

Theory Efforts

(a biased selection)



(Justus) Tobias Tsang
j.t.tsang@liverpool.ac.uk



2026 HEP meeting — UoL

21 May 2026

For the Theory Group

Outline

- 1 the theory group
- 2 a (very brief) introduction to “the lattice”
- 3 some selected applications
 - dark matter
 - precision quark-flavour
 - $g - 2$
 - finite temperature and density
- 4 summary

the theory group

Theoretical Physics @ UoL: who we are

Strings

Alon



Radu



Susha



Thomas M



QFT+(astro-)particle pheno

John



Juri



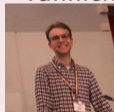
Martin



Thomas T



Yannick

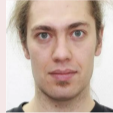


Lattice QFT

David



Pavel



Simon



Tobi



+ ~ 8 postdocs, ~ 20 PhD students and several emeriti

- Section lead:
Martin Gorbahn

- Deputy HoD Research:
Thomas Teubner
- Departmental AI lead:
Radu Tatar

Weekly events:

- theory colloquium
- lattice/string/pheno meeting
- student-led seminar

Theoretical Physics @ UoL: (some of) what we do

(some) interaction points with DoPhysics

- $g - 2$:
 - ⇒ “data-driven” (Thomas T, Yannick)
 - ⇒ “lattice” (Tobi)
 - precision QFT & Flavour:
John, Martin, Thomas T, Tobi, Yannick,
 - Quantum technologies:
 - ⇒ Q sensing (Juri)
 - ⇒ Q computing (David, Pavel)
 - Cosmo/Astro and Dark Matter:
David, Juri, Martin, Radu, Susha
- ...strings & pheno, BSM models, condensed matter, QCD phase diagram, future colliders...

Disclaimers

- wide spectrum of topics with fluid boundaries
- I am *not* an expert in most of these
- far too much to cover in 17+3 minutes

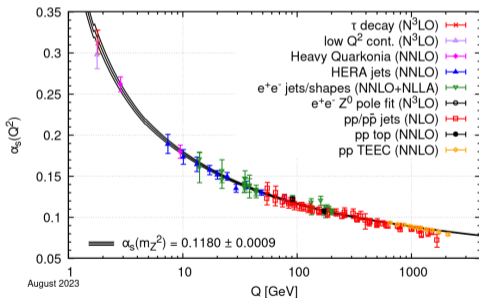
a (very brief) introduction to “the lattice”

The challenge: Quantum Chromodynamics and colour confinement

Theory

quarks and gluons

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} G_{\mu\nu}^a G^{\mu\nu,a}$$



[PDG]

Lattice QCD simulations provide **first principle predictions**.

Experiment

hadronic bound states



perturbative at high Q
 \Rightarrow “standard” methods work

non-perturbative at low Q
 \Rightarrow “standard” methods fail

Consequence: hadronic uncertainties
dominate in many situations

Lattice QCD in a nutshell

Based on the **Path Integral** formulation.

$$\langle \mathcal{O} \rangle_M = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \bar{\psi}, U] \mathcal{O}[\psi, \bar{\psi}, U] e^{iS[\psi, \bar{\psi}, U]}$$

Minkowski: Highly oscillatory, infinite dimensional integral.

✗

⇒ Wick rotate to Euclidean (i.e. imaginary) time ($t \rightarrow i\tau$).

$$\langle \mathcal{O} \rangle_E = \frac{1}{\mathcal{Z}} \int \mathcal{D}[\psi, \bar{\psi}, U] \mathcal{O}_E[\psi, \bar{\psi}, U] e^{-S_E[\psi, \bar{\psi}, U]}$$

Euclidean: Exponentially decaying, infinite dimensional integral.

