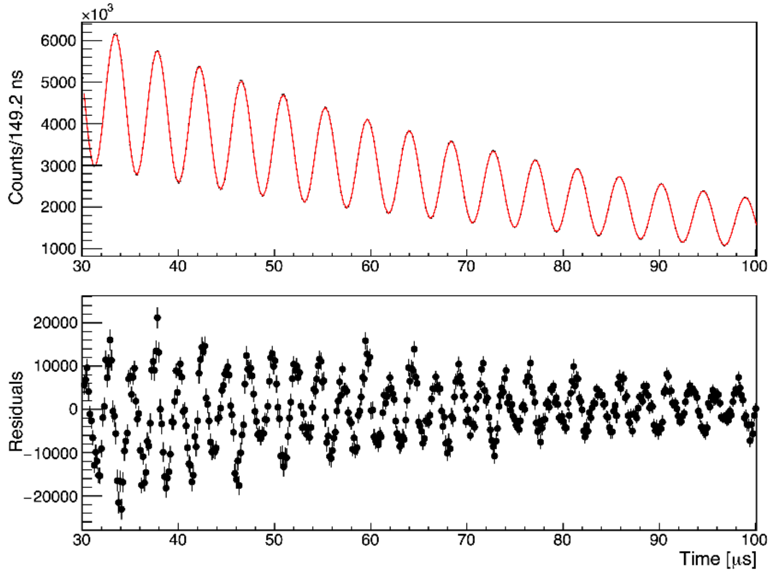
R: $\{V(t); U(t)\}$ → **RA**: $\{\bar{V}(t) = \sum_E A(E)\, V(E,t); \bar{U}(t) = \sum_E A(E) U(E,t)\}$

R: $\frac{V(t)-U(t)}{V(t)+U(t)}$ → **RA**: $\frac{\bar{V}(t)-\bar{U}(t)}{\bar{V}(t)+\bar{U}(t)}$

- Statistical uncertainty on ω_a is minimized
- Exponential due to muon lifetime is cancelled
- Reduces the sensitivity of ω_a to most «slow effects», such as **SiPM gain fluctuations**!



