Status of MUonE theory

F. Piccinini

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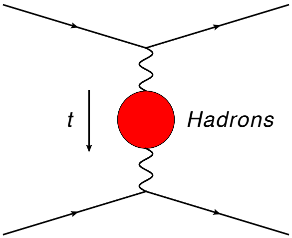
- ⋆ G. Abbiendi, C.M. Carloni Calame, U. Marconi, C. Matteuzzi, G. Montagna, O. Nicrosini, M. Passera, F. Piccinini, R. Tenchini, L. Trentadue, G. Venanzoni,
  Measuring the leading hadronic contribution to the muon g-2 via μe *scattering*
  Eur. Phys. J. C **77** (2017) no.3, 139 - arXiv:1609.08987 [hep-ph]
- ⋆ C. M. Carloni Calame, M. Passera, L. Trentadue and G. Venanzoni,
  A new approach to evaluate the leading hadronic corrections to the muon g-2
  Phys. Lett. B **746** (2015) 325 - arXiv:1504.02228 [hep-ph]

Master formula

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



⇝ The hadronic VP correction to the running of α enters

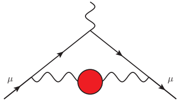
- ⋆ **$\Delta\alpha_{\rm had}(t)$ can be directly measured in a (single) experiment involving a space-like scattering process and $a_\mu^{\rm HLO}$ obtained through numerical integration**

  Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

- ⋆ **A data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

Kernel functions for a_μ^{HVP}

- LO: $\frac{\alpha}{\pi}(1-x)$

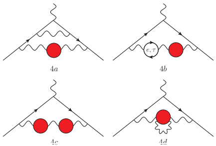


- NLO

  E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462

  A.V. Nesterenko, J. Phys. G49 (2022) 5, 055001;

  J. Phys. G50 (2022) 2, 029401



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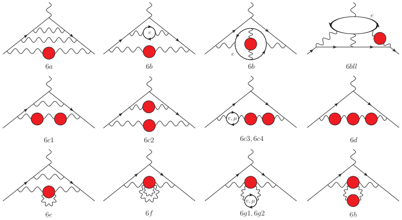
  E. Balzani, S. Laporta, M. Passera, Phys. Lett. B834 (2022) 137462



Main challenge: precision on shapes of differential distributions at the 10ppm level

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Main sources of systematics on the theory side

- **Radiative corrections to the Signal, the elastic process $\mu e \to \mu e$**

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- **Predictions for Background processes**

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Main sources of systematics on the theory side

- **Radiative corrections to the Signal, the elastic process $\mu e \to \mu e$**
- **Predictions for Background processes**

High precision Monte Carlo simulation tools required

Two completely independent fixed order Monte Carlo codes under development

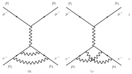
- **Mesmer** Pavia team
  - approximate NNLO calculation at $\mathcal{O}\left(\left[\frac{\alpha}{\pi}\ln\frac{m_\mu^2}{m_e^2}\right]^2\right)$

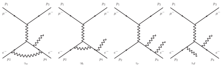
github.com/cm-cc/mesmer

- **McMule** PSI/Bern/Liverpool...
  - more refined approximation to NNLO: only terms of $\mathcal{O}(m_e^2/Q^2)$ neglected

gitlab.com/mule-tools/mcmule

- **exact calculation of corrections along one lepton line with all finite mass effects**





  - two independent calculations, with different IR singularities handling procedures (slicing (**Mesmer**) and subtraction **McMule**))

Carloni Calame et al., JHEP 11 (2020) 028,

P. Banerjee, T. Engel, A. Signer, Y. Ulrich, SciPost Phys. 9 (2020) 027

  - implemented in **Mesmer** and **McMule**, perfect numerical agreement

approximate *photonic* radiative corrections at **NNLO** in **`Mesmer`**

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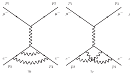

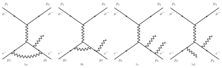

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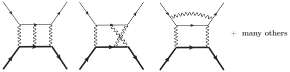
  - implemented in **`Mesmer`** and **`McMule`**, perfect numerical agreement

- **NNLO with finite mass effects and approximate up-down interference in `Mesmer`**
  - interference of LO $\mu e \to \mu e$ amplitude with



⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism, starting from the exact one-loop, to catch the IR divergent structure

- **NNLO calculation neglecting terms of $\mathcal{O}(m_e^2/Q^2)$ in `McMule`**

A. Broggio et al., JHEP 01 (2023) 112

photonic radiative corrections at **NNLO** in `McMule`

- **NNLO calculation neglecting terms of $\mathcal{O}(m_e^2/Q^2)$ in `McMule`**

A. Broggio et al., JHEP 01 (2023) 112

- this is based on several achievements
  - **complete calculation of the amplitude $f^+f^- \to F^+F^-$ with $m_f = 0$, $m_F \neq 0$** R. Bonciani *et al.*, PRL 128 (2022)
  - **"massification" to recover the leading m_e terms**, i.e. neglecting powers of m_e^2/Q^2

T. Engel, C. Gnendiger, A. Signer and Y. Ulrich, JHEP 02 (2019) 118

Y. Ulrich, *PoS* RADCOR2023 (2024) 077

  - **FKS$^\ell$ subtraction scheme**

T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

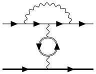
  - **Next-to-soft stabilisation**, to obtain numerical stability in real-virtual corrections with soft and/or collinear photon configurations

T. Engel, A. Signer, Y. Ulrich, JHEP 04 (2022) 097; T. Engel, JHEP 07 (2023) 177

NNLO virtual leptonic and hadronic pairs (vacuum polarization insertion)

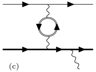
- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes

(a) (b)



(a)

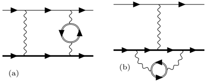
$+ \quad \cdots$



(c)

$+ \quad \cdots$

- interfered with $\mu e \to \mu e$ amplitude



(a) (b)

$+ \quad \cdots$

- 2-loop integral evaluated with dispersion relation techniques in **`Mesmer`**

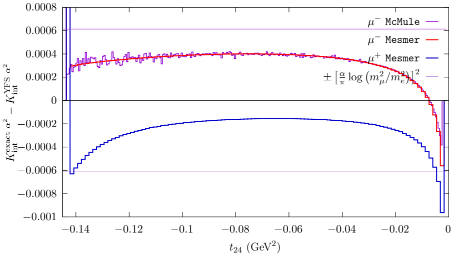
used e.g. in the past for Bhabha: Actis et al., Phys. Rev. Lett. 100 (2008) 131602; Carloni Calame et al., JHEP 07 (2011) 126

$$\frac{g_{\mu\nu}}{q^2 + i\epsilon} \to g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{R_\ell(z)}{q^2 - z + i\epsilon} = g_{\mu\nu} \frac{\alpha}{3\pi} \int_{4m_\ell^2}^{\infty} \frac{dz}{z} \frac{1}{q^2 - z + i\epsilon} \left(1 + \frac{4m_\ell^2}{2z}\right) \sqrt{1 - \frac{4m_\ell^2}{z}}$$

  - also independent investigation for hadronic corrections to μe scattering — M. Fael and M. Passera, Phys. Rev. Lett. 2019 19, 192001

- 2-loop integral evaluated (also) with hyperspherical method in **`McMule`** — M. Fael, JHEP02 (2019) 027

On the phenomenological side



- agreement (for the up-down (photonic) interference) within the estimated uncertainties

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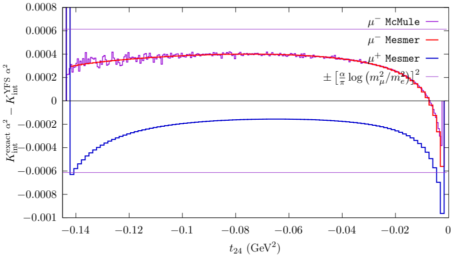
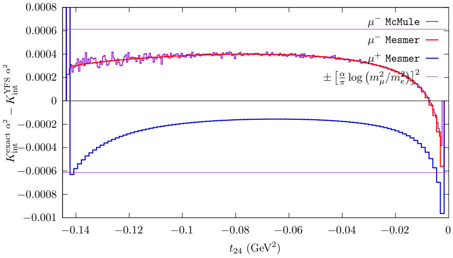


