

Status of radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

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on behalf of the RMCL2 Working Group

Liverpool Workshop on Muon Precision Physics 2025

Liverpool, 12 November 2025



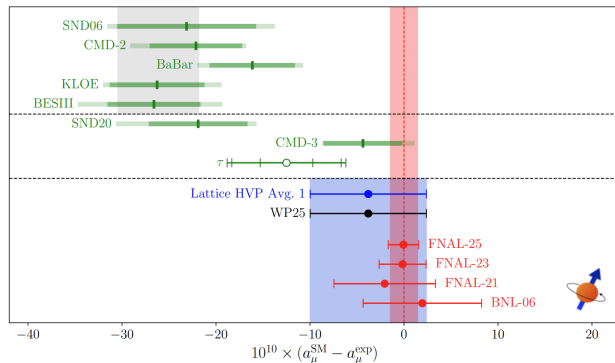
- **RadioMonteCarLow 2** is a community effort focused on Monte Carlo tools and radiative corrections for e^+e^- collisions at low energies ($\sqrt{s} < \text{few GeV}$)
- Collect, make accessible, keep alive, assess, and further improve the MC codes
↪ A **living repository** of MC generators and benchmark results
- Close collaboration between theorists and experimental collaborations
↪ BESIII, CMD-3, KLOE ...
- 7 codes: AFKQED, BABAYAGA@NLO, KKMC, MCGPJ, McMULE, PHOKHARA, SHERPA
- 3+3 processes (both for energy scan and radiative return):
 - $e^+e^- \rightarrow e^+e^-(\gamma)$
 - $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$
 - $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$



radiomontecarlow2.gitlab.io

The motivation: muon $g - 2$ puzzle

- MC generators and radiative corrections have a crucial role in the measurement of the $e^+e^- \rightarrow \text{hadrons}$ cross section
 $\hookrightarrow$ HVP contribution to $(g - 2)_\mu$
- Other precision measurements are expected to join the game, not only at flavour factories (e.g. ℓp scattering)
- Large overlap (people+physics) with MUonE theory effort (μe scattering)
 $\hookrightarrow$ See Fulvio's talk this afternoon



Quest for precision in hadronic cross sections at low energy: Monte Carlo tools vs. experimental data

Working Group on Radiative Corrections and Monte Carlo Generators for Low Energies

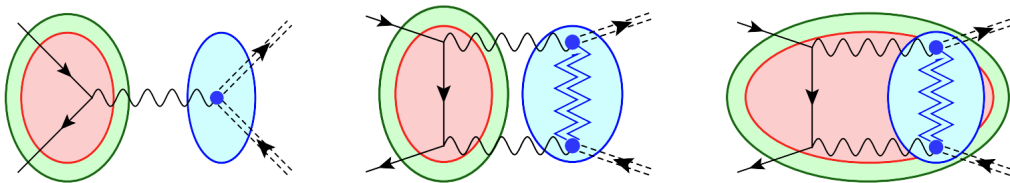
S. Actis³⁸, A. Arbuzov^{9,e}, G. Balossini^{32,33}, P. Beltrame¹³, C. Bignamini^{32,33}, R. Bonciani¹⁵, C.M. Carloni Calame³⁵, V. Cherepanov^{25,26}, M. Czakon¹, H. Czyż^{19,a,f,i}, A. Denig²², S. Eidelman^{25,26,g}, G.V. Fedotovitch^{25,26,e}, A. Ferroglia²³, J. Gluza¹⁹, A. Grzelińska⁸, M. Gunia¹⁹, A. Hafner²², F. Ignatov²⁵, S. Jadach⁸, F. Jegerlehner^{3,19,41}, A. Kalinowski²⁹, W. Kluge¹⁷, A. Korchin²⁰, J.H. Kühn¹⁸, E.A. Kuraev⁹, P. Lukin²⁵, P. Mastrolia¹⁴, G. Montagna^{32,33,b,d}, S.E. Müller^{22,f}, F. Nguyen^{34,d}, O. Nicrosini³³, D. Nomura^{36,h}, G. Pakhlova²⁴, G. Pancheri¹¹, M. Passera²⁸, A. Penin¹⁰, F. Piccinini³³, W. Placzek⁷, T. Przedzinski⁶, E. Remiddi^{4,5}, T. Riemann⁴¹, G. Rodrigo³⁷, P. Roig²⁷, O. Shekhovtsova¹¹, C.P. Shen¹⁶, A.L. Sibidanov²⁵, T. Teubner^{21,h}, L. Trentadue^{30,31}, G. Venanzoni^{11,c,i}, J.J. van der Bij¹², P. Wang², B.F.L. Ward³⁹, Z. Was^{8,g}, M. Worek^{40,19}, C.Z. Yuan²

Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions

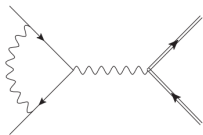
✉ Riccardo Aliberti¹, ✉ Paolo Beltrame², ✉ Ettore Budassi^{3,4},
✉ Carlo M. Carloni Calame⁴, ✉ Gilberto Colangelo⁵, ✉ Lorenzo Cotrozzi²,
✉ Achim Denig¹, ✉ Anna Driutti^{6,7}, ✉ Tim Engel⁸, ✉ Lois Flower^{2,9},
✉ Andrea Gurgone^{3,6,7}, ✉ Martin Hoferichter⁵, ✉ Fedor Ignatov²,
✉ Sophie Kollatzsch^{10,11}, ✉ Bastian Kubis¹², ✉ Andrzej Kupść^{13,14*},
✉ Fabian Lange^{10,11}, ✉ Alberto Lusiani^{7,15}, ✉ Stefan E. Müller¹⁶, ✉ Jérémy Paltrinieri²,
✉ Pau Petit Rosàs², ✉ Fulvio Piccinini⁴, ✉ Alan Price¹⁷, ✉ Lorenzo Punzi^{7,15},
✉ Marco Rocco^{10,18}, ✉ Olga Shekhovtsova^{19,20}, ✉ Andrzej Siódmok¹⁷,
✉ Adrian Signer^{10,11*}, ✉ Giovanni Stagnitto²¹, ✉ Peter Stoffer^{10,11},
✉ Thomas Teubner², ✉ William J. Torres Bobadilla²,
✉ Francesco P. Ucci^{3,4}, ✉ Yannick Ulrich^{2,5*} and ✉ Graziano Venanzoni^{2,7*}
(RadioMonteCarLow 2 working group)

Organisation in Working Packages

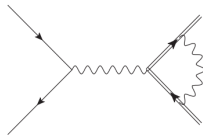
- **WP1 & WP2**: fixed-order massive QED
- **WP3**: hadronic final states (mainly pions)
- **WP3**: all-order QED (resummation)
- **WP5**: experimental inputs (pion form factor and cuts)



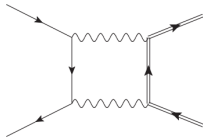
- **LO, NLO, NNLO**: Fixed-order contributions written in powers of α
↪ Each order contains *all* diagrams, both real and virtual
- **ISC, FSC**: Initial-State and Final-State Contributions
- **VPC**: Vacuum Polarisation Contributions, both leptonic and hadronic
- **CS**: Collinear Structures, resummation of collinear logs via structure functions
- **PS**: Parton Shower, exclusive resummation of collinear logs with angular effects
- **YFS**: Yennie-Frautschi-Suura exponentiation, exclusive resummation of soft logs