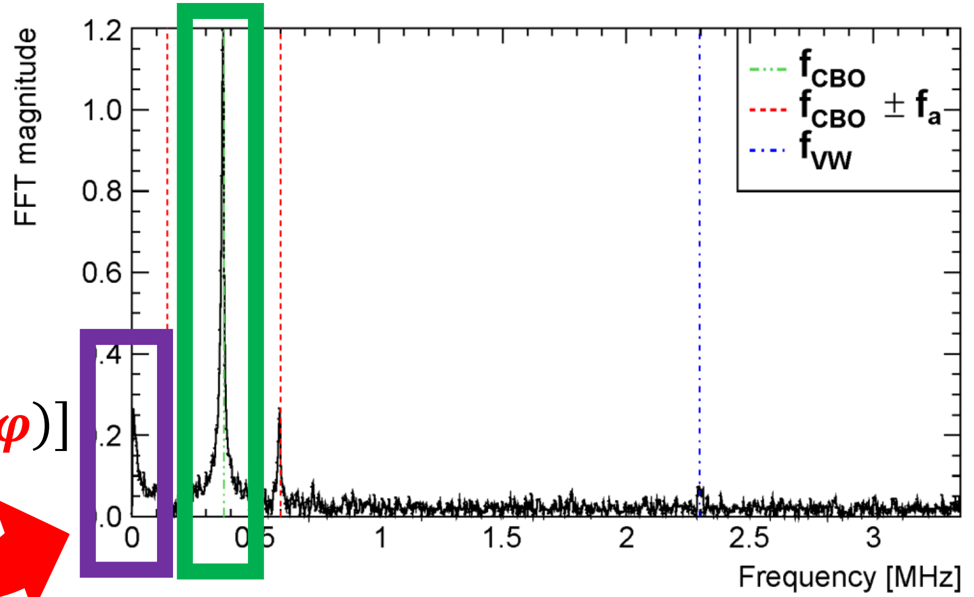
ω_a fit

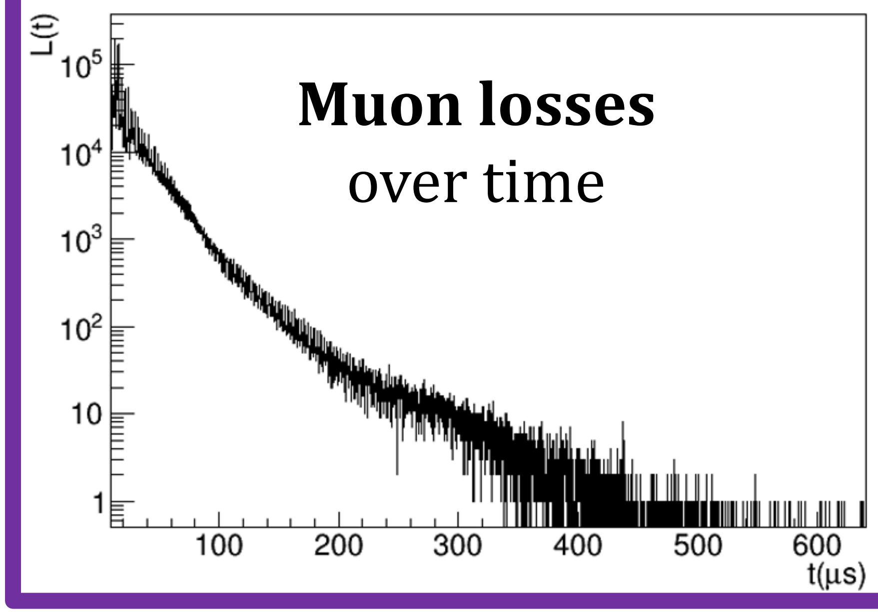


5-parameter fit

$$N(t) = N_0 e^{-\frac{t}{\gamma\tau_\mu}}[1 + A\cos(\omega_a t + \varphi)]$$

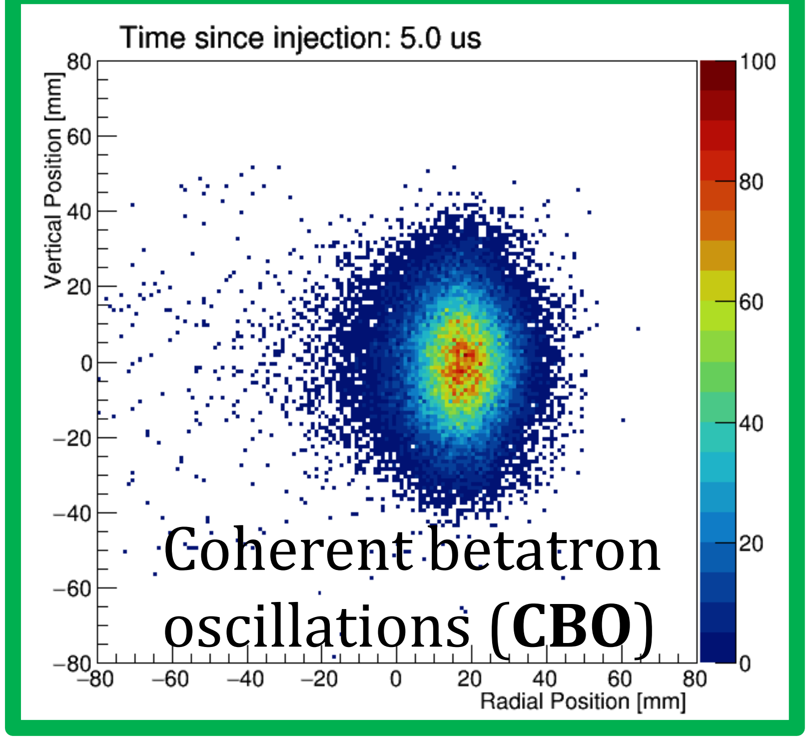
FFT of residuals





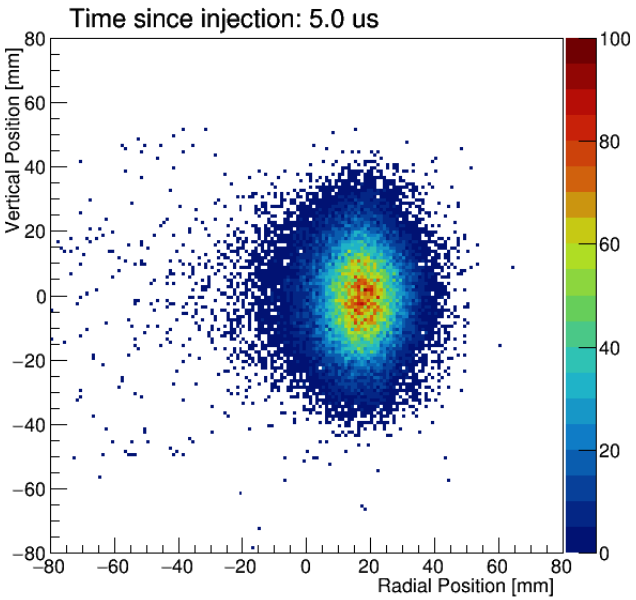
$$\Lambda(t) = 1 - k_{LM} \cdot J(t)$$

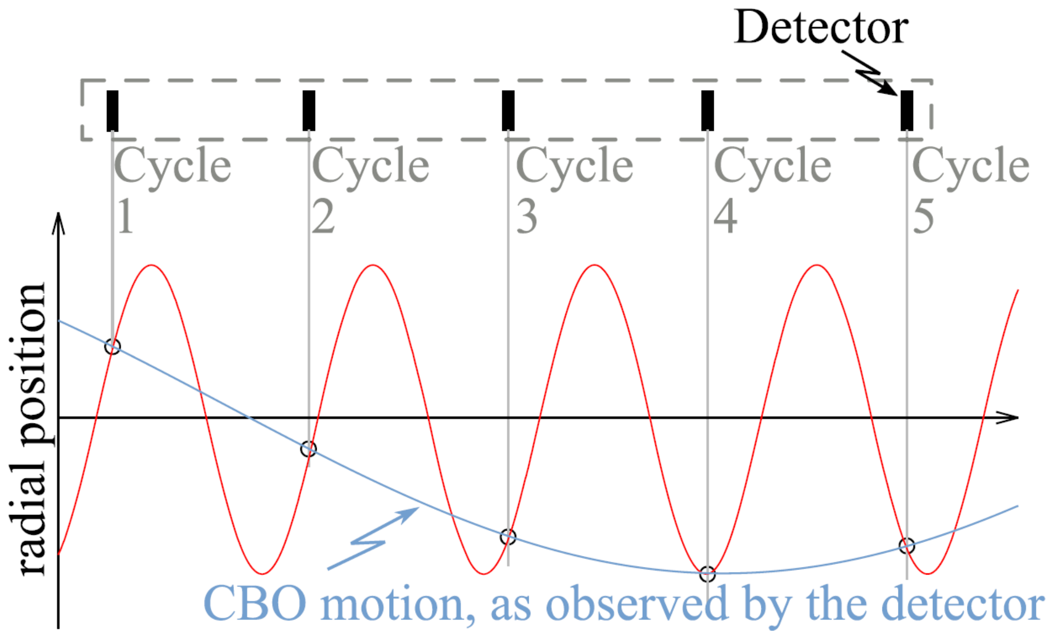
$$J(t) = \frac{\int_{t_0}^{t} L(t') e^{t'/\gamma\tau} dt'}{\int_{t_0}^{t_{end}} L(t) e^{t/\gamma\tau} dt}$$



Radial and vertical motion of the beam

- Field index: n (quad voltages)
- Radial motion of the beam: $\omega_x = \omega_c\sqrt{1-n}$
- CBO is the aliased frequency $\omega_{CBO} = \omega_c - \omega_x$
- CBO period of about 2.7 μs





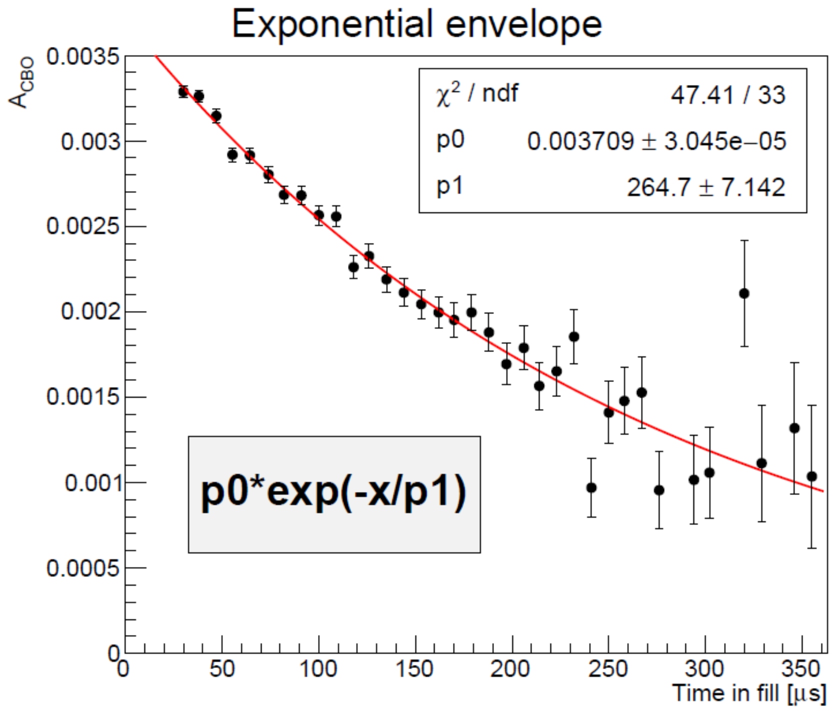
Quantity	Expression	Frequency [MHz]	Frequency [rad/μs]	Period [ns]
ω_a	$ea_\mu B/m$	0.23	1.439	4365
ω_C	v/R_0	6.7	42.0	149.2
ω_x	$\omega_C\sqrt{1-n}$	6.3	39.7	158.0
ω_y	$\omega_C\sqrt{n}$	2.2	13.8	454.2
ω_{CBO}	$\omega_C - \omega_x$	0.37	2.33	2686
ω_{VW}	$\omega_C - 2\omega_y$	2.3	14.4	435.3

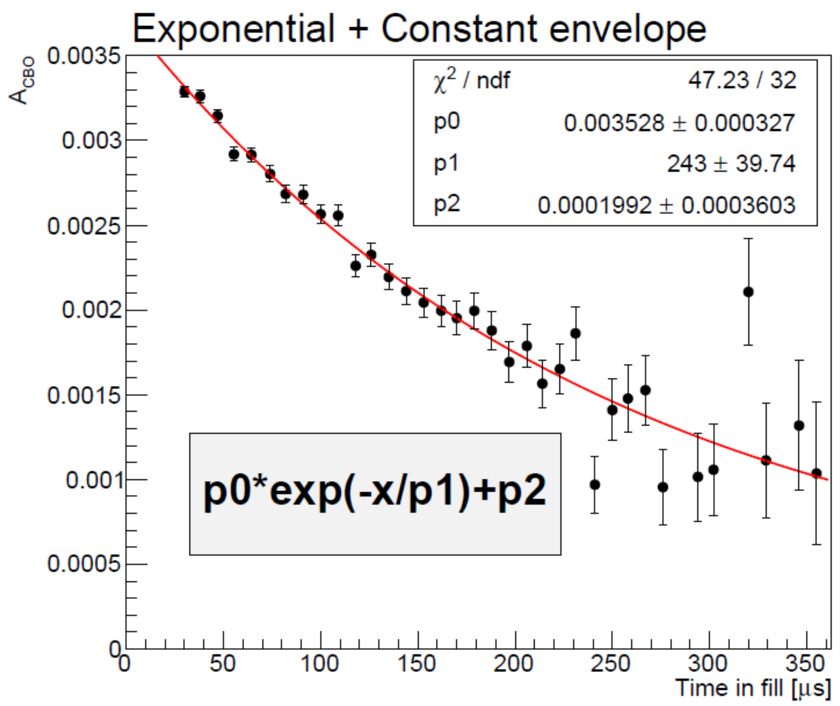
CBO model: amplitude vs time

CBO dominated Run-1 systematics: 38 ppb. Around 21 ppb on average after Run-1!

