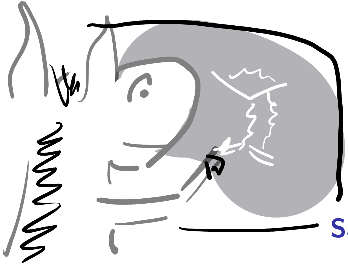


RMCL2 Meeting Liverpool 2025

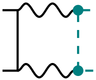
Update: McMule

Sara Gündogdu & Sophie Kollatzsch
on behalf of the McMule team

Paul Scherrer Institute / University of Zurich

example: $ee \to \pi\pi$ @ NLO in Form Factor Scalar QED (FsQED) [Colangelo et al 22]

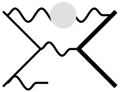
(also calculated in [Budassi et al 24] and [Holz, Cottini])



$$\ni \frac{F_\pi(k^2)}{k^2} = \frac{1}{k^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S\, \frac{\mathrm{Im}F_\pi(S)}{S(k^2-S)}$$

example: $ee \to \mu\mu\gamma$ @ NNLO with dispersive HVP

(also hyperspherical method [Fael 18] for t-channel)



$$\ni \frac{\Pi(k^2)}{k^2} = -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S\, \frac{\mathrm{Im}\Pi(S)}{S(k^2-S)}$$

Disperon QED

example: $ee \to \pi\pi$ @ NLO in Form Factor Scalar QED (FsQED) [Colangelo et al 22]

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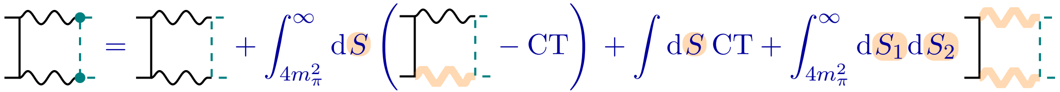

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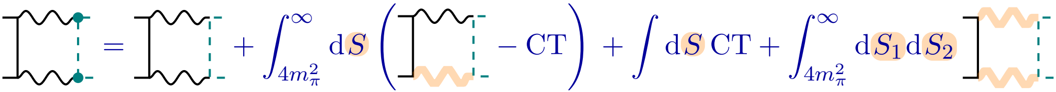

- tedious expressions
- additional integral
- threshold divergence

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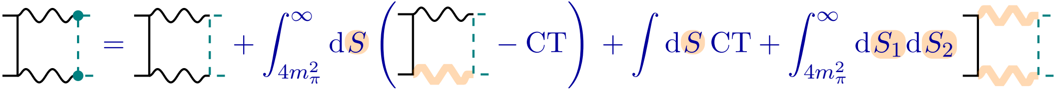

- tedious expressions
  $\Longrightarrow$ OpenLoops with disperons
- additional integral
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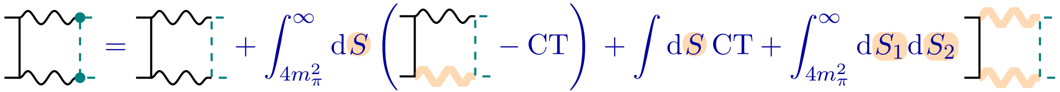
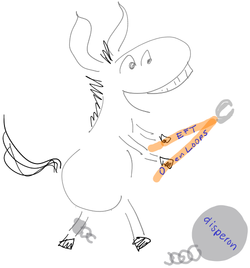


- tedious expressions
  $\Longrightarrow$ OpenLoops with disperons
- additional integral
  $\Longrightarrow \int_{4m_\pi^2}^{\Lambda(s)}$ OpenLoops $+ \int_{\Lambda(s)}^{\infty}$ Disperon EFT
- threshold divergence



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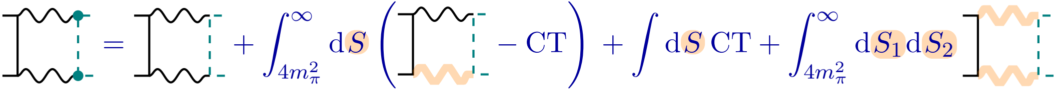

- tedious expressions
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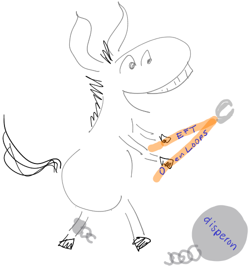
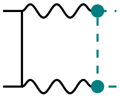
  $\Longrightarrow$ $\int_{4m_\pi^2}^{\Lambda(s)}$ OpenLoops + $\int_{\Lambda(s)}^{\infty}$ Disperon EFT
- threshold divergence
  $\Longrightarrow$ CT = $\begin{pmatrix} \text{process}- \\ \text{dependent} \end{pmatrix}$ $\times$ universal div.

