



Colourless processes at EW NLO+Parton Shower

Muon Group Meeting – Theory

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Introduction and motivation

QED parton shower

The electron structure function

Regulating the electron structure function for a parton shower

Parton shower dependence on technical parameters

NLO matching

Case study: $e^+e^- \rightarrow \nu_\mu\bar{\nu}_\mu$ at 500 GeV

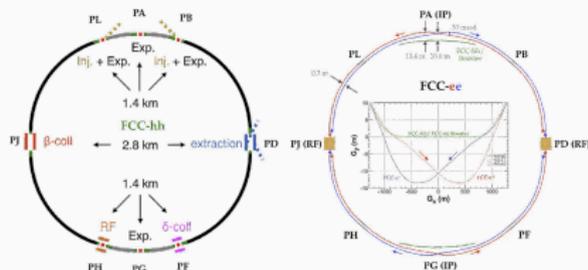
Results: ZH production at 365 GeV

Conclusions

Introduction and motivation

Introduction and motivation

- Precision QED needed for low-energy e^+e^- colliders (see [RadioMonteCarLow2](#), PHOKHARA, BABAYAGA developments)
- Future electron-positron collider now the top priority of collider community
- FCC-ee will run at 5 different centre-of-mass energies
- ZH production will be studied at
 - 240 GeV – dedicated run just above threshold
 - 365 GeV – produced during the $t\bar{t}$ run



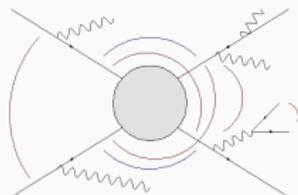
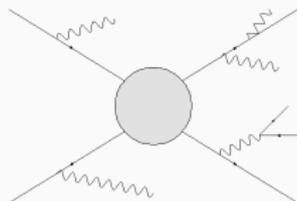
- NNLO QED for massive 4-fermion processes e.g. Broggio et al. '22
- YFS soft-photon resummation Yennie, Frautschi, Suura '61
 - Can be systematically matched to higher orders
 - Implemented in SHERPA Krauss, Schönherr '08, Krauss, Lindert, Linten, Schönherr '18, Krauss, Price, Schönherr '22, LF, Schönherr '22, Krauss, Price '25 and as CEEX in KKMC Jadach, Ward, Was '00
- Alternatively, NLO-matched parton shower for QED
 - Initial-state QED parton shower matched to NLO for $e^+e^- \rightarrow \ell^+\ell^-(\gamma)$ and recently $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ implemented in BABAYAGA Carloni Calame et al. '06, '25, '26
- Here we will introduce a process-independent solution based on MC@NLO Frixione, Webber '02

QED parton shower

Objective: to construct a universal approximation for higher-order corrections

Needs to:

- be suitable for implementation in a Monte Carlo event generator
- reproduce the leading soft and collinear limits of the $(n + 1)$ -particle XS
- regulate IR singularities w/ approximate virtual corrections
- generate emissions in the new one-particle phase space according to this approximation



- For MC@NLO, need to use the backward-evolution paradigm s.t. initial and final evolution are on equal footing
- We use SHERPA's dipole shower based on Catani-Seymour factorisation
[Schumann, Krauss '07](#)
- When an IS particle is either **emitter** or **spectator**, it changes its momentum fraction x and the associated PDF or structure function,

$$J_{\text{SF}} = \frac{f_a\left(\frac{x}{z}, t\right)}{f_{\tilde{a}j}(x, t)}$$

- For a proton, all f_a regular (and even small) as $x \rightarrow 1$
- For an electron, $f_e(x) \rightarrow \infty$ as $x \rightarrow 1$

