

Theory review of Lepton Flavour Violation

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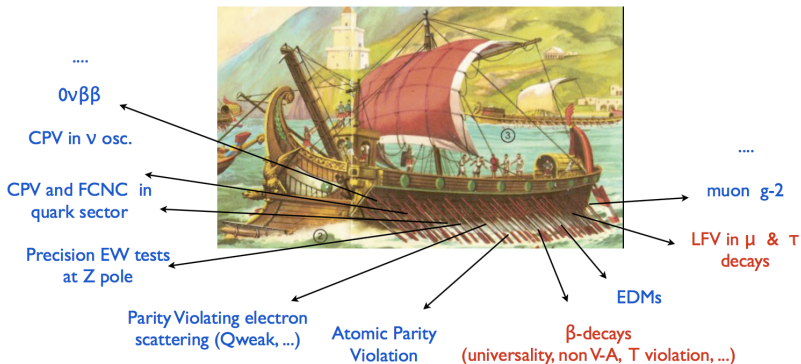
University of Padova and INFN

Workshop on Muon Precision Physics,
Liverpool, 13th November 2025

- 1 **Strategies to look for New Physics at low-energy**
- 2 **Current status of LFV**
- 3 **LFV from heavy NP**
- 4 **EDMs, $g-2$ and LFV interrelationship**
- 5 **LFV from light NP**
- 6 **LFV @ FCC-ee & Muon Collider (MuC)**
- 7 **Conclusions and future prospects**

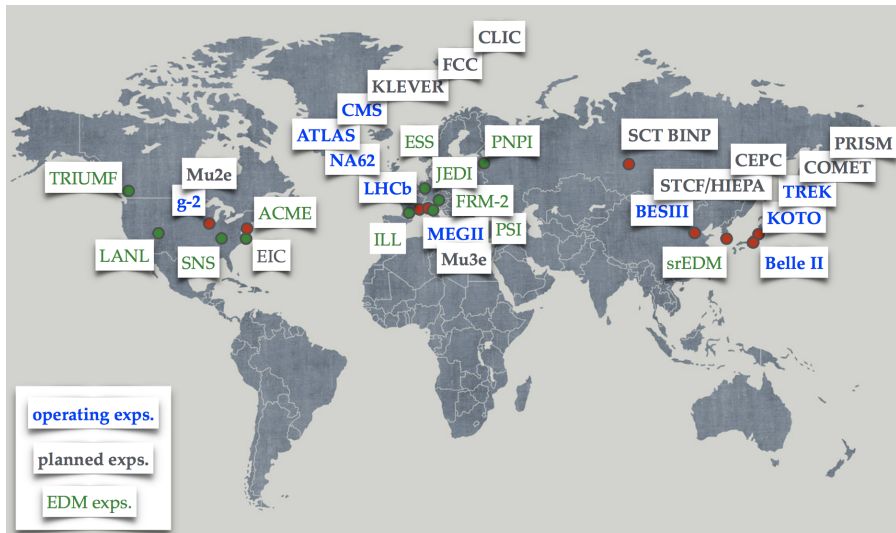
Where to look for New Physics at low-energy?

- Processes very **suppressed** or even **forbidden** in the SM
- Processes predicted with **high precision** in the SM



High-intensity frontier: A collective effort to determine the NP dynamics

Experimental status



Process	Present	Experiment	Future	Experiment
$\mu \rightarrow e\gamma$	1.5×10^{-13}	MEG	$\approx 6 \times 10^{-14}$	MEG II
$\mu \rightarrow 3e$	1.0×10^{-12}	SINDRUM	$\approx 10^{-16}$	Mu3e
$\mu^- \text{ Au} \rightarrow e^- \text{ Au}$	7.0×10^{-13}	SINDRUM II	?	
$\mu^- \text{ Ti} \rightarrow e^- \text{ Ti}$	4.3×10^{-12}	SINDRUM II	?	
$\mu^- \text{ Al} \rightarrow e^- \text{ Al}$	—		$\approx 10^{-16}$	COMET, MU2e
$\tau \rightarrow e\gamma$	3.3×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \rightarrow \mu\gamma$	4.4×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \rightarrow 3e$	2.7×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$\tau \rightarrow 3\mu$	2.1×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$d_e(\text{e cm})$	1.1×10^{-29}	ACME	$\sim 3 \times 10^{-31}$	ACME III
$d_\mu(\text{e cm})$	1.8×10^{-19}	Muon (g-2)	$\sim 10^{-22}$	PSI

Table: Present and future experimental sensitivities for relevant low-energy observables.