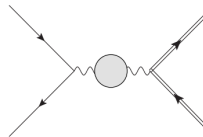
ISC



FSC



mixed



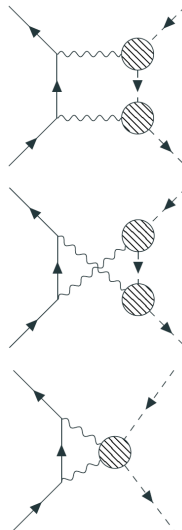
VPC

- **F $\times$ sQED**: Diagrams are computed in scalar QED and multiplied by a global form factor $F_\pi(q^2)$, where q^2 is chosen to ensure the cancellation of IR divergences $\hookrightarrow q^2 = m_{\pi\pi}^2$ for ISC, $q^2 = s$ in all other cases
- **GVMD**: The form factor is written as a sum of Breit-Wigner functions. The propagator-like form allows one to solve the loop integral with standard techniques

$$F_\pi(q^2) = \sum_{v=0}^N c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2} \quad \text{with} \quad \Lambda_v^2 = m_v^2 - im_v \Gamma_v$$

- **FsQED**: Under the general assumptions unitarity and analyticity, the form factor is decomposed using the dispersion relation

$$\frac{F_\pi(q^2)}{q^2} = \frac{1}{q^2 - \lambda^2} - \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{ds'}{s'} \frac{\text{Im} F_\pi(s')}{s'(q^2 - s')}$$



Phase 1 setup

- 3 channels $e^+e^- \rightarrow X^+X^-(\gamma)$
- 5 experimental scenarios
- 7 Monte Carlo codes

- 3 channels $e^+e^- \rightarrow X^+X^-(\gamma)$, both in **S**can and **R**adiative-return mode:

S: $e^+e^- \rightarrow e^+e^-$ $e^+e^- \rightarrow \mu^+\mu^-$ $e^+e^- \rightarrow \pi^+\pi^-$

R: $e^+e^- \rightarrow e^+e^-\gamma$ $e^+e^- \rightarrow \mu^+\mu^-\gamma$ $e^+e^- \rightarrow \pi^+\pi^-\gamma$

- 5 experimental scenarios
- 7 Monte Carlo codes

- 3 channels $e^+e^- \rightarrow X^+X^-(\gamma)$

- 5 experimental scenarios

CMD: $e^+e^- \rightarrow X^+(p_+)X^-(p_-)$ at $\sqrt{s} = 0.7$ GeV

cuts on $p_{\pm}, \theta_{\pm}, ||\phi^+ - \phi^-| - \pi|, |\theta_+ + \theta_- - \pi|$

KLOE-SA (untagged): $e^+e^- \rightarrow X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 1.02$ GeV

cuts on $p_{\pm}, \theta_{\pm}, M_{XX}$, if $\vec{p}_{\tilde{\gamma}} \equiv -(\vec{p}_+ + \vec{p}_-)$ then $\theta_{\tilde{\gamma}} \leq 15^\circ$ or $\theta_{\tilde{\gamma}} \geq 165^\circ$

KLOE-LA (tagged): $e^+e^- \rightarrow X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 1.02$ GeV

cuts on $p_{\pm}, \theta_{\pm}, M_{XX\gamma}$ and the photon

BESIII: $e^+e^- \rightarrow X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 4$ GeV

cuts on $p_{\pm}, \theta_{\pm}, M_{XX\gamma}$ and the photon

B-factory: $e^+e^- \rightarrow X^+(p_+)X^-(p_-)\gamma$ at $\sqrt{s} = 10$ GeV

cuts on $p_{\pm}, \theta_{\pm}, M_{XX\gamma}$ and the photon

- 7 Monte Carlo codes

- 3 channels $e^+e^- \rightarrow X^+X^-(\gamma)$
- 5 experimental scenarios
- 7 Monte Carlo codes

AFKQED **R**: LO+ISC with CS and FSC with Photos [only $X = \mu, \pi$]

BABAYAGA **S**: NLO+PS, **R**: LO + PS [F $\times$ sQED for $X = \pi$]

KKMC **S**: LO+YFS, **R**: LO+YFS [only $X = \mu$]

MCGPJ **S**: NLO+CS, **R**: LO+CS [GVMD for $X = \pi$]

McMULE **S**: NNLO, **R**: NLO [only ISC for $X = \pi$]

PHOKHARA **S**: LO, **R**: NLO [F $\times$ sQED for $X = \pi$]

SHERPA **S, R**: LO+YFS for $X = \mu$, LO for $X = e$

This was the situation in September 2024, some codes have been improved since then!

