Current Status of muon catalyzed fusion (μCF) theory

Department of Chemistry, Tohoku University
Yasushi Kino

y.k@tohoku.ac.jp

Collaboration (theory)

Tohoku U.

YK, Takuma Yamashita, Hidekazu Nihei, Kenichi Okutsu, Naoki Kamiya, Masahiro Taguchi, Yotaro Kawakami, Ran Murata, Arata Neki, Naoko Hayashi, Kai Shimizu, Tomotaka Inoue, Takuma Nakaminami, Shoma Nohara, Takaya Urabe, Kota Maruyama, Kazuhiro Yasuda, Motoaki Niiyama,
Taishi Sano, Konan Miyashita, Ryota Nakashima, Ren Konishi, Kyosuke Sasaki

Chubu U.

Shinji Okada, Yuuichi Toyama, Motoyasu Sato, Kimitaka Ito, Yoshiharu Tanahashi,
Jyunji Miyamato, Masanori Yamamoto

Kyushu U.

Masayasu Kamimura

JAEA

Toshitaka Oka

J-PARC/KEK

Patrick Strasser, Hiroaki Natori, Yukinori Nagatani, Yasuhiro Miyake, Narishige Kawamura,
Kanetada Nagamine, Soutaro Kanda, Kouichi Shimomura, Shougo Doteuchi, Motonobu Tampo,
Katsuhiko Ishida

KEK

Toshiuki Azuma

Tokyo Metropolitan U.

Takuma Okumura

Rikyo

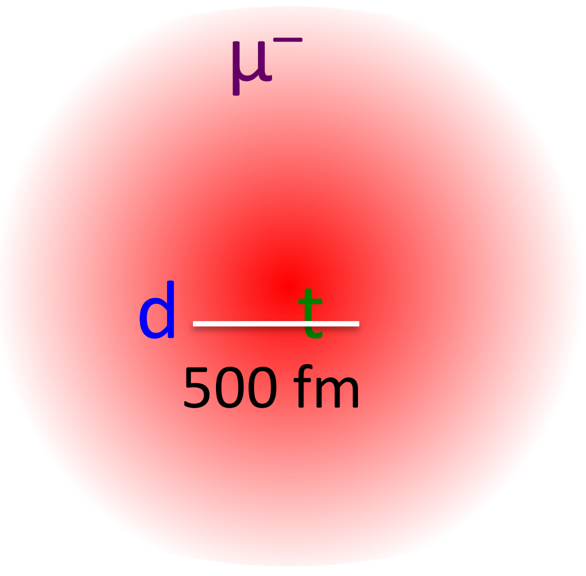
Yuto Ichinohe

Introduction

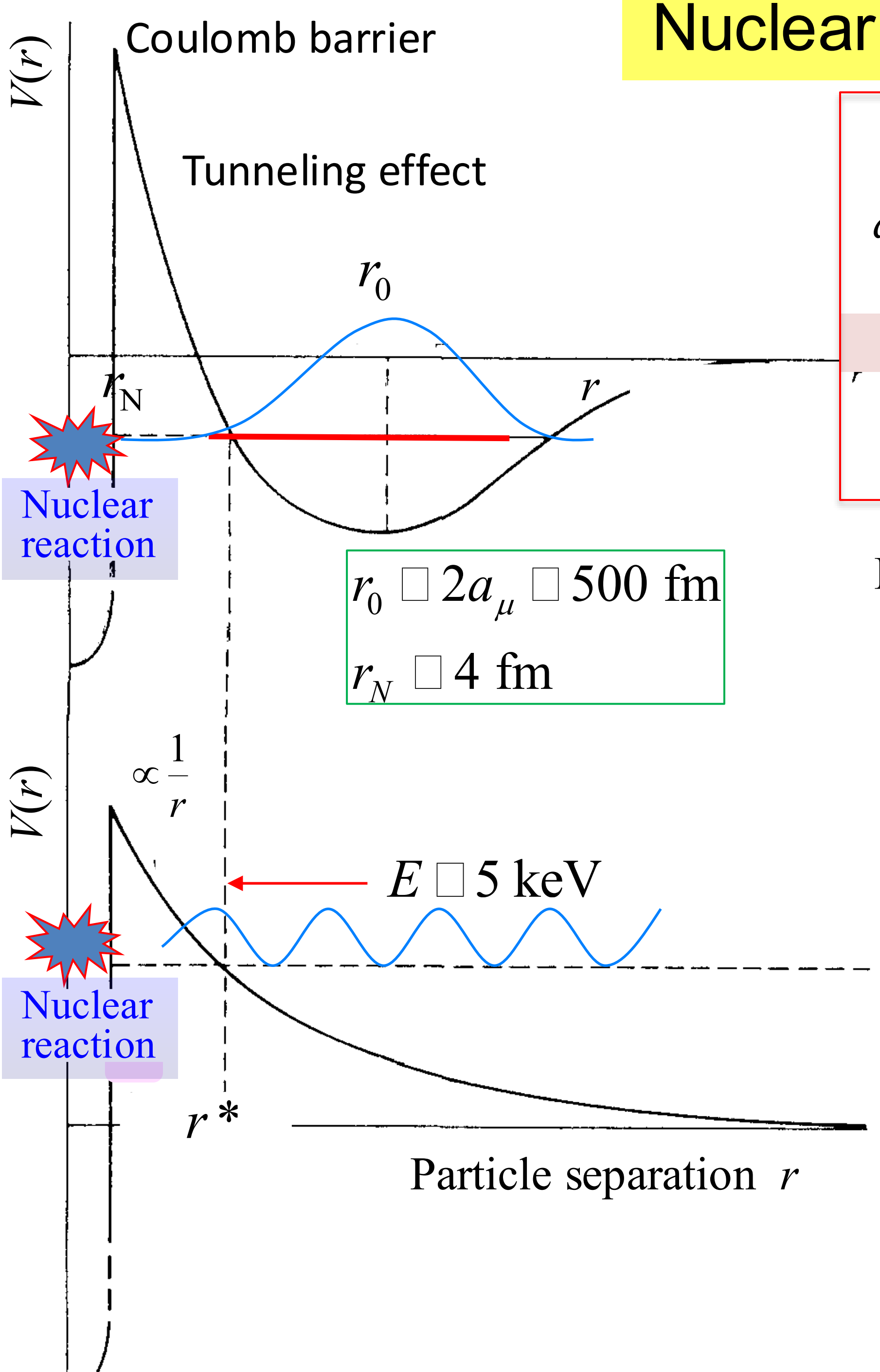
muon catalyzed fusion (μCF)

Nuclear fusion reaction in Muonic molecule

Muonic molecule dtμ



(p-p distance in H_2^+) ≈ 0.1 nm



		rate
$dd\mu \rightarrow$	$^3He + n + \mu + 3.3$ MeV / $t + p + \mu + 4.0$ MeV	$\lambda_f = 4.3 \times 10^8\ s^{-1}$
$dt\mu \rightarrow$	$^4He + n + \mu + 17.6$ MeV	$\lambda_f = 1.2 \times 10^{12}\ s^{-1}$
$tt\mu \rightarrow$	$^4He + 2n + \mu + 11.3$ MeV	$\lambda_f = 1.5 \times 10^7\ s^{-1}$