$$CBO(t) = 1 + A_{CBO}\cos(\omega_{CBO}t + \varphi_{CBO}) \times e^{-t/\tau}$$ →Decoherence

- Muons are an ensemble: betatron oscillations decohere over time
- Sliding window fits to determine good or bad envelopes: more statistics→more studies than Run-1; also input from tracker data



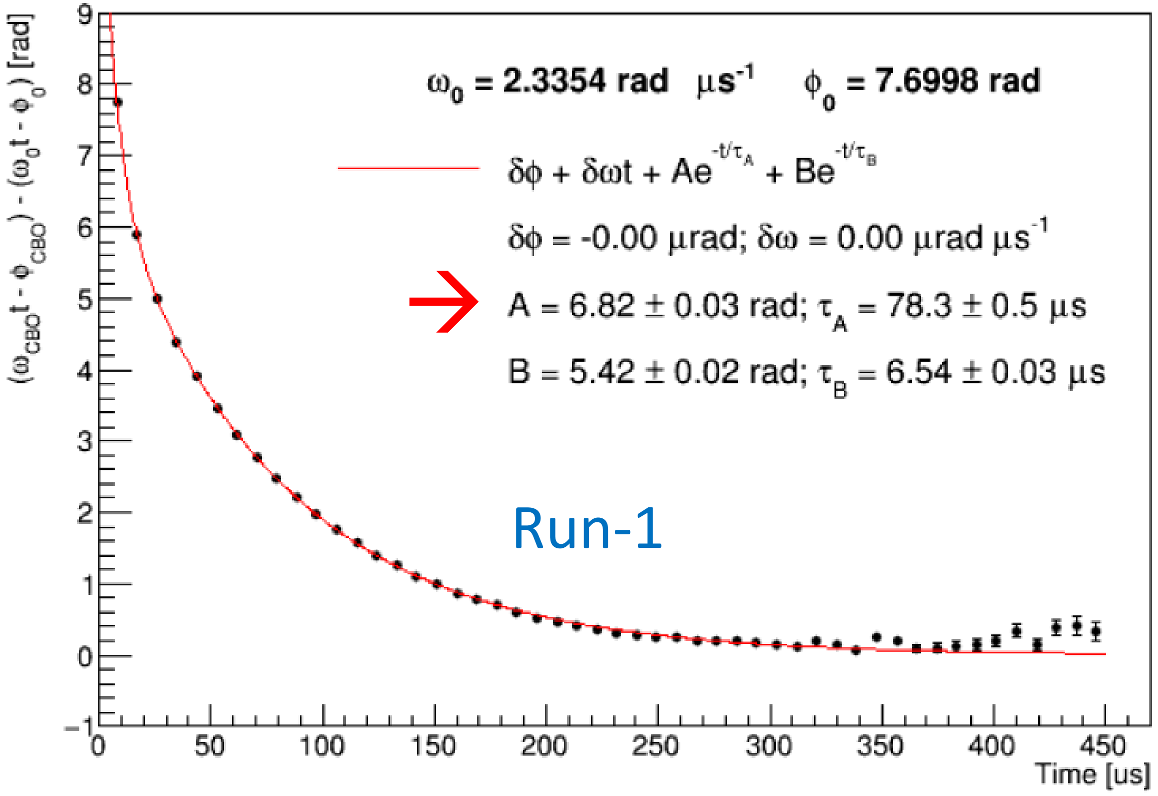


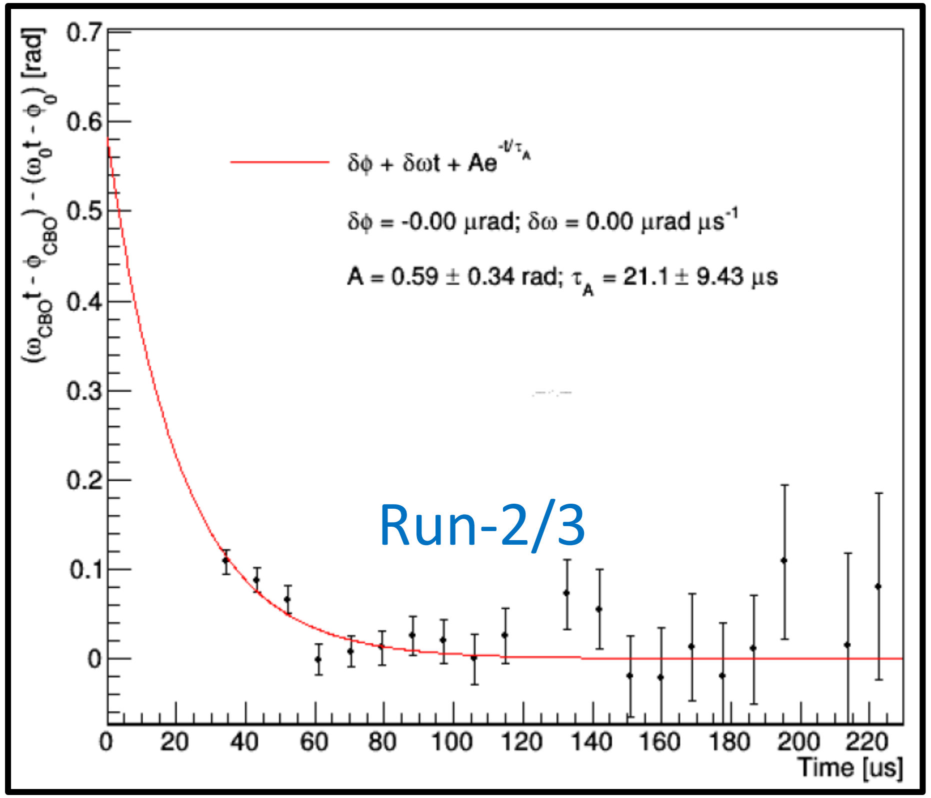
… and many other models tested in dedicated task force; different methods explored, etc…

CBO model: frequency vs time

CBO dominated Run-1 systematics (38 ppb). Now reduced to 21 ppb!

- Exponential relaxation of CBO frequency
- Run-1: faulty ESQ resistors enhanced this effect 10 times!
- Sliding window fits to determine lifetime and constrain it in ω_a fits