- 3 channels $e^+e^- \rightarrow X^+X^-(\gamma)$
- 5 experimental scenarios
- 7 Monte Carlo codes

AFKQED **R**: LO+ISC with CS and FSC with Photos [only $X = \mu, \pi$]

BABAYAGA **S**: NLO+PS, **R**: NLO+PS [$F \times s$ QED, **GVMD**, **FsQED** for $X = \pi$]

KKMC **S**: LO+YFS, **R**: LO+YFS [only $X = \mu$]

MCGPJ **S**: NLO+CS, **R**: LO+CS [GVMD for $X = \pi$]

McMULE **S**: NNLO, **R**: NLO and **NNLO for ISC** [**FsQED** for $X = \pi$]

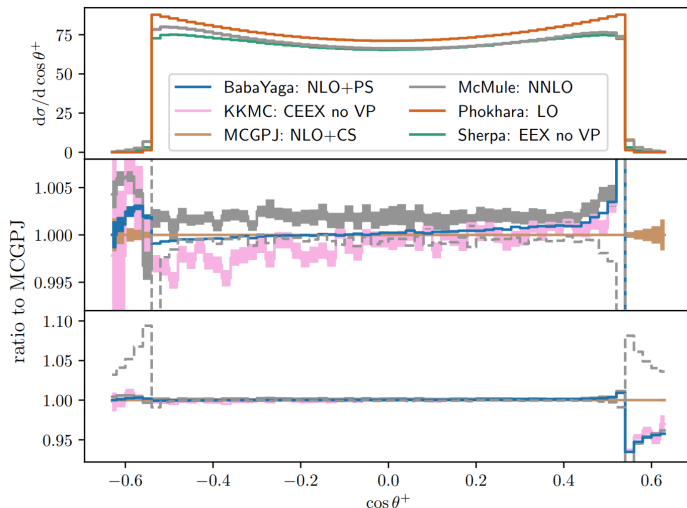
PHOKHARA **S**: LO, **R**: NNLO [**GVMD** for $X = \pi$]

SHERPA **S, R**: NLO+YFS [only $X = e, \mu$]

See Marco's talk for BabaYaga at NLOPS for radiative processes

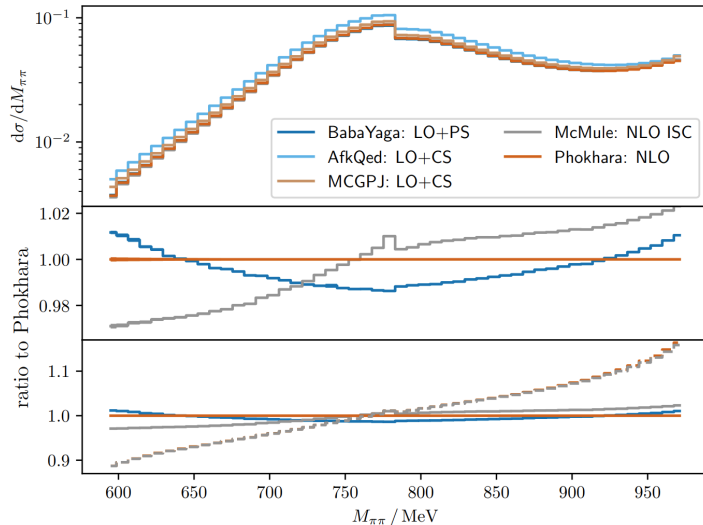
CMD-like scenario: $e^+e^- \rightarrow \mu^+\mu^-$