The electron structure function

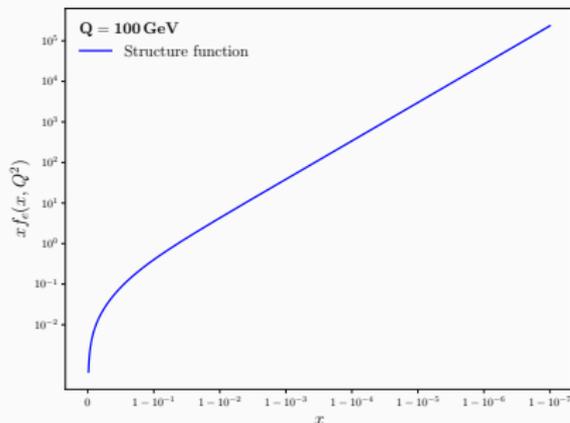
The electron structure function

Regulating the electron structure function
for a parton shower

The electron structure function

LL solution to DGLAP eq. with initial condition $f_e(x, m_e^2) = \delta(1 - x)$

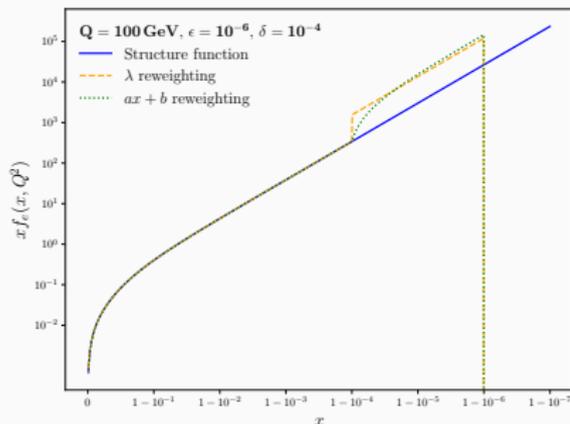
$$f_e(x, Q^2) = \beta \frac{\exp\left(-\gamma_E \beta + \frac{3}{4}\beta\right)}{\Gamma(1 + \beta)} (1 - x)^{\beta-1} + \beta \sum_{n=0}^{\infty} \beta^n \mathcal{H}_n(x)$$



where $\beta = \frac{\alpha}{\pi} \left(\ln \left(\frac{Q^2}{m_e^2} \right) - 1 \right)$

- Ongoing work towards higher orders [Frixione '19,'23](#); [Stahlhofen '25](#); [Schnubel, Szafron '25](#)
- **Integrable singularity at $x = 1$ at any order**

Integrable singularity at $x = 1$ is problematic for Monte Carlo...



Solution: reweight near $x = 1$

$$W_e = \begin{cases} f_e(x) & x \in [0, 1 - \delta] \\ w(x) f_e(x) & x \in [1 - \delta, 1 - \epsilon] \\ 0 & x \in [1 - \epsilon, 1] \end{cases}$$

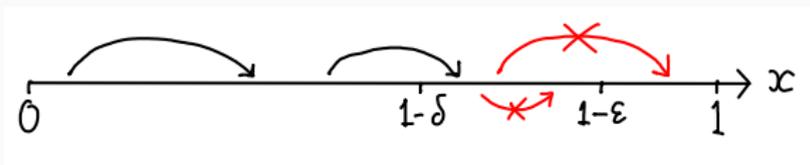
Determine $w(x)$ such that

$$\int_{1-\delta}^1 dx W_e = \int_{1-\delta}^1 dx f_e$$

- W_e no longer solves DGLAP equations – probably irrelevant
- Choosing $w(x) = \text{const.}$ means W_e not continuous – problematic for parton showers

Interplay with initial state QED shower

- Initial-state parton shower evolves backwards from hard process
- Backward evolution involves event moving in phase space to larger x
- Electron SF enhances this, $f_e(\text{large } x, Q^2)/f_e(\text{small } x, Q^2) \gg 1$
- Overestimate ratio of SFs using $k/(z-x)$, derived using $x \rightarrow 1$ limit
- Existence of ϵ, δ imply parton shower $x_{\max} < 1$
 - Choose global $x_{\max} = 1 - \epsilon$
 - Allow at most 1 emission into $[1 - \delta, 1 - \epsilon]$