- So far, only upper bounds. Still excellent prospects for exp. improvements.
- We can expect a NP signal in all above observables below the current bounds.

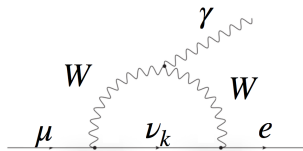
- GIM mechanism very effective in LFV transitions
- amplitude proportional to $A(\mu \rightarrow e\gamma) \propto m_\nu^2$

Very small !!!

$$\text{BR}(\mu \rightarrow e\gamma) \simeq \frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U_{ek}^* m_{\nu_k}^2}{M_W^2} \right|^2.$$

$$\text{BR}(\mu \rightarrow e\gamma) = 10^{-55} \div 10^{-54}$$

- similar suppressions for $\mu \rightarrow 3e, \tau \rightarrow 3\mu, \mu \rightarrow e, \dots$



Why flavor violation is visible in neutrino oscillation while it's not in charged LFV?
 The uncertainty principle sets the oscillation time for $\mu \rightarrow e\gamma$ to be $t \sim h/M_W!$

Message: Any evidence for LFV would be an unambiguous signal of NP!

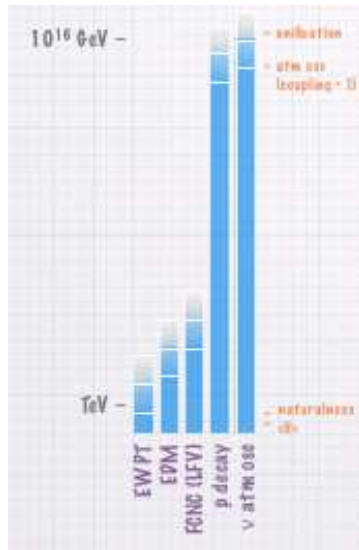
Why do we need New Physics (NP)?

- **Gravity** $\Rightarrow \Lambda_{\text{Planck}} \sim 10^{18-19} \text{ GeV}$
- **Neutrino masses** $\Rightarrow \Lambda_{\text{see-saw}} \lesssim 10^{15} \text{ GeV}$
- **BAU**: evidence of CPV beyond SM
 - ▶ Electroweak Baryogenesis $\Rightarrow \Lambda_{\text{NP}} \lesssim \text{TeV}$
 - ▶ Leptogenesis $\Rightarrow \Lambda_{\text{see-saw}} \lesssim 10^{15} \text{ GeV}$
- **Dark Matter (WIMP)** $\Rightarrow \Lambda_{\text{NP}} \lesssim \text{TeV}$
- **Hierarchy problem**: $\Rightarrow \Lambda_{\text{NP}} \lesssim \text{TeV}$

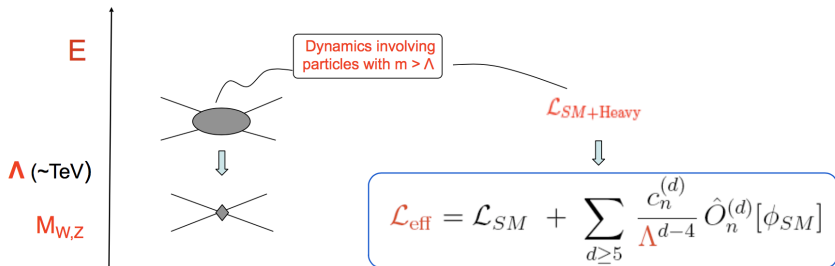
SM = effective theory at the EW scale

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{d \geq 5} \frac{C_{ij}^{(d)}}{\Lambda_{\text{NP}}^{d-4}} O_{ij}^{(d)}$$

- $\mathcal{L}_{\text{eff}}^{d=5} = \frac{y_{\nu}^{ij}}{\Lambda_{\text{see-saw}}} L_i L_j \phi \phi,$
- $\mathcal{L}_{\text{eff}}^{d=6}$ generates FCNC operators