✗

⇒ Discretise space-time and **interpret as a probability distribution.**

- Lattice spacing a (UV regulator)
- Box of length L (IR regulator)
- $\int \rightarrow \sum$, $\partial \rightarrow$ finite differences
- Evaluate **numerically**

Lattice: Exponentially decaying and finite dimensional

✓

Parameters of QCD and scale setting

Lattice QCD \equiv **first principles QCD**. Defined by parameters in the action:

- bare **coupling constant** g
- bare **quark masses** m_f

$$S_{\text{QCD}}[\psi, \bar{\psi}, U] = S_G[U] + S_F[\psi, \bar{\psi}, U]$$
$$= \int d^4x \frac{1}{2g^2} \text{Tr} [F_{\mu\nu} F_{\mu\nu}] + \sum_{N_f} \bar{\psi}_f (\gamma_\mu D_\mu + m_f) \psi_f$$

Simulations takes bare quantities, but want to reproduce real world (example $N_F = 2 + 1$):

- want to determine 3 parameters: $m_l (= m_u = m_d)$, m_s and $a(g)$
- need to sacrifice 3 predictions: for example m_π , m_K and m_Ω

Vary the quark masses m_l and m_s until

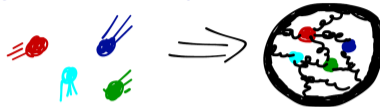
Require $(m_\Omega)_{\text{lat}} = (m_\Omega)_{\text{PDG}}$

$$\left(\frac{am_\pi}{am_\Omega} \right)_{\text{lat}} \equiv \left(\frac{m_\pi}{m_\Omega} \right)_{\text{PDG}} \quad \text{and} \quad \left(\frac{am_K}{am_\Omega} \right)_{\text{lat}} \equiv \left(\frac{m_K}{m_\Omega} \right)_{\text{PDG}} \quad a^{-1} = \frac{(m_\Omega)_{\text{PDG}}}{(am_\Omega)_{\text{lat}}}$$

Everything else is a (QCD) prediction

some selected applications

Lattice field theory studies of composite dark sectors



Combine early-universe interactions for relic density & thermalization
with present-day experimental constraints

Many novel features & signatures [cf. [arXiv:2602.23002](https://arxiv.org/abs/2602.23002) & refs therein]:

Direct detection via EM polarizability $\rightarrow \sigma \propto Z^4$ of nuclear target

Collider signatures beyond \cancel{E}_T + jets (e.g., $t\bar{t}b + t\bar{t}b\bar{b}$, [arXiv:2405.20061](https://arxiv.org/abs/2405.20061))

Stochastic gravitational waves from early-universe phase transition

Indirect detection via radiative decays

Diverse galactic sub-structure from self-interactions

- Distinct from more traditional WIMP and axion possibilities
- Lattice can make predictions relevant to all of the above!

\Rightarrow **lattice talk soon**

Over-constraining the Standard Model: CKM

- Determine individual elements from (many) different decays \Rightarrow Different determinations compatible?
- SM CKM-matrix is unitary \Rightarrow Does unitarity hold?

CKM = theory \otimes experiment

$$\text{experiment} \approx |V_{qq'}|^n \sum_i \text{kinematic} \times \text{non-perturbative}$$

$$\Gamma(B \rightarrow \ell \nu) \approx |V_{ub}|^2 \mathcal{K} f_B$$

$$\Delta m_d \approx |V_{tb}^* V_{td}|^2 \mathcal{K} f_{B_d}^2 \hat{B}_{B_d}^{(1)}$$

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} \approx |V_{ub}|^2 \left(\mathcal{K}_1 f_+^{B \rightarrow \pi}(q^2) + \mathcal{K}_2 f_0^{B \rightarrow \pi}(q^2) \right)$$

“best” channel depends on availability of experimental and theoretical precision.

Preferred (lattice) determinations

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{ud} \frac{K \rightarrow l\nu}{\pi \rightarrow l\nu} (+ V_{us})$$

$$V_{us} K \rightarrow \pi l\nu$$

$$V_{cd} D \rightarrow l\nu, D \rightarrow \pi l\nu$$

$$V_{cs} D_s \rightarrow l\nu, D \rightarrow K l\nu$$

$$V_{ub} B \rightarrow \pi l\nu$$

$$V_{cb} B \rightarrow D^{(*)} l\nu$$

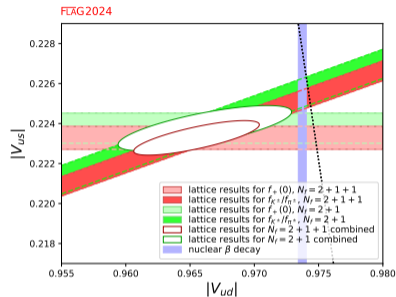
$$V_{td} B^0 \rightarrow \bar{B}^0 (+ V_{tb})$$

$$V_{ts} B_s^0 \rightarrow \bar{B}_s^0 (+ V_{tb})$$

1st place to look: FLAG [Flavour Lattice Averaging Group]

