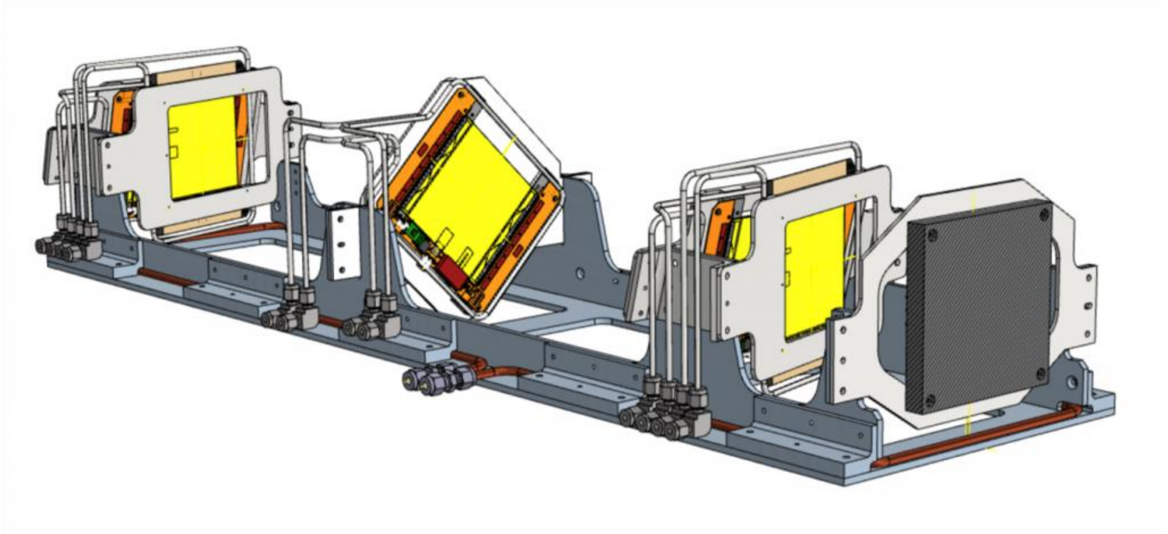
Determination of the muon beam energy at the MUonE experiment

Tom Lenane | University of Liverpool

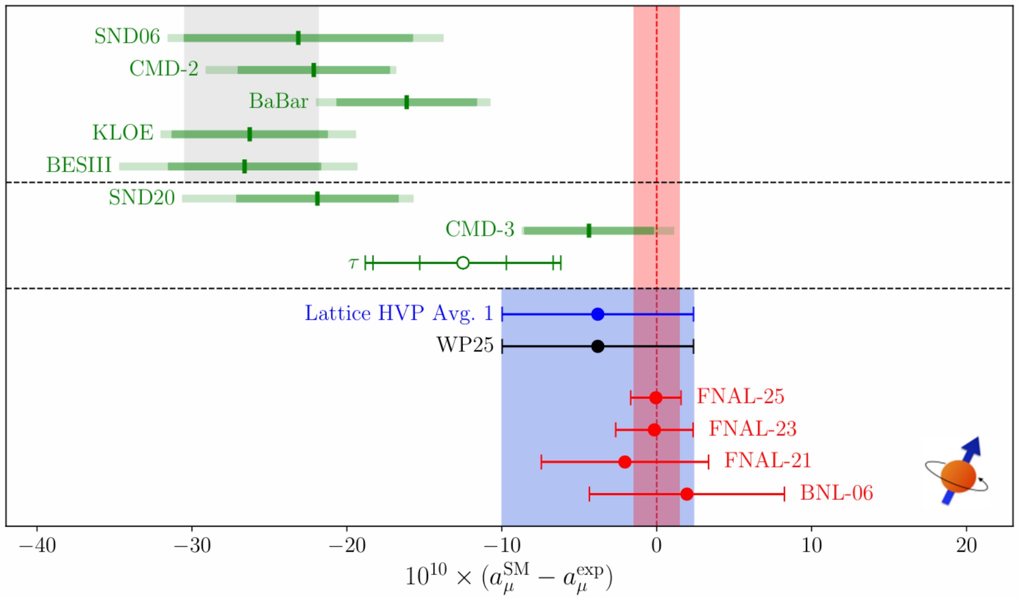
Supervisors: Saskia Charity, Riccardo Pilato, Graziano Venanzoni

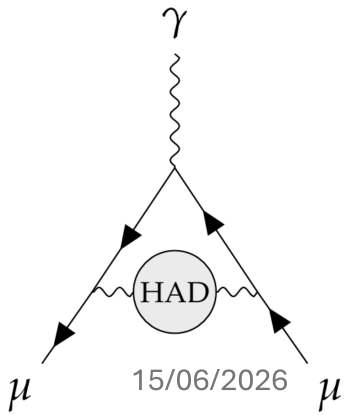


MUonE: Motivation

Tension in theory

- HVP contribution to a_μ non perturbative.
- Tension between lattice QCD and data-driven dispersive methods.
- Also a tension between different dispersive groups (KLOE/BaBAR/CMD3) calculating the $e^+e^- \to \pi^+\pi^-$ channel.

