

BabaYaga@NLO: update

RMCL2 Satellite Meeting - Liverpool

Marco Ghilardi

On behalf of the BabaYaga team

November 14, 2025



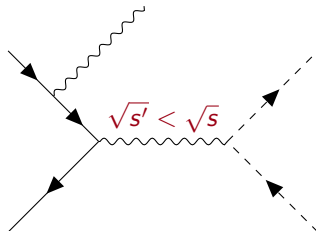
The radiative return



$$\sigma(e^+e^- \rightarrow \pi^+\pi^-, s') \iff \sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma)$$

The radiative return

$$\sigma(e^+e^- \rightarrow \pi^+\pi^-, s') \iff \sigma(e^+e^- \rightarrow \pi^+\pi^-\gamma)$$



Codes currently available:

- McMule ($\mu\mu\gamma$ @NLO and $\pi\pi\gamma$ @NLO-ISC)
- Phokhara ($\mu\mu\gamma$, $\pi\pi\gamma$ @NLO)
- AfkQED ($\mu\mu\gamma$, $\pi\pi\gamma$ @LO + Collinear SF)

Cfr. A. Denig's talk.

Radiative processes



- An important ingredient: NLOPS approach
 - Exact calculation up to NLO.
 - Exclusive LL resummation to all orders with explicit photon kinematics.

Radiative processes

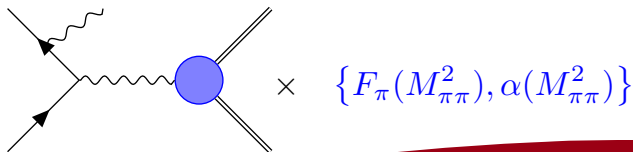


- An important ingredient: NLOPS approach
 - Exact calculation up to NLO.
 - Exclusive LL resummation to all orders with explicit photon kinematics.
- NLO accuracy for radiative channels
 - $e^+e^- \rightarrow \mu^+\mu^-\gamma$
 - $e^+e^- \rightarrow \pi^+\pi^-\gamma$ (With $F \times sQED$)



Radiative processes

- An important ingredient: NLOPS approach
 - Exact calculation up to NLO.
 - Exclusive LL resummation to all orders with explicit photon kinematics.
- NLO accuracy for radiative channels
 - $e^+e^- \rightarrow \mu^+\mu^-\gamma$
 - $e^+e^- \rightarrow \pi^+\pi^-\gamma$ (With $F \times sQED$)
- Non perturbative contributions



LL approximation



$$d\sigma_{\text{LOPS}} = \sum_{n=1}^{\infty} \Pi(\varepsilon, Q^2) \frac{1}{n!} |\mathcal{M}_n^X|^2 d\Phi_n(\{p\}, \{k\})$$

where

- $\Pi(\varepsilon, Q^2)$ is the Sudakov Form factor where ε is the soft-hard separator.
- $\mathcal{M}_n^X$ stands for $\mathcal{M}^{\text{ex}}$ for $n = 1, 2$ and $\mathcal{M}^{\text{LL}}$ for $n \geq 3$.



LL approximation

$$d\sigma_{\text{LOPS}} = \sum_{n=1}^{\infty} \Pi(\varepsilon, Q^2) \frac{1}{n!} |\mathcal{M}_n^X|^2 d\Phi_n(\{p\}, \{k\})$$

where

- $\Pi(\varepsilon, Q^2)$ is the Sudakov Form factor where ε is the soft-hard separator.
- $\mathcal{M}_n^X$ stands for $\mathcal{M}^{\text{ex}}$ for $n = 1, 2$ and $\mathcal{M}^{\text{LL}}$ for $n \geq 3$.

$$|\mathcal{M}_n^{\text{LL}}|^2 = \prod_{i=1}^{n-2} \left[-\frac{\alpha}{2\pi} P(z_i) \mathcal{I}_{\text{SF}}(k_i, z_i) \frac{4\pi^2 (1-z_i)}{z_i \omega_i^2} \right] \underbrace{\left| \mathcal{M}_2^{\text{ex}}(\{\tilde{p}\}, \{\tilde{k}\}) \right|^2}_{e^+ e^- \rightarrow XX \gamma \gamma}$$