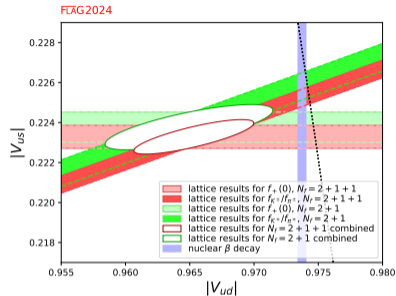
V_{ud} , V_{us} and isospin breaking corrections

- $|V_{ud}| = 0.97373(31)$ [nuclear β decay]
- $|V_{us}| f_+^{K \rightarrow \pi \ell \nu}(0) = 0.21654(41)$
- $\frac{|V_{us}|}{|V_{ud}|} \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(41)$ (IB-breaking correction from χ PT [Cirigliano, Neufeld'11])



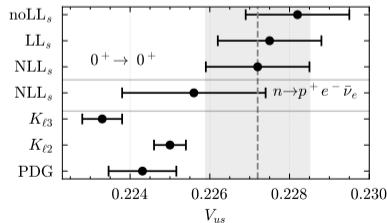
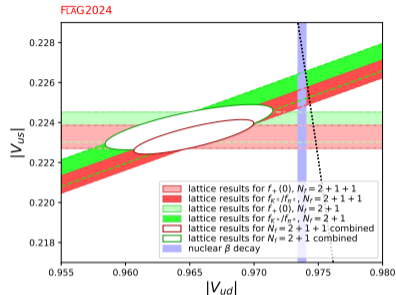
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 $\frac{|V_{us}|}{|V_{ud}|} \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27683(29)_e(20)_t[35]$
(IB-correction from lattice [Di Carlo, ETMC '19])
- A lot of lattice activity for QED corrections



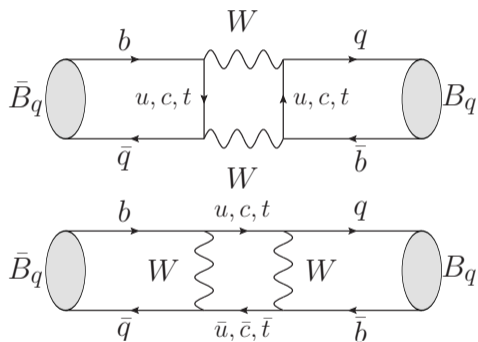
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- A lot of lattice activity for QED corrections
- [Moretti, Gorbahn, Jäger arXiv:2510.27648] \longrightarrow systematic inclusion of higher order corrections for V_{ud} , using nuclear input from Hardy, Towner '20



Neutral meson mixing - background e.g. $B_{(s)}^0$

Neutral mesons oscillate:



where $q = d, s$

mass eigenstate \neq flavour eigenstate

$$|B_{L,H}\rangle = p |B_q^0\rangle \pm q |\bar{B}_q^0\rangle$$

\Rightarrow **splittings** in mass eigenstates:

- mass splitting $\Delta m_q \equiv m_H - m_L$
- width splitting $\Delta \Gamma_q \equiv \Gamma_L - \Gamma_H$
- in particular Δm_q experimentally very precisely measured!

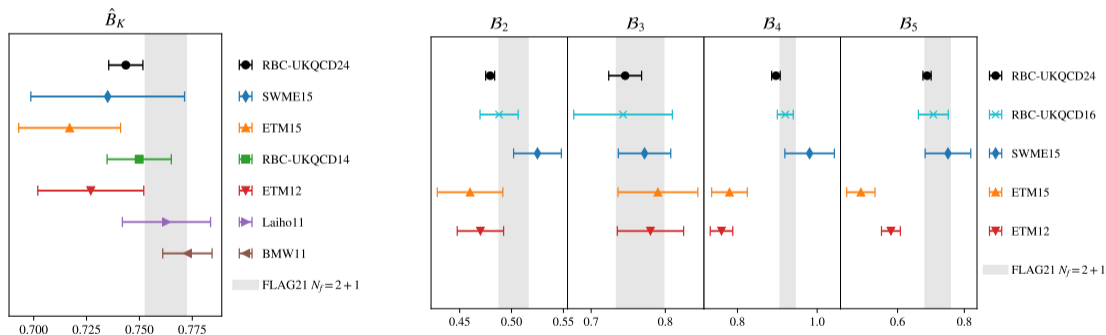
Time dependence:

$$|B_q^0(t)\rangle = g_+(t) |B_q^0\rangle + \frac{q}{p} g_-(t) |\bar{B}_q^0\rangle$$

$$|\bar{B}_q^0(t)\rangle = \frac{p}{q} g_-(t) |B_q^0\rangle + g_+(t) |\bar{B}_q^0\rangle$$