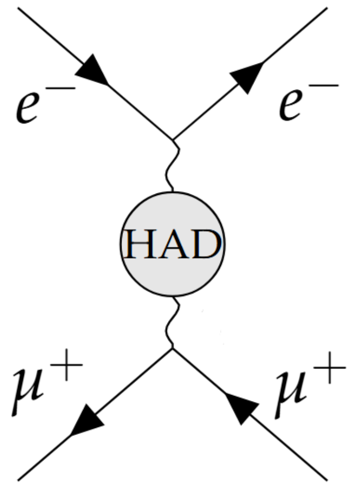




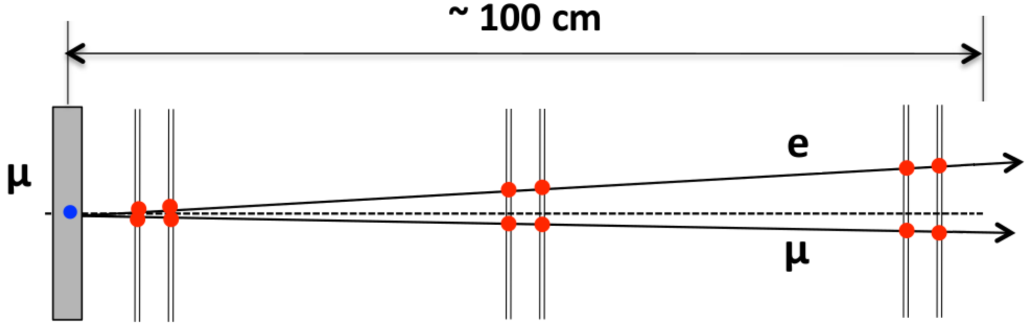
MUonE: Experimental Goal

MUonE Goal

- Calculate the LO HVP contribution in the space-like region.
- Completely different systematics to lattice/dispersive methods. Provides an independent calculation of LO HVP contribution.
- Measure elastic scattering of 160GeV muons on atomic electrons.
- From scattering angles we determine the differential cross-section and $\Delta\alpha_{\mathrm{had}}$.
- MUonE requires 0.3% precision on $\alpha_{\mu}^{\mathrm{HVP,\,LO}}$.
- Need to control shape of $\mu - \mathrm{e}$ differential cross section with 10ppm systematic error to achieve this.



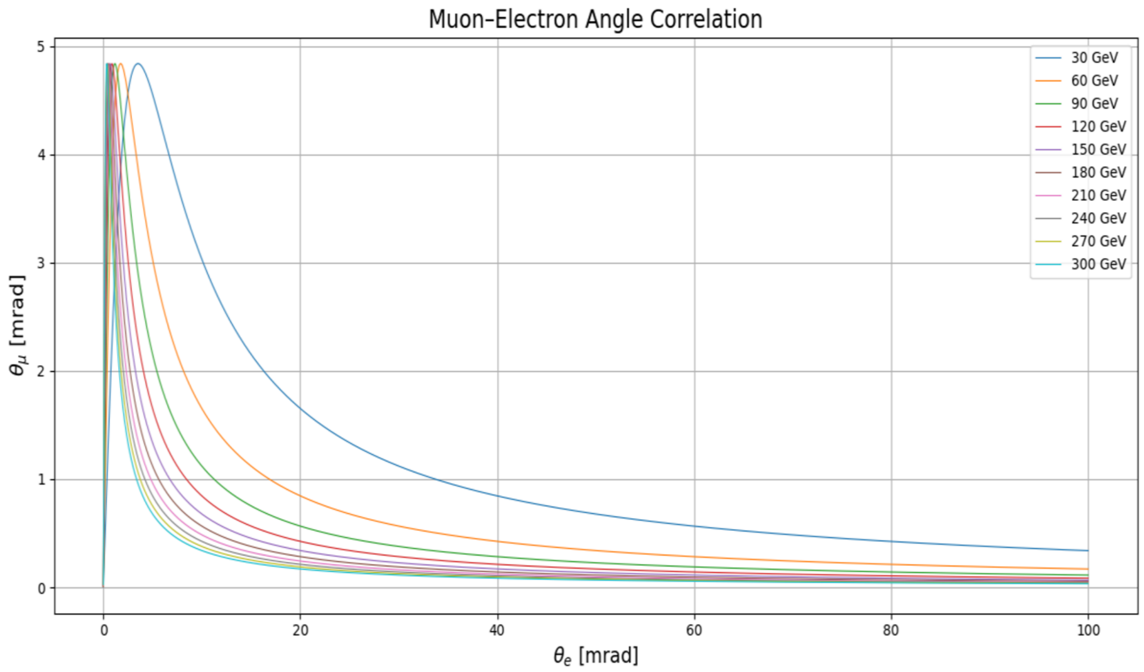
$$a_{\mu}^{\mathrm{HVP,\,LO}} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\mathrm{had}}[t(x)]$$



Beam Energy Requirements

Beam Energy

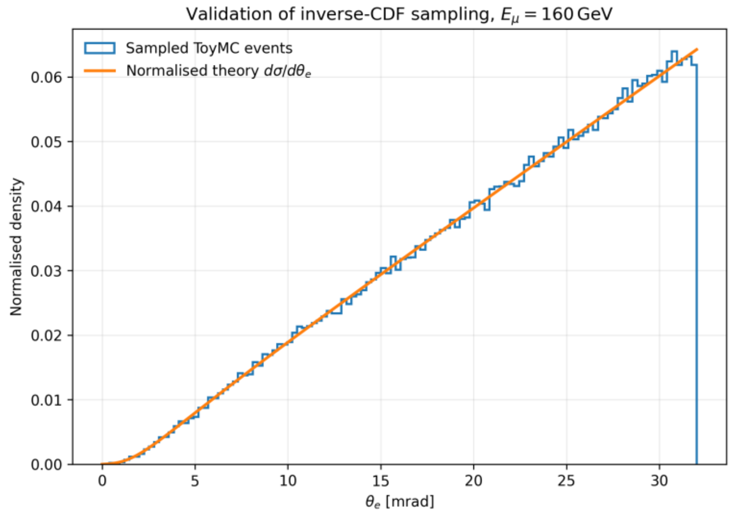
- We need knowledge of the beam energy to ~5MeV for a 160 GeV muon beam.
- The shape of the differential cross section is dependent on the shape of the elastic scattering correlation curve.
- The elastic scattering curve changes with the the incoming beam energy.



Toy MC for Investigating Beam Energy Extraction

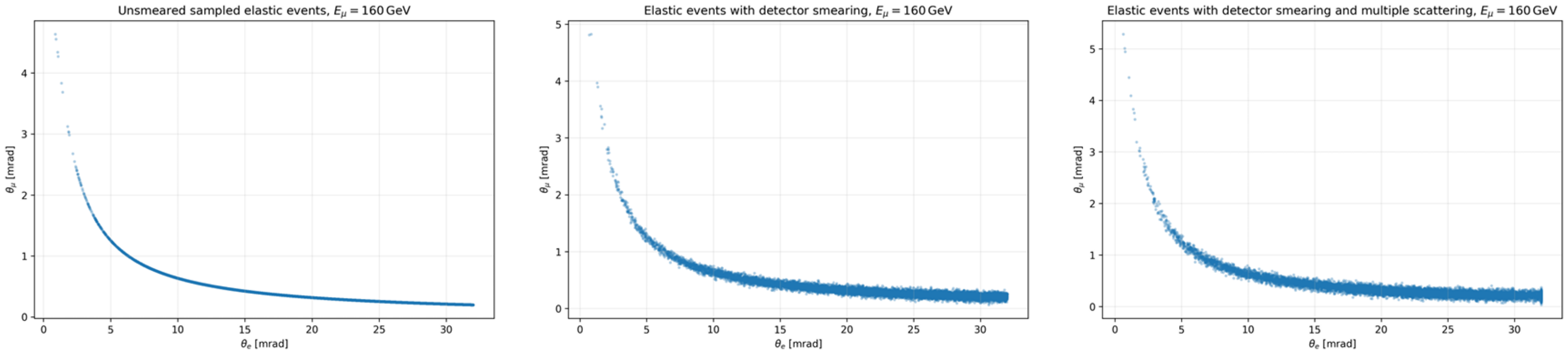
Event Generation from toy MC

- For a monoenergetic or gaussian beam energy, events are generated from the LO differential cross section.
- Angles are smeared to account for detector resolution and multiple scattering in the target.