$DT^+ \rightarrow {}^4He + n + e + 17.6$ MeV $\quad \lambda_f < 10^{-70}\ s^{-1}$

13.8 Billion y × 6×10^{23} s ~ 10^{41} s

1 fusion event in 10^{29} mol since Big Bang

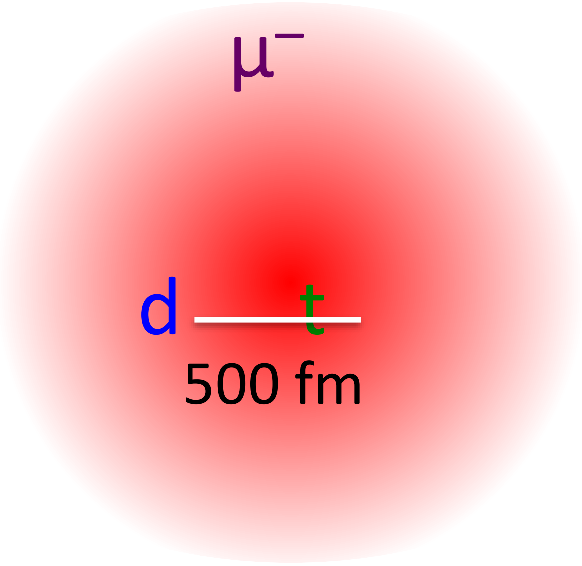
Thermal fusion in plasma

$$d + d \rightarrow \begin{cases} {}^3He + n + 3.3\ \text{MeV} \\ t + p + 4.0\ \text{MeV} \end{cases}$$

$$d + t \rightarrow {}^4He + n + 17.6\ \text{MeV}$$

$$t + t \rightarrow {}^4He + 2n + 11.3\ \text{MeV}$$

muon catalyzed fusion



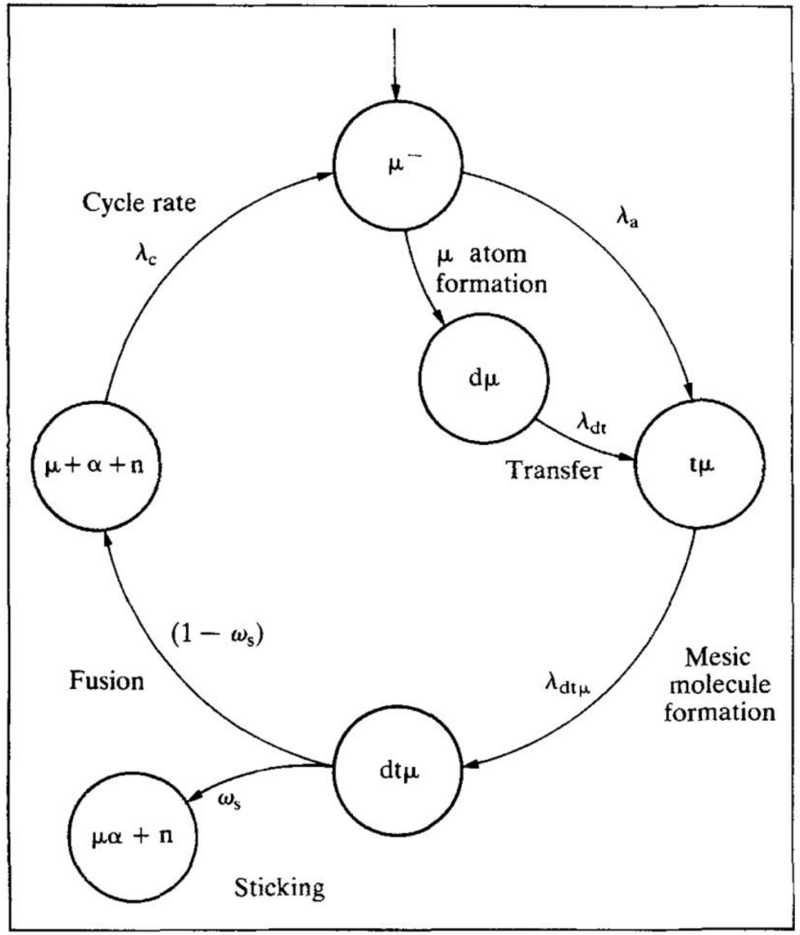
$10^{12}\ \mathrm{s}^{-1}$

$$dt\mu \square\ \mu + {}^{4}\mathrm{He} + n + 17.6\ \mathrm{MeV}$$

$10^{8}\ \mathrm{s}^{-1}$

$$dt\mu \square\ \mu + {}^{4}\mathrm{He} + n + 17.6\ \mathrm{MeV}$$

$$dt\mu \square\ \mu + {}^{4}\mathrm{He} + n + 17.6\ \mathrm{MeV}$$



Energy source — Out put (3.5 GeV/muon) vs. input (5.0 GeV/muon)

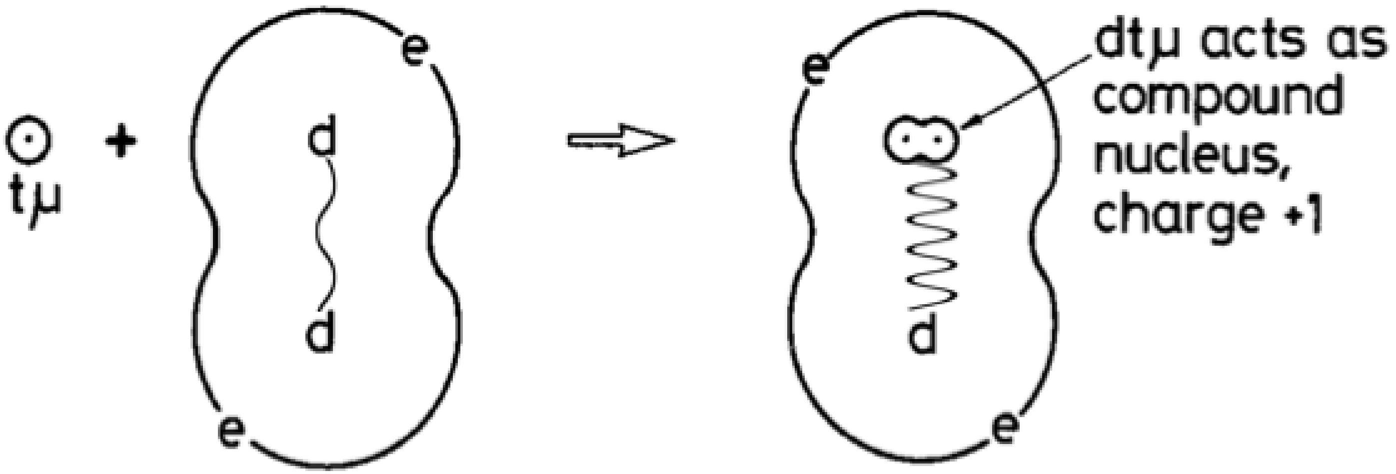
Neutron source — Compact 14.1 MeV monoenergetic neutron (point source)