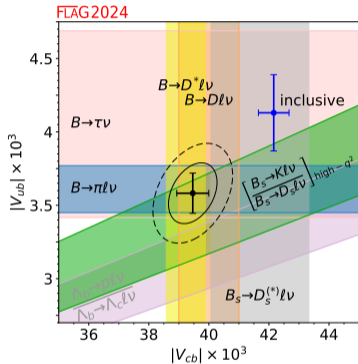
Neutral Kaon mixing

- 5 independent (parity even) short distance operators (“bag parameters”)
- Tensions for BSM bag parameters B_4 , B_5 persist [JTT, RBC/UKQCD, PRD, 2404.02297]
- Matching RI/(S)MOM $\leftrightarrow \overline{\text{MS}}$ needed. Now available at 2-loops [Gorbahn et al, JHEP, 2411.19861]



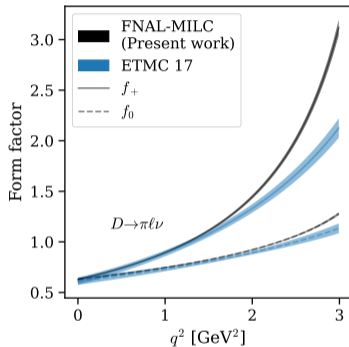
⇒ Foundation for $B_{(s)} - \overline{B}_{(s)}$ mixing programme [JTT, RBC/UKQCD + JLQCD PoS LATTICE2021 (2022) 224]

Status of heavy quark lattice physics?



Several tensions:

- Th-Th B -decays (\leftarrow mostly χ^2 -rescaled...)
- Th-Th D -decays (\rightarrow)
- Th-Ex $R(D)$, $R(D^*)$?
- Th-Ex $b \rightarrow s l l$?
- Th/Ex inclusive-exclusive?



Theory – Theory tensions \Leftrightarrow underestimated uncertainties somewhere

Novel methods to scrutinise results

Tensions \equiv overlooked systematics?

Suggestion of simplified benchmark quantities [\[JTT & Della Morte, EPJ-ST 2310.02705\]](#)

discretisation effects

- large quark masses and/or momenta
- renormalisation constants
- HQ-effective actions

massive NPR

[\[Boyle, Del Debbio, Khamseh, PRD, 1611.06908\]](#)

[\[JTT, RBC/UKQCD, PRD, 2407.18700\]](#)

inter- and extrapolations

- in quark masses
- EFT prescriptions (e.g. $\text{HM}\chi\text{PT}$)
- continuous description of momentum transfer

form factor bases

[\[JTT, RBC/UKQCD, PRD, 2303.11280\]](#)

enforcing unitarity constraints

[\[Flynn, Jüttner, JTT, JHEP, 2303.11285\]](#)

ground state parameters

- excited state contamination
- neglecting systematics
- neglecting/tampering with correlations

Laplace Filtering

[\[JTT & Portelli, 2508.11541, PRD 2508.01900\]](#)

and its applications

[\[JTT & Zuchanke, in preparation\]](#)

The anomalous magnetic moment of the muon

- Spin magnetic moment μ is related to spin \mathbf{S} by g :

$$\mu = g \frac{e}{2m} \mathbf{S}$$

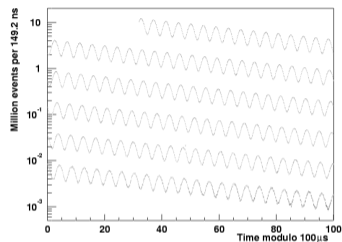
- At tree-level: $g = 2$
- $g \neq 2$ due to quantum corrections. Define

$$a_\mu \equiv \frac{g - 2}{2}$$

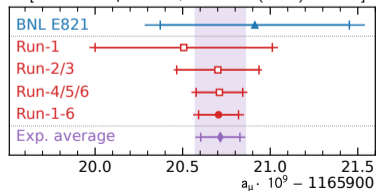
- Sensitive to effects due to QED, weak and strong interactions (and NP?)
- precisely measurable as a frequency:

$$a_\mu^{\text{exp,combined}} \times 10^{11} = 1165920715(145) \text{ [124 ppb]}$$