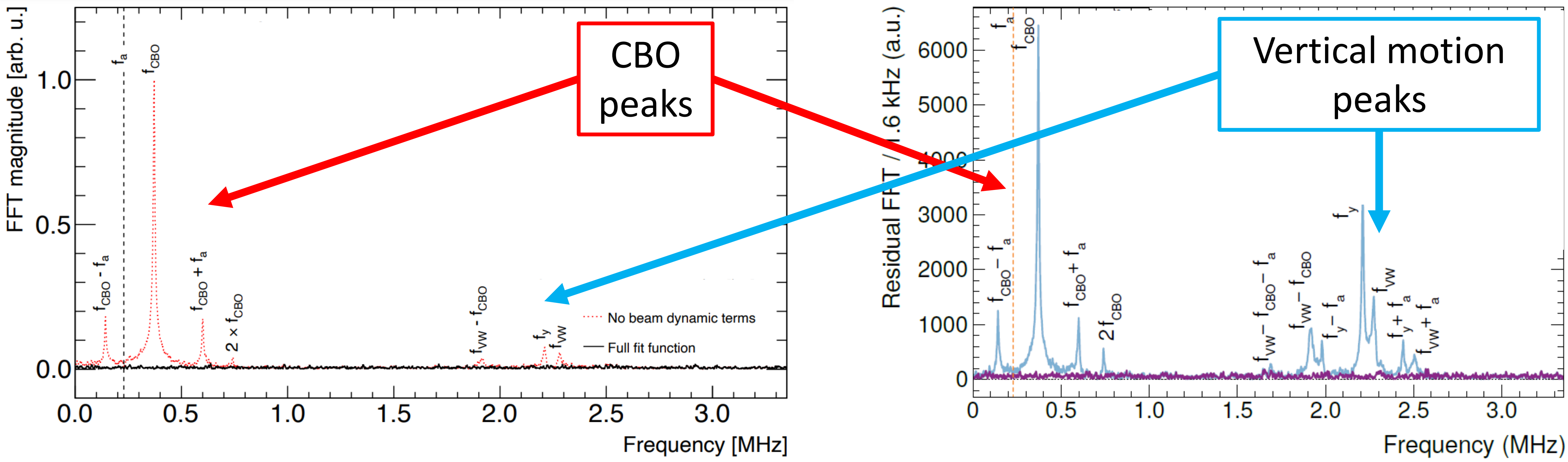




«Challenges» from Run-2/3 to Run-4/5/6

ω_a fit and FFT of residuals

Run-2/3   Run-4/5/6



Final fits include all oscillations

$$N(t) = \boldsymbol{N_0} e^{-\frac{t}{\gamma\boldsymbol{\tau_\mu}}}[1 + \boldsymbol{A}\cos(\boldsymbol{\omega_a} t + \boldsymbol{\varphi})] \times CBO(t) \times VW(t) \times \cdots$$

ω_a analysis teams (red = new methods w.r.t. Run-2/3)

Run-2/3	I	II	III	IV	V	VI	VII
Pulse fitting & clustering	Local	Local $\Delta t'$	Local «ITA1»	Local $\Delta t'$	Global	Global	Q
Pileup subtraction	Shadow	Empirical	«Semi» empirical	Empirical	Empirical	Empirical	-
Analysis methods	T, A	T, A	T, A, RT, RA	T, A, RT, RA	T, A, RT	T, A	Q, RQ

Run-4/5/6	I	II	III	IV	V
Pulse fitting & clustering	Local $\Delta t'$	Local «ITA2»	Local	Global	Q
Pileup subtraction	Empirical	«Semi» empirical	Empirical	Empirical	-
Analysis methods	T, A, RT, RA	T, A, RT, RA	T, A	T, A, RT, RA, ST, SA	Q, RQ

Two Run-4/5/6 "Task forces"

To address major Run-2/3 ω_a systematics

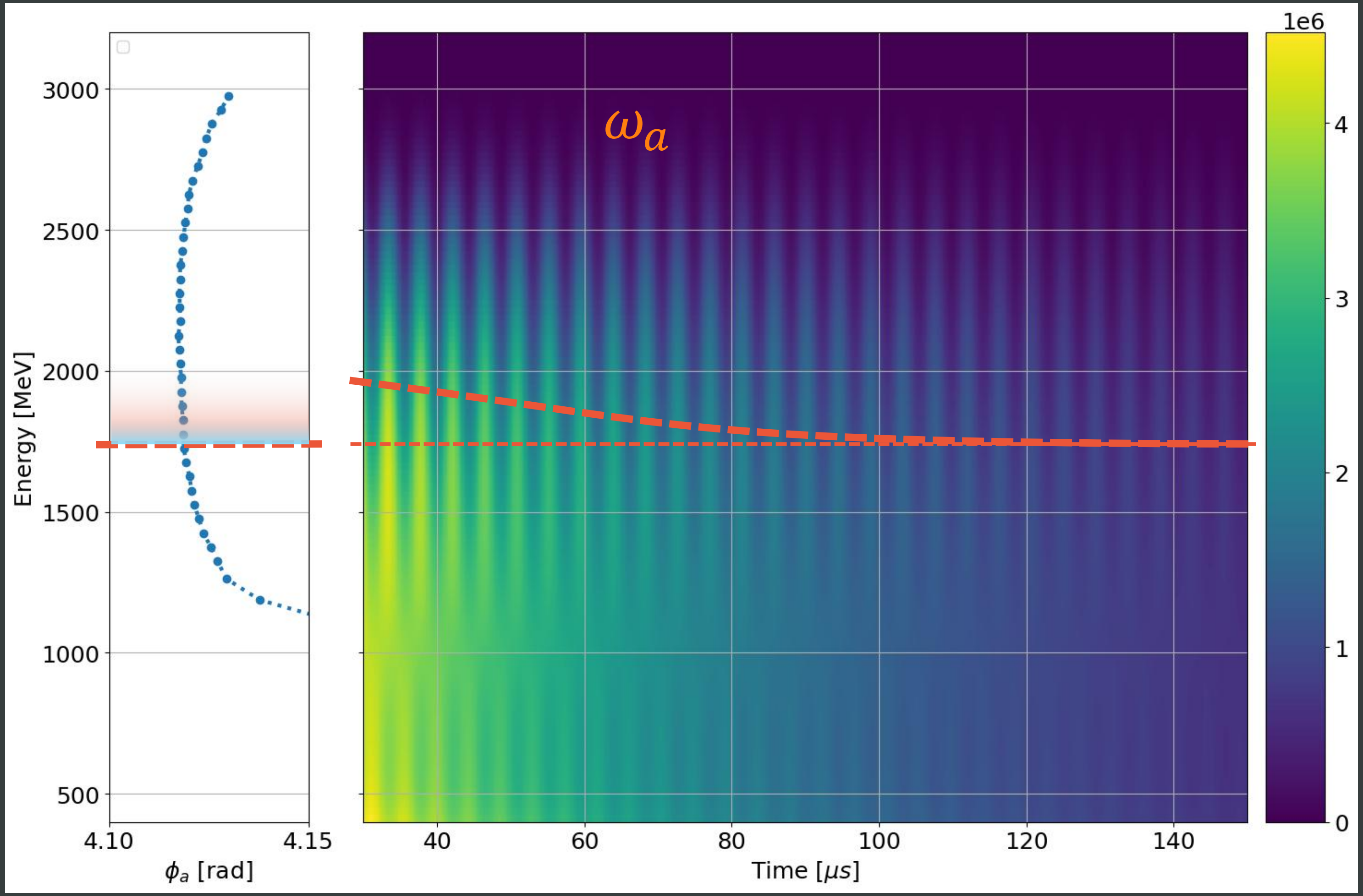
1. CBO treatment and systematics

- Quad RF off → on: CBO much reduced, but higher statistics helped us to parametrize it better
- New frequency components showed up in FFT of wiggle plot residuals
- New models, new methods to parametrize CBO or the entire fit function, new studies

2. Residual slow term

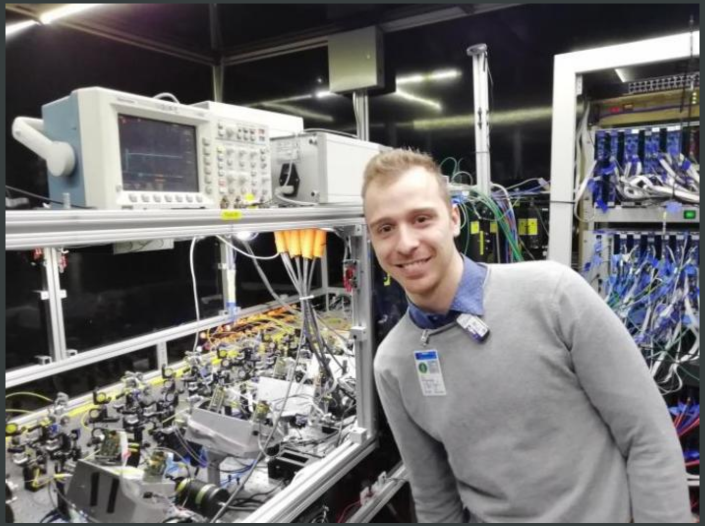
- Early-to-late effect in ω_a fits, ~ 20 ppb systematic in previous results
- Effect smaller than 10^{-4}(laser calibration level) but due to **phase-shifted** oscillation at ω_a it led to **larger sensitivity** than orig. estimated → Physics issue identified! And studied in UW labs
- Applied new correction to take this into account