1 Muon injected into D_2/T_2 mixture forms muonic atom (tμ, dμ) and muonic molecular ion dtμ

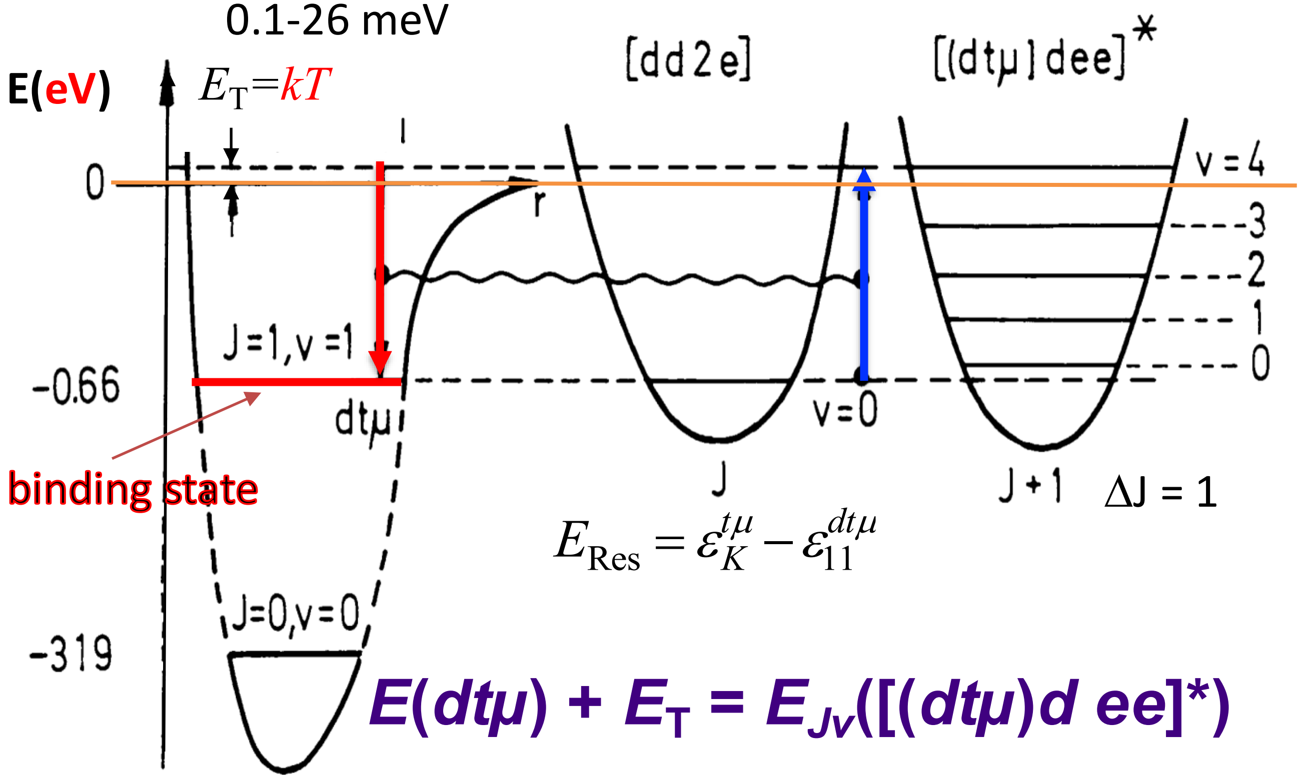
2 Coulomb barrier shrinks in the small dtμ molecule (internuclear distance ~ 1/200 of DT molecule)

3 Muons released after d-t fusion find another d-t pair (Muon works as a catalyst of the d-t fusion)

Compound molecular formation $[(dt\mu)dee]$



Feshbach resonance between tμ atom and D_2 molecule



$$E(dt\mu) + E_T = E_{Jv}([(dt\mu)d\, ee]^*)$$

Vesman mechanism (1967)

prediction

If the muon molecule has a shallow bound state, dtμ is resonantly generated by transferring the binding and kinetic energy to the vibrational and rotational excitation energy of the D_2 molecule.

For ddμ, dtμ, swallow binding states exist ! !

kT should be theoretically determined

-2711.2415 eV

0.6602 eV

-2711.9017 eV

meV accuracy

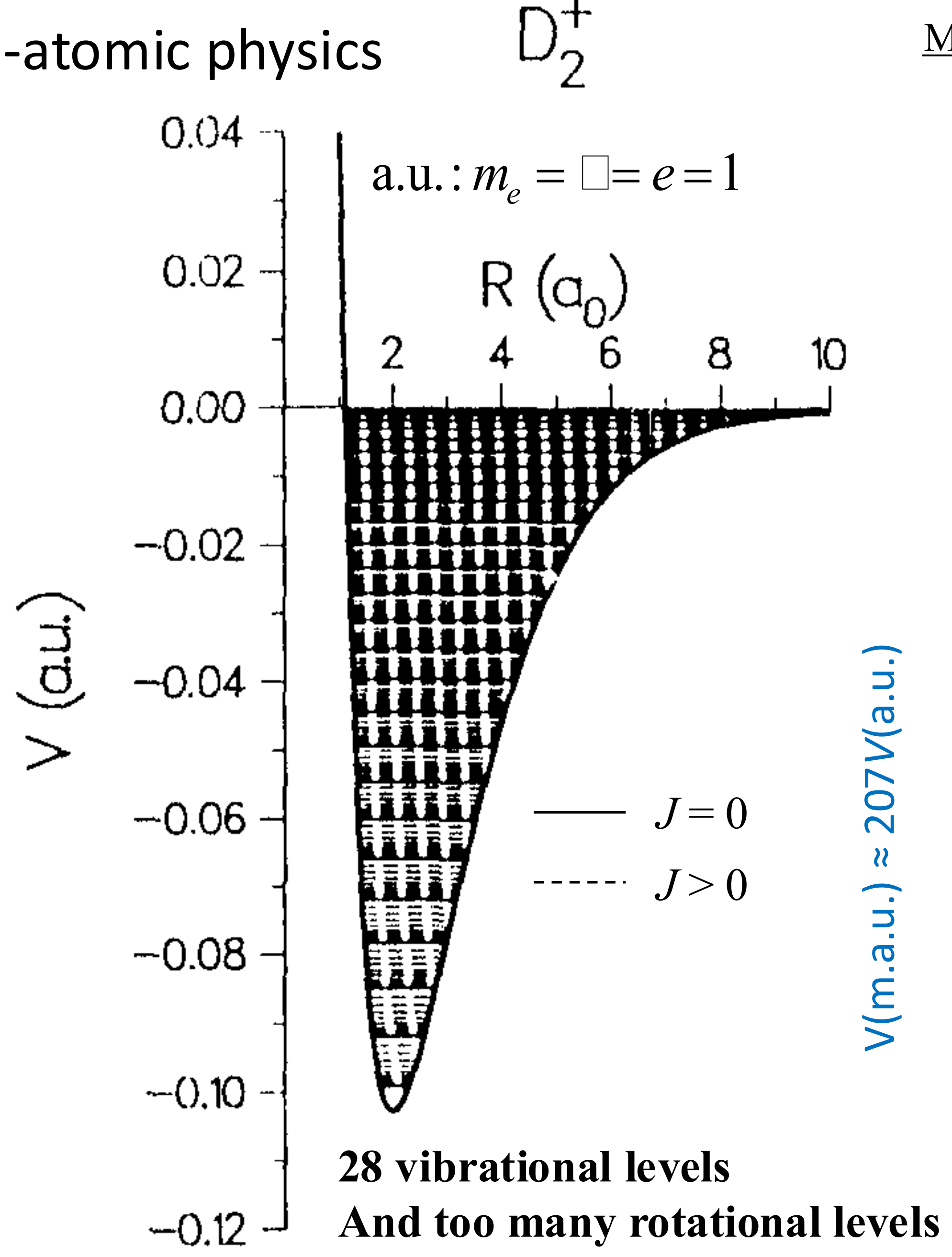
7 digits significant numbers

High precision three-body calculation

dtμ、ddμ Precise three body calculation : < meV (10 K)