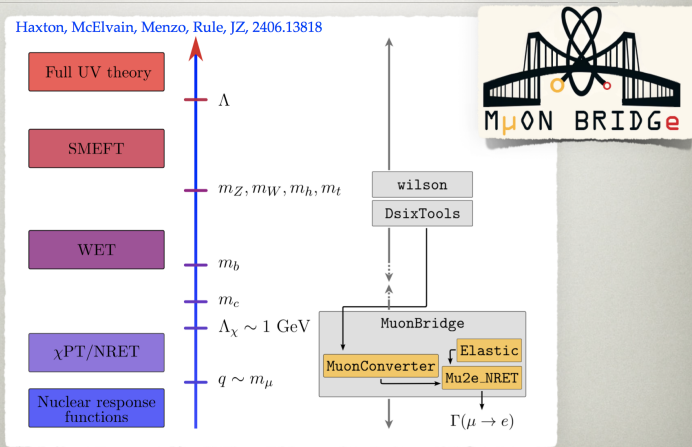
- Dynamics below the scale Λ [$\sim$ mass of new particles] is described by $\mathcal{L}_{\text{eff}}$



- $\mathcal{L}_{\text{eff}}$ is built out of relevant low-energy degrees of freedom (SM fields)
 - $\mathcal{L}_{\text{eff}}$ respects the SM gauge symmetries $G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
 - $\mathcal{L}_{\text{eff}}$ is organized in inverse powers of Λ (amplitudes suppressed by powers of E/Λ)
- Experiments at the precision frontier probe energy scale Λ and symmetries of the new interactions (coeff. & structure of $\hat{O}_n^{(d)}$)

TOWER OF EFTs ⇒ MUONBRIDGE CODE

Haxton, McElvain, Menzo, Rule, JZ, 2406.13818

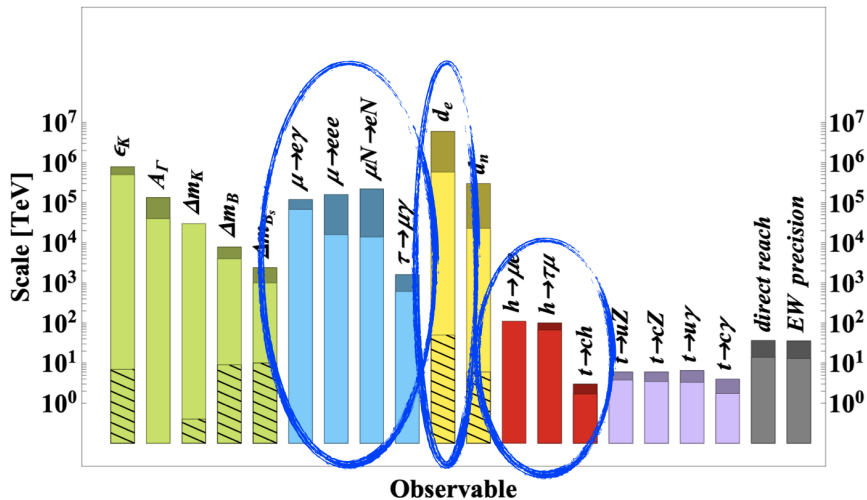


4-leptons operators		Dipole operators	
$Q_{\ell\ell}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L)$	Q_{eW}	$(\bar{L}_L \sigma^{\mu\nu} e_R) \tau_I \Phi W_{\mu\nu}^I$
Q_{ee}	$(\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$	Q_{eB}	$(\bar{L}_L \sigma^{\mu\nu} e_R) \Phi B_{\mu\nu}$
$Q_{\ell e}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{e}_R \gamma^\mu e_R)$		
2-lepton 2-quark operators			
$Q_{\ell q}^{(1)}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{Q}_L \gamma^\mu Q_L)$	$Q_{\ell u}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{u}_R \gamma^\mu u_R)$
$Q_{\ell q}^{(3)}$	$(\bar{L}_L \gamma_\mu \tau_I L_L)(\bar{Q}_L \gamma^\mu \tau_I Q_L)$	Q_{eu}	$(\bar{e}_R \gamma_\mu e_R)(\bar{u}_R \gamma^\mu u_R)$
Q_{eq}	$(\bar{e}_R \gamma^\mu e_R)(\bar{Q}_L \gamma_\mu Q_L)$	$Q_{\ell edq}$	$(\bar{L}_L^a e_R)(\bar{d}_R Q_L^a)$
$Q_{\ell d}$	$(\bar{L}_L \gamma_\mu L_L)(\bar{d}_R \gamma^\mu d_R)$	$Q_{\ell equ}^{(1)}$	$(\bar{L}_L^a e_R) \epsilon_{ab} (\bar{Q}_L^b u_R)$
Q_{ed}	$(\bar{e}_R \gamma_\mu e_R)(\bar{d}_R \gamma^\mu d_R)$	$Q_{\ell equ}^{(3)}$	$(\bar{L}_i^a \sigma_{\mu\nu} e_R) \epsilon_{ab} (\bar{Q}_L^b \sigma^{\mu\nu} u_R)$
Lepton-Higgs operators			
$Q_{\Phi\ell}^{(1)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{L}_L \gamma^\mu L_L)$	$Q_{\Phi\ell}^{(3)}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu^I \Phi)(\bar{L}_L \tau_I \gamma^\mu L_L)$
$Q_{\Phi e}$	$(\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R \gamma^\mu e_R)$	$Q_{e\Phi 3}$	$(\bar{L}_L e_R \Phi)(\Phi^\dagger \Phi)$