Gain-Like Detector Effects

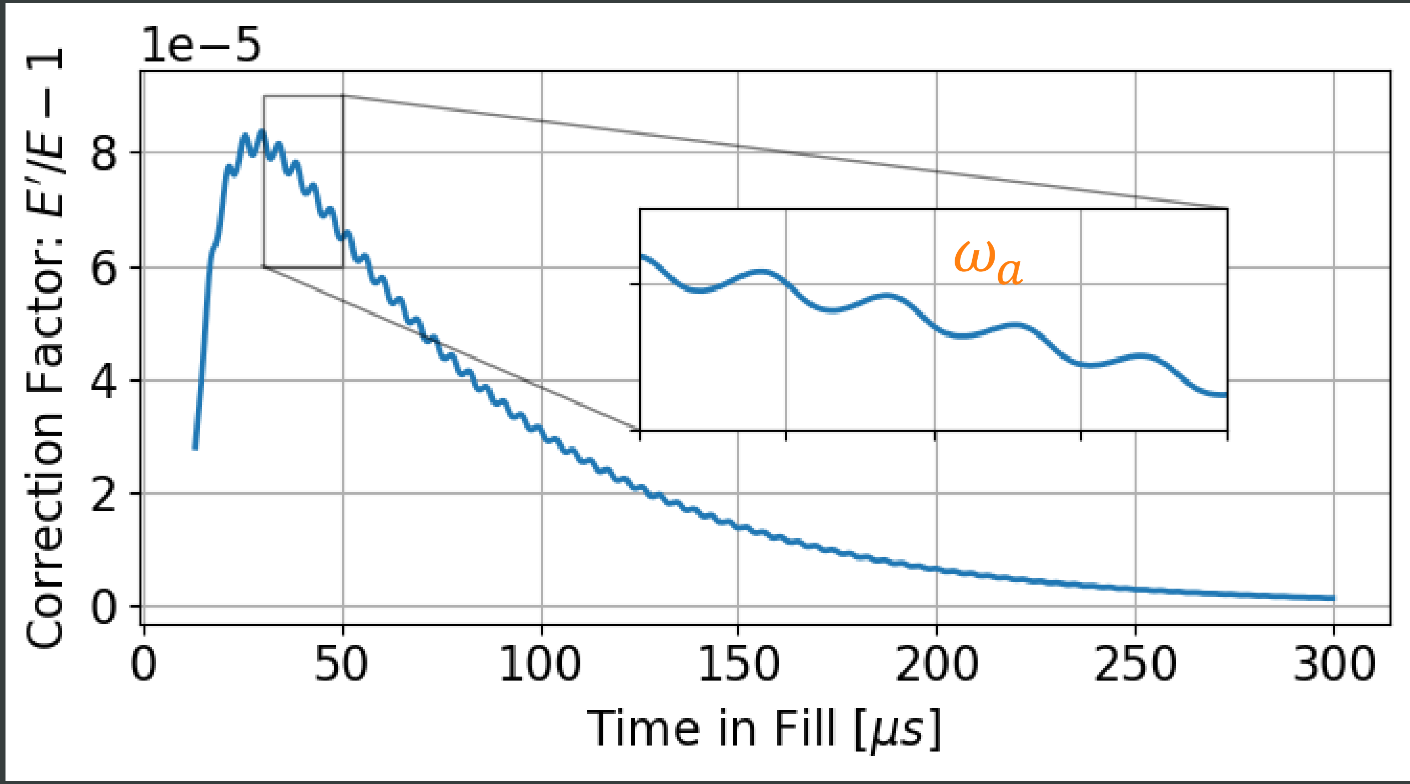


Effective **threshold changes** over time

Laser system to correct the gain on 10^{-4} –level



Gain-Like Detector Effects



*noRF dataset

New! Sensitive also below 10^{-4} if

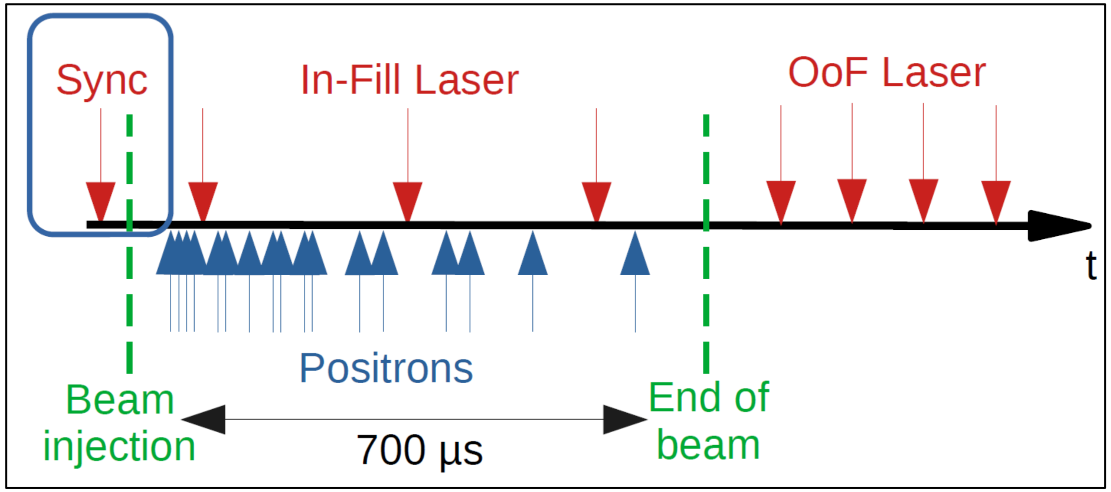
- Rate & Energy dependent
- Time constant $\sim 1/\omega_a$
- Correction shows ω_a-behavior but **out of phase**
- Time-dependent phase-change
- Fitted ω_a sensitive to such effects

Laser-based gain monitoring system

Built by INFN/CNR-INO: time synchronization and calibration of 1296 SiPMs on timescales from ns to days/weeks. Gain changes dominated ω_a systematics at BNL: exceeded goal of 20 ppb at FNAL.

Standard operating mode:

- **Sync pulse**: time synchronization at $\sim$ 50 ps
- **In-Fill pulses**: monitor rate-dependent gain changes at 10^{-4} during 700 µs of μ^+ beam