LL approximation



If we define

$$\mathcal{I}(k) = \sum_{i,j=1}^4 \mathcal{I}_{ij}(k) = \sum_{i,j=1}^4 \eta_i \eta_j \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} k_0^2,$$

then

$$\mathcal{I}_{\text{SF}}(k, z) = - \sum_{ij} \mathcal{I}_{ij}(k) \cdot F_{ij}(k, z)$$

where

$$\begin{cases} F_{ij}(k, z) = 1 - \delta_{ij} \frac{(1-z)^2}{1+z^2} & i = \text{fermion} \\ F_{ij}(k, z) = 1 & i = \text{scalar}. \end{cases}$$

LL approximation



If we define

$$\mathcal{I}(k) = \sum_{i,j=1}^4 \mathcal{I}_{ij}(k) = \sum_{i,j=1}^4 \eta_i \eta_j \frac{(p_i \cdot p_j)}{(p_i \cdot k)(p_j \cdot k)} k_0^2,$$

then

$$\mathcal{I}_{\text{SF}}(k, z) = - \sum_{ij} \mathcal{I}_{ij}(k) \cdot F_{ij}(k, z)$$

where

$$\begin{cases} F_{ij}(k, z) = 1 - \delta_{ij} \frac{(1-z)^2}{1+z^2} & i = \text{fermion} \\ F_{ij}(k, z) = 1 & i = \text{scalar}. \end{cases}$$

REMARK: In the limit $k \cdot p \rightarrow 0$ we correctly reproduce the **massive** unpolarized splitting function of *S. Dittmaier, arxiv:9904440*

Matching at NLO



$$d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}] |\mathcal{M}_1^{\text{ex}}|^2 d\Phi_1 + |\mathcal{M}_2^{\text{ex}}|^2 d\Phi_2$$

$$d\sigma^{\alpha} = [1 + C_{\alpha}] |\mathcal{M}_1^{\text{ex}}|^2 d\Phi_1 + |\mathcal{M}_2^{\text{ex}}|^2 d\Phi_2$$

Matching at NLO



$$d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}] |\mathcal{M}_1^{\text{ex}}|^2 d\Phi_1 + |\mathcal{M}_2^{\text{ex}}|^2 d\Phi_2$$

$$d\sigma^{\alpha} = [1 + C_{\alpha}] |\mathcal{M}_1^{\text{ex}}|^2 d\Phi_1 + |\mathcal{M}_2^{\text{ex}}|^2 d\Phi_2$$

- The hard contribution $|\mathcal{M}_2|^2$ is already **exact**.

Matching at NLO



$$d\sigma_{LL}^{\alpha} = [1 + C_{\alpha,LL}] |\mathcal{M}_1^{\text{ex}}|^2 d\Phi_1 + |\mathcal{M}_2^{\text{ex}}|^2 d\Phi_2$$

$$d\sigma^{\alpha} = [1 + C_{\alpha}] |\mathcal{M}_1^{\text{ex}}|^2 d\Phi_1 + |\mathcal{M}_2^{\text{ex}}|^2 d\Phi_2$$

- The hard contribution $|\mathcal{M}_2|^2$ is already **exact**.
- To match the exact NLO soft – virtual correction we introduce

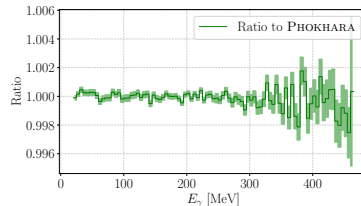
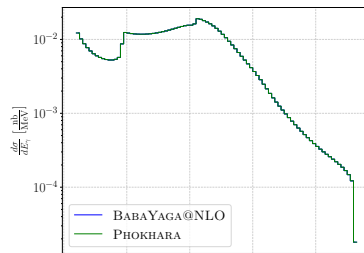
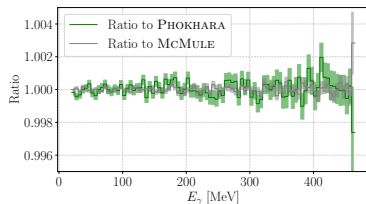
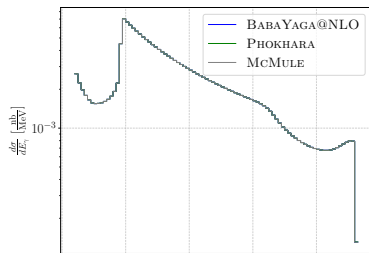
$$F_{SV} = 1 + (\mathcal{C}_{\alpha} - \mathcal{C}_{\alpha,LL})$$