Energy Extraction

- Toy MC pseudo-data used to develop methods for extracting beam energy.
- **Template fit, CNF, equal angle methods**.



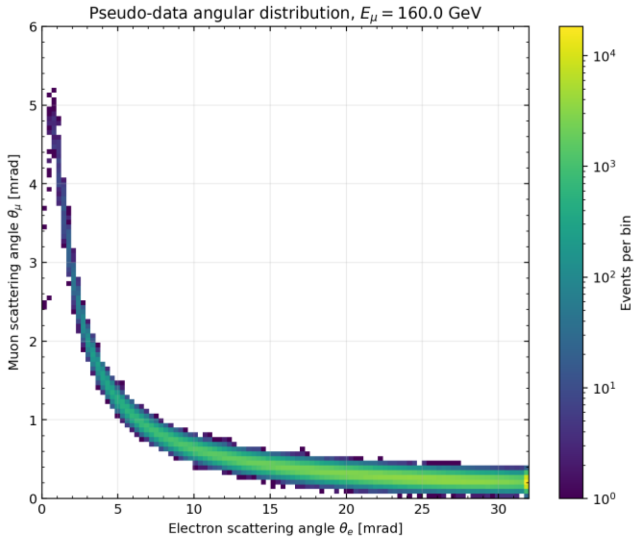
(a) No smearing (b) Detector resolution (c) Detector resolution + MS

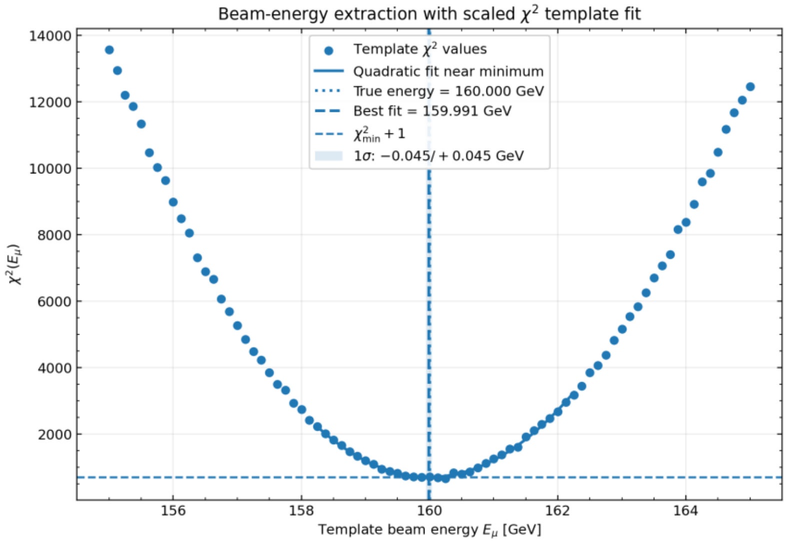
χ^2 Template Fit Method

Template Fit

- 2D (θ_e, θ_μ) histogram at a known true beam energy of 160 GeV.
- Toy MC generates template datasets from a range of energies (154-166 GeV).
- Binned χ^2 statistic plotted as a function of the template energy.
- Beam energy taken as minimum of parabolic fit.
- χ^2 vs template energy plot with fitted parabola from 500k events. Results for monoenergetic 160 GeV beam energy were $\mathbf{159.99 \pm 0.05\ GeV}$.

$$\chi^2(E_\mu) = \sum_i \frac{\left(H_{\mathrm{data},i} - H_{\mathrm{MC},i}(E_\mu)\right)^2}{H_{\mathrm{data},i} + H_{\mathrm{MC},i}(E_\mu)}$$

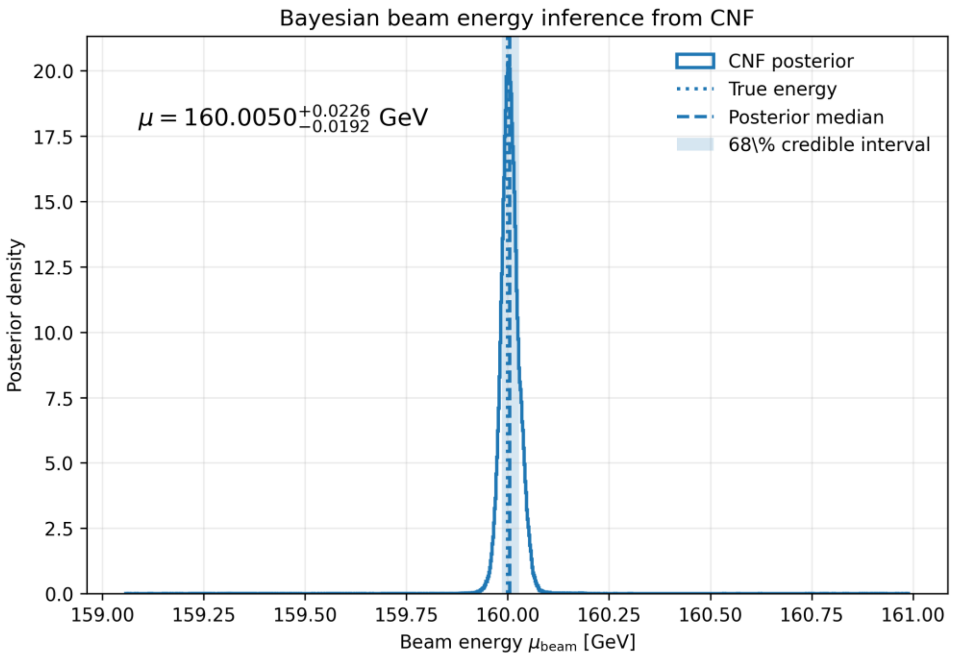




Conditional Normalising Flow (CNF) Method

CNF Method

- The CNF aims to learn the posterior probability density $p(\mu_{beam}|\theta_e, \theta_\mu)$.
- Trained flow produces posterior. Inferred beam energy is mean of posterior and uncertainty is the width.
- Posterior for 500k events from a monoenergetic 160 GeV beam energy gives a measured beam energy of $\mathbf{160.00 \pm 0.02\ GeV}$.



