4th Workshop on Muon Precision Physics The Spine, Liverpool, 11th Nov 2025

Status of the puzzles of g-2

Thomas Teubner

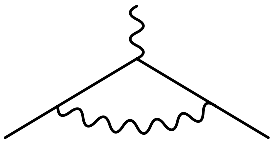
- Introduction & (not always so) recent history
- Main changes in the a_μ^{SM} prediction, WP25
  - Data and data-driven dispersive HVP evaluations
  - vs Lattice QCD+QED
- Current & future efforts, pathways to solving the puzzles
- Outlook

Disclaimer: Many more detailed/specialist talks later at this workshop

Introduction: it all started with the electron…

- 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with g = 2.00229 ± 0.00008

- 1948: Schwinger calculates the famous radiative correction:



➟ g = 2 (1+a), with the **anomaly**

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi} \approx 0.001161$$

This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

``If you can't join 'em, beat 'em"

- In terms of an effective Lagrangian, the anomaly is from the Pauli term:

$$\delta\mathcal{L}_{\rm eff}^{\rm amm} = -\frac{Qe}{4m}\, a\, \bar\psi_L \sigma^{\mu\nu}\psi_R F_{\mu\nu} \ + \ ({\rm L} \leftrightarrow {\rm R})$$

Note: This is a dimension 5 operator and NOT part of the fundamental (QED) Lagrangian, but occurs through radiative corrections and is **calculable in** (Standard Model) **theory**:

$$\boxed{a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadronic}}$$

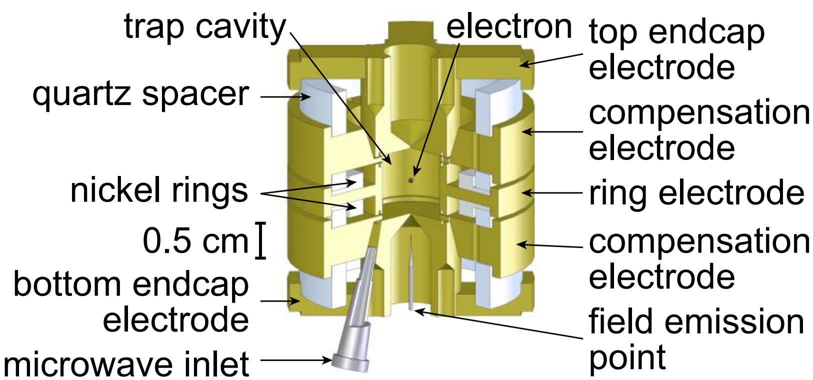
Short detour: a_e vs. a_μ and why we want to study the muon

a_e= 1 159 652 180.73 (0.28) 10^{-12} [0.24ppb]

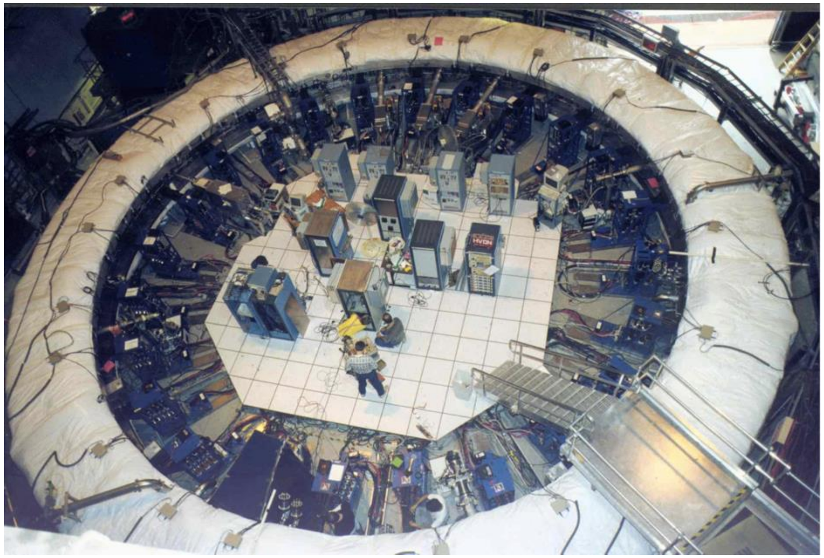
Hanneke et al., *PRL* 100(2008)120801 @ Harvard

a_μ= 116 592 089(63) 10^{-11} [0.54ppm]

Bennet et al., *PRD* 73(2006)072003 @ BNL



one-electron quantum cyclotron



- a_e^{EXP} more than 2000 times more precise than a_μ^{EXP}, but for e^- loop contributions come from very small photon virtualities, whereas muon `tests' higher scales
- dimensional analysis: sensitivity to NP (at high scale Λ_{NP}): $a_\ell^{\mathrm{NP}} \sim \mathcal{C}\, m_\ell^2/\Lambda_{\mathrm{NP}}^2$

→ μ wins by $m_\mu^2/m_e^2 \sim 43000$ for NP, a_e `determines' α, tests QED & low scales

[Note: τ too short-lived for storage-rings, hard to get precision at colliders]

a_e current status (exp @ Northwestern): *PRL* 130 (2023) 7, 071801

Measurement of the Electron Magnetic Moment

[arXiv:2209.13084]

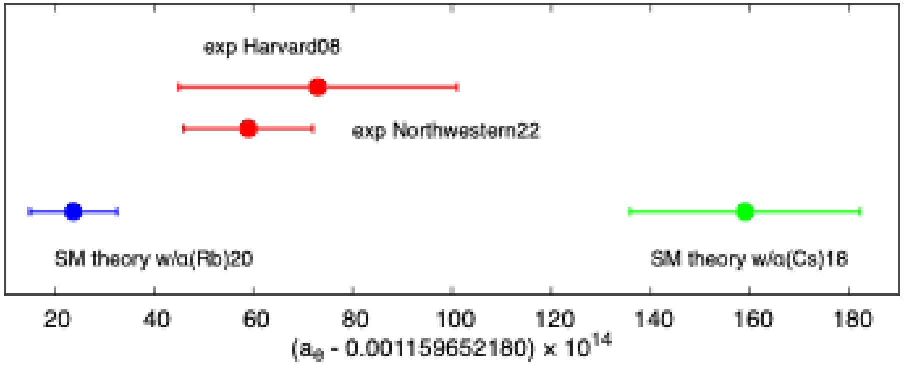
X. Fan,[1,2,*] T. G. Myers,[2] B. A. D. Sukra,[2] and G. Gabrielse[2,†]

[1] *Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*
[2] *Center for Fundamental Physics, Northwestern University, Evanston, Illinois 60208, USA*

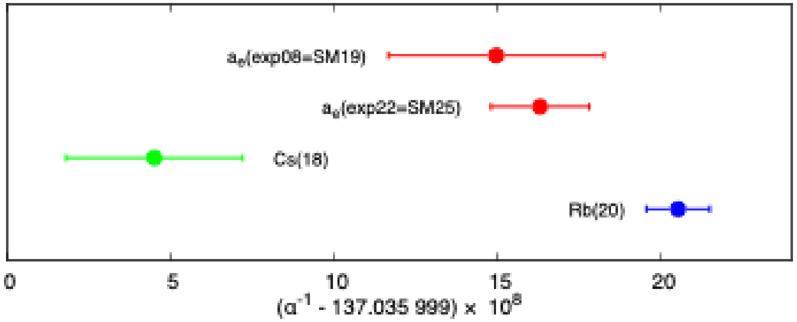
(Dated: September 28, 2022)

The electron magnetic moment in Bohr magnetons, $-\mu/\mu_B = 1.001\,159\,652\,180\,59\,(13)$ [0.13 ppt], is consistent with a 2008 measurement and is 2.2 times more precise. The most precisely measured property of an elementary particle agrees with the most precise prediction of the Standard Model (SM) to 1 part in 10^{12}, the most precise confrontation of all theory and experiment. The SM test will improve further when discrepant measurements of the fine structure constant α are resolved, since the prediction is a function of α. The magnetic moment measurement and SM theory together predict $\alpha^{-1} = 137.035\,999\,166\,(15)$ [0.11 ppb]

SM theory prediction depends strongly on **α, but measurements with Cs and Rb disagree by 5.4σ :**



Figures from Muon g-2 Theory Initiative **WP25** [Phys. Rept. 1143 (2025) 1-158]



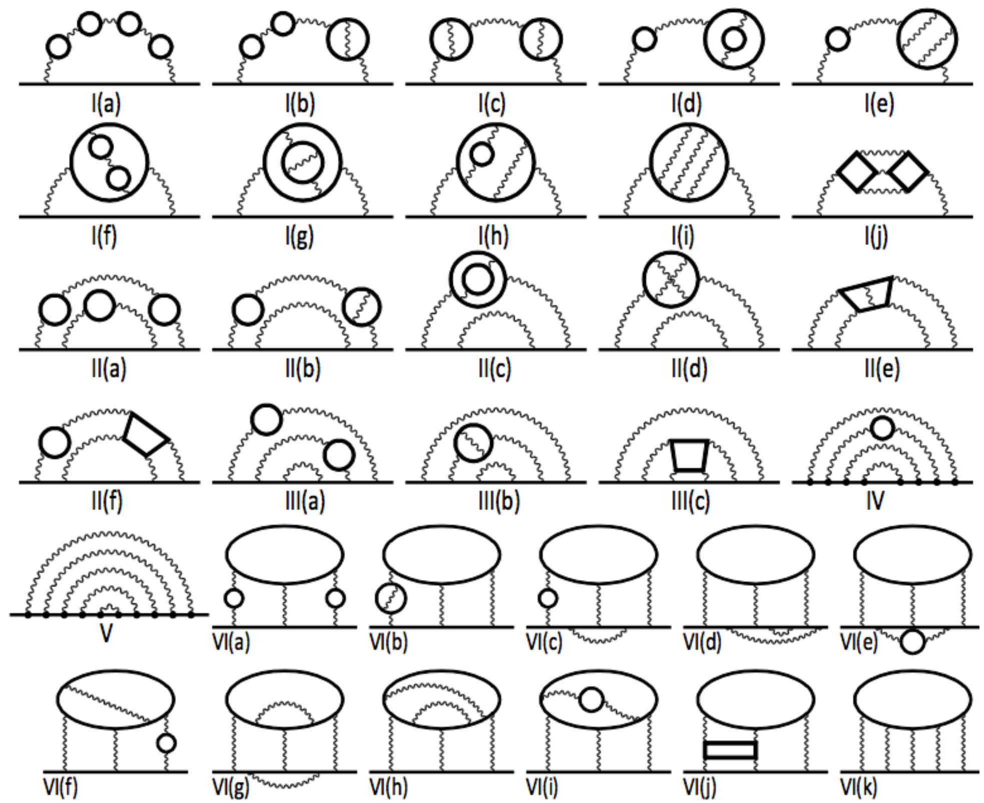
⟵ Translation to derived value of **α**

a_μ^{QED} & a_μ^{weak} : a triumph for perturbative QFT

QED: Kinoshita et al. + many tests

- g-2 @ 1, 2, 3, 4 & 5 loops
- Subset of 12672 5-loop diagrams:
- code-generating code, incl renormalisation
- multi-dim. numerical integrations
- including now **consolidation of numerical 5-loop** (analytical: Stefano Laporta's talk)
- latest update (**WP25**):