- agreement (for the up-down (photonic) interference) within the estimated uncertainties
- **NNLO corrections at the $10^{-4} - 10^{-3}$ level**

- **eventually fixed order calculations need to be matched to resummation of higher order corrections, through PS techniques (e.g. BaBayaga) or YFS techniques (e.g. KKMC/SHERPA)**

On the phenomenological side



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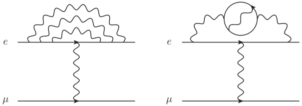
- **also next perturbative order should be estimated**

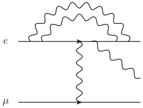
Towards N^{3}LO on the electron line

Y. Ulrich, N^{3}LO kick-off workstop/thinkstart, Durham, 3-5 August 2022

- All virtual (three loops)



- Single real emission (two loops)



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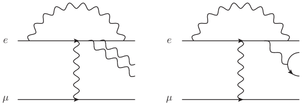
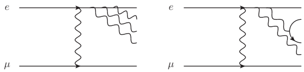


- Triple real



M. Fael, MUonE Collaboration Meeting, 16/05/2023, CERN

- **this contribution will allow improved perturbative predictions and more reliable theoretical uncertainty estimates**

Recent progress

- **the three-loop form factor with finite fermion mass is now available**

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev. Lett 128 (2022) 172003

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev.D 106 (2022) 034029

M. Fael, F. Lange, K. Schönwald, M. Steinhauser, Phys. Rev.D 107 (2023) 094017

- **all order subtraction scheme FKS$^\ell$ availale**

T. Engel, A. Signer, Y. Ulrich, JHEP 01 (2020) 085

- **generalisation of the LBK theorem to multi-photon emission** $\Longrightarrow$ extension of next-to-soft stabilisation to multiple radiation

T. Engel, JHEP 03 (2024) 004

- **real-virtual-virtual corrections recently recalculated with** $m_e \to 0$

S. Badger, J. Krys, R. Moodle, S. Zoia, JHEP 11 (2023) 041

V.S. Fadin, R.N. Lee, JHEP 11 (2023) 148

- **helicity amplitudes in massless QED to higher orders in** ϵ

T. Dave, W.J. Bobadilla, Phys. Rev. D111 (2025) 036024

Fixed target experiment $\Longrightarrow$ bound electron effects

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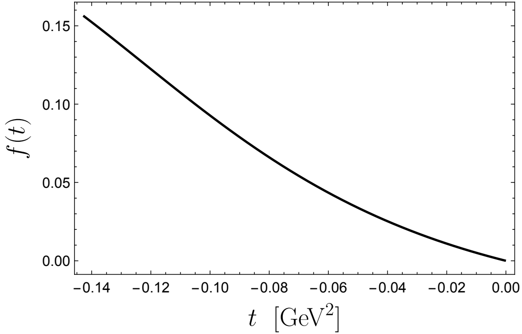
- **estimated**

R. Plestid and M.B. Wise, Phys.Rev.D 110 (2024), 056032



- for C

$$\frac{1}{\sigma}\frac{d\sigma}{dt} = \frac{1}{\sigma^0}\frac{d\sigma^0}{dt}\left(1 - K f(t)\right)$$

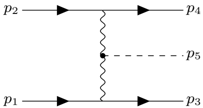
  - $K = 4.5 \cdot 10^{-4}$, scaling as $1/Z_A$

- preliminary investigations of the interactions between outgoing electrons and the residual charged debris in the final state

R. Plestid and M.B. Wise, Phys. Rev. D110 (2024), 113007

Backgrounds

- **pion pair production forbidden** kinematically with the available $\sqrt{s}$
- **single π^0 production possible**



- **π^0 production** calculated and shown to be **well below 10^{-5}** w.r.t. $\mu e \to \mu e$

E. Budassi et al., PLB 829 (2022) 137138

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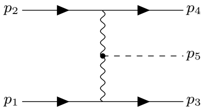