$$\mu \rightarrow e\gamma$$

$$\mu \rightarrow 3e$$

$$\mu \rightarrow e$$



[Physics Briefing Book, 1910.11775]

- **LFV operators @ dim-6**

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda_{\text{LFV}}^2} \mathcal{O}^{\text{dim-6}} + \dots$$

$$\mathcal{O}^{\text{dim-6}} \ni \bar{\mu}_R \sigma^{\mu\nu} H e_L F_{\mu\nu}, (\bar{\mu}_L \gamma^\mu e_L) (\bar{f}_L \gamma^\mu f_L), (\bar{\mu}_R e_L) (\bar{f}_R f_L), f = e, u, d$$

- $\ell \rightarrow \ell' \gamma$ probe ONLY the dipole-operator (at tree level)
- $\ell_i \rightarrow \ell_j \bar{\ell}_k \ell_k$ and $\mu \rightarrow e$ in Nuclei probe dipole and 4-fermion operators
- When the dipole-operator is dominant:

$$\text{BR}(\ell_i \rightarrow \ell_j \bar{\ell}_k \ell_k) \approx \alpha \times \text{BR}(\ell_i \rightarrow \ell_j \gamma)$$

$$\text{CR}(\mu \rightarrow e \text{ in N}) \approx \alpha \times \text{BR}(\mu \rightarrow e \gamma)$$

$$\frac{\text{BR}(\mu \rightarrow 3e)}{3 \times 10^{-15}} \approx \frac{\text{BR}(\mu \rightarrow e \gamma)}{5 \times 10^{-13}} \approx \frac{\text{CR}(\mu \rightarrow e \text{ in N})}{3 \times 10^{-15}}$$

- Ratios like $\text{Br}(\mu \rightarrow e \gamma) / \text{Br}(\tau \rightarrow \mu \gamma)$ probe the NP flavor structure
- Ratios like $\text{Br}(\mu \rightarrow e \gamma) / \text{Br}(\mu \rightarrow eee)$ probe the NP operator at work

Polarising the muon to distinguish operators

Kuno, Okada hep-ph/9909265

$$\frac{dB(\mu \rightarrow e\gamma)}{d(\cos\theta_e)} = 192\pi^2 \left(\frac{v}{\Lambda}\right)^4 \left[|C_{D,R}|^2 (1 - P_\mu \cos\theta_e) + |C_{D,L}|^2 (1 + P_\mu \cos\theta_e) \right]$$

Muon polarization vector

Angle between e momentum and $\vec{P}_\mu$

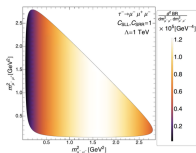
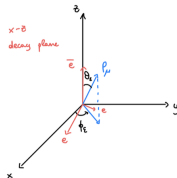


Petcov, Bolton 2204.03468

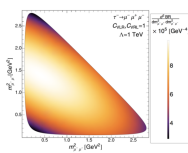
$$\frac{dB_{\mu \rightarrow 3e}}{dx_1 dx_2 d\Omega_e} = \frac{3}{2\pi} [C_1(x_1, x_2) + C_2(x_1, x_2) P_\mu \cos\theta_e + C_3(x_1, x_2) P_\mu \sin\theta_e \cos\phi_e + C_4(x_1, x_2) P_\mu \sin\theta_e \sin\phi_e]$$

Can distinguish $C_{V,LX}, C_{V,LX}, C_{S,R}$ from $C_{V,RX}, C_{V,RX}, C_{S,L}$ but not scalars from vectors
CP asymmetries are also measurable (phase between dipoles and vectors)

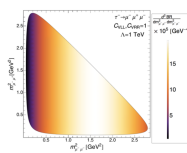
- Dalitz plots for the three body also possible to distinguish operators (vector vs scalar)