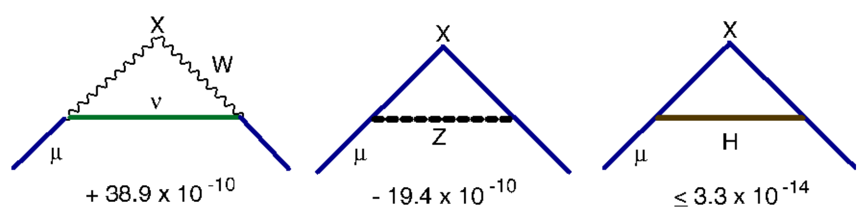
$$a_\mu^{QED} = \mathbf{116\ 584\ 718.8\ (2) \times 10^{-11}} \ \checkmark$$



Weak: (several groups agree)

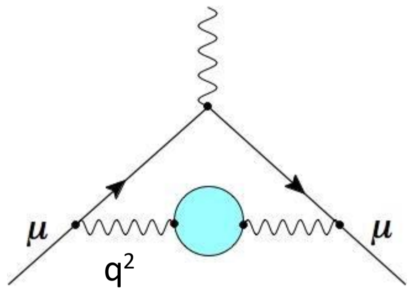
- done to 2-loop order, 1650 diagrams
- the first full 2-loop weak calculation
- latest update (**WP25**):

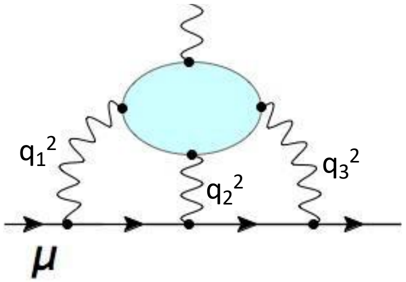
$$a_\mu^{weak} = \mathbf{154.4\ (4) \times 10^{-11}} \ \checkmark$$



SM weak 1-loop diagrams

a_μ^{hadronic} : non-perturbative, the limiting factor of the SM prediction





- **Q:** What's in the hadronic (**Vacuum Polarisation** & **Light-by-Light scattering**) blobs?

  A: Anything `**hadronic**' the virtual photons couple to, i.e. **quarks + gluons** + **photons**

  But: low q^2 photons dominate loop integral(s) ➠ cannot calculate blobs with perturbation theory

- **Two very different** (model independent) **strategies:**

  1. use wealth of hadronic data, `**data-driven dispersive methods**':
     - data combination from many experiments, **radiative corrections** required
  2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `**lattice QCD**':
     - finite size, finite lattice spacing, artifacts from lattice actions, **QCD + QED** needed
     - numerical Monte Carlo methods require large computer resources

Muon g-2 Theory Initiative est. 2017

https://muon-gm2-theory.illinois.edu

- Organised 13 int. workshops in 2017-2024
- **White Paper 2020** posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

  ``The anomalous magnetic moment of the muon in the Standard Model'' [*Phys. Rept.* 887 (2020) 1-166]
- **White Paper 2025** **``… : an update''** [*Phys. Rept. 1143* (2025) 1-158]

Mission: ``… map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental result.''

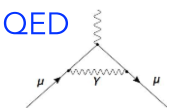
Group photo from the Orsay workshop in September 2025

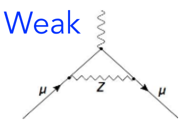
SM (WP20) vs. **Exp**eriment, after FNAL run2+3 results

$$a_\mu = a_\mu^{\rm QED} + a_\mu^{\rm weak} + a_\mu^{\rm hadronic} + a_\mu^{\rm NP?}$$

White **P**aper [T. Aoyama et al., *Phys. Rept.* 887 (**2020**) 1-166]

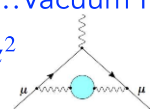
0.37 ppm

QED  +… $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

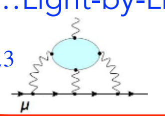
Weak  +… $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

α^2  +… $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

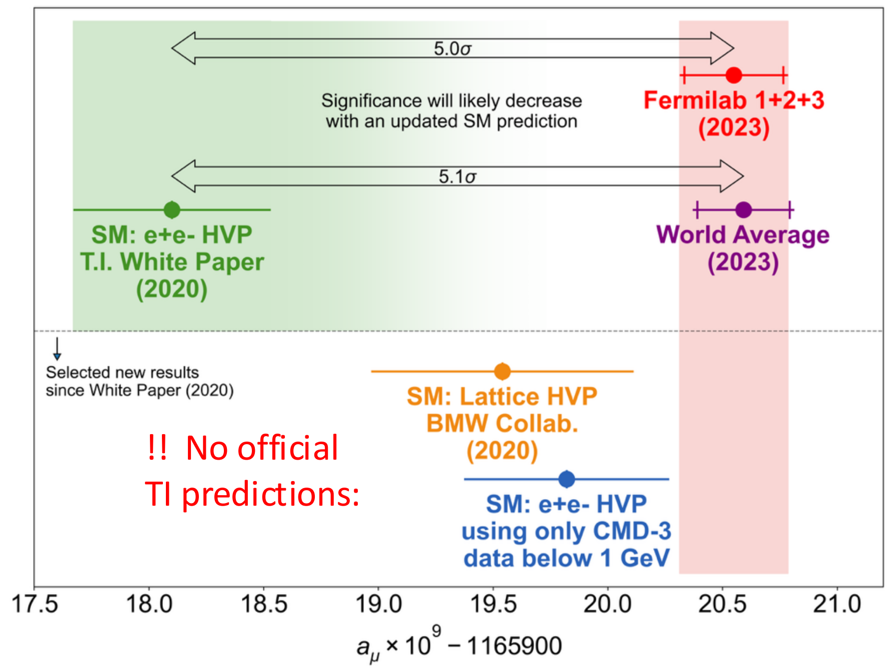
…Light-by-Light (HLbL)

α^3  +… $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

- SM uncertainty dominated by hadronic contributions, now with δ HVP > δ HLbL

Measurement of the Positive Muon Anomalous Magnetic Moment to **0.46 ppm** [*Phys. Rev. Lett.* 126 (**2021**) 14, 141801]

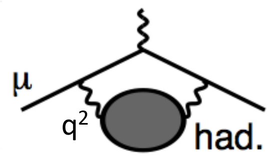
… **to 0.20 ppm** [*PRL* 131 (**2023**) 16, 161802]



!! No official TI predictions:

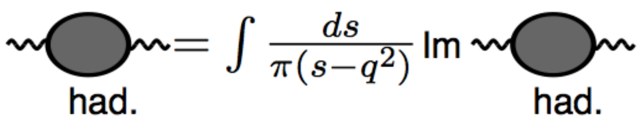
… seemed close to a NP discovery?!

a_μ^{HVP} : Basic principles of **dispersive** data-driven method



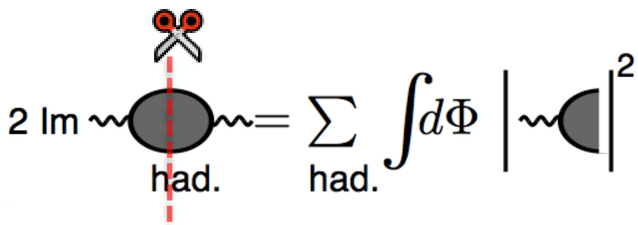
One-loop diagram with hadronic blob =

integral over q^2 of virtual photon, 1 HVP insertion



Causality ➠ analyticity ➠ dispersion integral:

obtain HVP from its imaginary part only



Unitarity ➠ Optical Theorem:

imaginary part (`cut diagram') =

sum over |cut diagram|², i.e.

∝ sum over all total hadronic cross sections

$$\boxed{a_\mu^{\mathrm{had,LO}} = \frac{m_\mu^2}{12\pi^3}\int_{s_{\mathrm{th}}}^{\infty} ds\, \frac{1}{s}\hat{K}(s)\sigma_{\mathrm{had}}(s)}$$

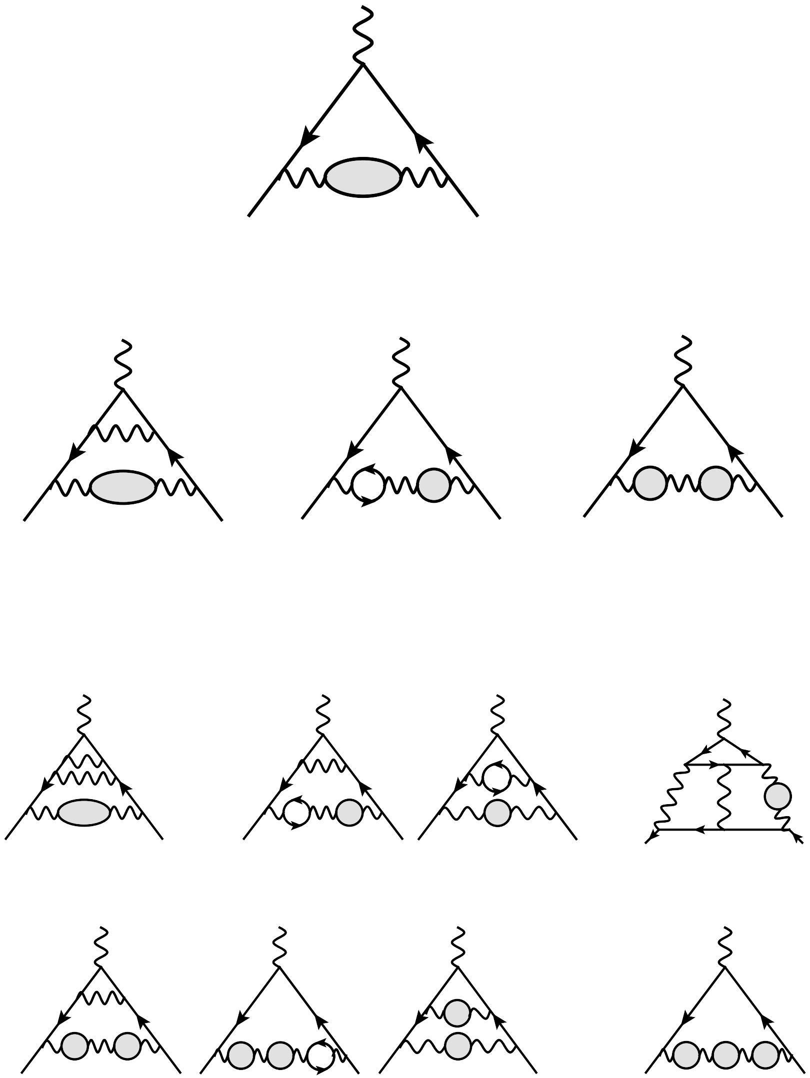
- Weight function $\hat{K}(s)/s = \mathcal{O}(1)/s$