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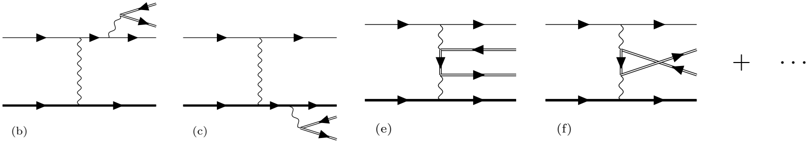
E. Budassi et al., PLB 829 (2022) 137138

- **lepton pair production**
  - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
  - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$
  - they can mimic the signal if one or two leptons are missed (or overlap)

$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ (same pert. order as virtual pairs)

E. Budassi et al., JHEP 11 (2021) 098

- it also contributes at NNLO accuracy w.r.t. $\mu e \to \mu e$



- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large background, because of the presence of "peripheral" diagrams which develop powers of collinear logarithms upon integration

G. Racah, Il Nuovo Cimento 14 (1937) 83-113; L.D. Landau, E.M. Lifschitz, Phys. Z. Sowjetunion 6 (1934) 244; H.J. Bhabha, Proc. Roy. Soc. Lond. A152 (1935) 559;

R.N. Lee, A.A. Lyubyakin, V.A. Smirnov, Phys. Lett. B 848 (2024) 138408

- **$\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$ calculated with finite mass effects and implemented in `Mesmer`**

Real pair emission from scattering on nucleus: $\mu^{\pm} N \to \mu^{\pm} N \ell^{+} \ell^{-}$

G. Abbiendi et al., Phys. Lett B854 (2024) 138720

- **cross section scaling $\sim Z^2$**

- **approximation: scattering on the external nucleus field**

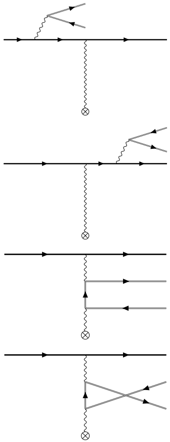
- **finite extension of the nucleus through a form factor**

$$F_Z(q) = \frac{1}{Ze} \int_0^{\infty} dr\, r^2 \rho_Z(r) \frac{\sin(qr)}{qr}$$

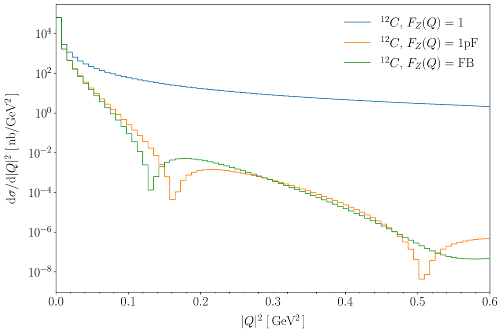
  - q : momentum transferred to the nucleus
  - ρ_Z : nuclear charged density

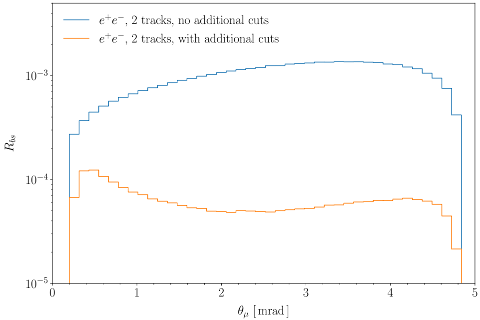
  - **different models for charge density**

J. Heeck, R. Szafron, Y. Uesaka, PRD 105 (2022) 053006



Background/signal ratio





G. Abbiendi, E. Budassi, C.M. Carloni Calame, A. Gurgone, F.P., Phys.Lett.B 854 (2024) 138720

For a safe background subtraction

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- **QED corrections to lepton pair production will be important**

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- **Interaction with an additional nucleus is of the same order, enhanced by Z**

Possible New Physics contamination in the $\Delta\alpha(t)$ determination?