Basics of muon-atomic physics

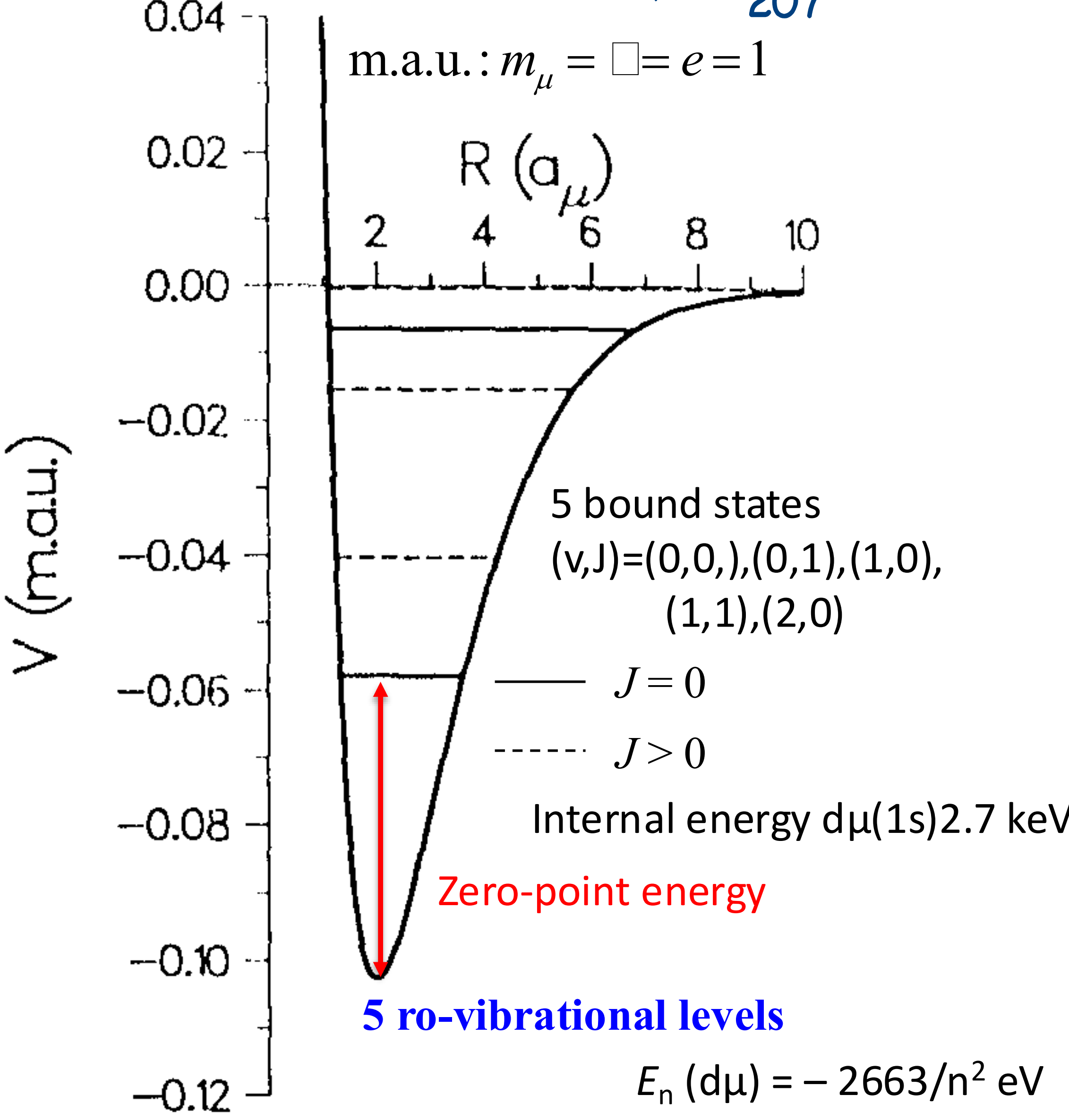
V: adiabatic potential energy for d-d distance



$V(\text{m.a.u.}) \approx 207\, V(\text{a.u.})$

Muon deuterium molecular ion $dd\mu$

$$a_\mu = \frac{a_0}{207}$$



$E_n\,(d\mu) = -2663/n^2$ eV

BUT, the adiabatic picture is unsuitable for precise calculations.

The difference becomes pronounced in low-energy atomic processes in μCF.

High-precision few-body calculation theory (non-adiabatic)

7-digits precision in binding energy of dtμ molecule (M. Kamimura, PRA 1987)

Muonic atom nucleus collision (YK, MK, 1990-2000)

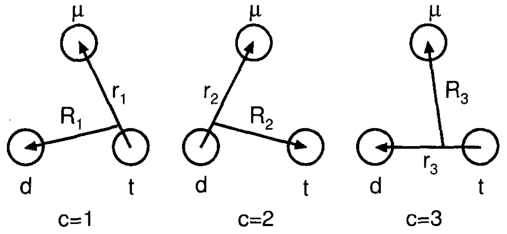


Figure 24: Three Jacobian coordinates of the $d + t + \mu$ system.

PERGAMON

Progress in Particle and Nuclear Physics 51 (2003) 223–307

Progress in Particle and Nuclear Physics

http://www.elsevier.com/locate/npe

Gaussian Expansion Method for Few-Body Systems

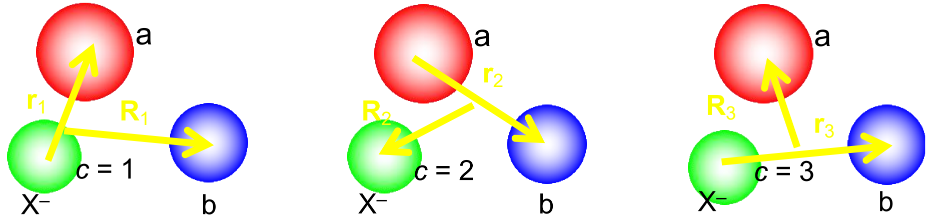
E. HIYAMA[1], Y. KINO[2] and M. KAMIMURA[3]

[1]High Energy Accelerator Research Organization (KEK), Tsukuba 308-0801, Japan
[2]Department of Chemistry, Tohoku University, Sendai 980-8578, Japan
[3]Department of Physics, Kyushu University, Fukuoka 812-8581, Japan