$\Longrightarrow$ Lower energies more important

$\Longrightarrow$ $\pi^+\pi^-$ channel: 73% of total $a_\mu^{\mathrm{had,LO}}$

- Total hadronic cross section $\boldsymbol{\sigma}_{\mathbf{had}}$ from ~ 250 data sets for **e^+e^- ➞ hadrons** in ~ 50 final states
- Uncertainty of a_μ^{HVP} prediction from statistical & systematic uncertainties of input data
- pQCD only at large s, **no modelling** of $\sigma_{had}(s)$, direct data integration
- **The method is solid beyond doubt**

a_μ^{HVP} : Higher orders & power counting; WP20 values in $\mathbf{10^{-11}}$



➤ All hadronic blobs also contain **photons**,
i.e. real + virtual corrections in $\sigma_{had}(s)$

- **LO: 6931(40)**

- **NLO: - 98.3(7)**

  from three classes of graphs:
  - 207.7(7) + 105.9(4) + 3.4(1)   [KNT19]

  (photonic, extra e-loop, 2 had-loops)

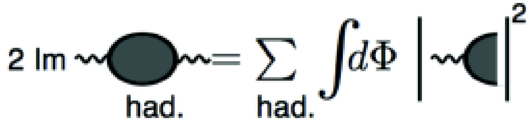
- **NNLO: 12.4(1)** [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]

  from five classes of graphs:
  8.0 - 4.1 + 9.1 - 0.6 + 0.005

➥ good convergence,
iterations of hadronic blobs _very_ small

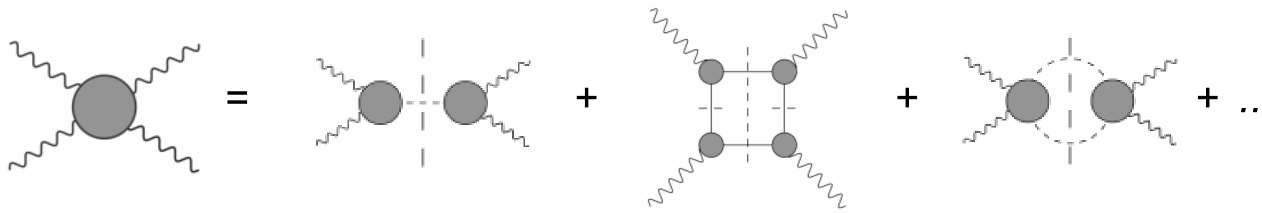
➠ `double-bubbles' very small

a_μ^{HLbL} : Hadronic Light-by-Light: **Dispersive** approach

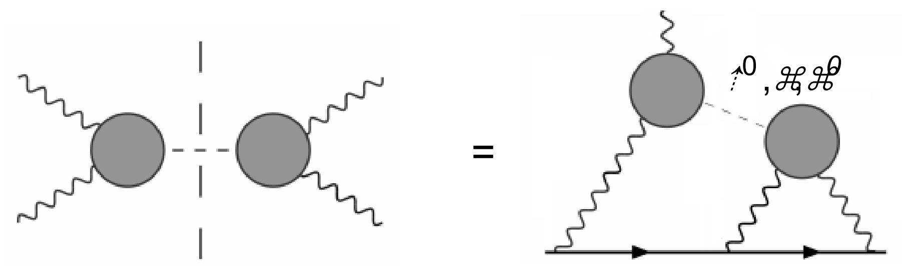
For HVP $\Rightarrow$ 

$\Rightarrow$ $\mathrm{Im}\,\Pi_{\mathrm{had}}(s) = \left(\frac{s}{4\pi\alpha}\right)\sigma_{\mathrm{had}}(s)$

For HLbL $\Rightarrow$ $\Pi_{\mu\nu\lambda\sigma} = \Pi^{\mathrm{pole}}_{\mu\nu\lambda\sigma} + \Pi^{\mathrm{box}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \ldots$

$\Rightarrow$ 

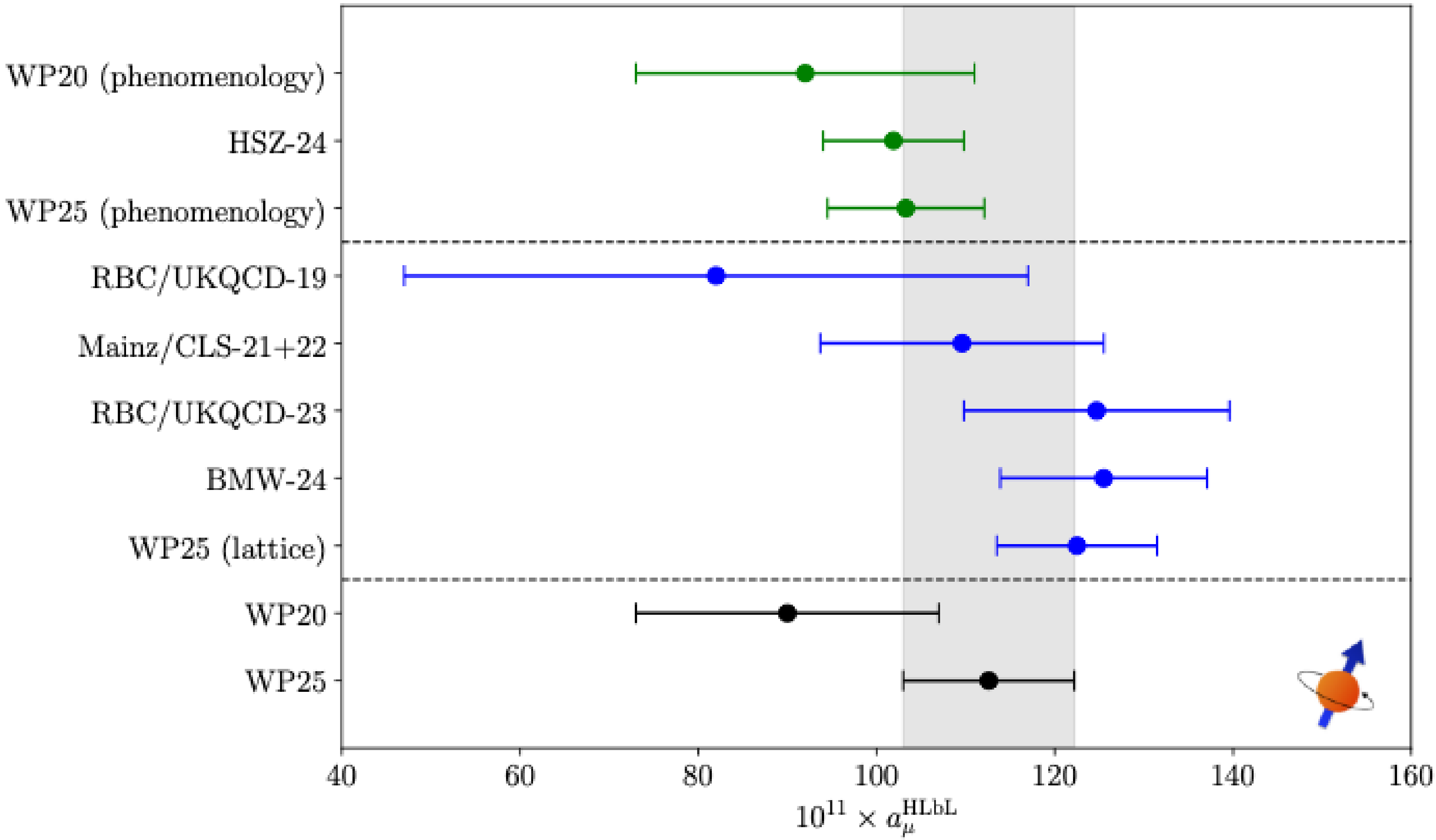
$\Rightarrow$ Dominated by pole (pseudoscalar exchange) contributions

$\Pi^{\mathrm{pole}}_{\mu\nu\lambda\sigma} =$ 

$\Rightarrow$ Sum all possible diagrams to get a_μ^{HLbL}

- Dispersive analyses have matured in recent years and provide precise results without relying on (and not contradicting earlier) hadronic model calculations

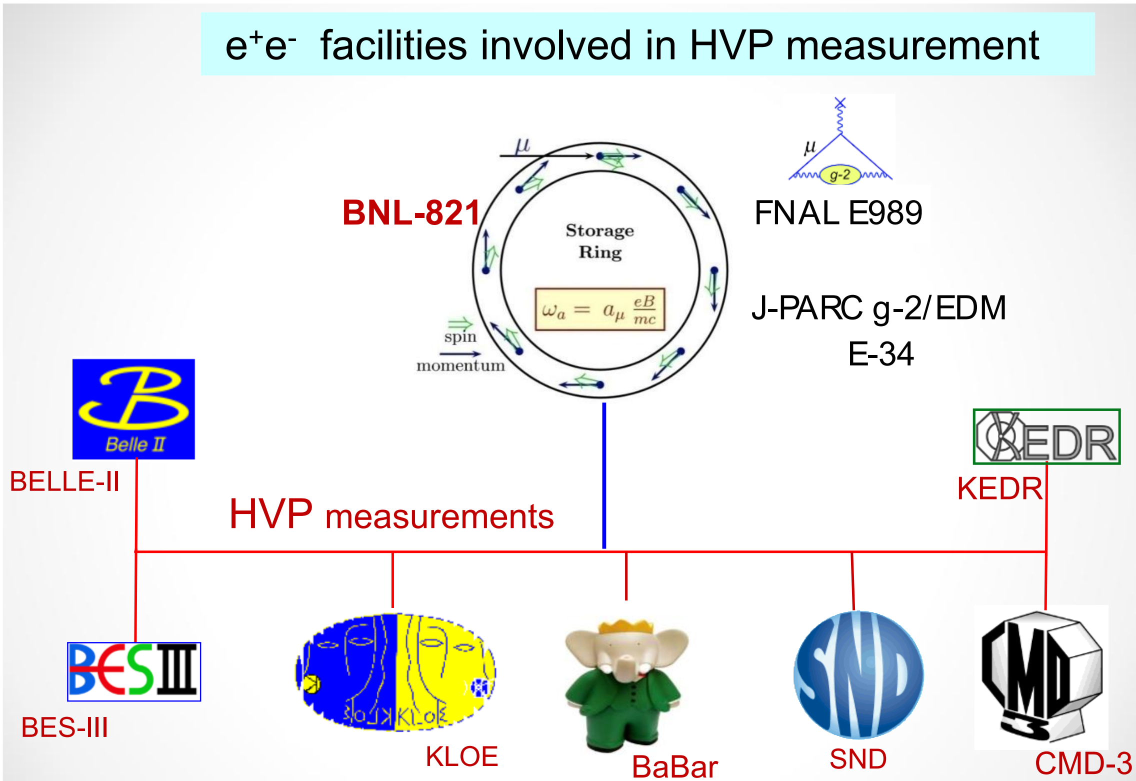
a_μ^{HLbL} : **WP25** Summary of Hadronic Light-by-Light contributions

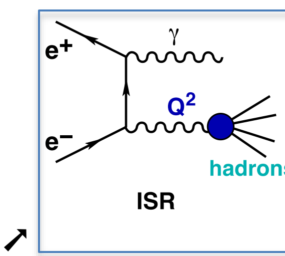


- New, precise **data-driven dispersive** (`phenomenology') & **lattice** results are consistent
- Their combination has improved the **WP25** prediction of $\mathbf{a_\mu^{HLbL}}$ to a precision of **8.5%** ✓