[E821 experiment at BNL, PRD 73 (2006) 072003]



[Fermilab experiment, PRL 135 (2025) 101802]



$$\frac{g - 2}{2} \equiv a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{strong,HLbL}} + a_\mu^{\text{strong,HVP}}$$

- QED (differing input for α) [PRL 109 111808, Atoms 7 28]

$$a_\mu^{\text{QED}} \times 10^{11} = 116584718.931(104)$$

$$a_\mu^{\text{QED}} \times 10^{11} = 116584718.842(106)$$

- Weak [PRD 67 073006, PRD 73 119901, PRD 88 053005]

$$a_\mu^{\text{weak}} \times 10^{11} = 153.6(1.0)$$

- Strong (Had. Light-by-Light) [PRD 70, 113006, PRD 95, 054026, JHEP 04 161, JHEP 10 141, PRD 100 034520, PLB 798 134994, JHEP 03 101, EPJC 74 3008, PRD 95 014019, STMP 274 1, PLB 787 111, PRD 101 054015, PRD 101 074019, PLB 735 90, PRL 124 132002, EPJC 81 (2021) 651; EPJC 82 (2022) 664,...] – hadronic models, data-driven & lattice: all consistent

$$a_\mu^{\text{HLbL}} \times 10^{11} = 92(18)$$

- Strong (Hadronic vacuum polarisation) ???

“data driven” vs “lattice” at WP1

data driven [dominated by KNT'19]

$$a_{\mu}^{HVP} = \left(\frac{\alpha m_{\mu}}{3\pi}\right)^2 \int_{m_{\pi^0}^2}^{\infty} ds \frac{R_{\text{had}}(s) \hat{K}(s)}{s^2}$$

- dominated by low s region
- $\hat{K}(s)$ - known analytic function
- $R_{\text{had}}(s) = \frac{3s}{4\pi(\alpha(s))^2} \sigma(e^+e^- \rightarrow \text{hadrons})$
- based on **experimental data**
- sum over exclusive channels

White paper 1 [taken for Theory prediction]:

$$a_{\mu}^{HVP} \times 10^{11} = \mathbf{6845(40)}$$

lattice

$$a_{\mu}^{HVP} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dt \tilde{K}(t) G(t)$$

- dominated by light-quark contributions
- $\tilde{K}(t)$ - known analytic function
- $G(t)$: lattice 2-point function(s)
- **Independent of experimental inputs** (other than m_{μ} and scale setting)
- sum l+s+c+disc.+slB + QED

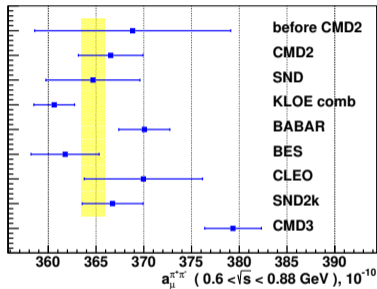
White paper 1:

$$a_{\mu}^{HVP} \times 10^{11} = 7116(184)$$

Two game-changing new results

Input for data-driven

[Ignatov et al. (CMD-3 Collab.), PRD 109 (2024) 11, 112002]

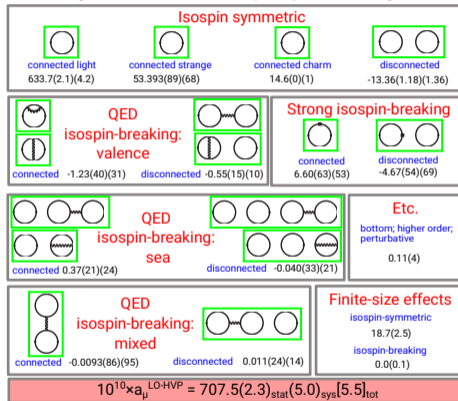


Tensions between experiments (**inputs for data driven method**) in dominant region

- **Kloe vs BaBar**
- **CMD3** tension yet to be understood

Complete lattice calculation

[BMWc 20, Nature **593** (2021) 7857, 51-55]



- Impressive result: sub-percent precision
- Needs independent verification

Particularly challenging:

small t region: discretisation effects

large t region: finite volume effects

$$a_{\mu}^{\text{HVP,X}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dt \quad \tilde{K}(t)G(t)$$

Q: How can we scrutinise these difficult calculations?