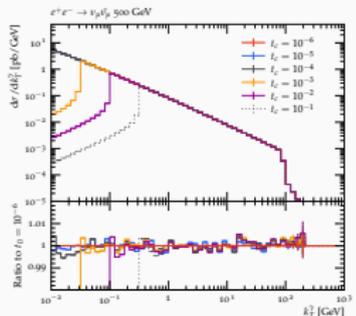
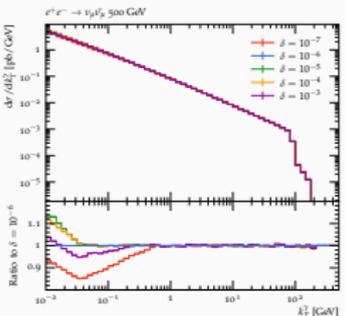
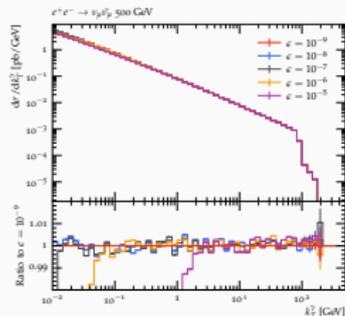
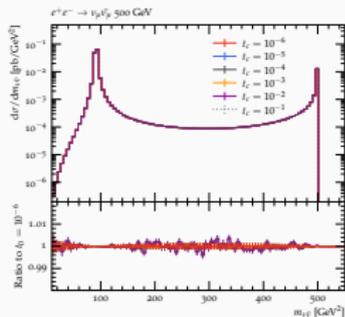
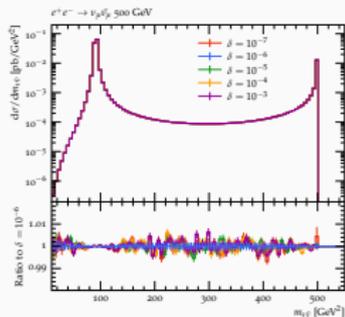
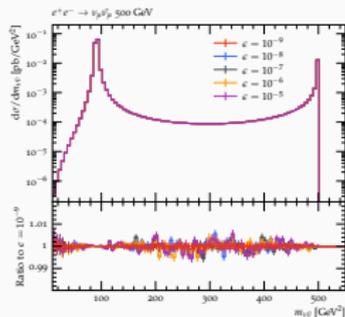


- Ideally ϵ, δ do not act as IR regulators, that is left to the parton shower IR cutoff $t_c \approx m_e^2$

The electron structure function

Parton shower dependence on technical parameters

Parton shower dependence on technical parameters



NLO matching

What is NLO matching?

NLO matching: producing a prediction for an IR-safe observable $\langle O \rangle$ which contains the parton shower resummation but which gives the correct NLO value for the observable:

$$\langle O \rangle^{\text{Matched}} = \langle O \rangle^{\text{NLO}} + \mathcal{O}(\alpha^{m+2}) \quad \text{if the LO process is } \mathcal{O}(\alpha^m)$$

Crucially, we must avoid double counting of the first emission.

MC@NLO gives us a process-independent way of constructing the calculation, assuming we have:

- ✓ A QED parton shower
- ✓ All the relevant amplitudes and subtraction terms at NLO

$$\begin{aligned}
 \langle O \rangle^{\text{MC@NLO}} = & \int d\Phi_n \bar{B} \left[\underbrace{\bar{\Delta}_n(t_c, \mu_Q^2) O(\{p_n\})}_{\text{unresolved, singular}} + \underbrace{\sum_{ij,k} \int d\Phi_1 O(\{p_{n+1}\}) \frac{D_{ij,k}^A}{B} \bar{\Delta}_n(t_c, \mu_Q^2)}_{\text{resolved, singular}} \right] \\
 & + \int d\Phi_{n+1} \left[\underbrace{R - \sum_{ij,k} D_{ij,k}^A}_{\text{resolved, non-singular}} O(\{p_{n+1}\}) \right]
 \end{aligned}$$

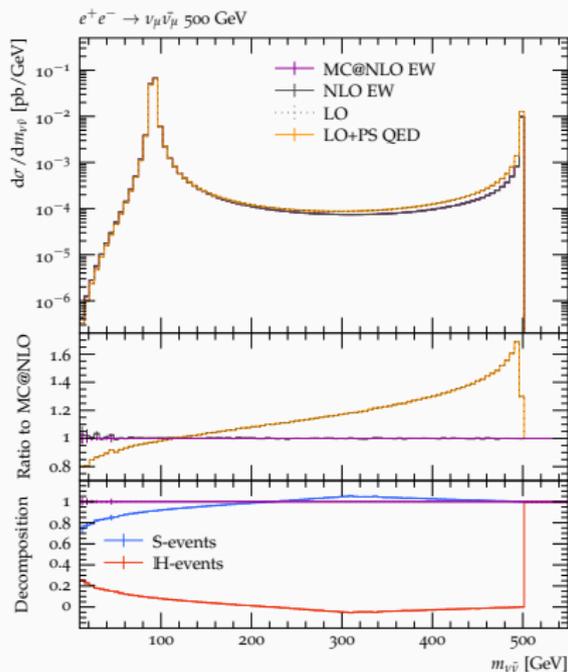
1. Generate \mathbb{S} -event (using the \bar{B} ME squared) or \mathbb{H} -event (using $R - D^A$) according to their relative probabilities
2. If \mathbb{H} -event, pass straight to standard parton shower
3. If \mathbb{S} -event, use one-step shower with D^A as kernels
 - If no emission, event has Born kinematics
 - If emission at $t_{ij,k}$, pass to standard shower for further showering $t < t_{ij,k}$