The master formula for NLOPS accuracy reads

$$d\sigma_{\text{NLOPS}} = \sum_{n=1}^{\infty} \Pi(\varepsilon, Q^2) F_{SV} \frac{1}{n!} |\mathcal{M}_n^X|^2 d\Phi_n(\{p\}, \{k\}).$$

Tuned comparison - E_γ

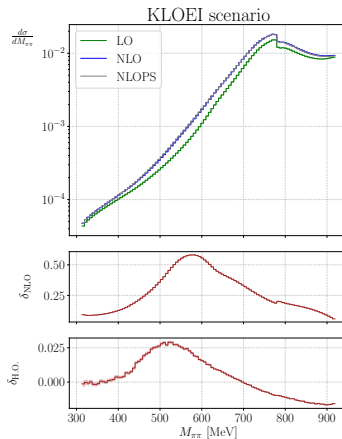
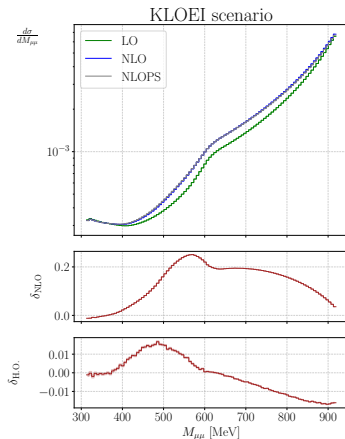
Scenario KLOE-LA @ 1.02 GeV



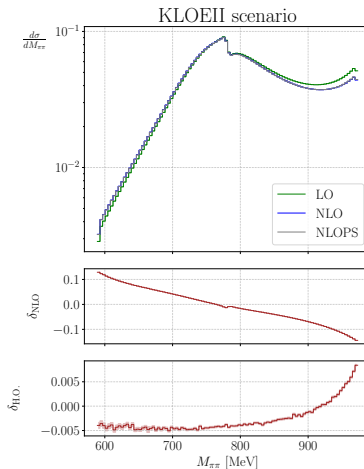
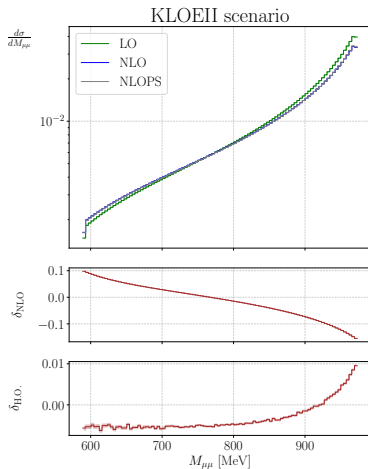
KLOE-LA scenario - $M_{\mu\mu,\pi\pi}$

$$\delta_{\text{NLO}} = \frac{d\sigma_{\text{NLO}} - d\sigma_{\text{LO}}}{d\sigma_{\text{LO}}}$$

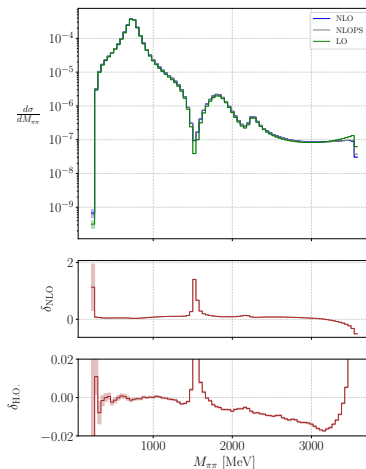
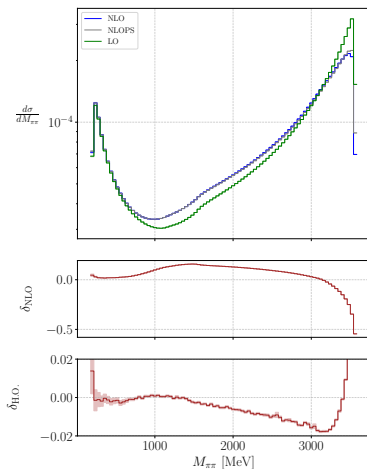
$$\delta_{\text{H.O.}} = \frac{d\sigma_{\text{NLOPS}} - d\sigma_{\text{NLO}}}{d\sigma_{\text{NLO}}}$$