April 8, 2003

Abstract

We review our method of calculation, Gaussian Expansion Method (GEM), for bound and scattering states of few-body systems. The method was proposed in 1988 and has been applied to a variety of few-body systems. The understanding on the structure and the mechanisms of reactions of those systems obtained from such applications is discussed together with some useful techniques for the calculations. A well-chosen set of Gaussian basis functions forms an approximate complete set in a finite coordinate space so that it can describe accurately short-range correlations and long-range asymptotic behavior as well as highly oscillatory character of wave functions in the bound and the scattering states of the systems. Examples of applications of GEM include i) the latest determination of antiproton mass by the analysis of laser spectroscopic data for antiprotonic helium atoms, ii) predictions and experimental verifications on the structure of hypernuclei and hyperon-nucleon interactions, iii) Coulomb three-body calculations of bound and resonant states of muonic molecules as well as muon transfer reactions in muon catalyzed fusion cycles, iv) a new treatment of CDCC (continuum-discretized coupled channels) method for three- and four-body breakup processes, and v) benchmark test calculations for three- and four-nucleon bound states using realistic interactions.



$$\psi_{JM} = \psi_{JM}^{[1]}(\mathbf{r}_1, \mathbf{R}_1) + \psi_{JM}^{[2]}(\mathbf{r}_2, \mathbf{R}_2) + \psi_{JM}^{[3]}(\mathbf{r}_3, \mathbf{R}_3)$$

E. Hiyama (Gauss lobe)

1) Three Jacobian coordinates (Exact scattering boundary condition)
2) Total wavefunction is describe as a sum of channel wavefunctions (interparticle correlation)
3) The channel wavefunctions are expanded in terms of Gaussian basis functions

The GEM has been successfully applied to

- muonic molecules, — Kino
- antiprotonic atoms,
- positronic atoms,
- Antihydrogen atoms
- 3-, 4- nucleon systems — Hiyama
- hyper nuclei
- five-quark systems

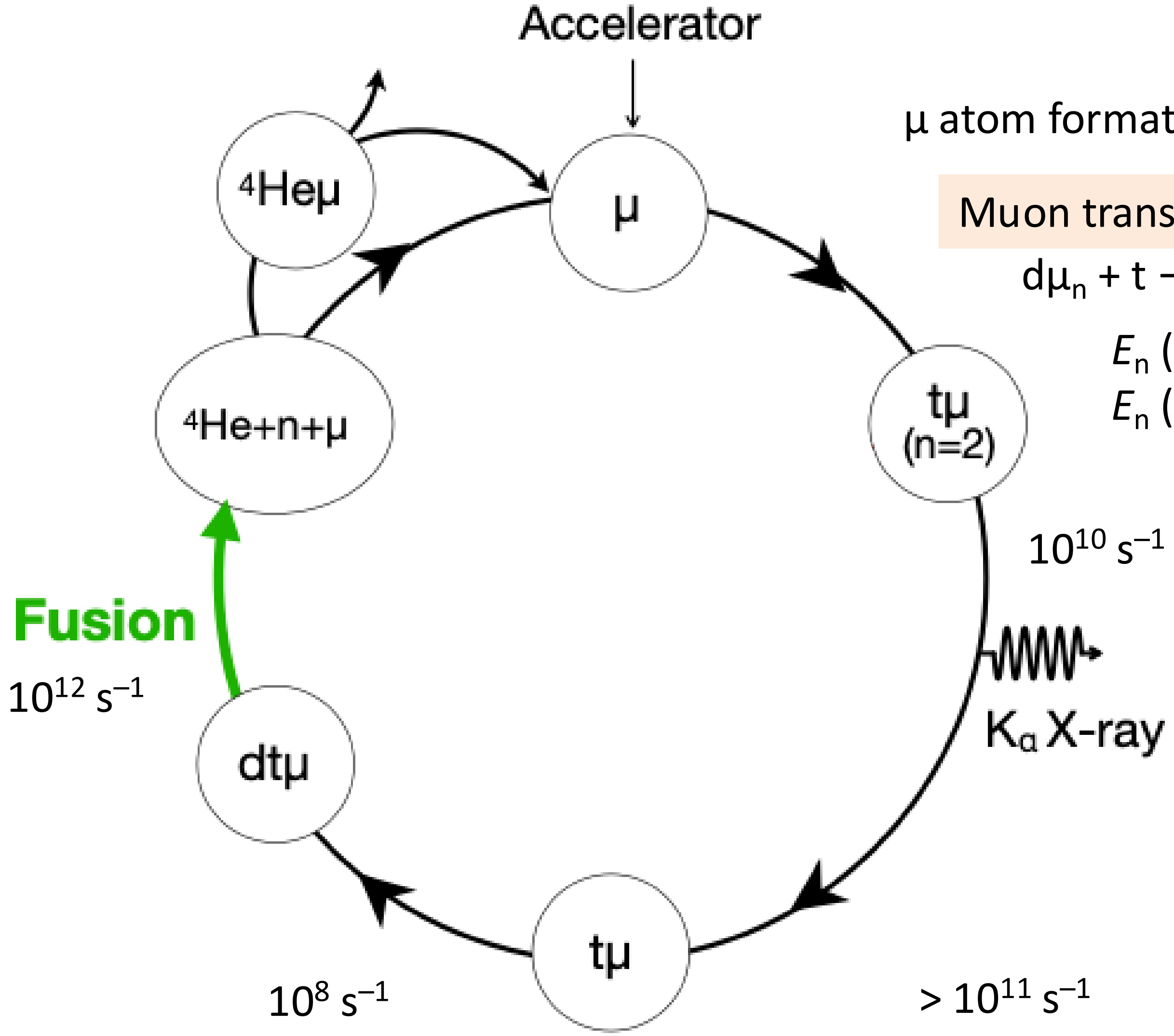
Recent study on muon catalyzed fusion (μCF) theory

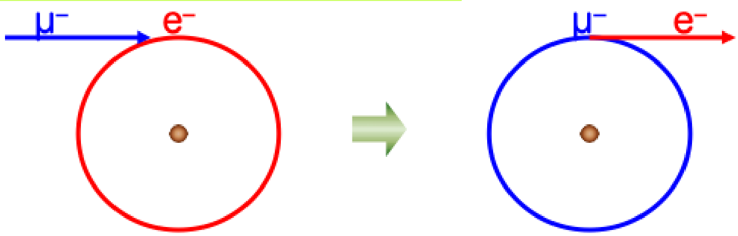
Based on precise few-body theory

T. Yamashita, Y. Kino, K. Okutsu, S. Okada, M. Sato,	Sci. Rep., 12, 6393 (2022).
M. Kamimura, Y. Kino, T. Yamashita,	Phys. Rev. C, 107, 034607 (2023).
T. Yamashita, K. Yasuda, Y. Kino,	Phys. Rev. A 111, 012811 (2025)
Y. Toyama et al.,	Sci. Adv. 12, eaed3321 (2026).