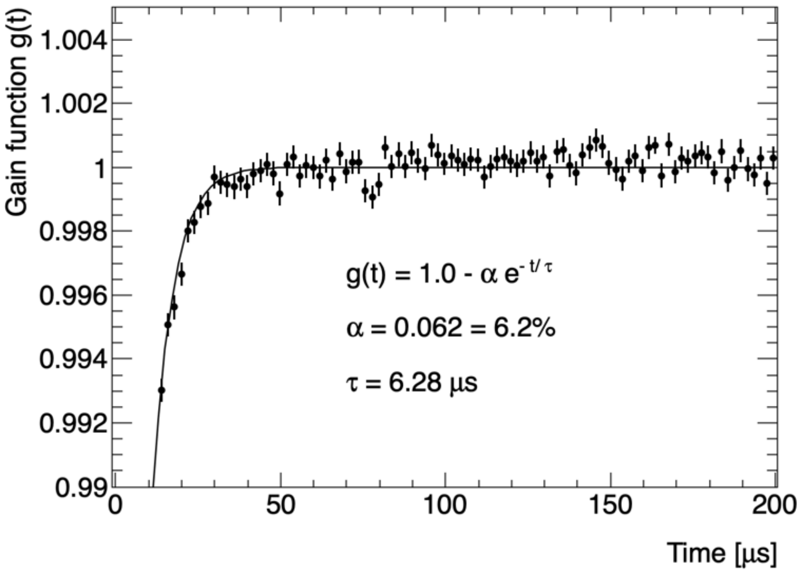


- **Out-of-Fill pulses**: monitor stability over days

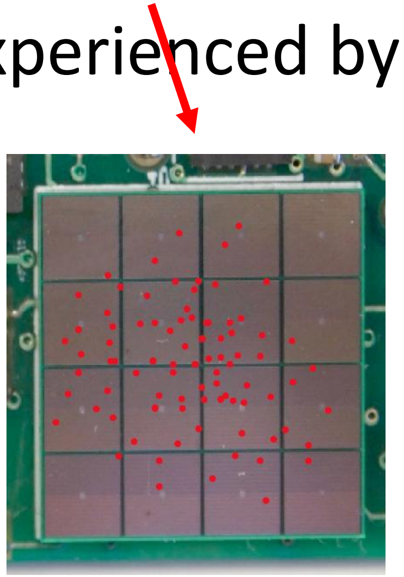
SiPM gain calibration

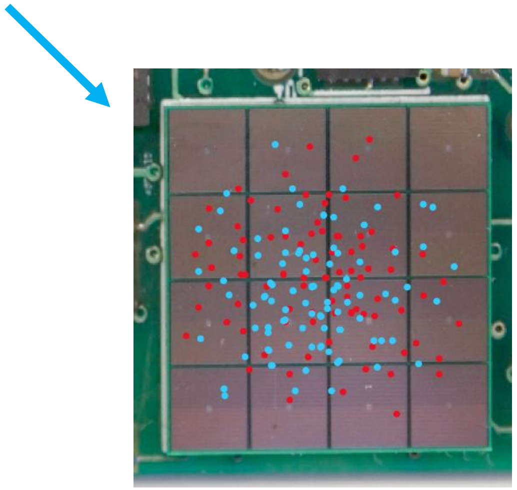
In-Fill: sag in power supply due to initial injection splash.

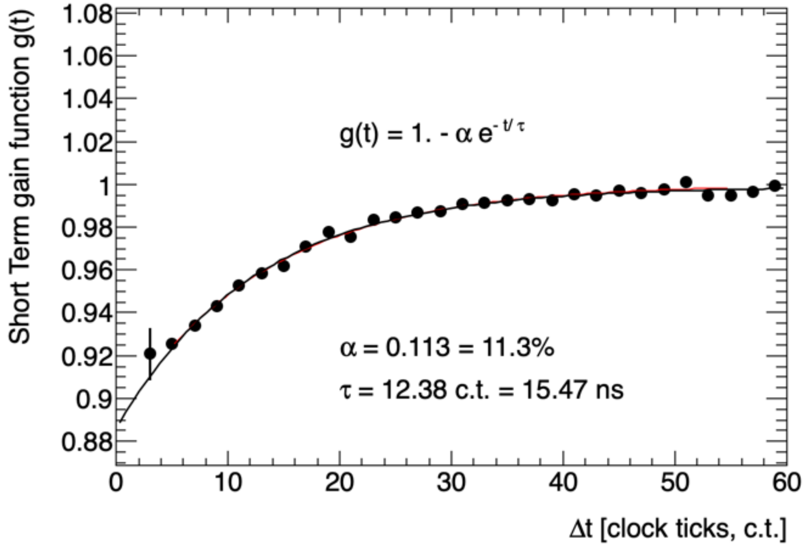
Recovery timescale of front-end electronics: $\mathcal{O}(10\ \mu s)$.



Short-term: consecutive positron hits within $\mathcal{O}(100\ ns)$.
After the first hit, the recovery time of pixels reduce the gain experienced by the second hit.

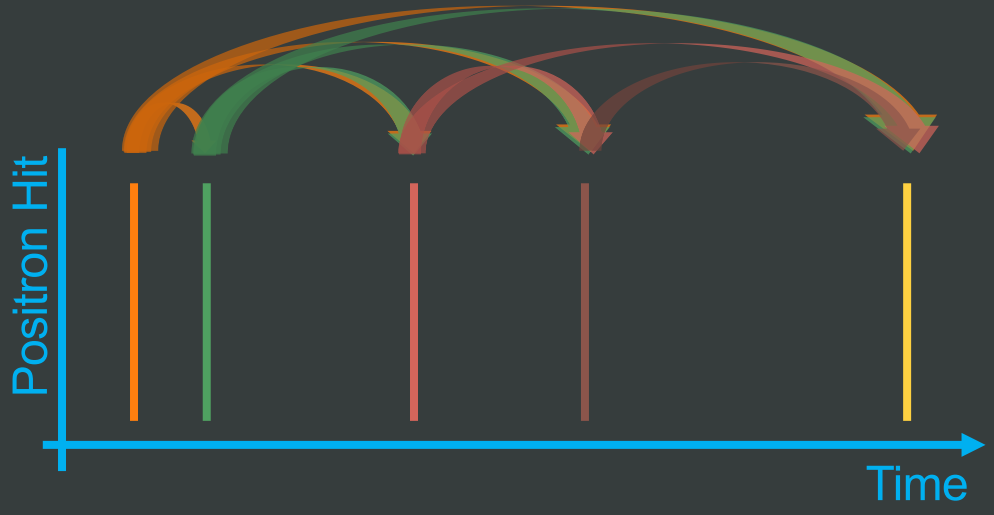




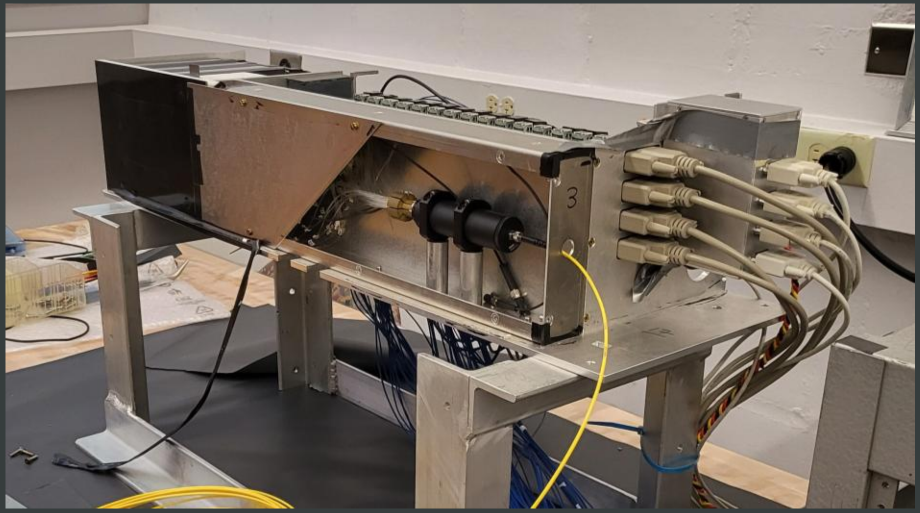


Residual Slow Term

- known in Run-1: “early-to-late effects” 10-22 ppb uncertainty
- better understood in Run-2/3: split into reconstruction & detector part with “Possible sources [...] changes in gain, acceptance, or reconstruction over the duration of a fill.” ~~5-14 ppb uncertainty~~ New! **36 pbb** uncertainty
- **Run-4/5/6: New! Identified physical explanation**
  - Detector effect due to preceding positron hits (rate dependent): 20-40 ppb effect, ~25ppb uncertainty