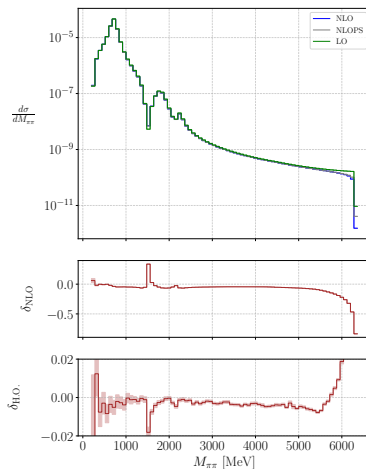
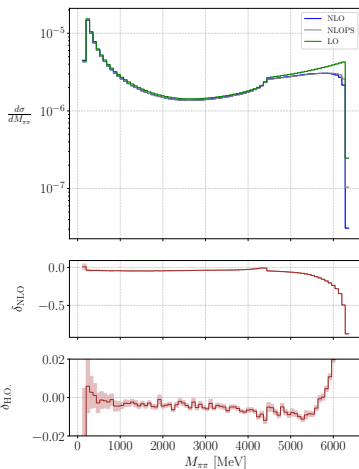
KLOE-SA scenario - $M_{\mu\mu,\pi\pi}$

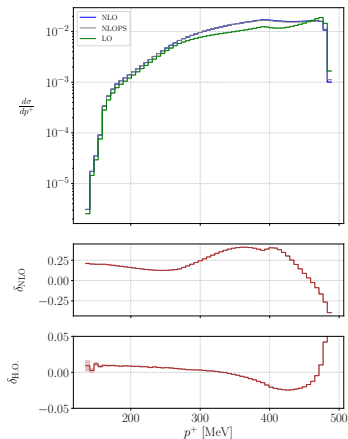
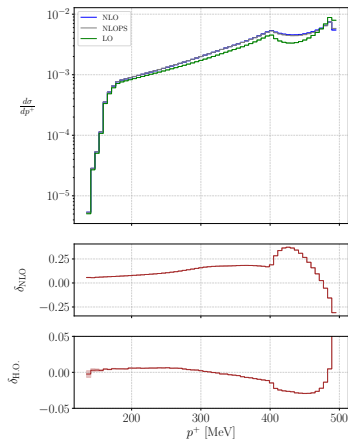


BES3 scenario - $M_{\mu\mu,\pi\pi}$



B scenario - $M_{\mu\mu,\pi\pi}$

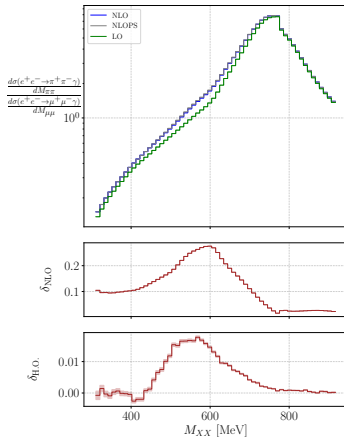


KLOE-LA scenario - p_+ 

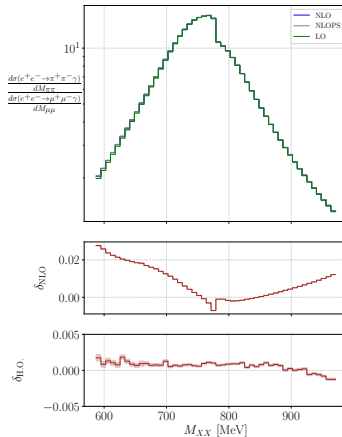
KLOE scenario - $d\sigma_{\pi\pi\gamma}/d\sigma_{\mu\mu\gamma}$



KLOE – LA



KLOE – SA



Summary and Outlook



- Multiple photon contributions are in general important for radiative return experiment.