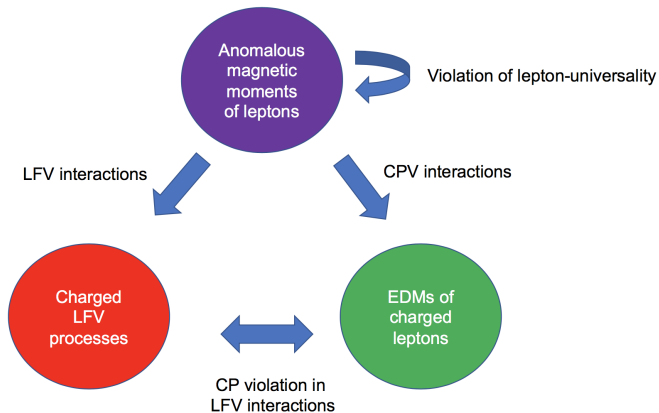
Scalars



Vectors



Probing NP in the leptonic sector



- NP effects are encoded in the effective Lagrangian

$$\mathcal{L} = e \frac{m_\ell}{2} \left(\bar{\ell}_R \sigma_{\mu\nu} \mathbf{A}_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} \mathbf{A}_{\ell\ell'}^* \ell_R \right) F^{\mu\nu} \quad \ell, \ell' = e, \mu, \tau,$$

- ▶ Branching ratios of $\ell \rightarrow \ell' \gamma$

$$\frac{\text{BR}(\ell \rightarrow \ell' \gamma)}{\text{BR}(\ell \rightarrow \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48\pi^3 \alpha}{G_F^2} \left(|\mathbf{A}_{\ell\ell'}|^2 + |\mathbf{A}_{\ell'\ell}|^2 \right).$$

- ▶ Δa_ℓ and leptonic EDMs

$$\Delta a_\ell = 2m_\ell^2 \text{Re}(\mathbf{A}_{\ell\ell}), \quad \frac{d_\ell}{e} = m_\ell \text{Im}(\mathbf{A}_{\ell\ell}).$$

- ▶ “Naive scaling”: a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \quad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}}.$$

- $\text{BR}(\ell_i \rightarrow \ell_j \gamma)$ vs. $(g - 2)_\mu$

$$\text{BR}(\mu \rightarrow e \gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}} \right)^2$$

$$\text{BR}(\tau \rightarrow \mu \gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}} \right)^2$$

- EDMs vs. $(g - 2)_\mu$

$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 10^{-29} \left(\frac{\phi_e^{CPV}}{10^{-5}} \right) e \text{ cm},$$

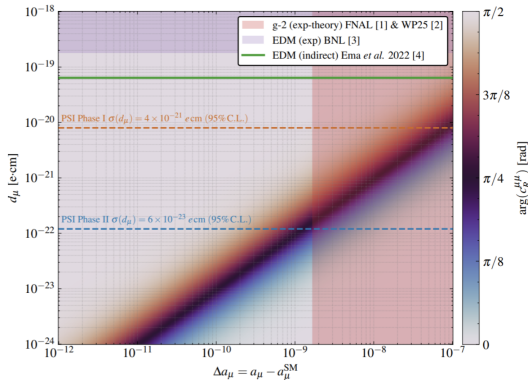
$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22} \phi_\mu^{CPV} e \text{ cm},$$

- Main messages:

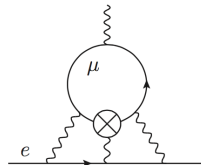
- ▶ $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ requires a nearly flavor and CP conserving NP
- ▶ Large effects in the muon EDM $d_\mu \sim 10^{-22} e \text{ cm}$ are still allowed!

[Giudice, P.P., & Passera, '12]

Experimental status of the muon EDM



[Crivellin, Hoferichter & Schmidt-Wellenburg, '18]



$$d_\mu \leq 10^{-21} \text{ e cm} \left(\frac{d_e}{10^{-31} \text{ e cm}} \right)$$

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}} \right) 2 \times 10^{-22} \phi_\mu^{CPV} \text{ e cm},$$

[Giudice, PP & Passera, '12]

Many Options, Much Fun?

How do I make sense of the landscape of singlet options?