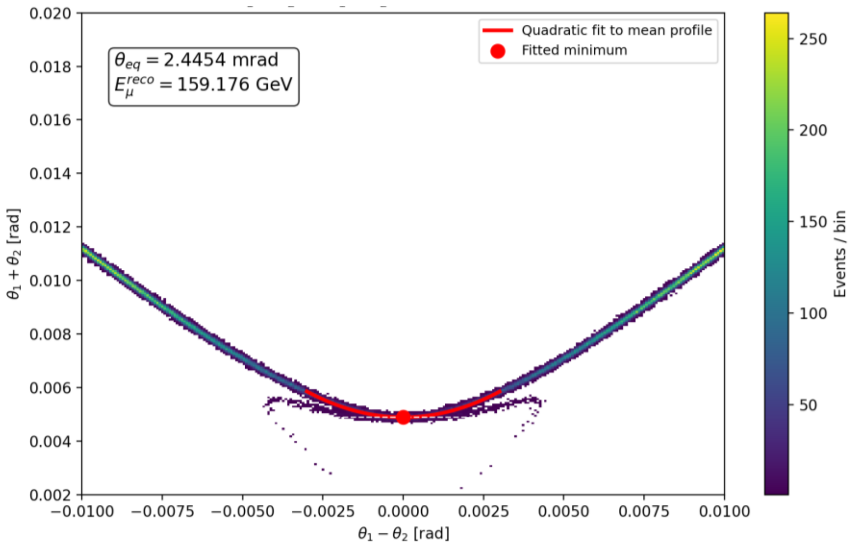
$$p(\mu_{beam}|\theta_e, \theta_\mu) = p_Z(f^{-1}(\mu_{beam}; \theta_e, \theta_\mu)) \left| \det \frac{\partial f^{-1}(\mu_{beam}; \theta_e, \theta_\mu)}{\partial \mu_{beam}} \right|$$

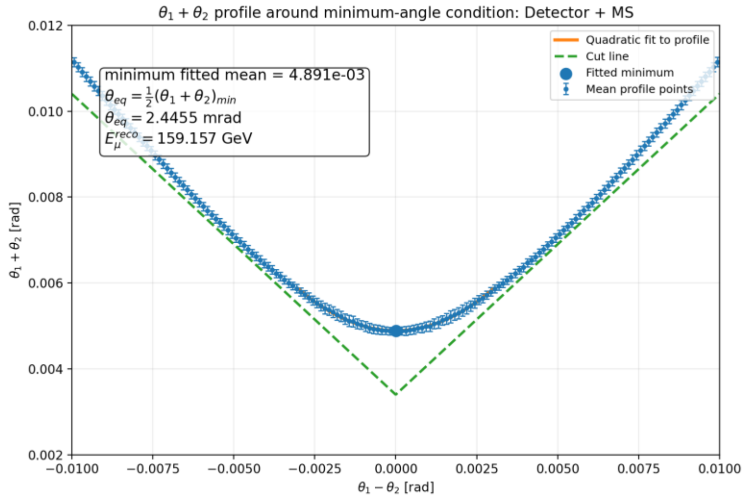
Equal Angles Method

Equal Angles Method

- $\theta_\mathrm{e} = \theta_\mu \equiv \theta_\mathrm{eq}$ region.
- Uses relation between the θ_eq and the beam energy E_μ.
- Plot $\theta_1 + \theta_2$ against $\theta_1 - \theta_2$ (PID not needed), bin in $\theta_1 - \theta_2$ and make profile histogram.
- The minimum point corresponds to $2\theta_\mathrm{eq}$ from which the energy can be extracted.
- To make profile plot we need to cut low θ_e tails.
- Profile for 1M events from a monoenergetic 160 GeV beam energy gave a beam energy of **159.157 GeV**.
- Work in progress to assign a systematic uncertainty to this method and improve robustness.

$$\cos\theta_\mathrm{eq} = \frac{E_\mu + m_e}{\sqrt{(E_\mu + 2m_e)^2 - m_\mu^2}}$$





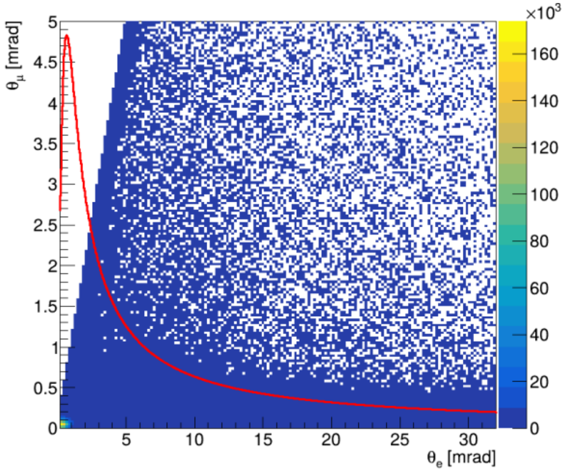
Elastic Event Selection Cuts on 2025 data