HVP: Recent (of >30 years) experiments providing input $\sigma_{had}(s)$ data

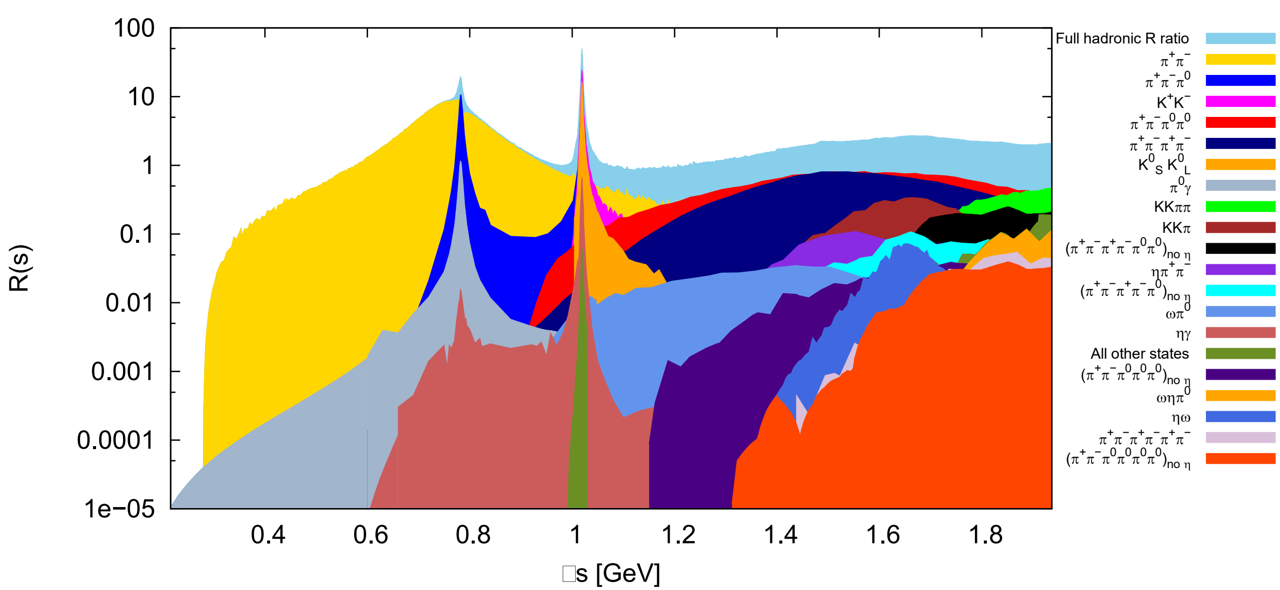
S. Serednyakov (for SND) @HVP KEK workshop





- Different methods: \`**Direct Scan**' (tunable e^+e^- beams) &
  \`**Radiative Return**' (Initial State Radiation scan at fixed cm energy) ↗
- Over last decades detailed studies of **radiative corrections** & **Monte Carlo Generators** for $\sigma_{had}(s)$
  - **RadioMonteCarLow** Working Group report: *Eur. Phys. J.* C66 (2010) 585-686
  - full NLO radiative corrections in ISR MC ***Phokhara***: Campanario et al, PRD 100(2019)7,076004

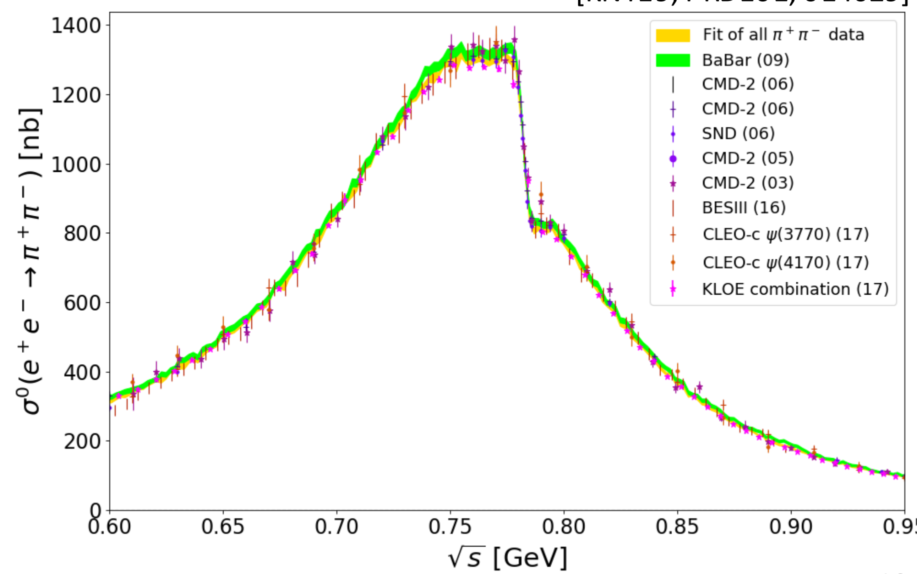
HVP disp: Landscape of $\sigma_{had}(s)$ data. Most important $\pi^+\pi^-$ channel



[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of 2π

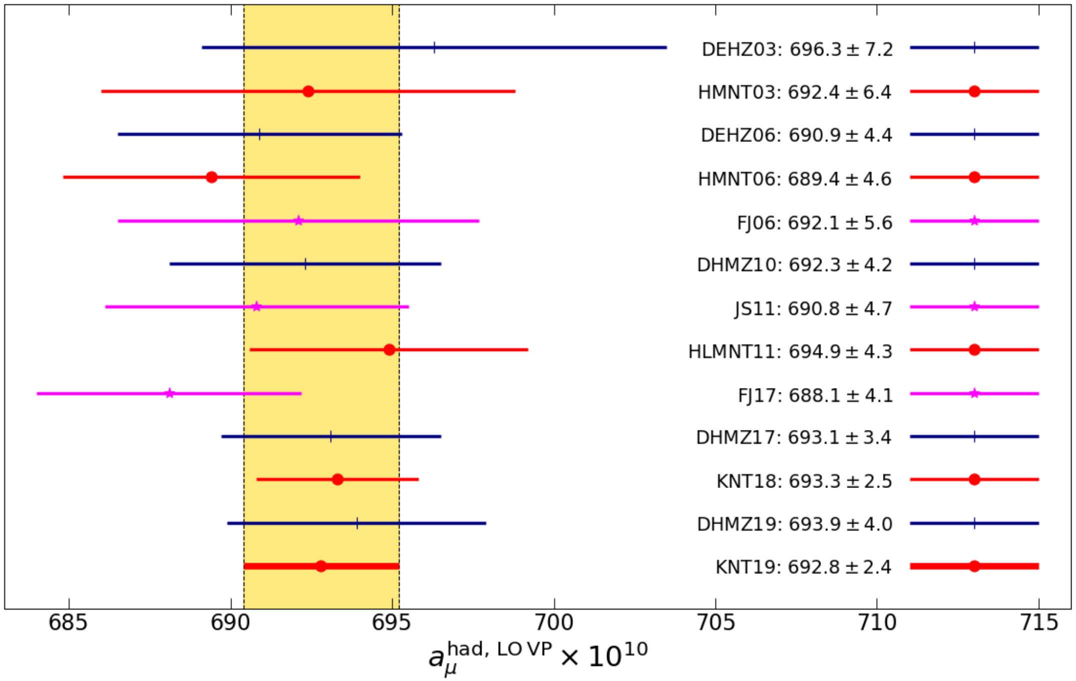
[KNT19, PRD101, 014029]



$\pi^+\pi^-$:

- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from **6 independent experiments** with different systematics and different radiative corrections
- Dominated by Radiative Return, **tensions could be accommodated by error inflation**
- **Until the CMD-3 result in 2023** (not in here →)

a_μ^{HVP} : > 20 years of data based predictions, `pies'



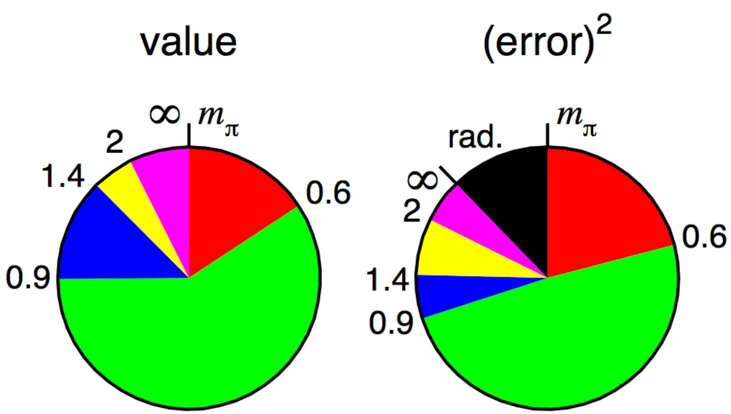
- **Stability** and **consolidation** over two decades thanks to more and better data input and improved compilation procedures

- Compare with **merged** DHMZ & KNT value in 2020 TI WP:

$a_\mu^{\text{had, LO VP}}$**(WP20) = 693.1(4.0)×10^{-10}**

Pie diagrams for KNT compilation:

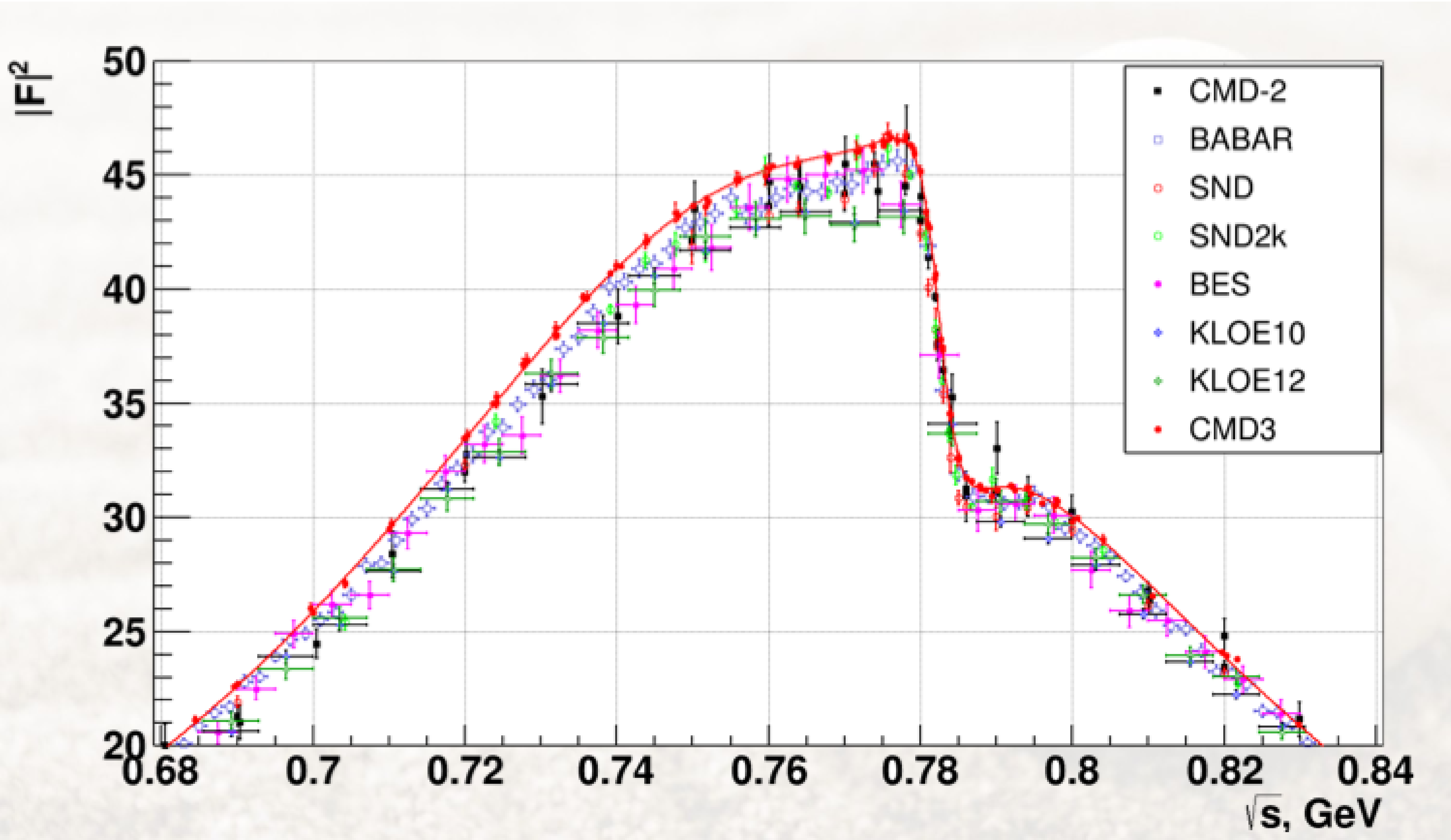
- error dominated by the two pion channel
- significant contribution to error from additional **uncertainty from rad**iative **corrections**
- **From 2023: Is all this invalidated by the CMD-3 data?**