We now have all the ingredients for the initial-state MC@NLO

- ✓ MC@NLO formalism worked out for EW
- ✓ Suitable identification of scales μ_F, μ_Q
- ✓ QED parton shower with all splitting functions and dipole efficiency options
- ✓ New regularisation scheme for electron structure function
- ✓ Good overestimate for ratio of initial luminosities
- ✓ Semi-analytic integration of KP subtraction terms
- ✓ Mixed EW scheme to absorb as many higher-order effects as possible

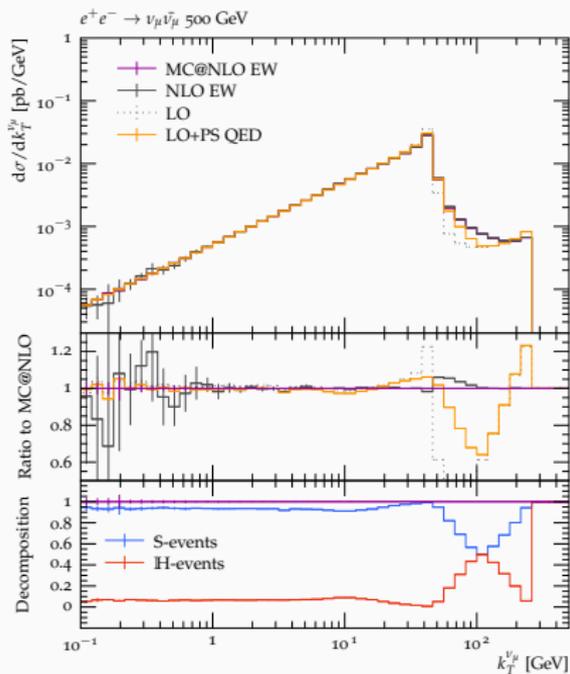
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$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV



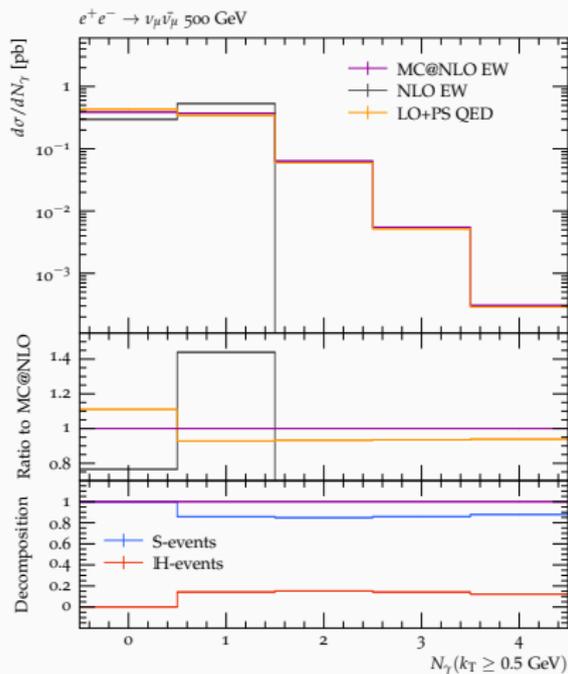
- Invariant mass is unchanged by initial-state shower emissions
- Shape changes quantify virtual and real NLO contributions
- Lower ratio plot shows makeup in terms of \mathbb{S} and \mathbb{H}