Summary and Outlook



- Multiple photon contributions are in general important for radiative return experiment.
- Relevance of **NLOPS MC** event generators.

Summary and Outlook



- Multiple photon contributions are in general important for radiative return experiment.
- Relevance of **NLOPS MC** event generators.

NLOPS **vs** NNLO $\Rightarrow$ Assessment of theoretical uncertainty

- A preliminary version of BabaYaga@NLO for $\mu\mu\gamma$ and $\pi\pi\gamma$ is working (with room for improvement in the near future).

Summary and Outlook



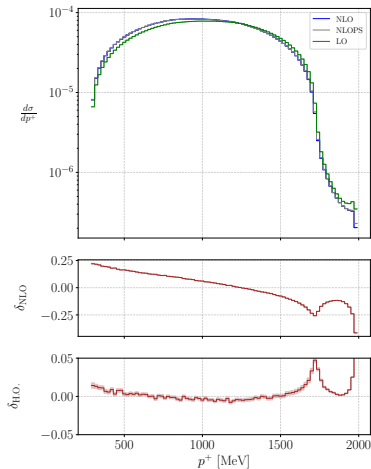
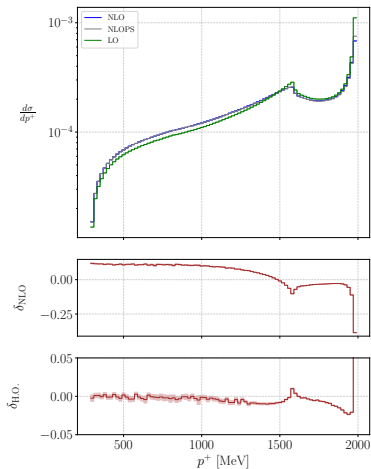
- Multiple photon contributions are in general important for radiative return experiment.
- Relevance of **NLOPS MC** event generators.

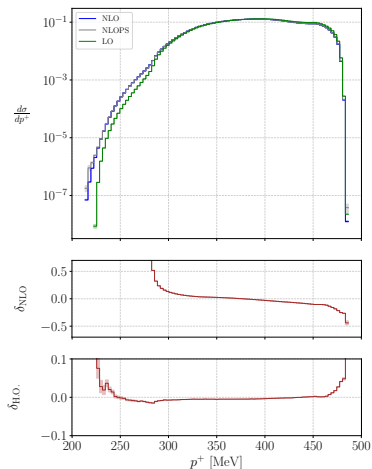
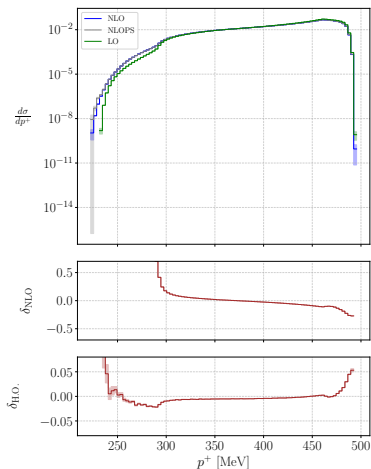
NLOPS **vs** NNLO $\Rightarrow$ Assessment of theoretical uncertainty

- A preliminary version of BabaYaga@NLO for $\mu\mu\gamma$ and $\pi\pi\gamma$ is working (with room for improvement in the near future).
- **Future developments**
 - $\pi\pi\gamma$ with FsQED and GVMD at NLOPS accuracy.
 - $ee\gamma$ at NLOPS accuracy.
 - Extension to other hadronic channels ($K^+K^- \dots$)

Thank you !

BES3 scenario - p_+



KLOE-SA scenario - p_+ 

**(a) KLOEII Large-Angle scenario**

$$\sqrt{s} = 1.02 \text{ GeV}$$

$$50^\circ \leq \vartheta_{\pm} \leq 130^\circ$$

$$|\mathbf{p}_{\pm}^z| \geq 90 \text{ MeV} \vee |\mathbf{p}_{\pm}^{\perp}| \geq 160 \text{ MeV}$$

$$50^\circ \leq \vartheta_{\gamma} \leq 130^\circ \wedge E_{\gamma} \geq 20 \text{ MeV}$$