EFT: $\mathcal{L}_{\text{portal}} \supset \frac{c_n}{\Lambda^{\Delta_{\text{dark}} + \Delta_{\text{SM}} - 4}} \mathcal{O}_{\text{dark}} \mathcal{O}_{\text{SM}}$

$\uparrow$ Any gauge invariant combination of SM fields

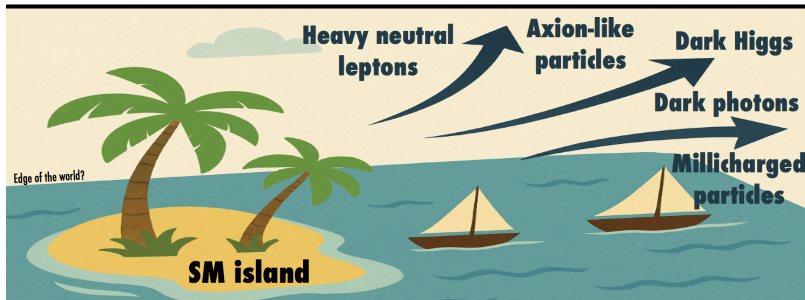
Lower dimensional portals

$$\mathcal{O}_{\text{dark}} = \{\phi, N, V_\mu, \dots\}$$

Scalar, Fermion, Vector



Largest rates

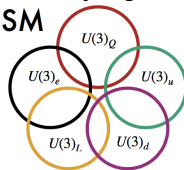


Flavoured Axion-like Portal

Accidental symmetries of the SM might be broken by light new particles feebly coupled to the SM

Example: Flavour dependent Peccei-Quinn charges

[Calibbi et al 1612.08040]
[Ema et al 1612.05492]



$$\mathcal{L} \supset \underbrace{\sum_i \frac{\partial_\mu a}{2f_a} \bar{f}_i C_{ii}^A \gamma_\mu \gamma_5 f_i}_{\text{Flavour conserving}} + \underbrace{\sum_{i \neq j} \frac{\partial_\mu a}{2f_a} \bar{f}_i \gamma^\mu \left(C_{ij}^V + C_{ij}^A \gamma_5 \right) f_j}_{\text{Flavour violating}} \quad [\text{Feng et al hep-ph/9709411}]$$

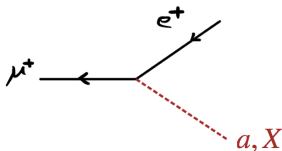
! Hierarchy between **flavour-conserving** and **flavour-violating** depends on the UV theory

Flavour anarchy: $C_{ij}^{A,V}(\Lambda_{UV}) \sim \mathcal{O}(1)$

Minimal flavour violation: $C_{ij}^{A,V}(\Lambda_{UV}) = 0$

What New Physics?

Lepton-flavour Violating

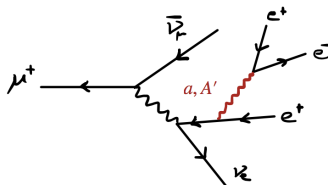


$$\mathcal{L}_{\text{FV-a}} = \frac{m_\mu}{2f_a} \frac{1}{|C_{e\mu}|} a \bar{\mu} (C_{e\mu}^V + C_{e\mu}^A \gamma_5) e$$

Bump hunt in electron momentum p_e

[Bayes et al (TWIST Collaboration) 1411.1770]
[Perrevoort et al (Mu3e Collaboration) 1812.00741]

Lepton-flavour Conserving



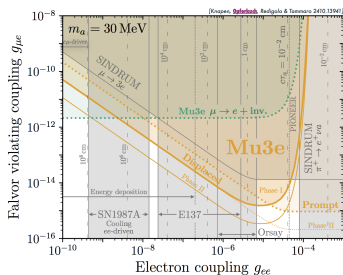
$$\mathcal{L}_0 \supset X \left[\bar{\mu} (g_S^\mu + g_P^\mu \gamma_5) \mu + \bar{e} (g_S^e + g_P^e \gamma_5) e \right]$$

$$\mathcal{L}_1 \supset X^\alpha \left[\bar{\mu} \gamma_\alpha (g_V^\mu + g_A^\mu \gamma_5) \mu + \bar{e} \gamma_\alpha (g_V^e + g_A^e \gamma_5) e \right]$$

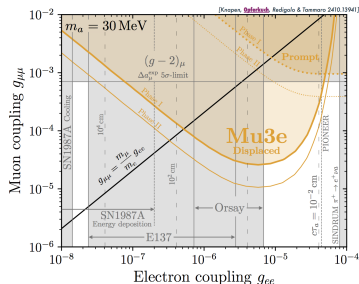
Bump hunt in e^+e^- -pair invariant mass