Muon scattering angle



- No VPC for KKMC and SHERPA, but present for all other codes
- BABAYAGA close to MCGPJ despite two different resummation procedures (PS vs. CS)
- PHOKHARA is designed for radiative return, only LO for scan setups
- Dashed grey line in lower panels: McMule at NLO
- Agreement at $\sim 0.2\%$ at the center, larger deviations at edges

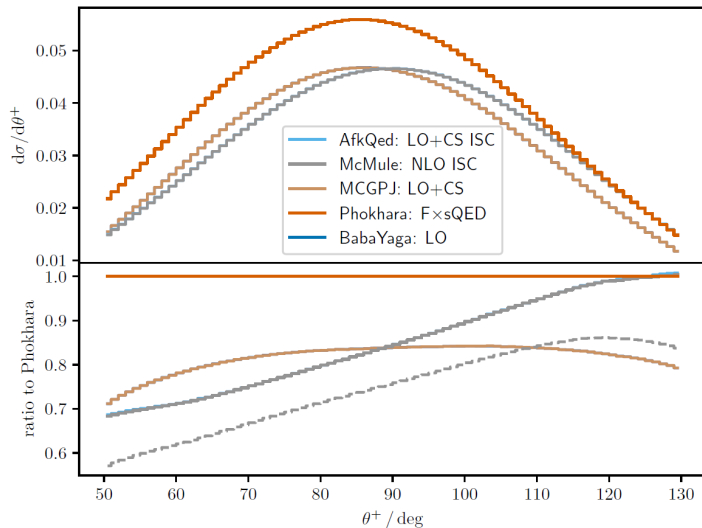
Two-pion invariant mass



- PHOKHARA territory (full NLO)
- NLO corrections are large (10%)
- AFKQED and MCGPJ are not suitable for radiative return (LO + collinear photons)
- BABAYAGA is designed for $2 \rightarrow 2$ processes but the photon can be tagged from the exclusive PS
- McMule has ISC only (no radiation off pions). At LO, the effect is negligible, but is of $\sim 2\%$ at NLO

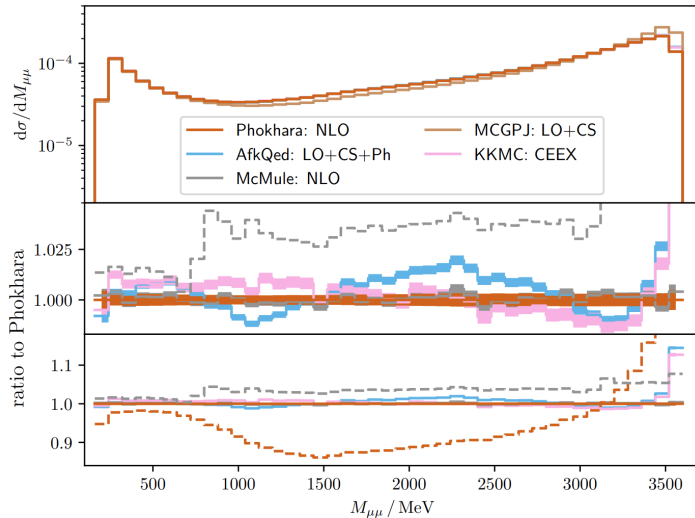
KLOE-like large-angle scenario: $e^+e^- \rightarrow \pi^+\pi^-\gamma$