$$0.1 \text{ GeV}^2 \leq M_{XX}^2 \leq 0.85 \text{ GeV}^2$$

(b) KLOEII Small-Angle scenario

$$\sqrt{s} = 1.02 \text{ GeV}$$

$$50^\circ \leq \vartheta_{\pm} \leq 130^\circ$$

$$|\mathbf{p}_{\pm}^z| \geq 90 \text{ MeV} \vee |\mathbf{p}_{\pm}^{\perp}| \geq 160 \text{ MeV}$$

$$\vartheta_{\tilde{\gamma}} \leq 15^\circ \vee \vartheta_{\tilde{\gamma}} \geq 165^\circ$$

$$0.35 \text{ GeV}^2 \leq M_{XX}^2 \leq 0.95 \text{ GeV}^2$$

(c) BES3 scenario

$$\sqrt{s} = 4 \text{ GeV}$$

$$|\cos \vartheta_{\pm}| \leq 0.93 \wedge |p_{\pm}^{\perp}| \geq 300 \text{ MeV}$$

$$[|\cos \vartheta_{\gamma}| \leq 0.8 \wedge E_{\gamma} \geq 25 \text{ MeV}] \vee$$

$$[0.86 \leq |\cos \vartheta_{\gamma}| \leq 0.92 \wedge E_{\gamma} \geq 50 \text{ MeV}]$$

$$\exists! \gamma \text{ s.t. } E_{\gamma} \geq 400 \text{ MeV}$$

(d) B scenario

$$\sqrt{s} = 10 \text{ GeV}$$

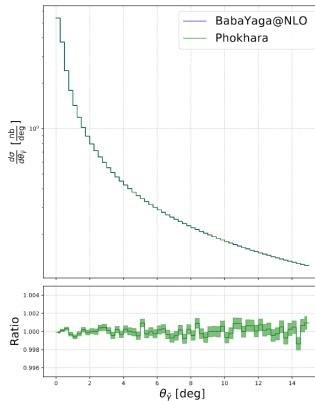
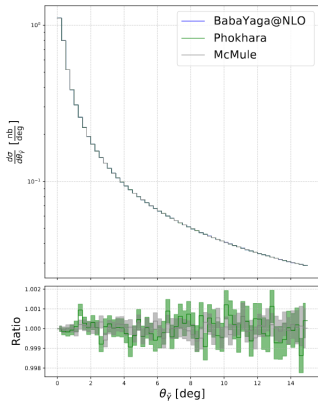
$$0.65 \text{ rad} \leq \vartheta_{\pm} \leq 2.75 \text{ rad} \wedge |\mathbf{p}_{\pm}| \geq 1 \text{ GeV}$$

$$0.6 \text{ rad} \leq \vartheta_{\gamma} \leq 2.7 \text{ rad} \wedge E_{\gamma} \geq 3 \text{ GeV}$$

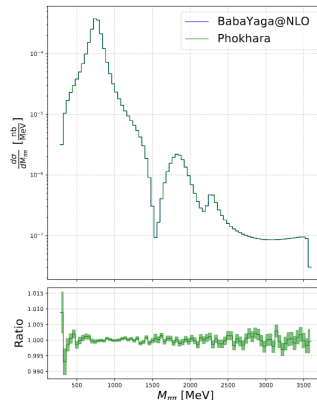
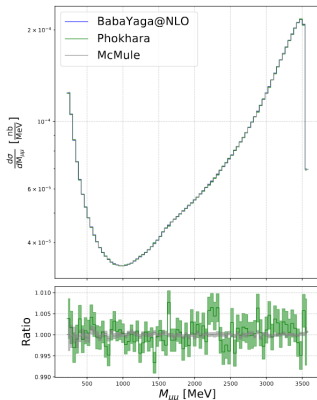
$$\vartheta_{\tilde{\gamma}, \gamma^{(h)}} \leq 0.3 \text{ rad}$$