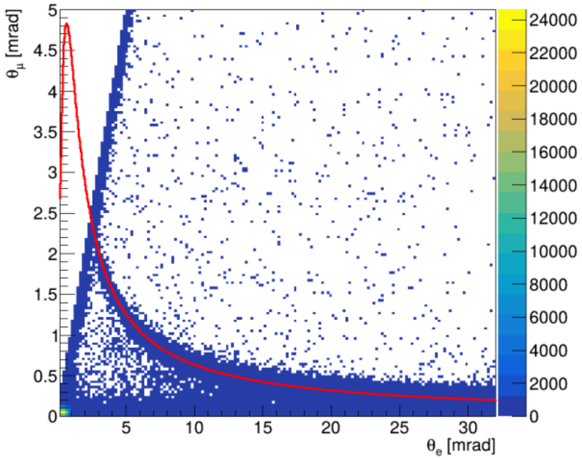
Event Selection

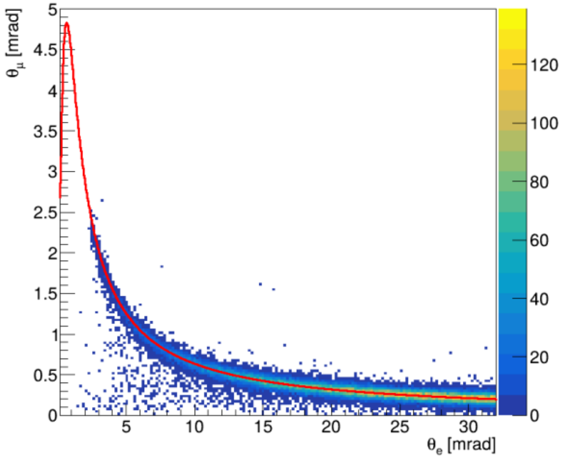
- Beam energy extraction methods will be applied to data and full MC.
- Work has been done on elastic event selection to remove backgrounds.

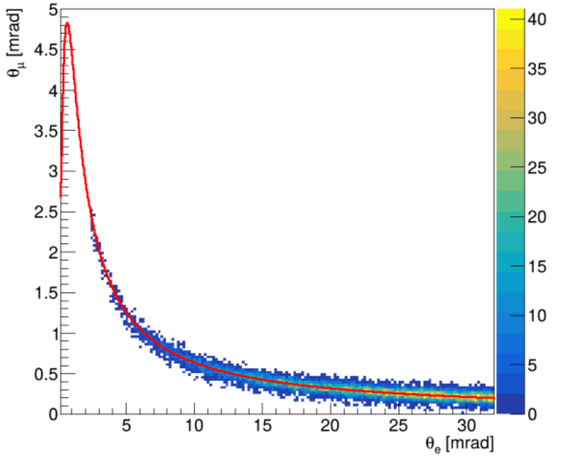
Cuts

1. Electron and muon angle range.
2. Number of hits in the trackers.
3. Acoplanarity.
4. Interaction position along beam axis.
5. χ^2 vertex cut.
6. Fiducial region on target.
7. Elastic cut.









Conclusions and Future Work

- The beam energy is the most important systematic to be understood in MUonE. Three methods are being developed (template fit, CNF, equal angle) to accurately measure the energy.

- Template fit and CNF methods: robust in extracting the beam energy from the scattering angle correlation. Further work needed to understand how far the precision of these methods can be pushed.

- Work in progress: using the CNF to extract the mean of a gaussian toy MC beam energy and a more realistic beam profile (see backup slides).

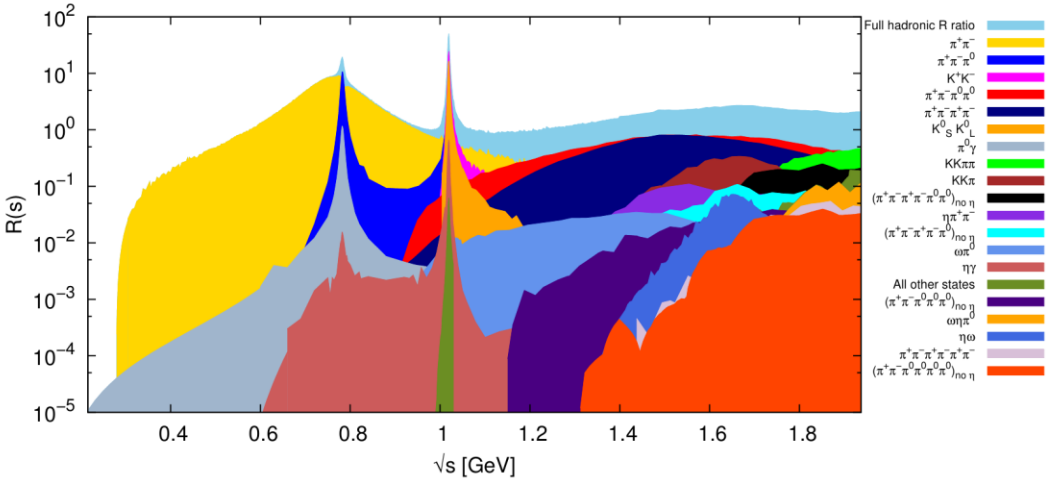
- Equal angle method: not yet successful for toy MC data, further work is needed to assign a systematic uncertainty to this method.

- The goal of this work is to be able to use these methods with the 2025 data to provide an energy measurement for the upcoming proof-of-principle measurement of $\Delta\alpha_{\text{lep}}$.

Backup: MUonE Advantage

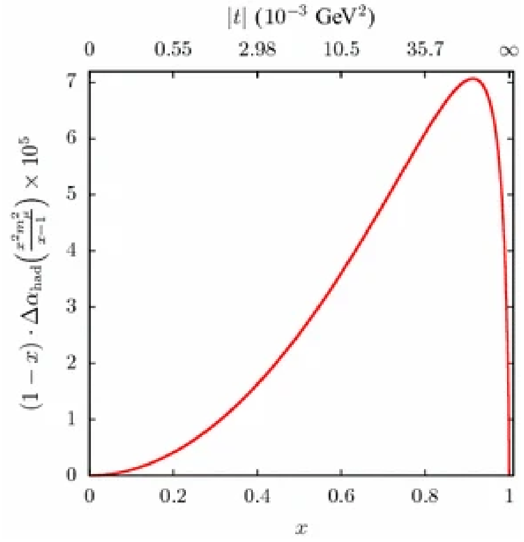
Difficulty with theory calculations

- The R-ratio plots from data-driven methods has resonant peaks.
- Resonances are hard to integrate.
- Systematics introduced when combining all channels.



MUonE Advantage

- Space-like integrand is smooth and free of resonances meaning it is easier to compute than the time-like dispersive integrals.
- One measurement.