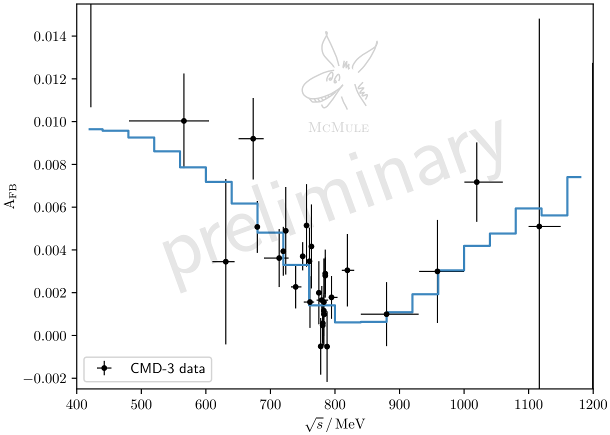




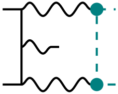
Pisa :: first results CMD-3 RMCL2 observables

now :: studies of $\Lambda(s)$ at different $s \implies A_{\text{FB}}$

future :: beyond FsQED ??



progress since Pisa :: $ee \to \pi\pi\gamma$

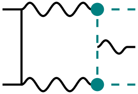


Pisa :: ‘proof of concept’

now :: validation of Disperon EFT against OpenLoops

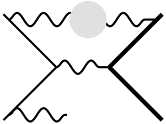
work in progress :: assembly of threshold CT

. . . waiting for $ee \to \gamma\gamma^*$ @ NNLO



Pisa :: ‘let’s do this in FsQED’

now :: ??



Pisa :: nothing but comittement

later ::

- change of renormalisation in OpenLoops

$$Z_{2,m}^{\mathrm{OS,QED}} \neq Z_{2,m}^{\mathrm{OS,DisperonQED}}$$

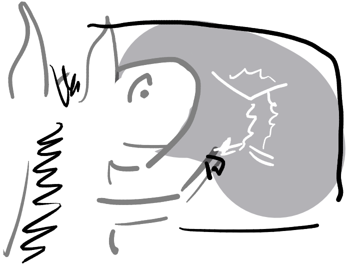
$\Longrightarrow$ OpenLoops enabled support of customised CT

- we can resumm the HVP

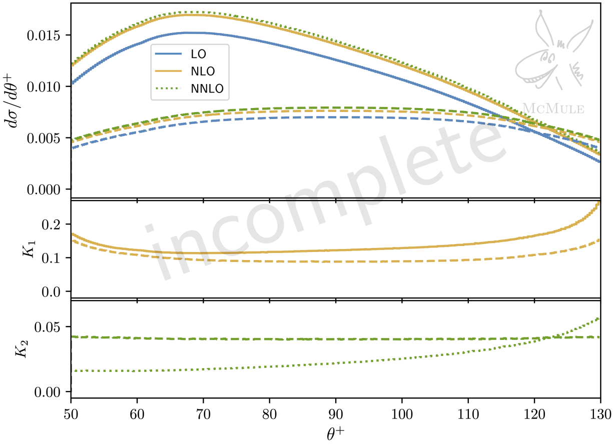
$$\frac{\Pi_{\mathrm{res}}(k^2)}{k^2} \equiv \frac{1}{1-\Pi(k^2)}\frac{1}{k^2} \quad \text{instead of} \quad \frac{\Pi(k^2)}{k^2}$$

currently :: implementing & cross checking

$ee \to \gamma^* \Rightarrow ee \to \mu\mu \Rightarrow ee \to \gamma\gamma^* \Rightarrow ee \to \mu\mu\gamma$



$$ee \to \mu\mu\gamma$$



KLOE-LA

$\sqrt{s} = 1.02\,\mathrm{GeV}\,;$

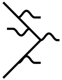
$50^\circ \le \theta^\pm \le 130^\circ\,;$

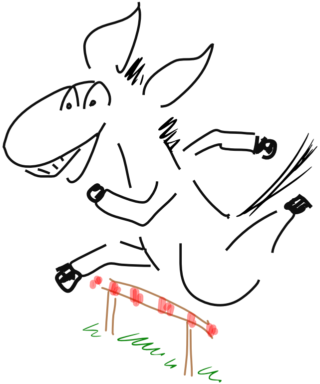
$|p_z^\pm| > 90\,\mathrm{MeV} \;\vee\; |p_\perp^\pm| > 160\,\mathrm{MeV}\,;$

$50^\circ \le \theta_\gamma \le 130^\circ \;\wedge\; E_\gamma > 20\,\mathrm{MeV}\,;$

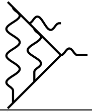
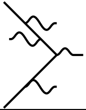
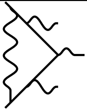
$0.1\,\mathrm{GeV}^2 \le M_{\mu\mu}^2 \le 0.85\,\mathrm{GeV}^2$

What are the challenges for $ee \to \gamma^*\gamma$ @NNLO?

1. virtual-virtual 
   - challenge (massification) $\forall\ \sqrt{s},\ \Theta \to 0$
2. real-real 
   - numerical instabilities for $\sqrt{s} > 4\text{GeV}$
   - helicity method
   - LBK



McMule goal: MuonE calculation @ N^3LO

	$ee \to \gamma^*\gamma$	methods (until now)	challenges	suggestions
VV		massification	KLOE SA ✗	fully massive calculation by volunteers (?) / jettification
RR		OL	B/BES ✗	collinear expansion/ helicity method/ LBK
RV		OL	KLOE SA/LA ✓ B/BES ✓	

McMule team

f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & Mainz), S.Gündogdu (Zurich & PSI), D.Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D.Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), P.Banerjee (Cosenza), M.Ronchi (Mainz), Y.Fang (PSI), G.Billis (PSI), J.Wilson (Liverpool)

McMule

`mule-tools.gitlab.io`