New muon catalyzed fusion cycle

Conventional μCF cycle





Muon mean lifetime 2.2 μs

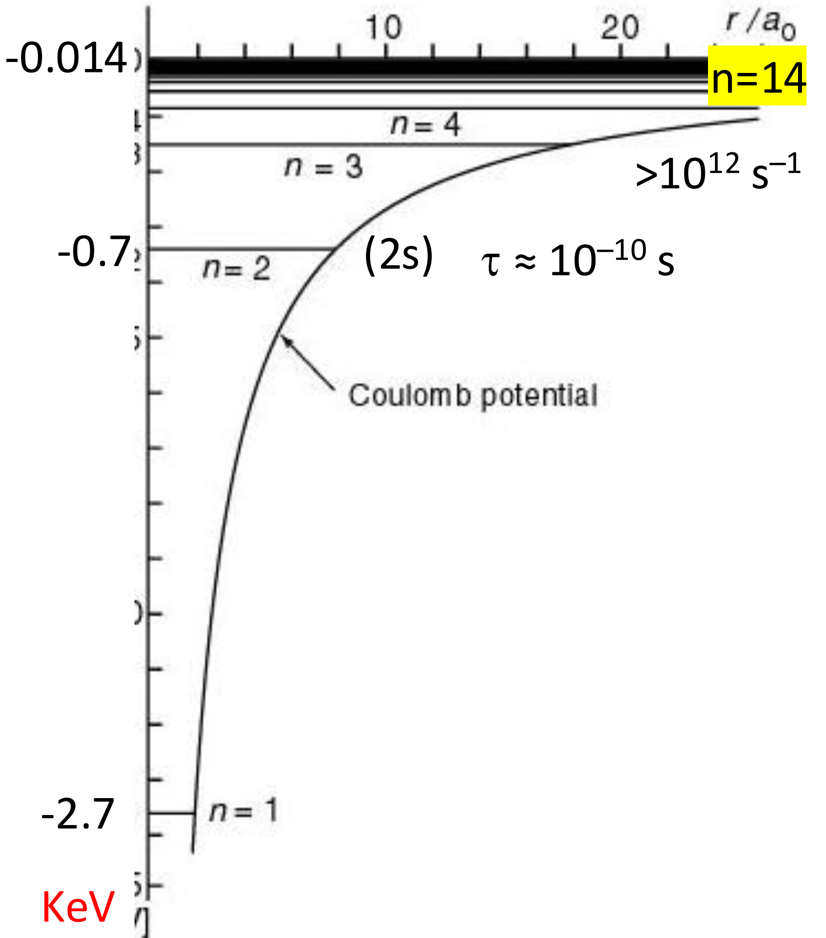
μ atom formation rate ≈ 10^{12} s^{-1}

Muon transfer reaction $>10^{12}$ s^{-1}

$$d\mu_n + t \rightarrow d + t\mu_n + 48/n^2 \text{ eV}$$

$$E_n (d\mu) = -2663/n^2 \text{ eV}$$

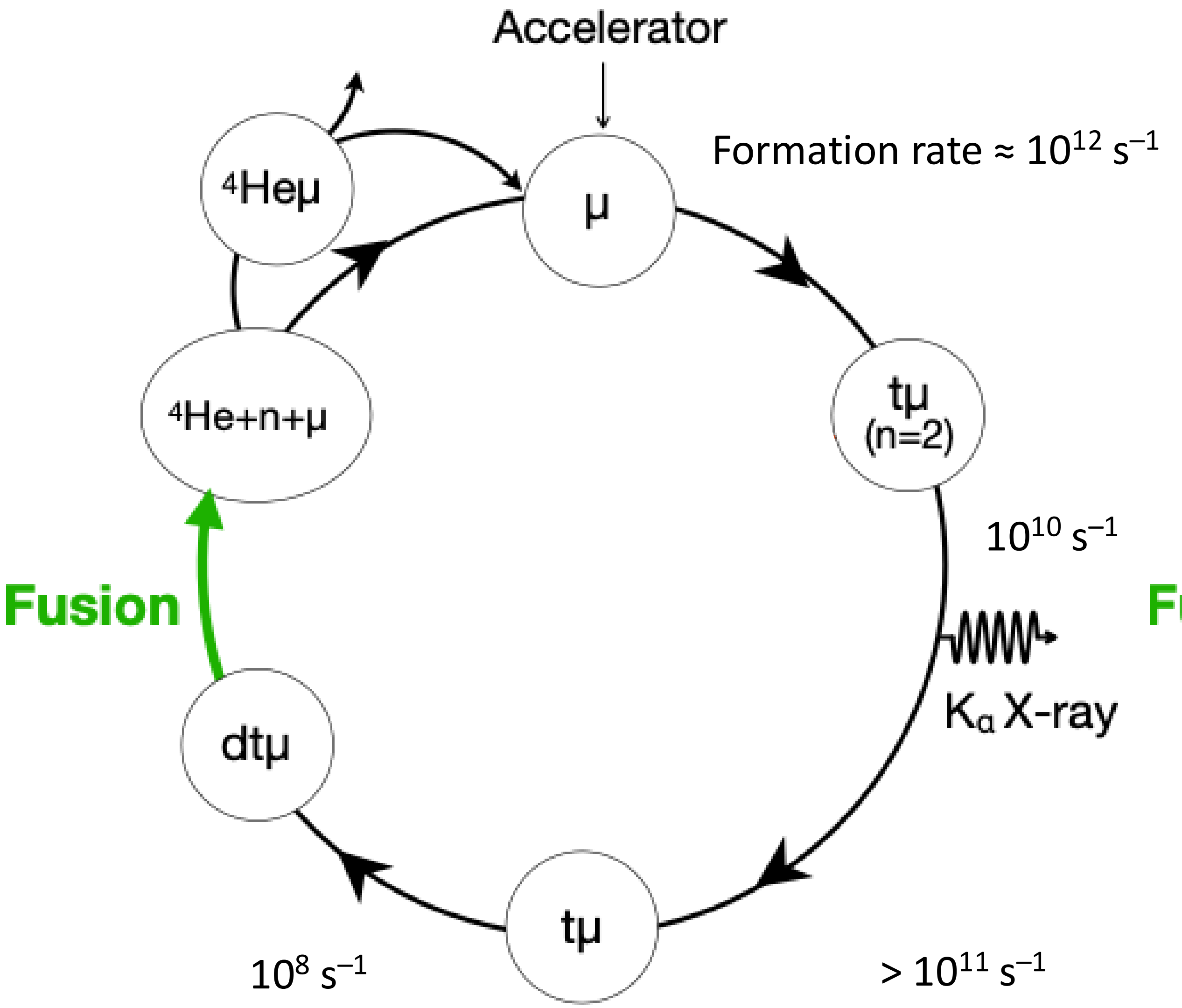
$$E_n (t\mu) = -2711/n^2 \text{ eV}$$



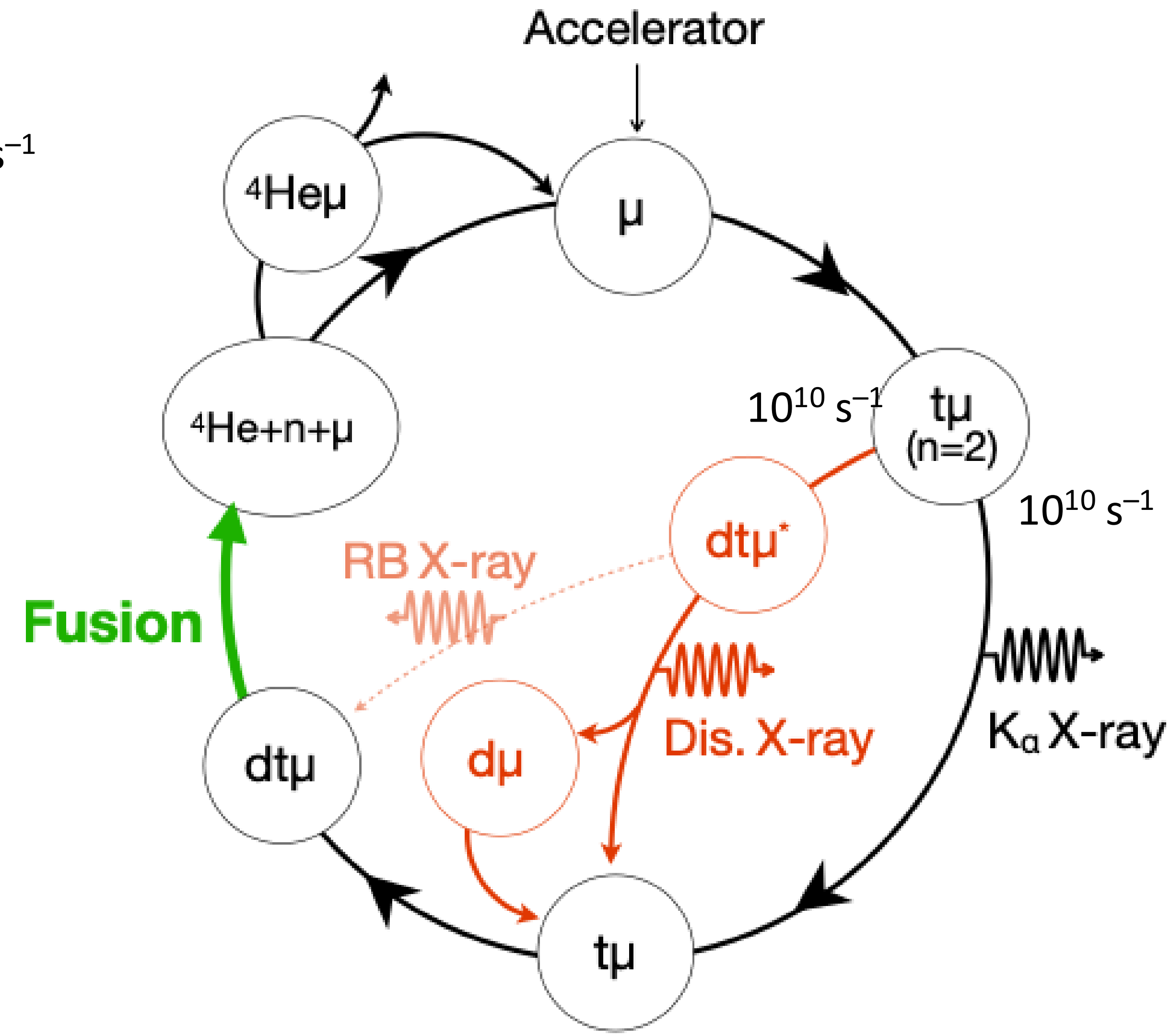
New muon catalyzed fusion cycle

dtμ* : muonic molecule resonance state