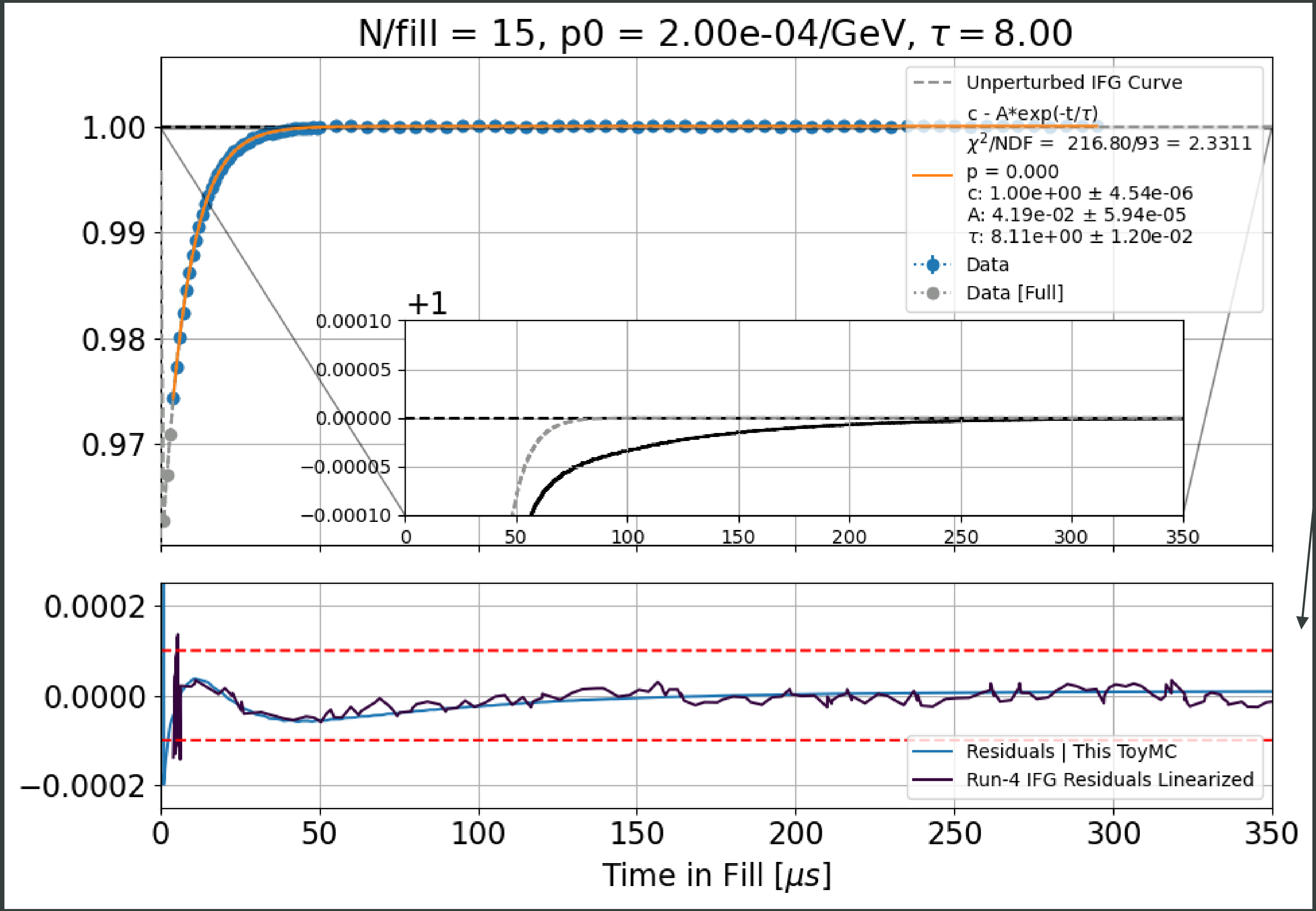


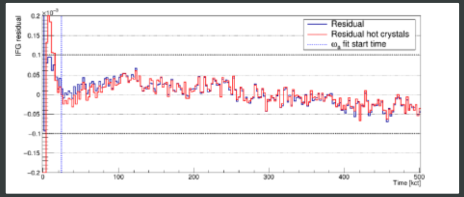
* electronics board dependent



Calorimeter in lab for dedicated measurements

Symptoms of Such An Effect | Toy MC Example





The shape induced by the ITDP effect matches the real IFG residuals

Extra: $g - 2$ blinding and unblinding procedures

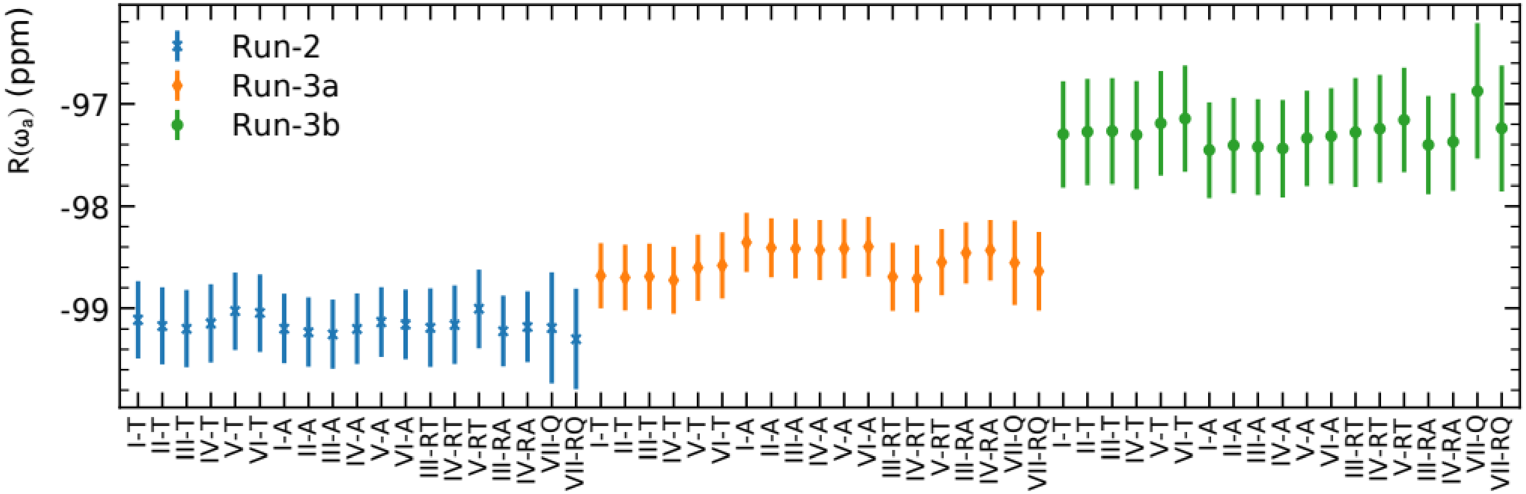
Blinded analysis

- **Hardware**: main clock is tuned at $(40 - \varepsilon)$ MHz
Offset only known to two scientists external to the collaboration

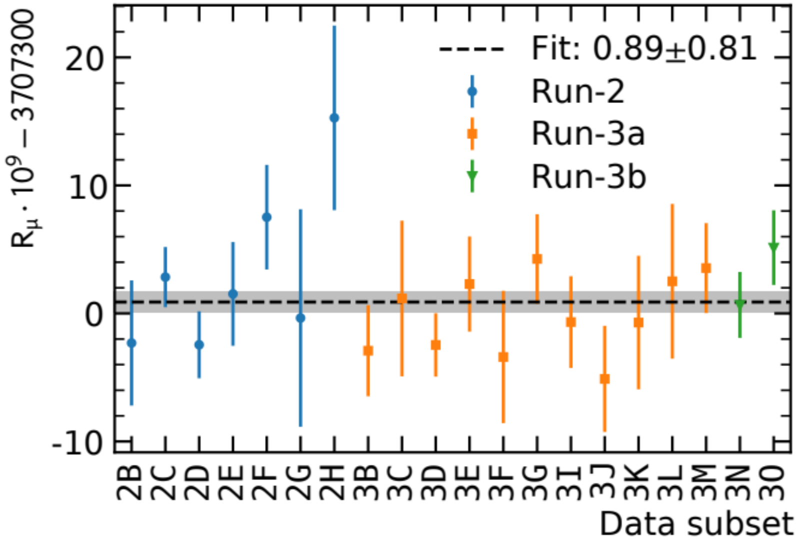
- **Software**: each ω_a analyzer applies their own, secret offset to their results