$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV



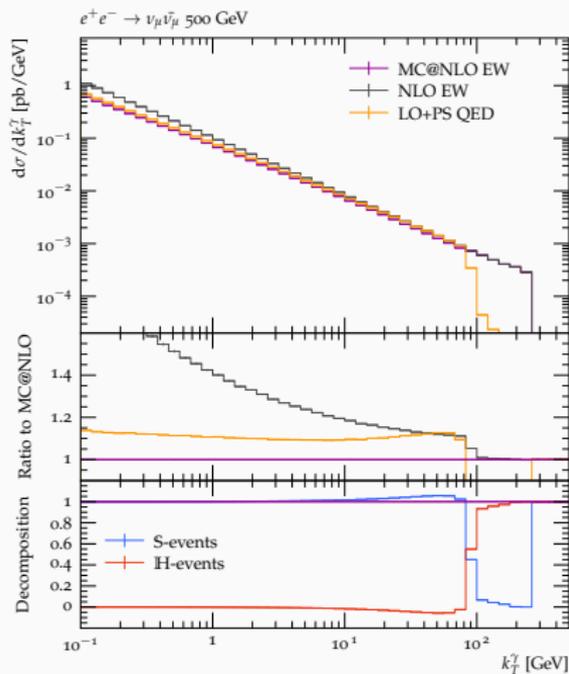
- $k_T^{\nu\mu}$ affected by shower at LO but much smaller effect at NLO
- Very good agreement with fixed-order in tail

$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV



- H events contribute positively to extra photon production
- LO+PS, MC@NLO and S-event ratio flat beyond first emission
- Cut used here is 0.5 GeV

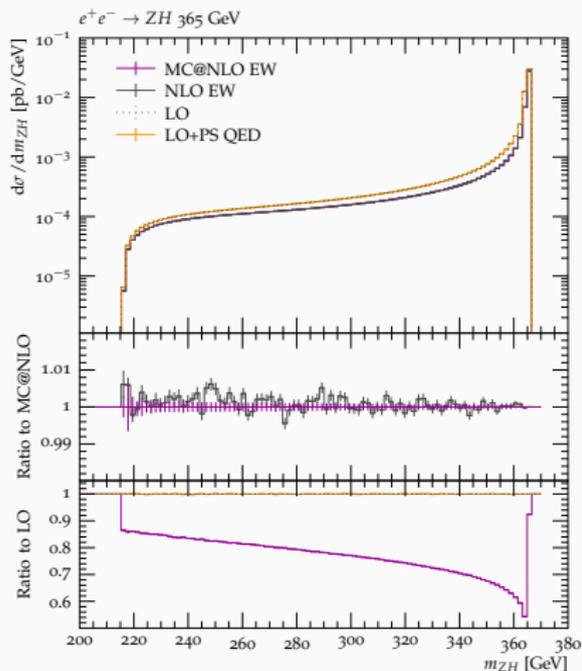
$e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ at 500 GeV



- Excellent agreement with NLO at high k_T^γ where only H-events contribute
- Shower-like behaviour at low k_T^γ
- Unlike QCD, Sudakov peak \ll detectable photon energy, but divergence of R still regularised