[Echenard et al 1411.1770]
[Perrevoort et al (Mu3e Collaboration) 1812.00741]

Lepton Flavour Violating ALP

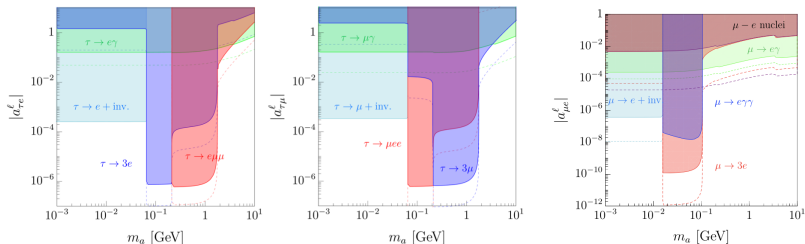


Lepton Flavour Conserving ALP

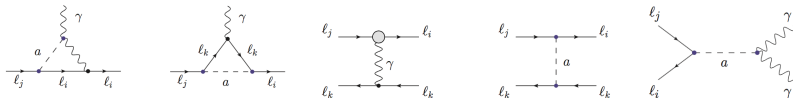


LFV from light NP: axionlike particles

$$\mathcal{L}_{\text{eff}}^{\text{d}\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2 a^2}{2} + e^2 c_{\gamma\gamma} \frac{a}{\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu} + g_s^2 c_{gg} \frac{a}{\Lambda} G_{\mu\nu} \tilde{G}^{\mu\nu} - \frac{\partial_\mu a}{\Lambda} \sum_{f,i,j} \bar{f}_i \gamma^\mu (v_{ij}^f - a_{ij}^f \gamma_5) f_j$$



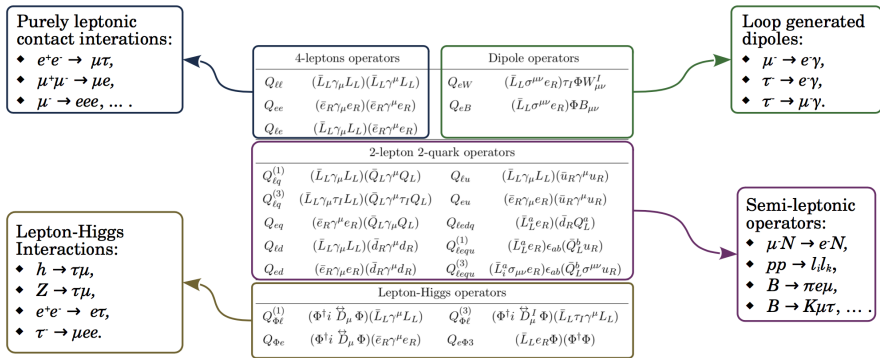
[Cornella, P.P. & Sumensari, '19.]



$$\mathcal{B}(\ell_i \rightarrow \ell_j \gamma \gamma) \approx \mathcal{B}(\ell_i \rightarrow \ell_j a) \times \mathcal{B}(a \rightarrow \gamma \gamma) \Rightarrow \text{BR}(\ell_i \rightarrow \ell_j \ell_k \bar{\ell}_k) \not\propto \alpha \times \text{BR}(\ell_i \rightarrow \ell_j \gamma)$$

Message: correlations among LFV signals discriminate heavy vs. light NP!

Low-energy vs high-energy LFV



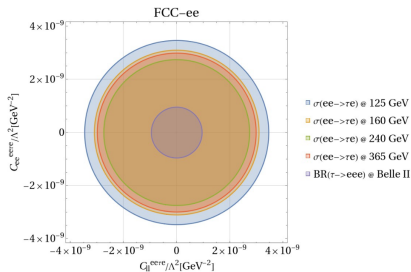
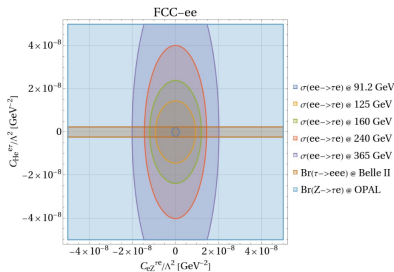
Energy dependence of LFV signals

$$\Gamma(\tau \rightarrow 3e) \sim \frac{m_\tau^5}{\Lambda^4}$$

$$\Gamma(Z \rightarrow \tau e) \sim \frac{m_Z^5}{\Lambda^4}$$

$$\sigma(ee \rightarrow \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

- # of Z events @ FCC-ee: $N_Z(e^+e^- \rightarrow Z) \sim 10^{11}$
- # of Z events @ MuC: $N_Z(\mu^+\mu^- \rightarrow \bar{\nu}\nu Z) \sim 10^{11} \left(\frac{\sqrt{s}}{30 \text{ TeV}} \right)^4$



$$(\bar{L}_L \sigma^{\mu\nu} e_R) \Phi Z_{\mu\nu} \\ \text{vs} \\ (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi) (\bar{e}_R \gamma^\mu e_R)$$