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A. Masiero, P. Paradisi and M. Passera, Phys. Rev. D102 (2020) 075013

P.S.B. Dev, W. Rodejohann, X.-J. Xu and Y. Zhang, JHEP 05 (2020) 053

- Effects of **heavy** $(M_{NP} \gg 1$ GeV$)$ NP mediators investigated through EFT with dim-6 operators
  - excluded (at the 10^{-5} level) by existing data
- Effects of **light** $(M_{NP} \leq 1$ GeV$)$ NP mediators investigated with spin-dependent general models
  - spin-0 NP mediators (ALPs)
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HVP determination with MUonE data will be robust against New Physics

Possible New Physics studies with MUonE (in complementary regions to $\Delta\alpha_h$)

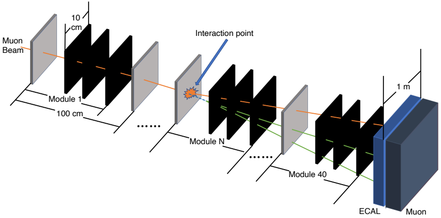
- **interesting proposals for NP searches at MUonE (new light mediators) in $2 \to 3$ processes**
  - invisibly decaying light Z' in $\mu e \to \mu e Z'$

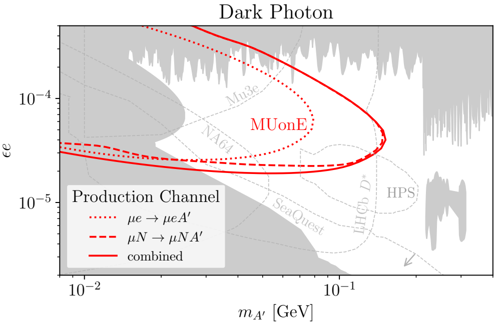
    Asai et al., Phys. Rev. D106 (2022) 5
    - a relevant background can be $\mu e \to \mu e \pi^0$, in addition to $\mu e \to \mu e \gamma$
  - long-lived mediators with displaced vertex signatures $\mu e \to \mu e A' \to \mu e e^+ e^-$

    Galon et al., Phys.Rev.D 107 (2023) 095003; G. Krnjaic, D. Rocha, I.R. Wang, Phys. Rev. Lett. 134 (2025) 161801
  - through scattering off the target nuclei $\mu N \to \mu N X \to \mu N e^+ e^-$

    Grilli di Cortona and E. Nardi, Phys. Rev. D105 (2022) L111701; D. Rocha, I.R. Wang, arXiv:2511.03222

Example of potential reach in dark photon searches





D. Rocha, I.R. Wang, arXiv:2511.03222

Summary

- **Given its precision requirements, MUonE represents a challenge (on the theory side) for**
  - **QED corrections**
  - **background calculation**

- **at present we have two independent Monte Carlo tools, `Mesmer` and `McMule` featuring**
  - NLO QED corrections
  - NNLO QED corections from single lepton legs
  - YFS inspired approximation to the full NNLO QED in **`Mesmer`**
  - full NNLO QED with electron “massification” in **`McMule`**
  - pair production in **`Mesmer`**
    - $\mu^{\pm} e^{-} \to \mu^{\pm} e^{-} \ell^{+} \ell^{-}$
    - $\mu^{\pm} N \to \mu^{\pm} N \ell^{+} \ell^{-}$

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    - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

- **enough to study the pre- LHC LS data**

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  - full NNLO QED with electron “massification” in **`McMule`**
  - pair production in **`Mesmer`**
    - $\mu^{\pm} e^{-} \to \mu^{\pm} e^{-} \ell^{+} \ell^{-}$
    - $\mu^{\pm} N \to \mu^{\pm} N \ell^{+} \ell^{-}$

- **enough to study the pre- LHC LS data**

- **ongoing efforts towards N^{3}LO and resummed multiphoton effects**

Summary

- **Given its precision requirements, MUonE represents a challenge (on the theory side) for**
  - **QED corrections**
  - **background calculation**

- **at present we have two independent Monte Carlo tools, `Mesmer` and `McMule` featuring**
  - NLO QED corrections
  - NNLO QED corections from single lepton legs
  - YFS inspired approximation to the full NNLO QED in **`Mesmer`**
  - full NNLO QED with electron "massification" in **`McMule`**
  - pair production in **`Mesmer`**
    - $\mu^\pm e^- \to \mu^\pm e^- \ell^+ \ell^-$
    - $\mu^\pm N \to \mu^\pm N \ell^+ \ell^-$

- **enough to study the pre- LHC LS data**
- **ongoing efforts towards N^{3}LO and resummed multiphoton effects**
- **worth to study the potential for BSM searches**

THANK YOU!