Backup: MUonE Calculation

MUonE calculation

- At LO the running of α can be factored out of the differential cross section and from this we can calculate $a_\mu^{\text{HVP, LO}}$ with the space-like integral.

$$\frac{d\sigma}{dt} = \frac{d\sigma_0}{dt}\left|\frac{\alpha(t)}{\alpha}\right|$$

$$\frac{d\sigma/dt}{d\sigma_0/dt} = \frac{1}{|1-\Delta\alpha(t)|} \simeq 1 + 2\Delta\alpha(t)$$

$$a_\mu^{\text{HVP, LO}} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\text{had}}[t(x)]$$

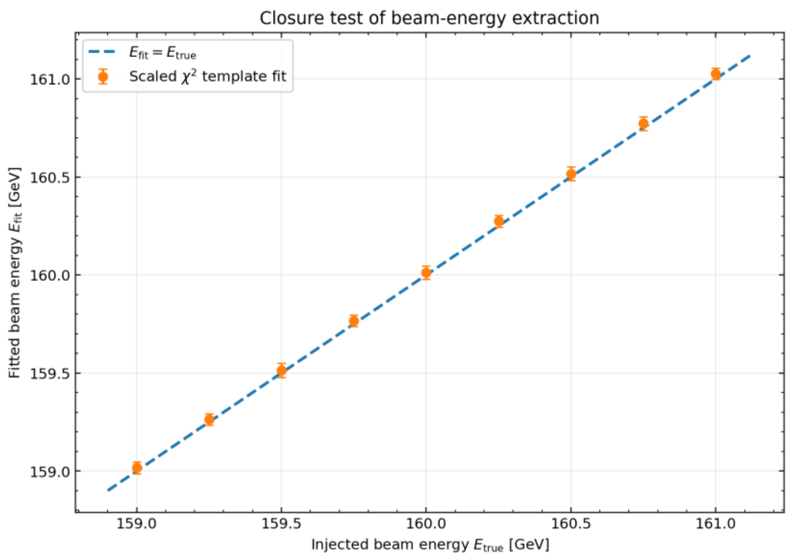
Backup: Toy MC event generation.

- Evaluates cross-section as a function of electron scattering angle, $d\sigma/d\theta_e$ over an angular grid up to 32mrad.
- Random samples of θ_e are generated by sampling from the differential cross section.
- θ_μ calculated analytically from θ_e.

$$\sin\theta_\mu = \sin\theta_e \sqrt{\frac{E_e^2(\theta_e) - m_e^2}{[E_\mu + m_e - E_e(\theta_e)]^2 - m_\mu^2}}$$

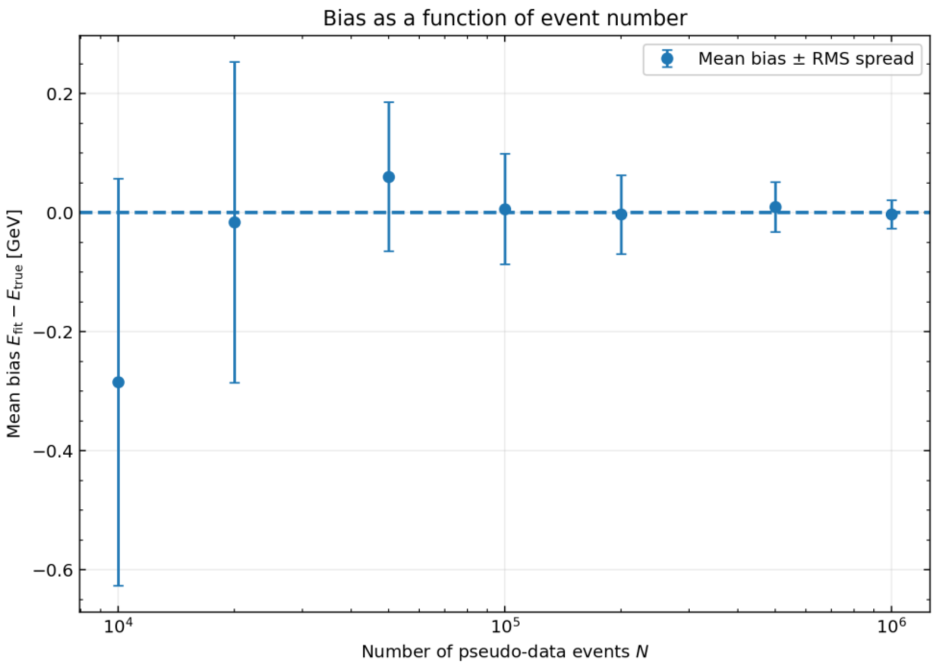
Backup: Template fit further analysis

- Results for 9 datasets with true beam energy between 159-161GeV shown to lie on expected diagonal line.
- Running 80 different pseudo experiments at the same beam energy resulted in a mean bias of +0.02 GeV and RMS of biases of 0.04 GeV.

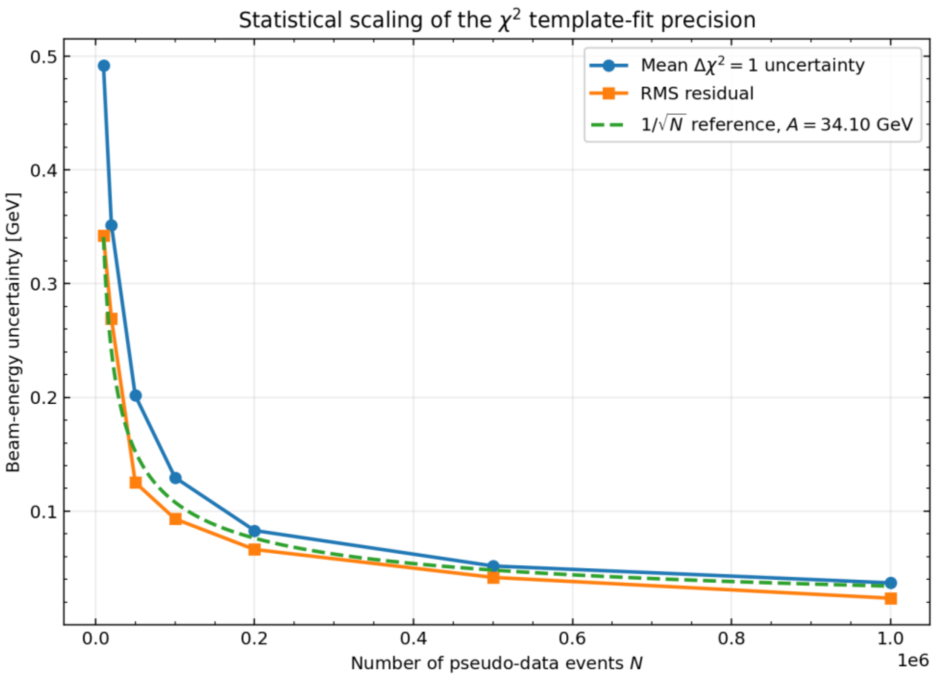


Backup: Template Fit Precision Scaling

Bias vs number of events



Uncertainties vs number of events



Backup: CNF Method Futher Detail

CNF Method

- The CNF aims to learn the posterior probability density $\mathrm{p}(\mu_{\mathrm{beam}}|\theta_{\mathrm{e}}, \theta_{\mu})$.
- The flow defines an invertible mapping between a variable from a simple normal distribution $\mathrm{z} \sim \mathrm{p_Z}(\mathrm{z})$ to the beam energy parameter, $\mu_{\mathrm{beam}} = f\left(z; \theta_{\mathrm{e}}, \theta_{\mu}\right)$.
- The transformation is conditioned on the vector s.
- Using the inverse transformation and change of variables formula we can define the conditional density:

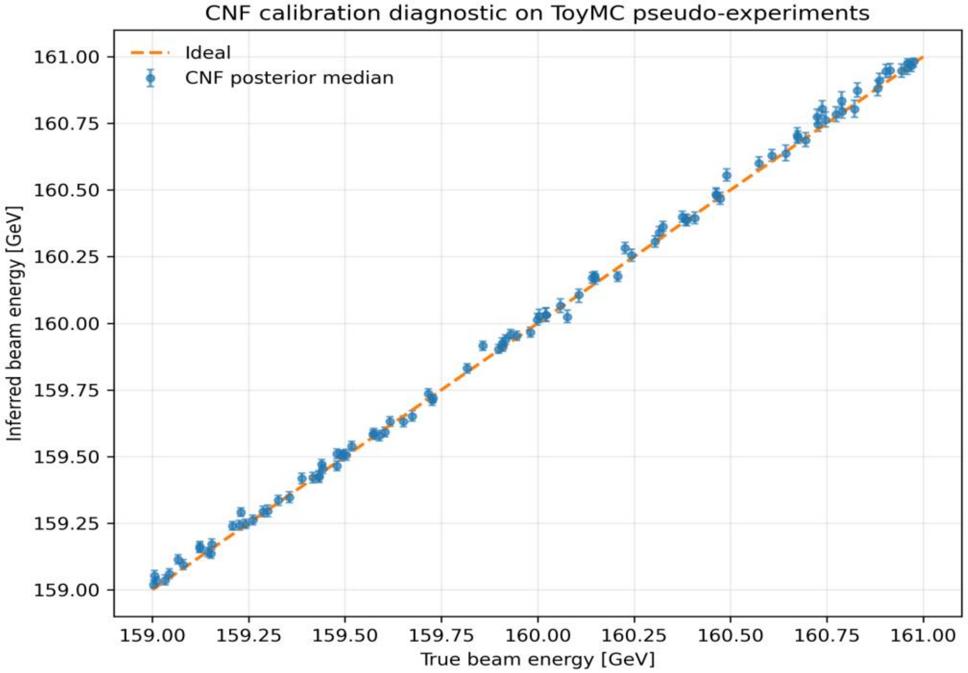
$$\mathrm{p}(\mu_{\mathrm{beam}}|\theta_{\mathrm{e}}, \theta_{\mu}) = \mathrm{p_Z}(\mathrm{f}^{-1}(\mu_{\mathrm{beam}}; \theta_{\mathrm{e}}, \theta_{\mu})) \left|\det \frac{\partial \mathrm{f}^{-1}(\mu_{\mathrm{beam}}; \theta_{\mathrm{e}}, \theta_{\mu})}{\partial \mu_{\mathrm{beam}}}\right|$$

- Taking logs simplifies to an addition and the flow is optimised by minimising the negative log likelihood over a set of toy MC experiments:

$$\mathcal{L} = -\sum_{\mathrm{i}=1}^{\mathrm{N}} \log \mathrm{p}(\mu_{\mathrm{beam,\,i}}|(\theta_{\mathrm{e}}, \theta_{\mu})_{\mathrm{i}})$$