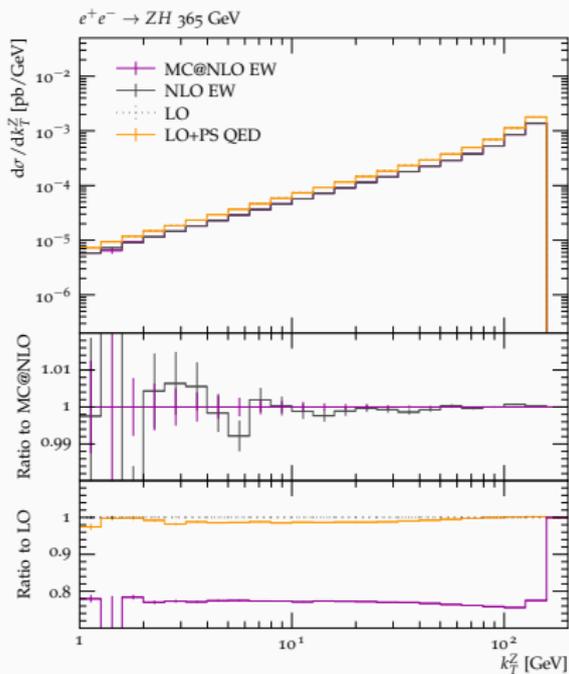
Results: ZH production at 365 GeV

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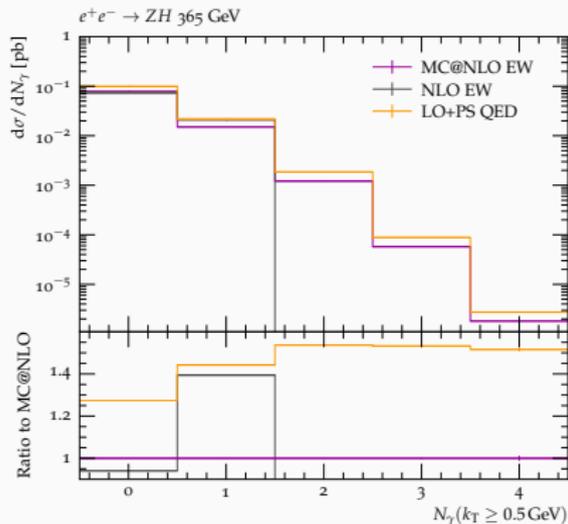
- PS does not change prediction
- Excellent agreement with fixed-order
- NLO correction large and negative
- Local K -factor up to 40% just below peak

Results: ZH production at 365 GeV



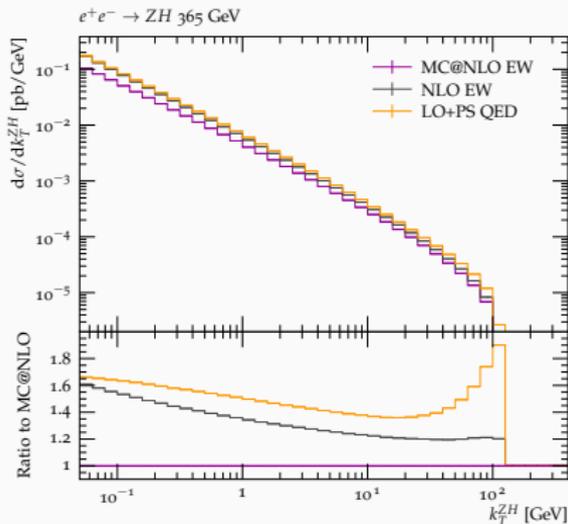
- Z p_T mostly unchanged by parton shower
- NLO correction is large but flat
- MC@NLO samples tail better

Results: ZH production at 365 GeV



- MC@NLO corrects overproduction of photons by LO+PS
- Ratio is again flat beyond 1 photon

Results: ZH production at 365 GeV



- ZH k_T highly correlated with hardest photon k_T
- Unlike $\nu\bar{\nu}$, no exact agreement between MC@NLO and NLO at high k_T
- This is due to different resonance structure & scales
- MC@NLO flattens out quicker than LO+PS at low k_T

Conclusions

What we did:

- Created an NLO+PS method that is largely process-independent and automated (applicable to future colliders *and* past/current low-energy colliders)
- Produced the first EW NLO+PS prediction for $e^+e^- \rightarrow ZH$

Next steps/work in progress:

- Apply to low-energy $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \pi^+\pi^-$
- Compare with BABAYAGA, YFS, fixed order
 - `rivet` interface to BABAYAGA thanks to Alan Price
 - Automated one-loop amplitudes for pions thanks to Jonas Lindert

Future-proofing our Monte Carlos: at the SHERPA collaboration meeting I learned that more of us than ever are working on QED/EW physics!

Thanks for listening!

arxiv:2603.05585

Backup

Backup: Dipole efficiency

- For efficiency in processes where there are many charged particles around
- Allow only the **opposite-sign-same-flavour particle** which is **closest in invariant mass** as spectator
- Not a perfect scheme, but pretty good e.g. in Higgs decay to 4 leptons
- Can separately turn off initial-final interference
- New, better scheme in the works