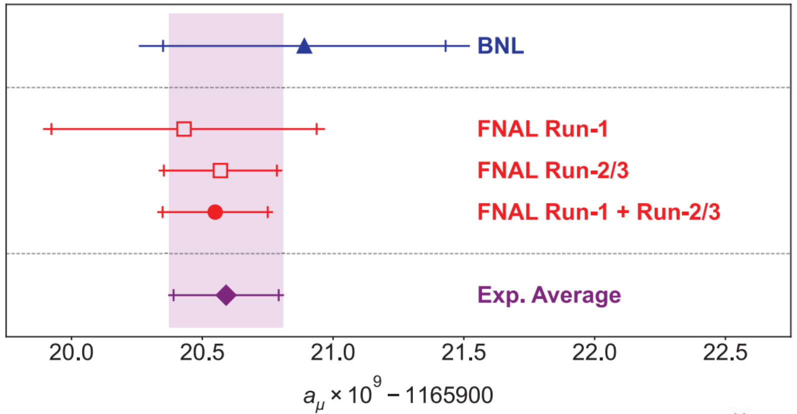
Example: Run-2/3 unblinding

- **Software unblinding**



- **Hardware unblinding**

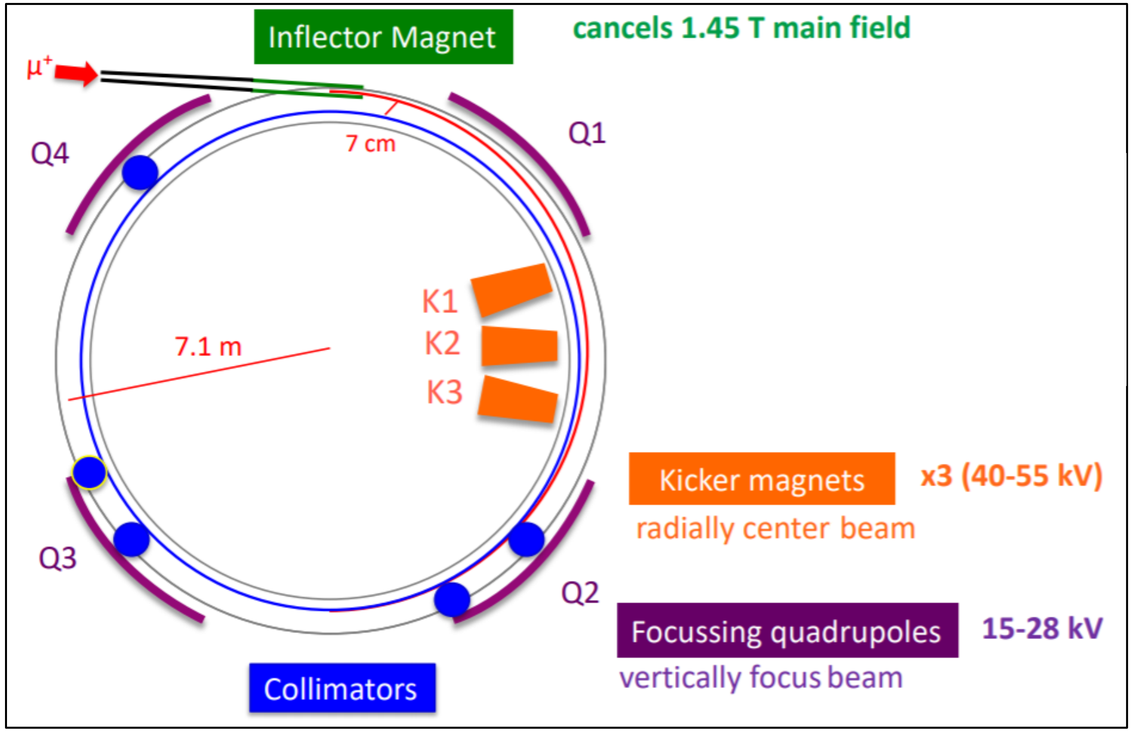


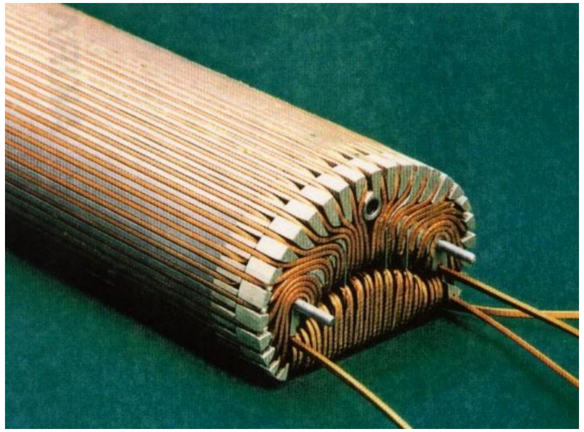


Extra: Run-1 → Run-2/3

Injection and muon storage

1. Inflector cancels main dipole field and injects at ~8 cm radially away from nominal orbit
2. 3 fast magnetic kickers provide 10 mrad kick and place muons in orbit
3. 8 Electrostatic Quadrupoles (ESQ) focus in the vertical direction



1.


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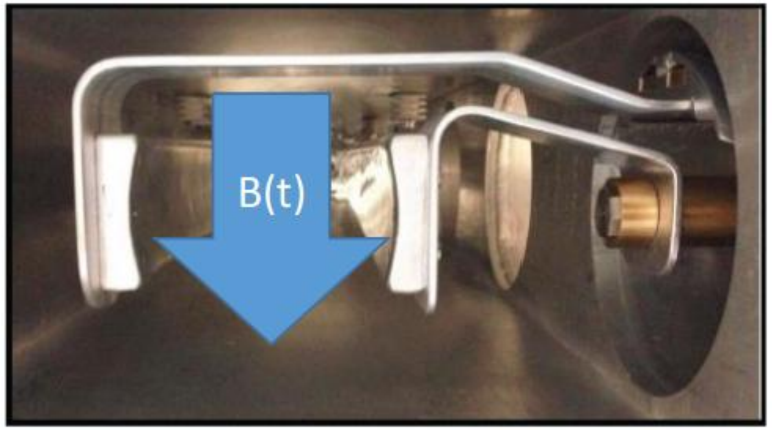

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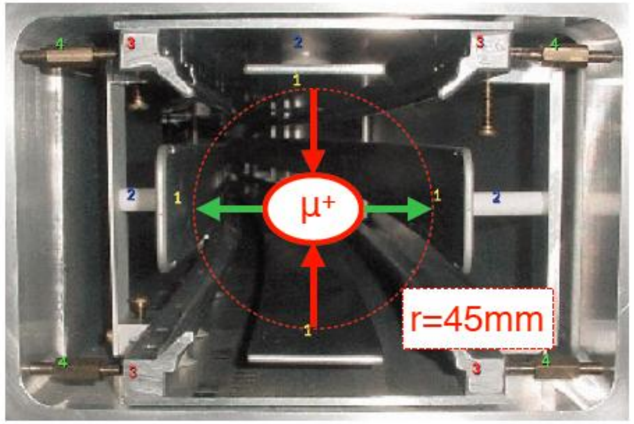

Run-2/3 improved running conditions

- Before Run-2: **fixed faulty resistors** in 2/32 quadrupole plates → better storage, more stable beam oscillations and reduced systematics
- After Run-2: added thermal insulation to ring → less variable magnetic field
- Mid Run-3: **upgraded kicker** cables for optimal kick → more centered beam