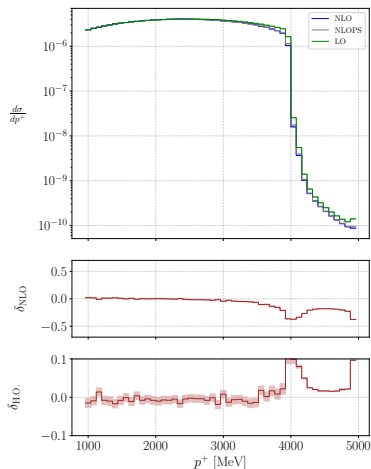
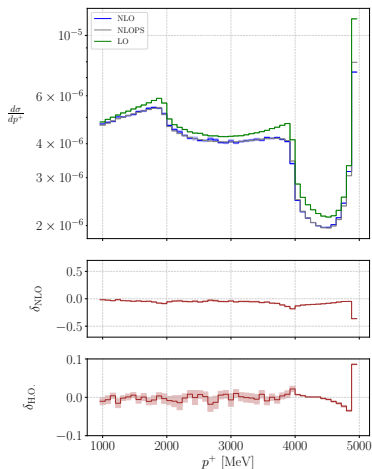
$$M_{XX\gamma} \geq 8 \text{ GeV}$$

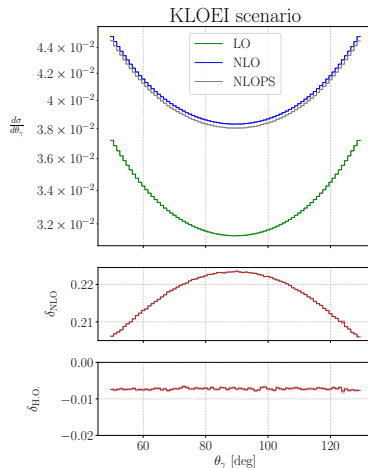
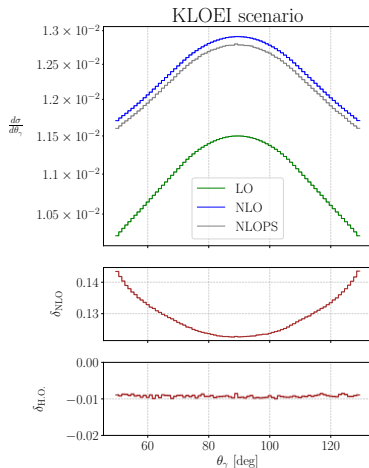
Tuned comparison - KLOE-SA