FCC-ee constraints on 2-dimensional parameter space:

- LFV Z couplings better probed by FCC-ee than BelleII,
- LFV 4f operators better probed by Belle II than FCC-ee.

$$(\bar{L}_L \gamma_\mu L_L) (\bar{L}_L \gamma^\mu L_L) \\ \text{vs} \\ (\bar{e}_R \gamma_\mu e_R) (\bar{e}_R \gamma^\mu e_R)$$

Energy dependence of LFV signals

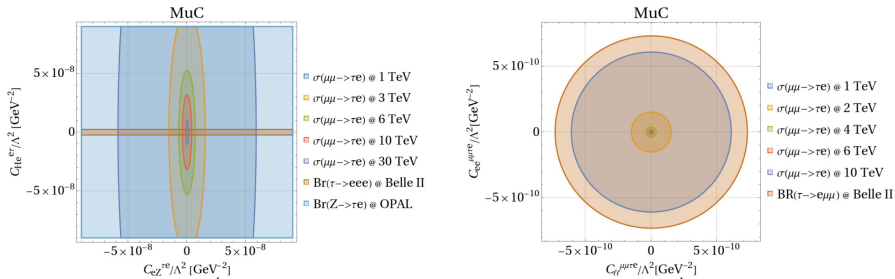
$$\Gamma(\tau \rightarrow 3e) \sim \frac{m_\tau^5}{\Lambda^4}$$

$$\Gamma(Z \rightarrow \tau e) \sim \frac{m_Z^5}{\Lambda^4}$$

$$\sigma(ee \rightarrow \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

Message: The high luminosity FCC-ee is competitive with Belle II to probe LFV

[Bartocci, Pagani, PP & Scantamburlo, to appear. See also Calibbi et al., '21 & Altmannshofer et al. '23]



$$(\bar{L}_L \sigma^{\mu\nu} e_R) \Phi Z_{\mu\nu} \\ \text{vs} \\ (\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi)(\bar{e}_R \gamma^\mu e_R)$$

MuC constraints on 2-dimensional parameter space:

- LFV Z couplings probed by both FCC-ee and BelleII,
- LFV 4f operators better probed by MuC than Belle II.

$$(\bar{L}_L \gamma_\mu L_L)(\bar{L}_L \gamma^\mu L_L) \\ \text{vs} \\ (\bar{e}_R \gamma_\mu e_R)(\bar{e}_R \gamma^\mu e_R)$$

Energy dependence of LFV signals

$$\Gamma(\tau \rightarrow 3e) \sim \frac{m_\tau^5}{\Lambda^4}$$

$$\Gamma(Z \rightarrow \tau e) \sim \frac{m_Z^5}{\Lambda^4}$$

$$\sigma(\mu\mu \rightarrow \tau e) \sim \frac{E_{CM}^2}{\Lambda^4}$$

Message: Energy helps accuracy!

[Bartocci, Pagani, PP & Scantamburlo, to appear.]

- **Important questions in view of ongoing/future experiments are:**
 - ▶ What are the expected deviations from the SM predictions induced by TeV NP?
 - ▶ Which observables are not limited by theoretical uncertainties?
 - ▶ In which case we can expect a substantial improvement on the experimental side?
 - ▶ What will the measurements teach us if deviations from the SM are [not] seen?
- **(Personal) answers:**
 - ▶ We can expect any deviation from the SM expectations below the current bounds.
 - ▶ LFV processes and leptonic EDMs do not suffer from theoretical limitations and there are still excellent prospects for experimental improvements.
 - ▶ FCC-ee and a high-energy Muon Collider are complementary with each other and also with Belle II and LHCb to probe LFV.

Message: an exciting program is in progress at the Intensity Frontier which would greatly benefit from the next high-energy frontier program (FCC & MuC)!