CMD-3 $\pi^+\pi^-$ data vs. other experiments

Figure from Fedor Ignatov's TI talk 27.3.2023

PRD 109(2024)11,112002 PRL 132(2024)23,231903



Theory Initiative: Sep. 2023 workshop at Bern

Michel Davier's summary of the `**49 Questions to CMD-3'** (all answered by Fedor Ignatov):

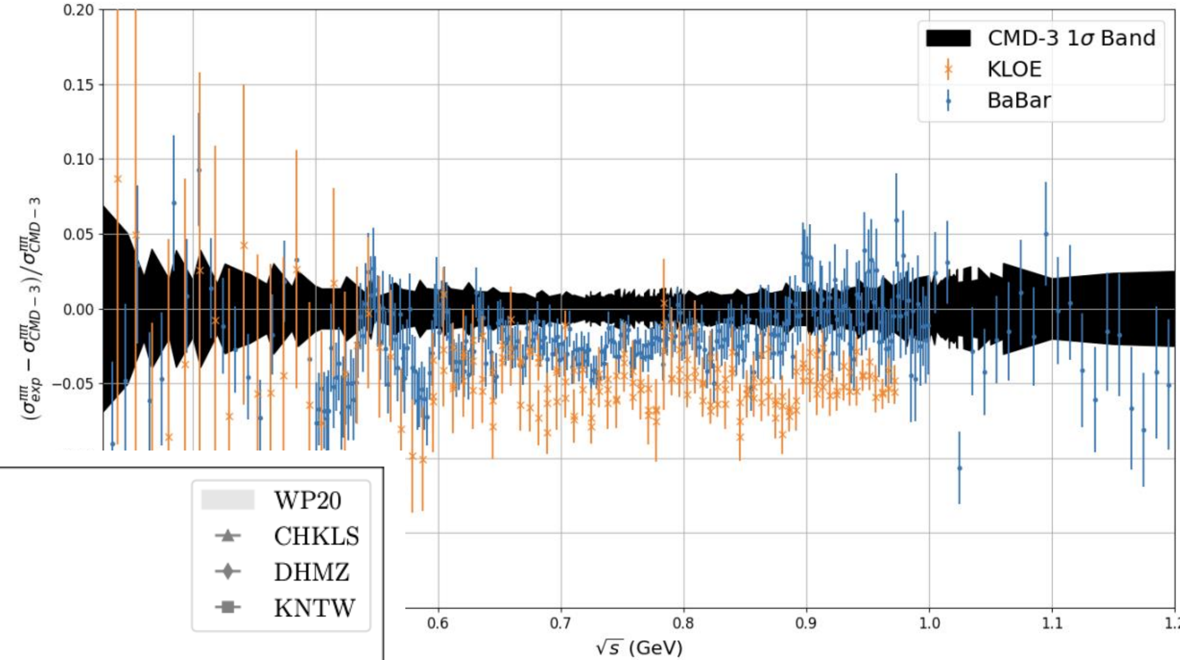
Conclusions

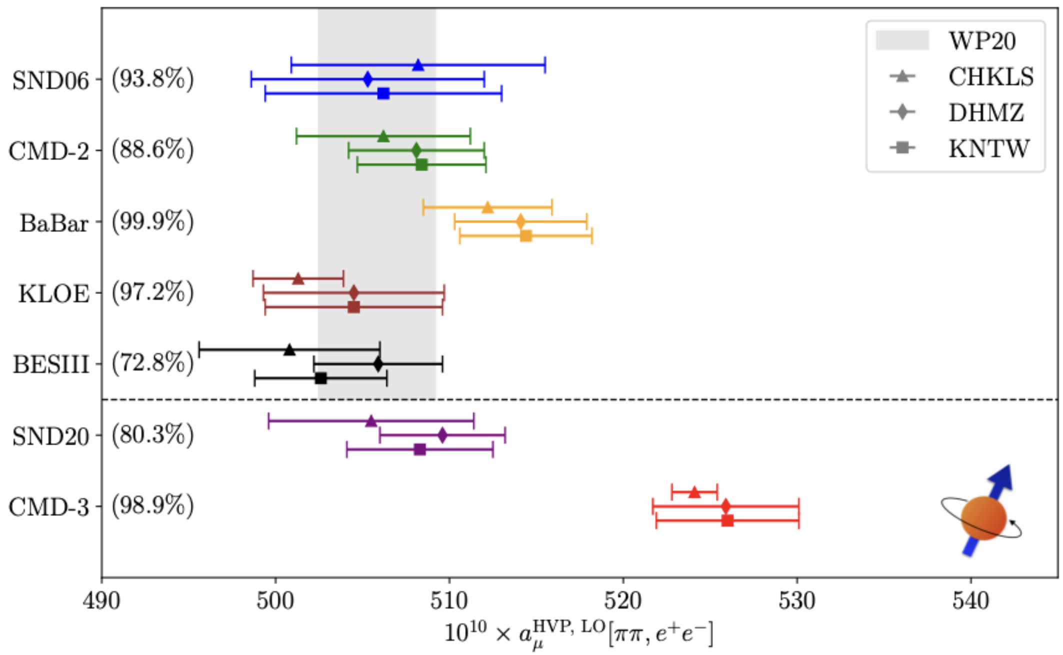
- Difficult exercise: sophisticated analyses are not easy to penetrate without access to the data
- However we got documented answers on detailed questions covering the important aspects of the analysis
- It is fair to say that no major issue significantly impacting the results has been identified
- The strength of the analysis lies in (1) the large statistics accumulated giving the possibility to perform systematic tests with high precision, (2) improved performance of the CMD-3 detector, and (3) the fact that two independent methods were used for channel separation
- Still several points remained unclear to us and /or not enough convincing with the information available
- Possible effects on the results from these minor issues need to be quantified with respect to the claimed accuracy
- Need guidance from CMD-2/3 on how to handle their data

In my opinion, these statements still stand.

a_μ^{HVP} : $\pi^+\pi^-$ data tensions: **KLOE vs. Babar vs. CMD-3 puzzle**

- CMD-3 spectrum much higher than all other previous data
- tensions with BaBar (~2.5σ) and KLOE (~5σ)
- no errors found despite significant efforts



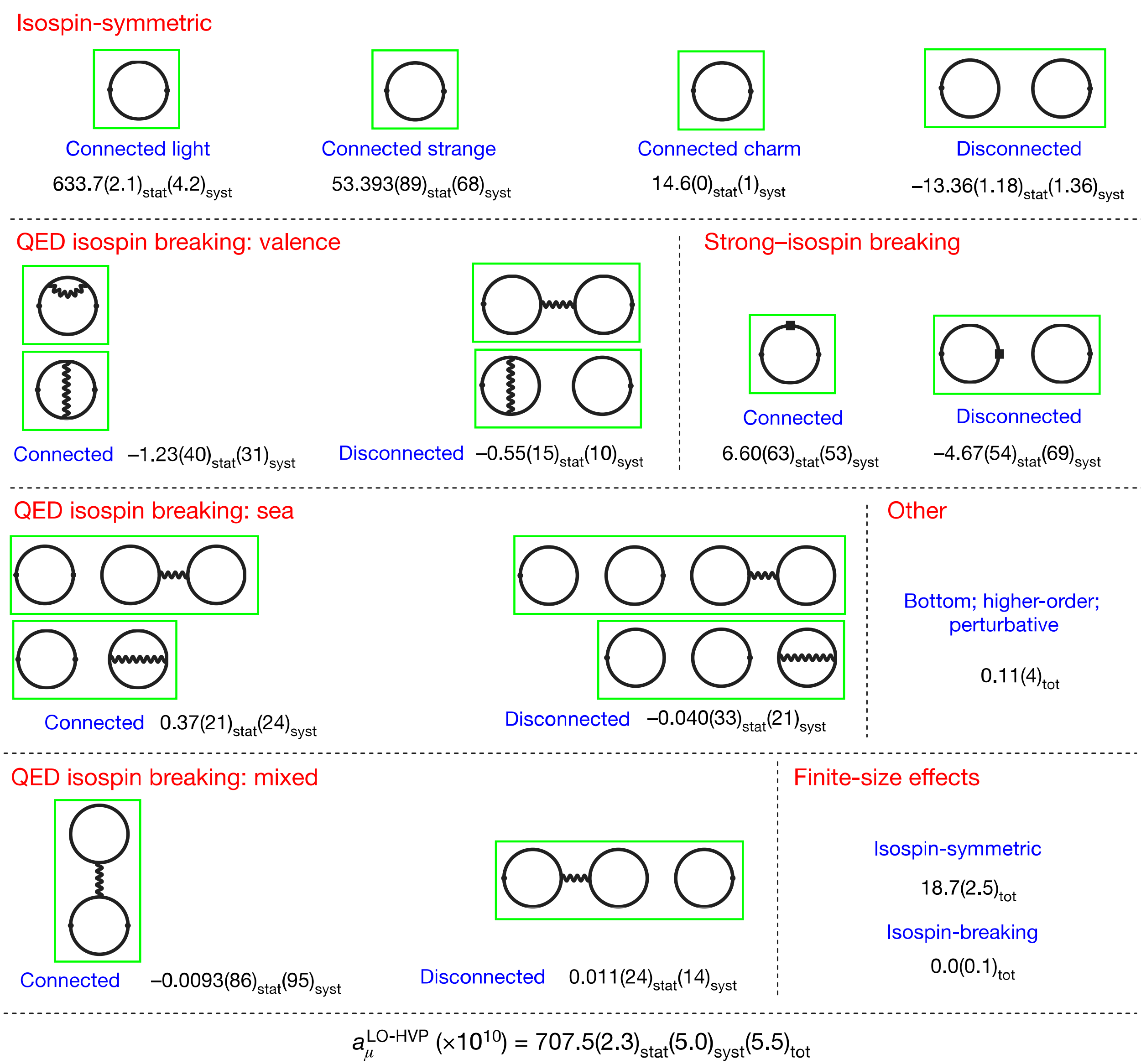


Theory Initiative's **WP25**:

- detailed discussion of data, combination methods and theory inputs, but
- **NO** data-driven a_μ^{HVP} \`average'

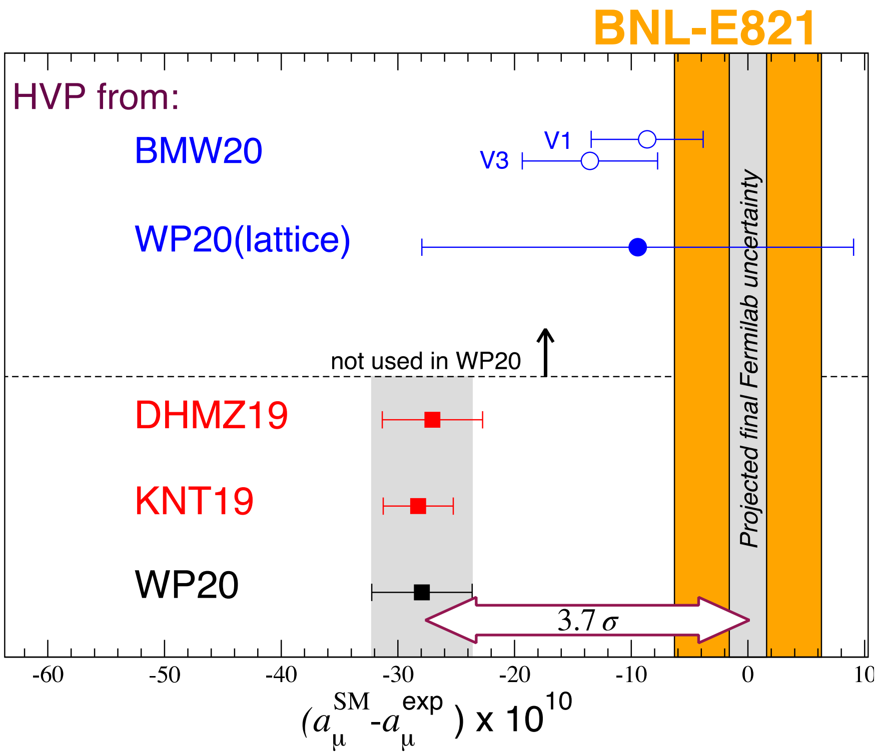
For more details on HVP dispersive see Aidan Wright's talk tomorrow morning

a_μ^{HVP} : Lattice result from BMW [Borsanyi et al., Nature 2021]

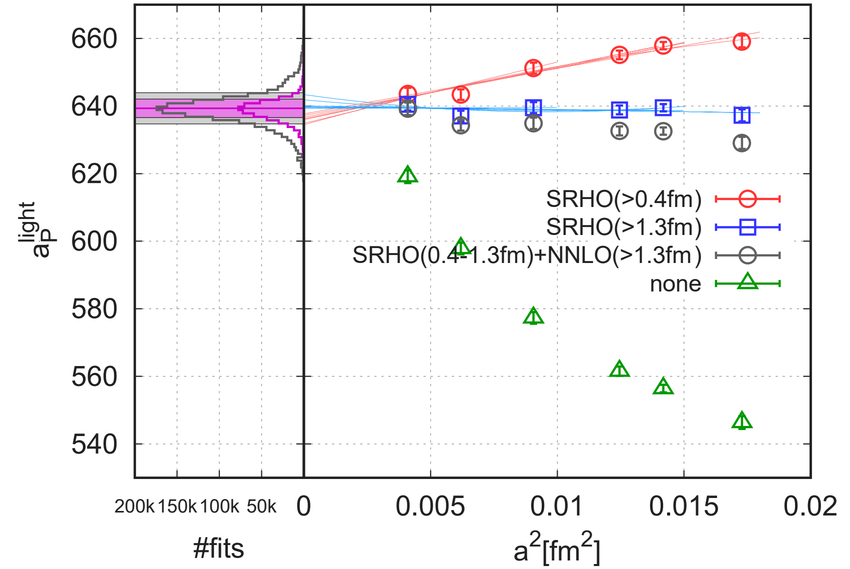


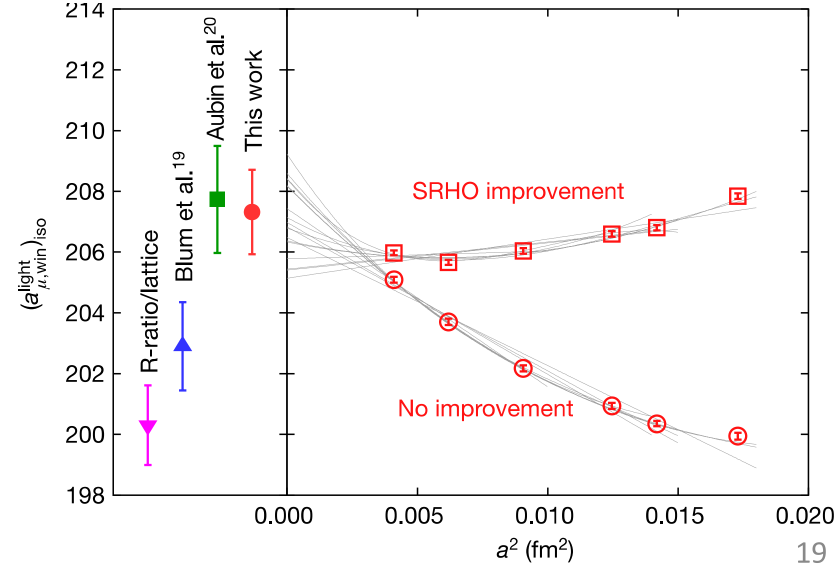
- First full lattice prediction with errors matching the data-driven approach
- Current-current correlators, summed over all distances and integrated over time (TMR)
- Using a L~6fm lattice (11fm for finite size corrections)
- Physical quark masses
- Strong + QED isospin breaking corrections

a_μ^{HVP} : Tension between **data-driven & BMW**. Systematics



BMW20 [Borsanyi et al, arXiv:2002.12347, 2021 Nature]





BMW20: large systematics from **continuum limit,**
large taste-breaking corrections (`SRHO')

- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central `**window**' compared to other lattice and data-driven results (**3.7σ** tension)

a_μ^{HVP} : **Window method** for more detailed comparison

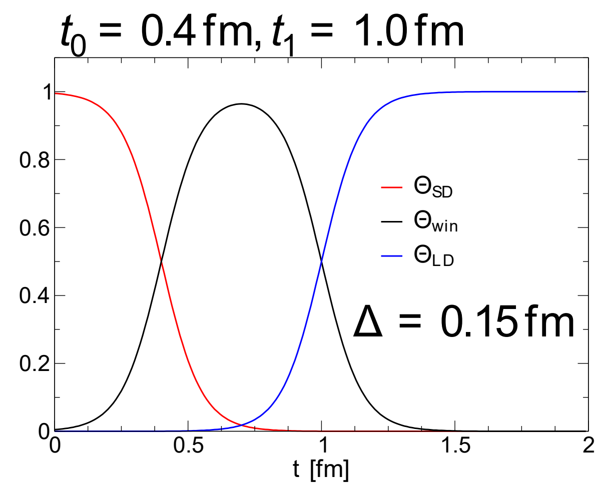
$$\boxed{a_\mu^{HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \tilde{w}(t)\, C(t)}$$

- Use windows in Euclidean time to consider the different time regions separately.

Short Distance (SD) $\quad t : 0 \rightarrow t_0$

Intermediate (W) $\quad t : t_0 \rightarrow t_1$

Long Distance (LD) $\quad t : t_1 \rightarrow \infty$



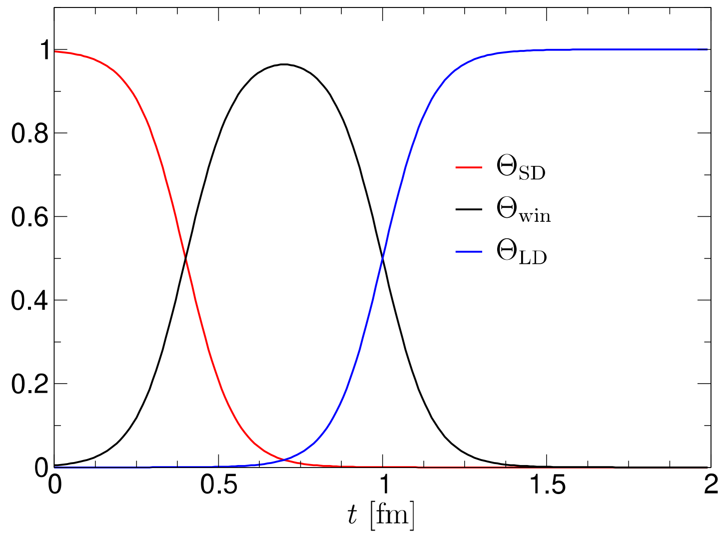
- Compute each window separately (in continuum, infinite volume limits,…) and combine

$$a_\mu = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

a_μ^{HVP}: **Window method** for more detailed comparison

Correspondence to kernels for comparison with (time-like) dispersive approach:

Windows for lattice ↔ **approximate, wide regions for dispersive**



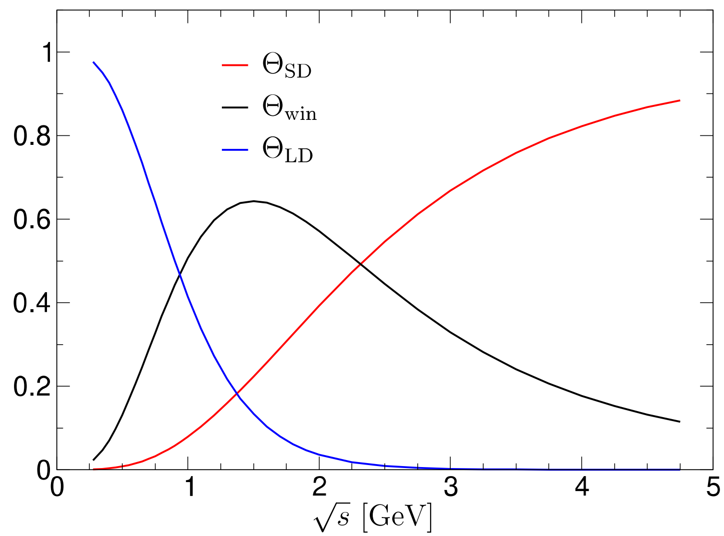
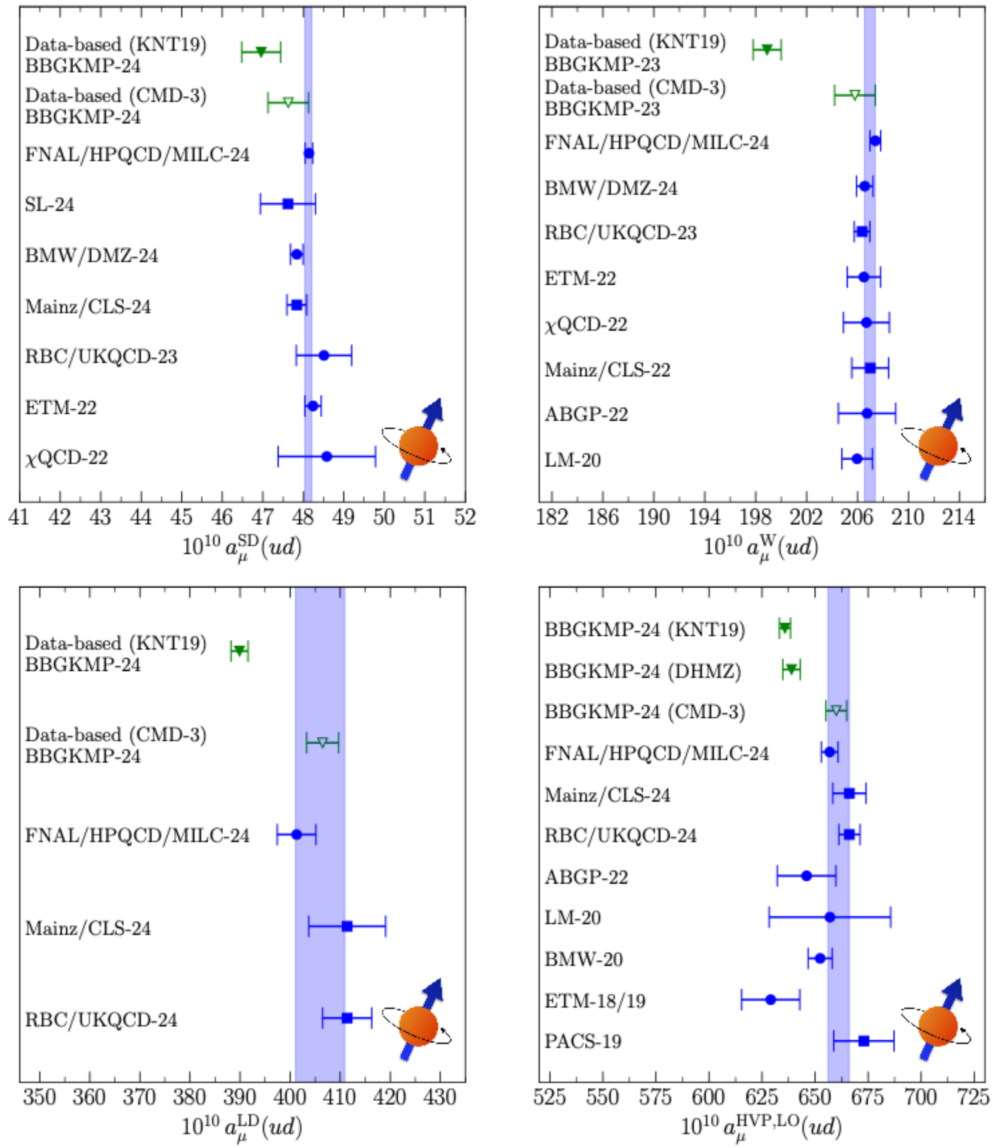


Fig.: G. Colangelo, PWA12/ATHOS7 2021

a_μ^{HVP} : **Window method** for detailed comparisons (WP25)

- Comparison of the leading **connected ud** quark contributions to $a_\mu^{HVP,\,LO}$, **in the three windows and** for the **total** contribution, between **data-driven** and **lattice** results

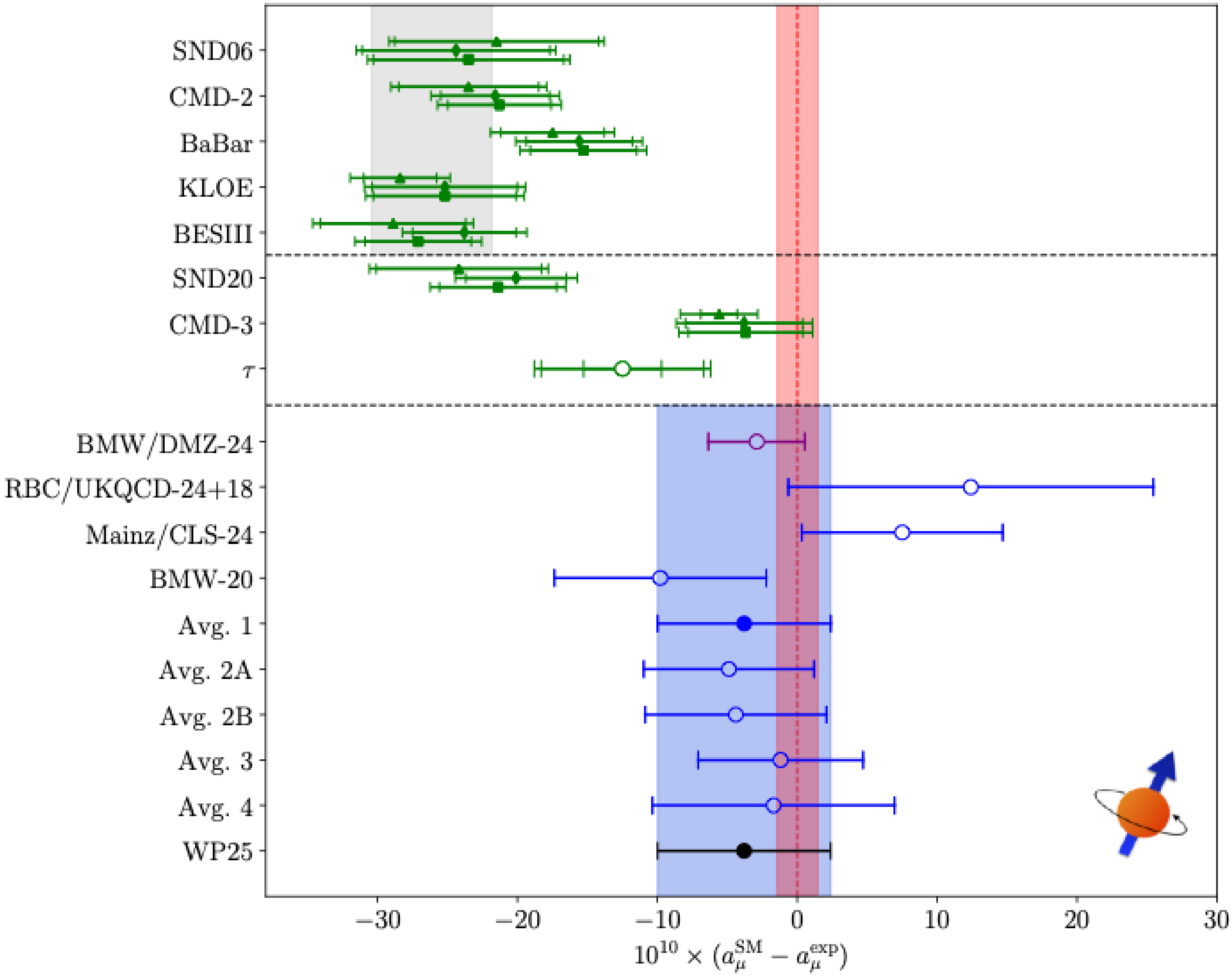
- **Discrepancies** are **clearly established**, *unless CMD-3 is used as input for the data-driven evaluation*



For more details on lattice HVP see Davide Giusti’s talk tomorrow morning

a_μ^{HVP} : Summary plot for HVP comparison from WP25

- Comparison of predictions for the full $a_\mu^{HVP, LO}$ from **data-driven dispersive** (including tau-based*) and **lattice** approaches
- The red band represents the current experimental uncertainty from the latest FNAL results
- Agreement between different averages for the lattice-based predictions
- **Th error bands :-!**



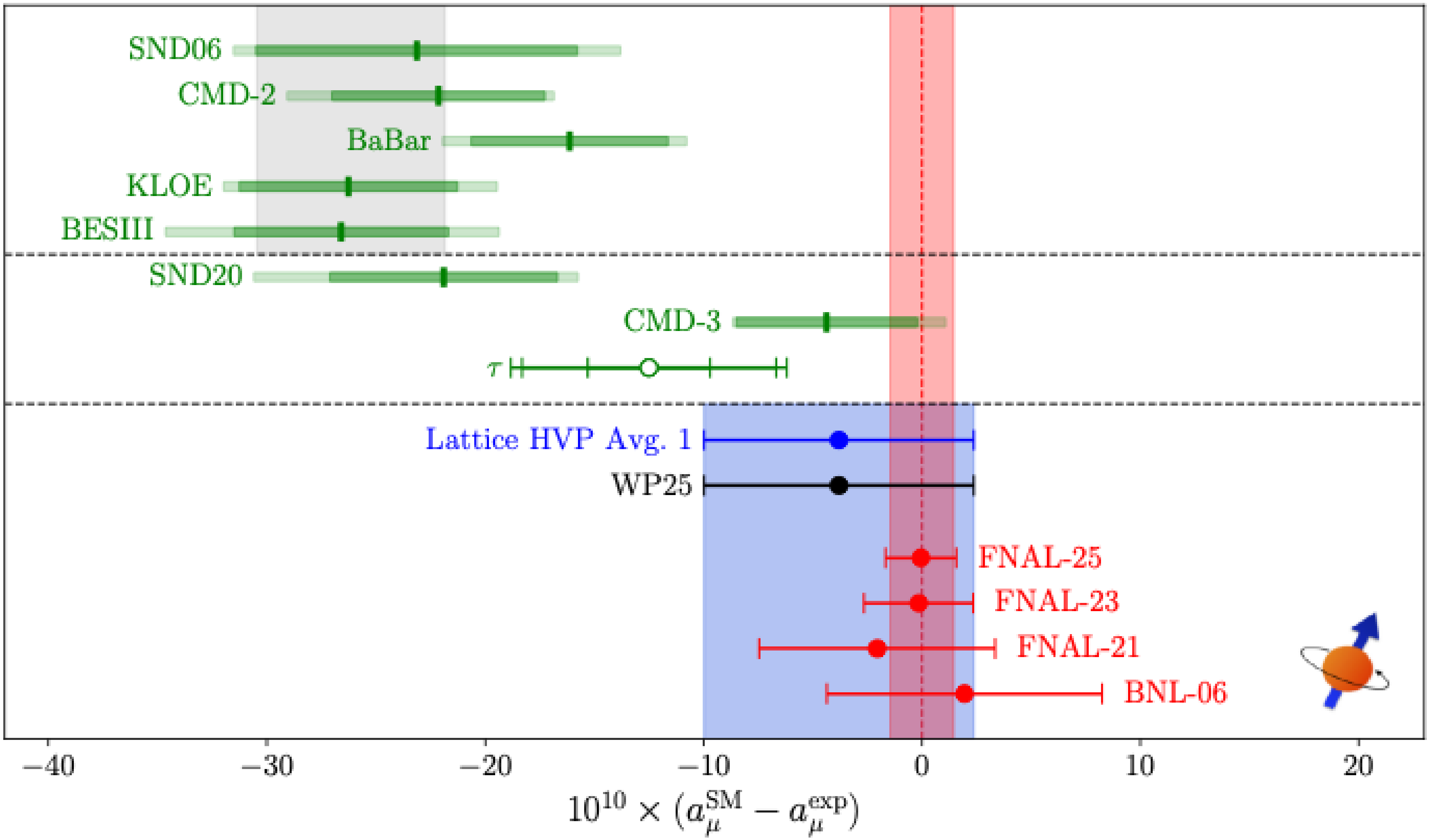
* See Martin Hoferichter's talk on Thursday for latest news on tau