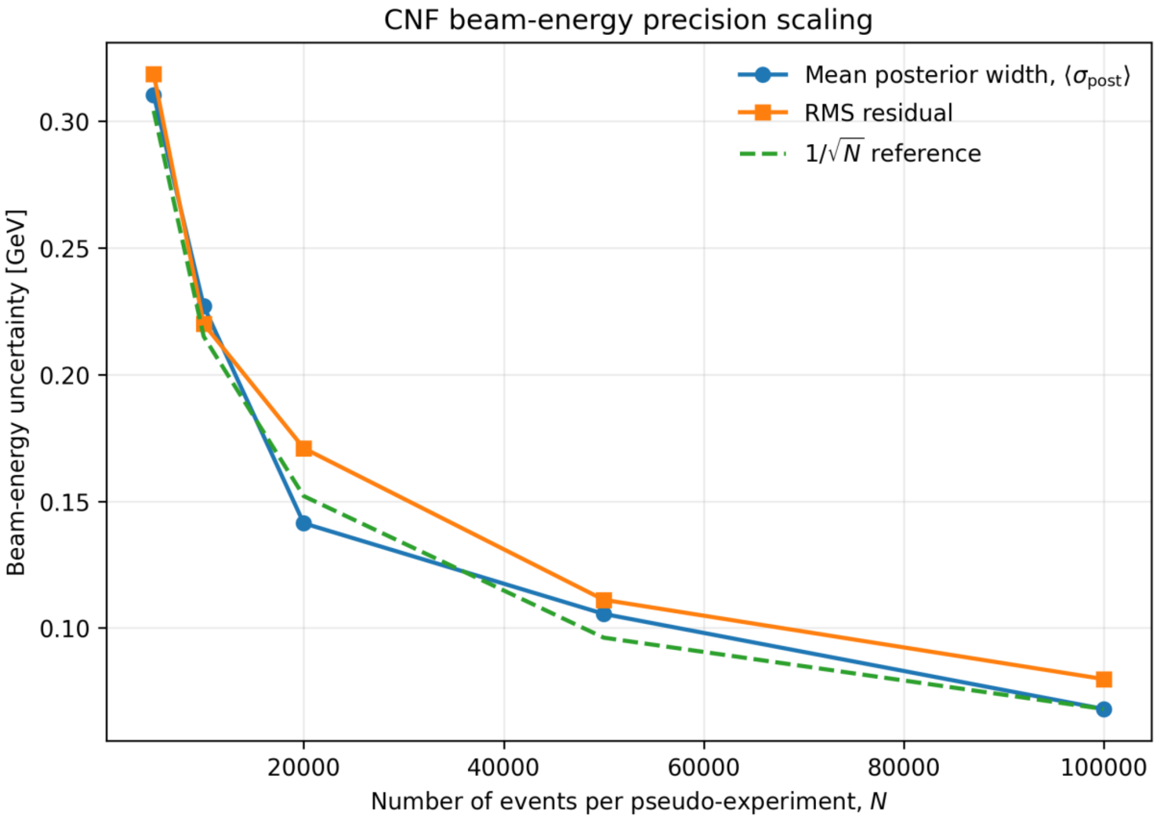
Backup: CNF Method Futher Analysis

- Results for many different toy MC experiments with beam energy between 159-161 shown to lie on the expected diagonal line.
- Running 100 different experiments at the same beam energy resulted in a mean bias of +3.9MeV and RMS of biases of 0.02 GeV.



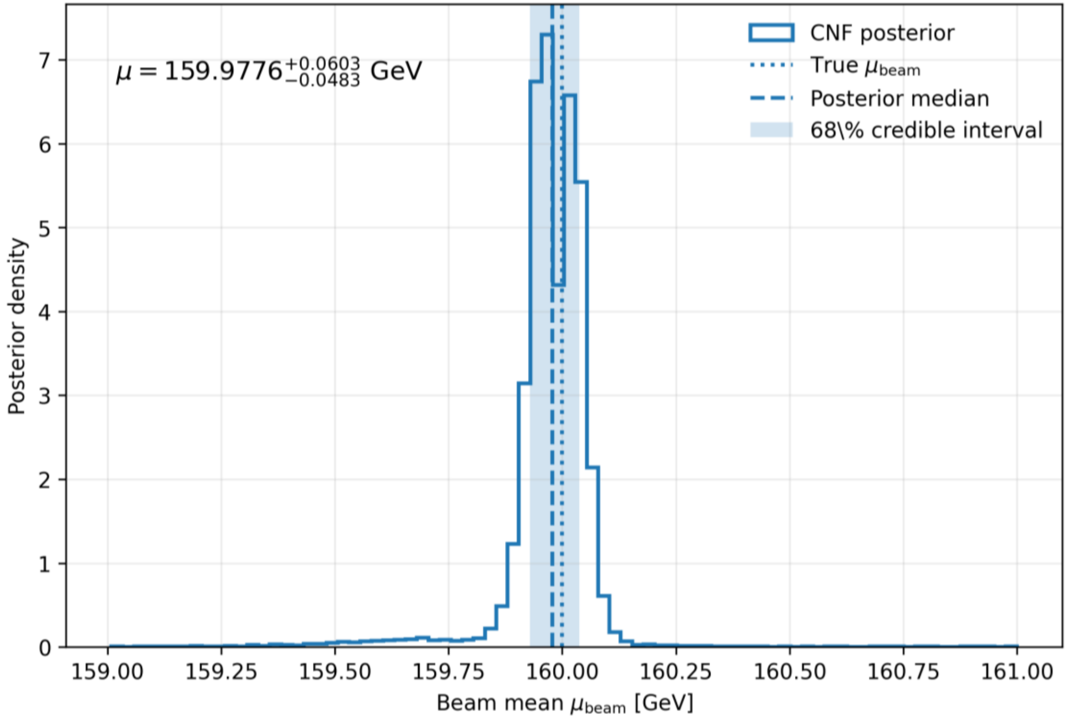
Backup: CNF Precision Scaling

Uncertainties vs number of events

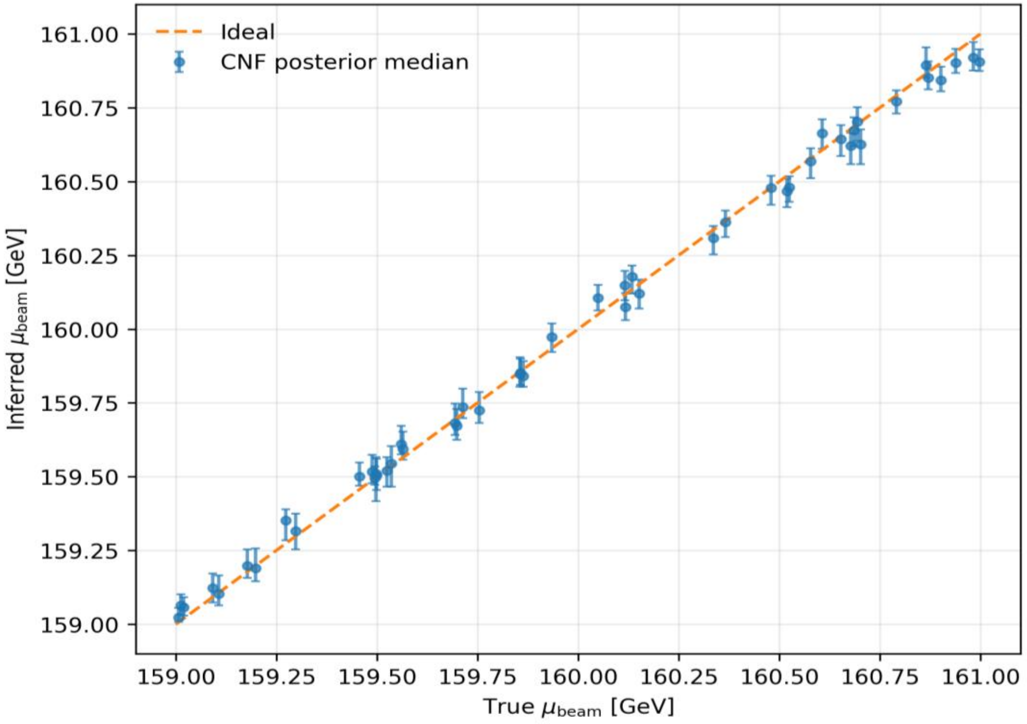


Backup: Gaussian Beam Energy

Posterior for Gaussian beam mean



Beam mean extracted for 159-161 GeV true beam means



Backup: Equal angle cut inefficiency

- Plotting the spread of the value in each bin shows that the tails from the low electron angles increase the spread despite the V-shaped cut.
- Further work is needed to reduce the increase in spread in these areas which effects the fit and therefore the inferred beam energy.