Tuned comparison - BES3

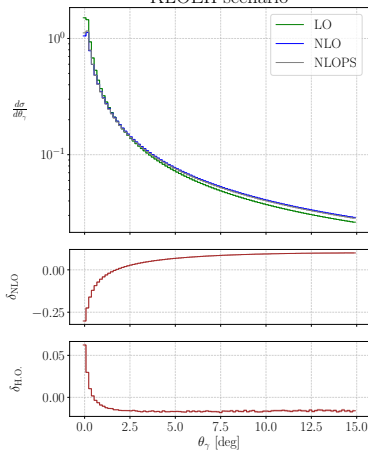


B scenario - p_+ 

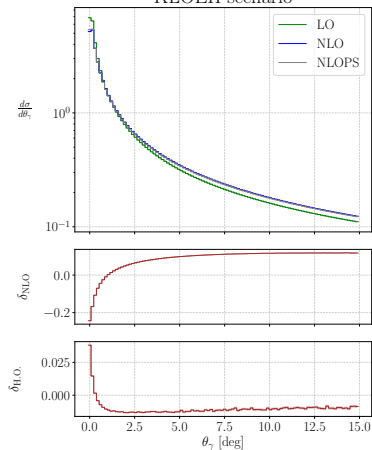
KLOE-LA scenario - θ_γ 

KLOE-SA scenario - $\theta_{\tilde{\gamma}}$

KLOEII scenario

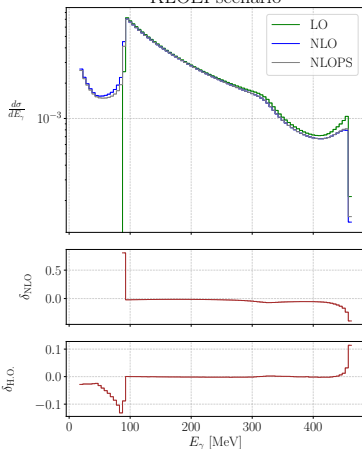


KLOEII scenario

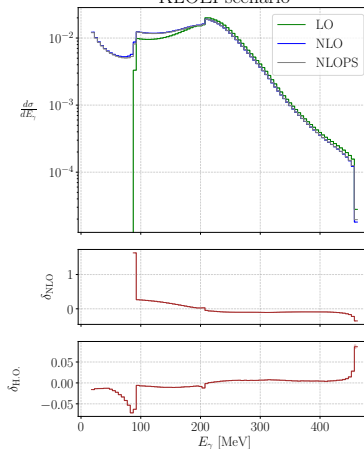


KLOE-LA scenario - E_γ 

KLOEI scenario

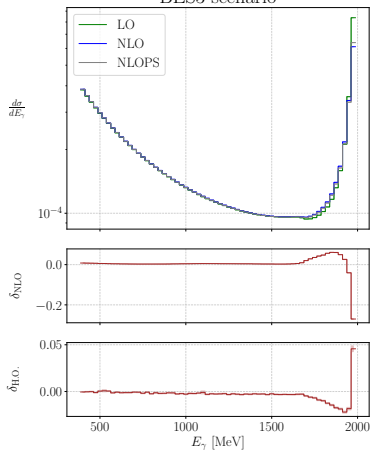


KLOEI scenario

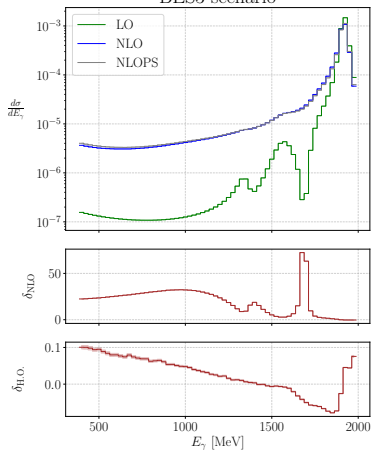


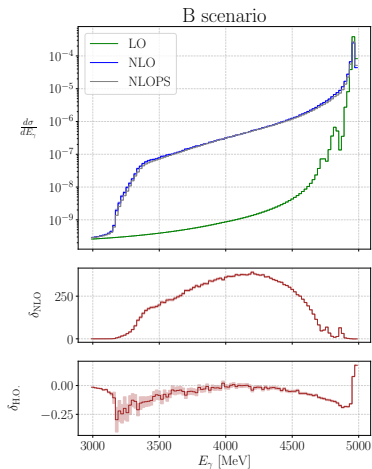
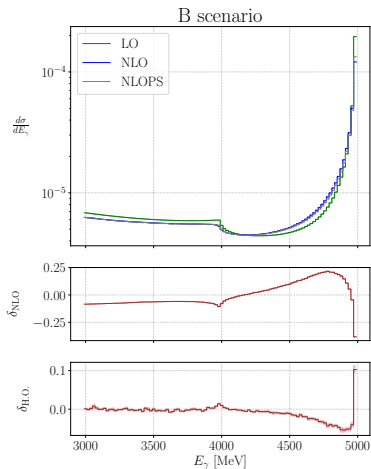
BES3 scenario - E_γ

BES3 scenario



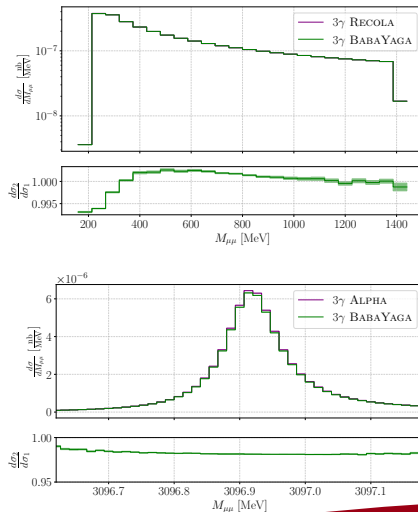
BES3 scenario



B scenario - E_γ 

Matrix element comparison for $n \geq 3$

Scenario B with 2 additional photons with $E_\gamma \geq 100$ MeV (Tree-level)



ALPHA [arXiv:hep-ph/9507237](https://arxiv.org/abs/hep-ph/9507237)
RECOLA [arXiv.org/abs/1211.6316](https://arxiv.org/abs/1211.6316)