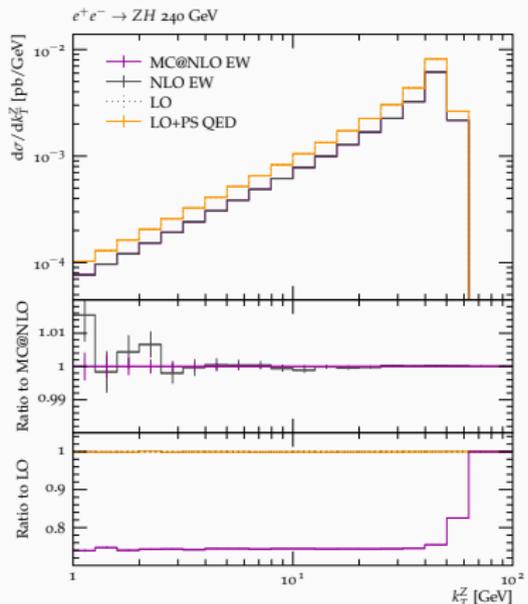
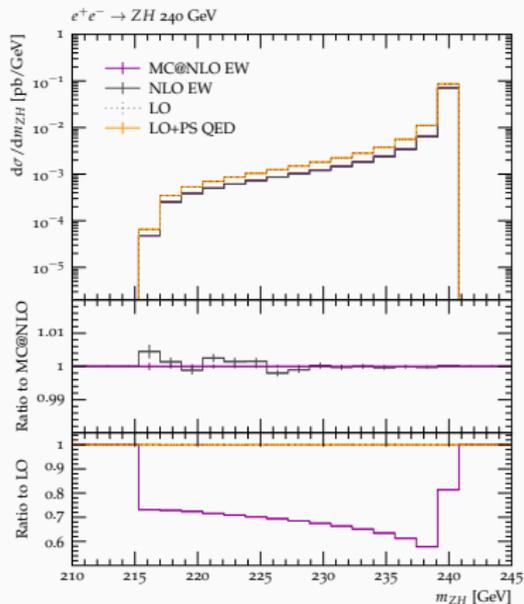
Backup: Scale identification

- 'Core scale' for Born process identified as shower starting scale μ_Q
- In $e^+e^- \rightarrow$ neutral, choose neutral invariant mass
- In $e^+e^- \rightarrow$ charged, not 100% clear, but in our implementation we choose $\sqrt{s'} = (p_{e^+} + p_{e^-})$
- Use MEPS-type clustering for identification of hard scale when $n > 4$ (e.g. in \mathbb{H} -events)
- Two possible cases:
 - $t_H < \mu_Q^2$ The shower should start from t_H to avoid an unordered configuration and double counting
 - $t_H > \mu_Q^2$ Not clear-cut, t_H is now the hardest scale, but do not want to extend parton shower beyond its region of validity
- We choose to start the shower from t_H in both cases

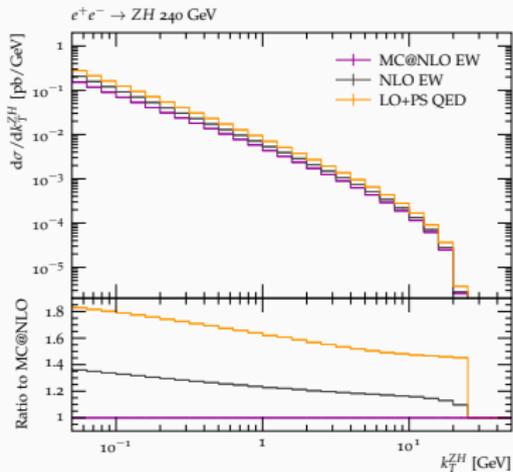
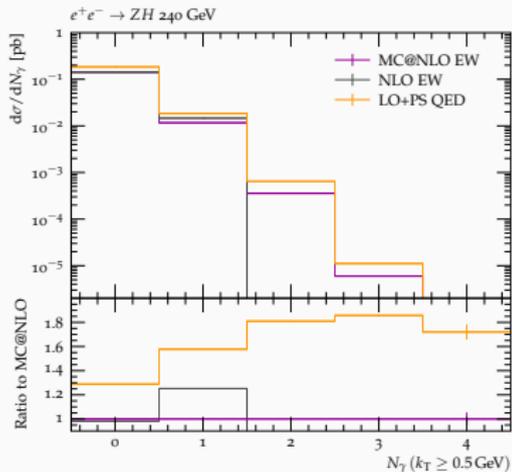
Backup: Mixed electroweak scheme

- For the hard scattering, helpful to use an EW scheme which resums higher-order corrections, e.g. G_μ scheme, $\alpha(m_Z)$ scheme
- Real (long-distance) photons should always couple with Thomson limit $\alpha(0)$ since their wavefunction renormalisation cancels logarithms in α renormalisation
- To reconcile this, define mixed scheme [Denner, Dittmaier '19](#)
 - All NLO parts of the calculation are defined in the $\alpha(0)$ scheme
 - All Born parts are defined in G_μ scheme

Backup: $e^+e^- \rightarrow ZH$ at 240 GeV



Backup: $e^+e^- \rightarrow ZH$ at 240 GeV



Backup: Validation of MC@NLO one-step shower