Conventional μCF cycle



New μCF cycle

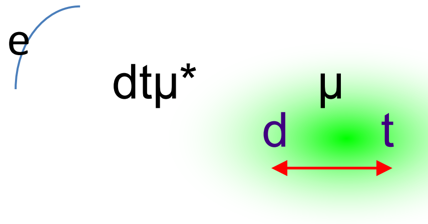


$\overline{\mathrm{H}}(NL) + \mathrm{Ps}(nl) \rightarrow \overline{\mathrm{H}}^{+} + e^{-}$ (⇦precise 4-body scattering calculation, T. Yamashita, YK, et al., NJP2021, PRA 2022)

$\bar{p}, e^{+}, e^{+}, e^{-}$

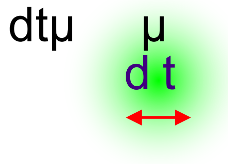
2020～

Muonic molecule and orbital electron

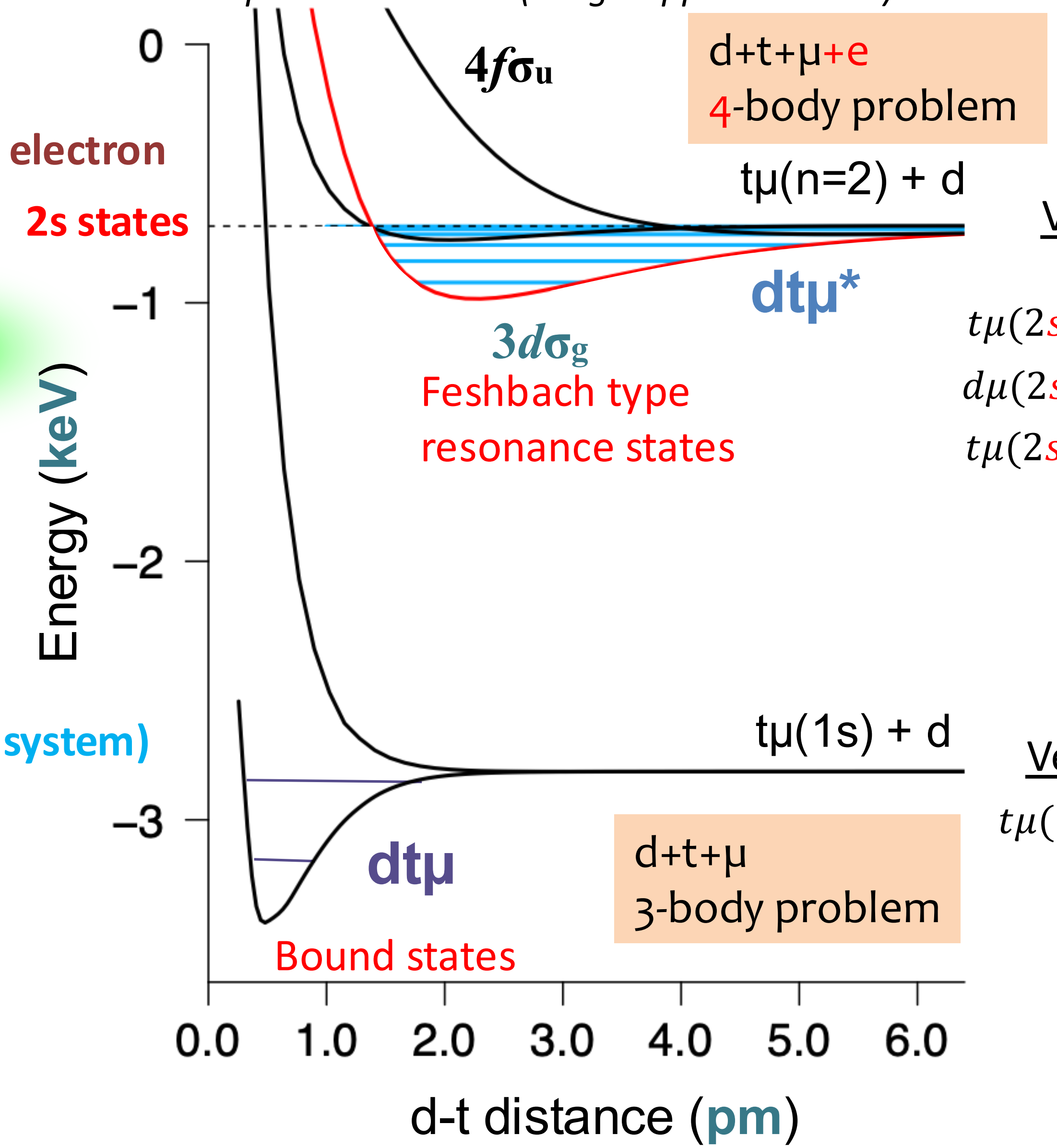


1990～

Muonic molecule (isolated system)