Particularly challenging:

small t region: discretisation effects

large t region: finite volume effects

$$a_{\mu}^{\text{HVP},X} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dt T_X(t; t_0, t_1, \Delta) \tilde{K}(t) G(t)$$

Q: How can we scrutinise these difficult calculations? **A:** Separate these regions!

Split integration range [RBC/UKQCD 18]

$$a_{\mu}^{\text{HVP}} = a_{\mu}^{\text{HVP},\text{SD}} + a_{\mu}^{\text{HVP},\text{W}} + a_{\mu}^{\text{HVP},\text{LD}}$$

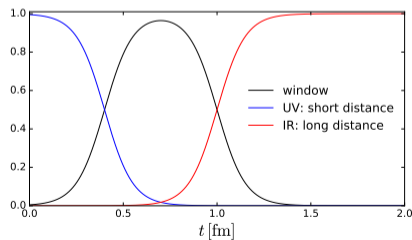
SD Short distance ($T_{\text{SD}} = 1 - \Theta(t, t_0, \Delta)$)

W Intermediate window

$$(T_{\text{W}} = \Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta))$$

LD Long distance ($T_{\text{LD}} = \Theta(t, t_1, \Delta)$)

$$\Theta(t_0, t_1, \Delta) = \frac{(1 + \tanh(t_0 - t_1)/\Delta)}{2}$$

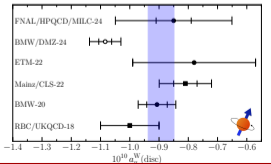
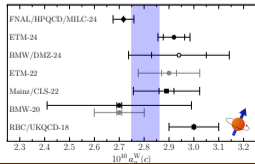
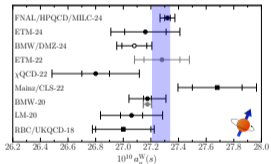
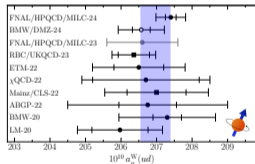


\Rightarrow **Use windows for scrutiny**

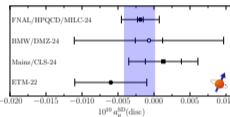
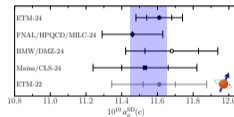
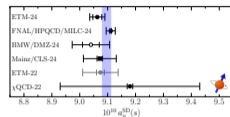
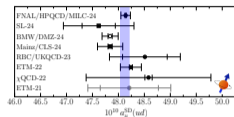
New lattice results for WP2 [inc. JTT, RBC/UKQCD, PRD, 2301.08696 & PRL, 2410.20590]

- Many new results
- Many of them from blinded analyses
- Many ways to build averages of these
- Highly complementary

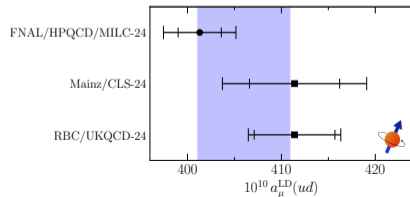
W



SD

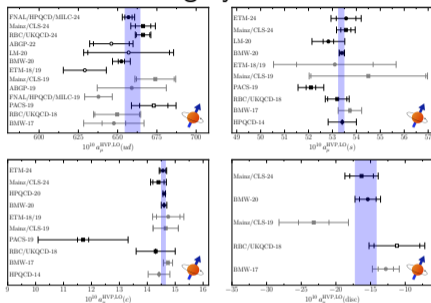


LD

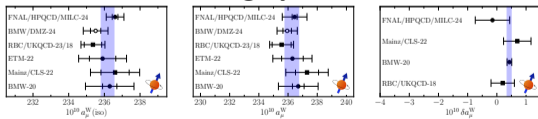


White Paper 2 - Consolidating lattice averages

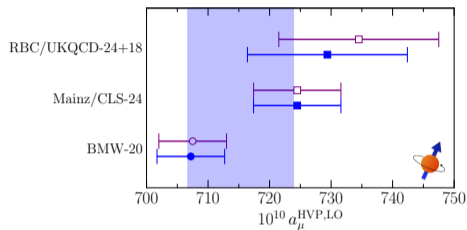
Summing by windows:



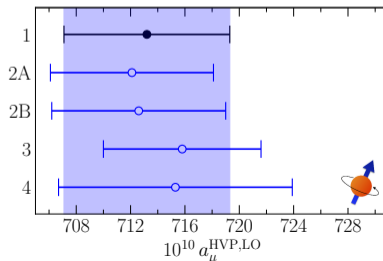
Summing by flavours:



“Complete” calculations

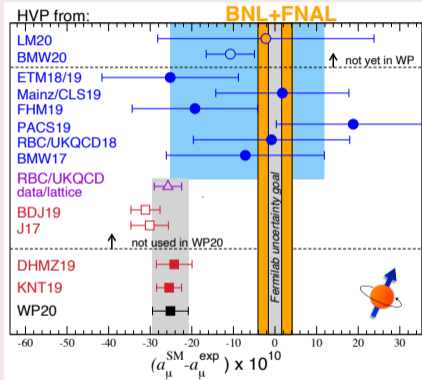


Final averages



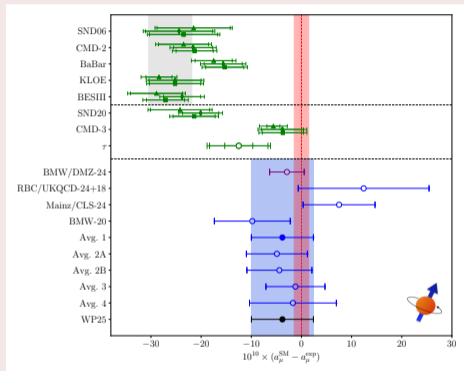
HVP through the (recent) ages

Snowmass (post WP1) 2203.15810



$$a_\mu^{HVP, \text{data driven}} \times 10^{11} = 6845(40)$$

2025 - WP2



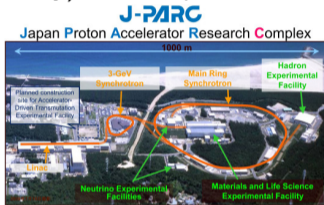
$$a_\mu^{HVP, \text{lattice}} \times 10^{11} = 7132(61)$$

a huge success for the lattice!

Outlook $g - 2$

A huge step forward, but more work needs to be done:

- (hopefully) more experimental **and highly complementary** results:



+



- Data driven vs Lattice tensions need to be understood!
- More experimental inputs for $e^+e^- \rightarrow$ hadrons to scrutinise data-driven predictions
- Hybrid approach by BMW+DMZ'24: Supplements very noisy large- t tail with experimental data.
- Further reduction from lattice expected: more collaborations, finer ensembles, larger volumes, ...

- conjectured charge asymmetry in heavy-ion collisions due to interplay between magnetic fields and QCD topology
- no statistically significant signal at RHIC and LHC despite extensive searches
- statistically significant CME signatures by STAR [2506.00275] in NARROW RANGE of beam energies ONLY
- Conjecture: CME enhancement near QCD tri-critical point/chiral crossover [2012.02926]
- Test of conjecture in finite-density two-colour QCD chiral crossover line of 2nd order phase transition [Buividovich, von Smekal, Smith, 2605.04743]
⇒ no statistically significant CME enhancement across the (μ, T) phase diagram

⇒ **lattice talk soon**

summary

Summary

Presented topics

- small (and biased) selection of topics
- 'lattice' is a first principle tool for QCD
- $g - 2$ from lattice and R-ratio
- method development and application, e.g. for many flavour observables

the TP group

- we are a friendly bunch (I think)
- there are **many** more topics...
- synergies and complementarities
- relevance for/interplay with experimental activities

Relationship with HEP in Physics

- we should have more TP \Leftrightarrow EP interactions!
- feel free to come along to our seminars/meetings!
- scope for more joint events/meetings/lunches/...?