Pion scattering angle



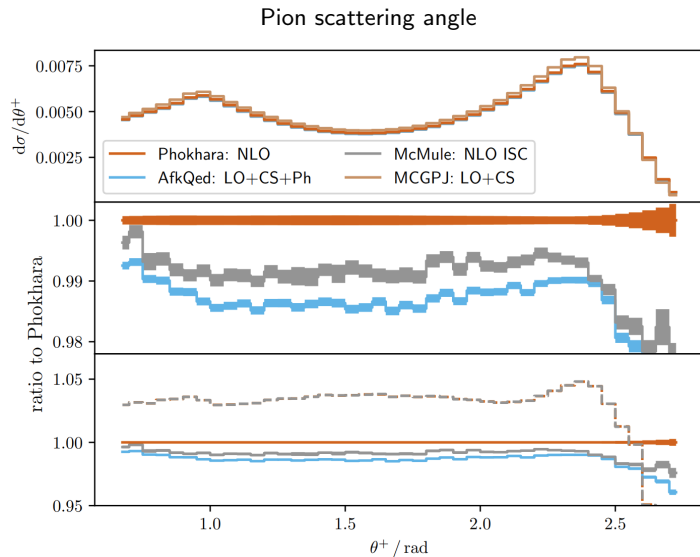
- PHOKHARA territory (full NLO)
- MCGPJ $\simeq$ PHOKHARA at LO
- AFKQED $\simeq$ McMULE at NLO
- FSR huge at LO (10 – 20%), see McMULE at LO (dashed grey line)
- Even larger at NLO (20 - 40%)
- A reliable implementation of FSC is crucial for this scenario

Two-muon invariant mass



- No VPC, technical comparison
- VPC > 3% (see McMULE at NLO with VPC, dashed grey line)
- NLO corrections are large ($\sim 10\%$)
- Deviations between codes for the non-VP part are within 2% except at the end of the distribution

B-factory scenario: $e^+e^- \rightarrow \pi^+\pi^-\gamma$



- LO: FSC $\sim 0.01\%$ (suppressed by form factor at $\sqrt{s} = 10$ GeV)
- NLO: FSC 1 - 2%
- 1% precision is a long way ahead
- At least NLOPS and NNLO needed

- The MC groups are working to further improve the QED accuracy of their codes:
 - ↪ Complete NNLO and NLOPS computations for $2 \rightarrow 3$ processes
 - ↪ Towards NNLOPS and N³LO for ISC for $2 \rightarrow 2$ processes
- Systematic implementation of $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ beyond F $\times$ sQED
 - ↪ Already done in BABAYAGA@NLO and McMULE
- More processes ($e^+e^- \rightarrow \gamma\gamma, \pi^+\pi^-\pi^0\dots$)
- Evaluate the theory error (e.g. comparison between NNLO and NLOPS)
- Strengthen collaboration with experiments to better understand the impact of radiative corrections on data analysis → No longer just a *theory exercise* but actual physics
- See RMCL2 Satellite Meeting on Friday for the latest updates!

RadioMonteCarLow 2 members for Phase 1

Riccardo Aliberti, Paolo Beltrame, Ettore Budassi, Carlo M. Carloni Calame, Gilberto Colangelo, Lorenzo Crottozzi, Achim Denig, Anna Driutti, Tim Engel, Lois Flower, Andrea Gurgone, Martin Hoferichter, Fedor Ignatov, Sophie Kollatzsch, Bastian Kubis, Andrzej Kupsc, Fabian Lange, Alberto Lusiani, Stefan E. Muller, Jeremy Paltrinieri, Pau Petit Rosas, Fulvio Piccinini, Alan Price, Lorenzo Punzi, Marco Rocco, Olga Shekhovtsova, Andrzej Siodmok, Adrian Signer, Giovanni Stagnitto, Peter Stoffer, Thomas Teubner, William J. Torres Bobadilla, Francesco P. Ucci, Yannick Ulrich, Graziano Venanzoni



RMCL2 Workshop in Pisa, May 2025

Contact Andrzej Kupsc, Adrian Signer, Yannick Ulrich, or Graziano Venanzoni if you are interested in joining the RadioMonteCarLow2 effort, have feedback or want to discuss