a_μ : Final WP25 summary table

Contribution	WP25	WP20
HVP LO (lattice)	7132(61)	7116(184)
HVP LO (e^+e^-, τ)	Table 5	6931(40)*
HVP NLO (e^+e^-)	−99.6(1.3)	−98.3(7)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)
HLbL (phenomenology)	103.3(8.8)	92(19)
HLbL NLO (phenomenology)	2.6(6)	2(1)
HLbL (lattice)	122.5(9.0)	82(35)
HLbL (phenomenology + lattice)	112.6(9.6)	90(17)
QED	116 584 718.8(2)	116 584 718.931(104)
EW	154.4(4)	153.6(1.0)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)	92(18)
Total SM Value	116 592 033(62)	116 591 810(43)

Table 33: Comparison of the key results from this work (WP25), as given in Table 1, to the corresponding numbers from WP20 [1] (in units of 10^{-11}). Note that the "HLbL (lattice)" result from WP20 has been adapted to include the charm-loop contribution. The entry "HVP (LO + NLO + NNLO)" derives from HVP LO (lattice) [WP25] and HVP LO (e^+e^-) [WP20], respectively. The asterisk indicates that the LO HVP value from WP20 was based on e^+e^- data only, while in Table 5 we also include the current status for τ-based evaluations.

a_μ : Final WP25 summary plot

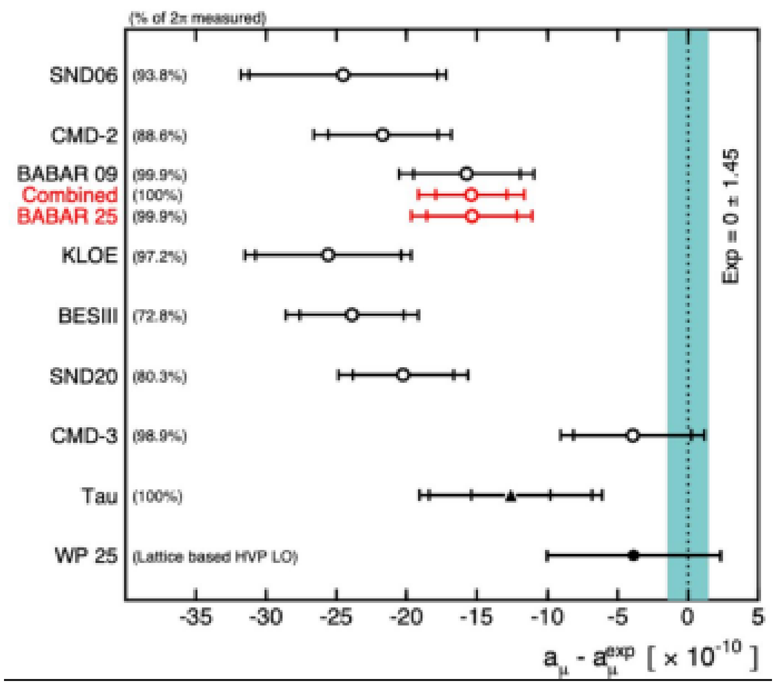


For WP25, the pendulum has swung. Will it stay? My opinion: too early to tell!

a_μ^{HVP} : latest news for $\pi^+\pi^-$

BaBar:

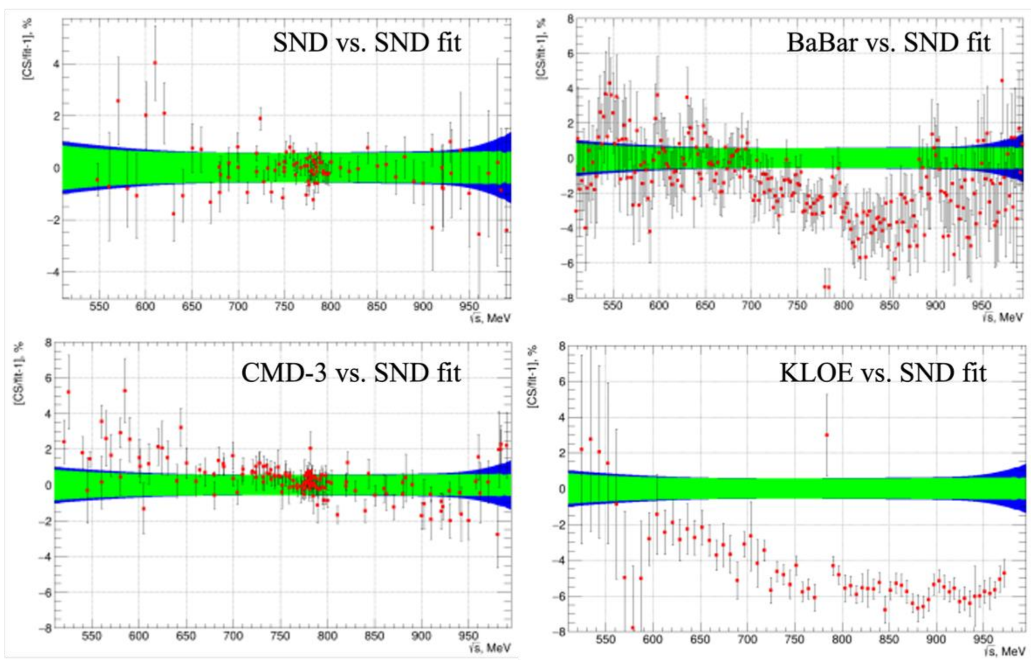
- Preliminary new analysis based on new method
- Reduced systematics for 0.5-1.4 GeV region
- Excellent agreement with previous analysis



A. Pinto, talk at the TI plenary 2025

SND:

- *Preliminary* new, unblinded analysis leads to cross section significantly higher than previously
- Results more in line with CMD-3



A. Kupich, talk at the TI plenary 2025

For more on HVP data, see Achim Denig's talk tomorrow morning

Pathways to solving the puzzles

- Situation complicated due to emerged puzzles in data, mainly (but not only) in the leading two pion channel

- Lattice predictions may further consolidate and improve, but full agreement between the methods and higher accuracy are needed to match the experimental precision, and to have full confidence in conclusions w.r.t. BSM

- **More & more precise data are needed and are already coming from:**
  BaBar, CMD-3, SND, BES III, Belle II, and KLOE

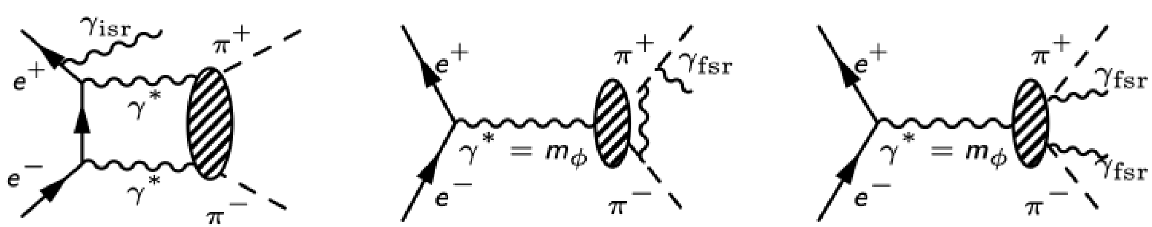
- For this, **improved Radiative Corrections & Monte Carlo generators** are needed, especially for Radiative Return

- To avoid any possible bias, **blinded analyses** are now the standard, for both experiments (g-2 and $\boldsymbol{\sigma_{had}}$) and lattice, also for the next KNTW compilation

- The third way: **MUonE** scattering experiment at CERN
  (see Wednesday afternoon talks by Giovanni Cantatore and Fulvio Piccinini)

KLOE 2π, RC & MC activities

- Liverpool+ effort to analyse the **full** KLOE **2π** statistics (**integrated *L* ~ 1.7 fb^{-1}**)
  - Graziano, with his Leverhulme grant, has brought together a team of **exp+Th+MC** in Liverpool (+ external collaborators), and the work is ongoing
- Goal: sub-percent accuracy for $e^+e^- \to \pi^+\pi^-(\gamma)$ from KLOE, i.e. aim at reduction of uncertainty in g-2 to $\Delta a_\mu^{\text{KLOE } 2\pi} \lesssim 0.4\%$
- This requires significant involvement from theoretical groups
  - improvement of MC(s) to better describe **ISR and FSR** (Phokhara,…)
  - main aim is NNLO+ (+resummation) for ISR and improvement/consistent FF treatment for FSR, e.g.



  - other MC groups are also concentrating on $e^+e^- \to \pi^+\pi^-$, $\mu^+\mu^-$, e^+e^- at higher order
  - International Working Group RadioMonteCarLow 2 has already published their phase 1 ➠

RadioMonteCarLow 2

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Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

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(RadioMonteCarLow 2 working group)

Abstract

We present the results of Phase I of an ongoing review of Monte Carlo tools relevant for low-energy hadronic cross sections. This includes a detailed comparison of Monte Carlo codes for electron–positron scattering into a muon pair, pion pair, and electron pair, for scan and radiative-return experiments. After discussing the various approaches that are used and effects that are included, we show differential cross sections obtained with AfkQed, BabaYaga@NLO, KKMC, MCGPJ, McMule, Phokhara, and Sherpa, for scenarios that are inspired by experiments providing input for the dispersive evaluation of the hadronic vacuum polarisation.

For more on RMCL2, see talk tomorrow's talks by Andrea Gurgone and Marco Ghilardi

Outlook / Conclusions

- The still unresolved **g-2 puzzles** have triggered a lot of experimental & theory activities, including experiments, the Muon g-2 Theory Initiative & lattice

- Progress in **lattice** methods and simulations has been impressive

- The **final FNAL result** will define the field for years to come
- … but to fully exploit it, much more effort will be needed to improve the SM prediction

- There is a hard but clear pathway for this, from theory, MC, via experimental analyses and completely new approaches including **MUonE**

- In the longer future, **J-PARC** can provide a completely independent measurement at low muon momentum

- Together with other lepton moment measurements (including the **muon EDM**) and **lepton flavour violation** searches, g-2 continues to be a testbed for new methods, and new physics.

- My view: the puzzles are very much \`**WIP**',  and g-2 is not \`**RIP**'

\`The closer you look the more there is to see' (Fred Jegerlehner)