Adiabatic potential curves (rough approximation)



Vesman formation

~10^{10} s^{-1}

$t\mu(2s) + \mathrm{D}_2 \rightarrow (dt\mu^{*})e - de$

$d\mu(2s) + \mathrm{D}_2 \rightarrow (dd\mu^{*})e - de$

$t\mu(2s) + \mathrm{T}_2 \rightarrow (tt\mu^{*})e - te$

Vesman formation

~10^{8} s^{-1}

$t\mu(1s) + \mathrm{D}_2 \rightarrow (dt\mu)e - de$

New model vs. experimental value

shadow : ambiguities in atomic physics

$0.1 \le \Upsilon_d \le 0.9$

Cycle rate $\lambda_c = \lambda_\mu\left[\varphi\left(Y_f^{-1} - W\right)\right]^{-1}$

Total sticking probability

$$W = \sum_F P_{d\mu(1s,F)} \frac{0.58 c_d \lambda_{dd\mu(F)} \omega_d}{c_d \lambda_{dd\mu(F)} + c_t \lambda_{dt}} + \sum_F P_{t\mu(1s,F)} \frac{c_d \lambda_{dt\mu(F)} \omega_s + c_t \lambda_{tt\mu} \omega_t}{c_d \lambda_{dt\mu(F)} + c_t \lambda_{tt\mu}}$$

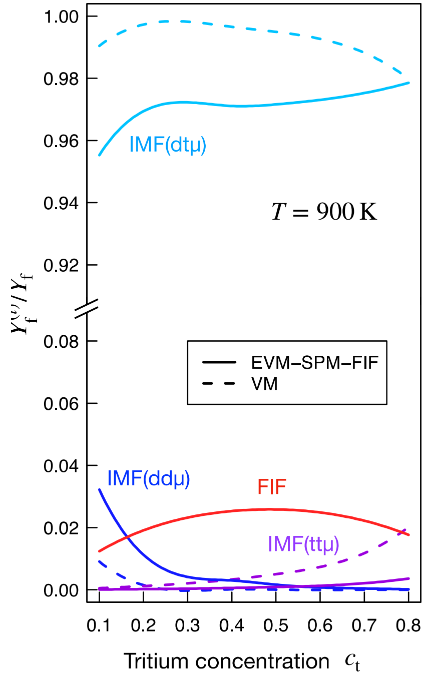
#(fusion)

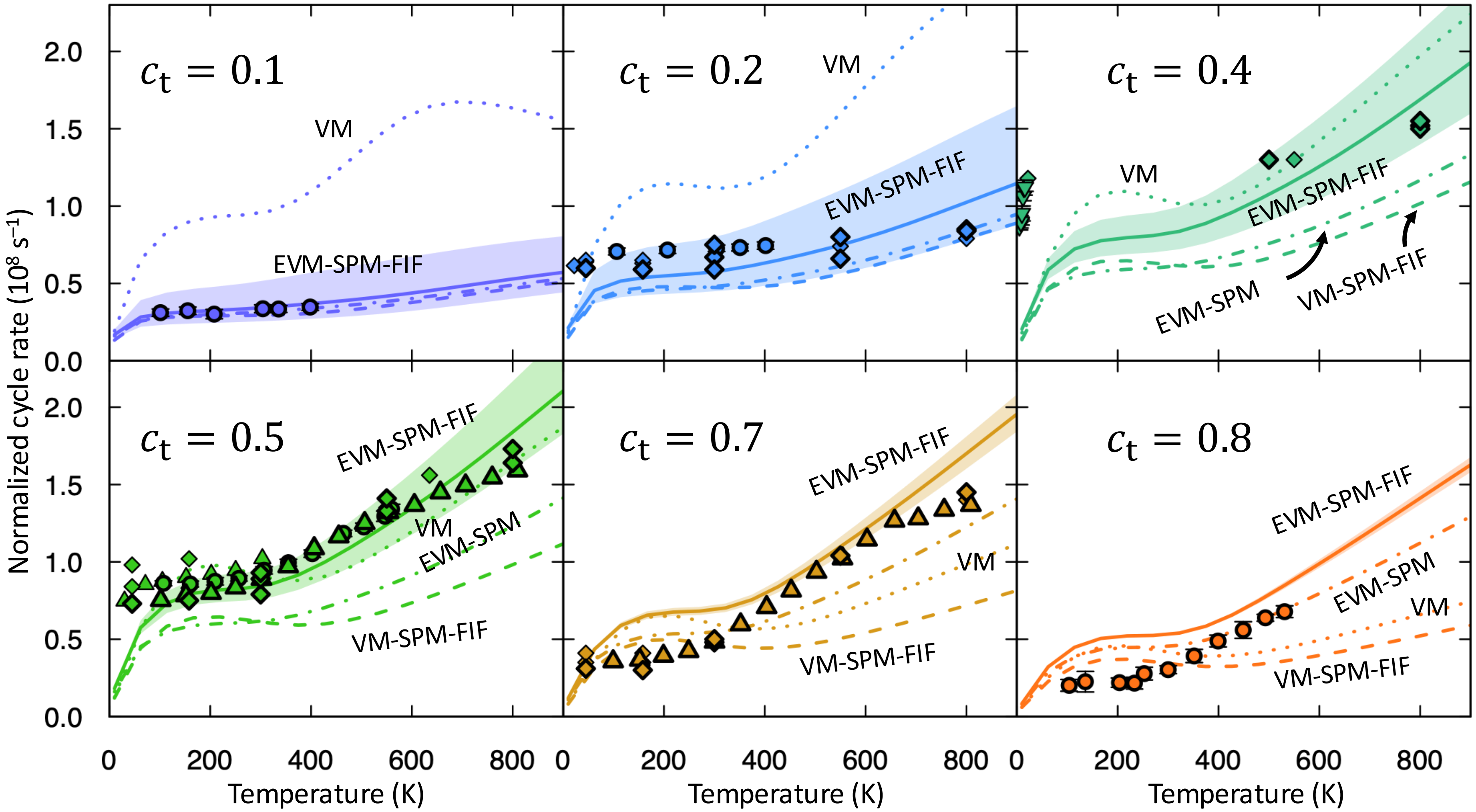
$$Y_f = \left[\frac{\lambda_\mu}{\varphi \lambda_c} + W\right]^{-1}$$

10^{-2} 10^{-2}

In-flight μCF

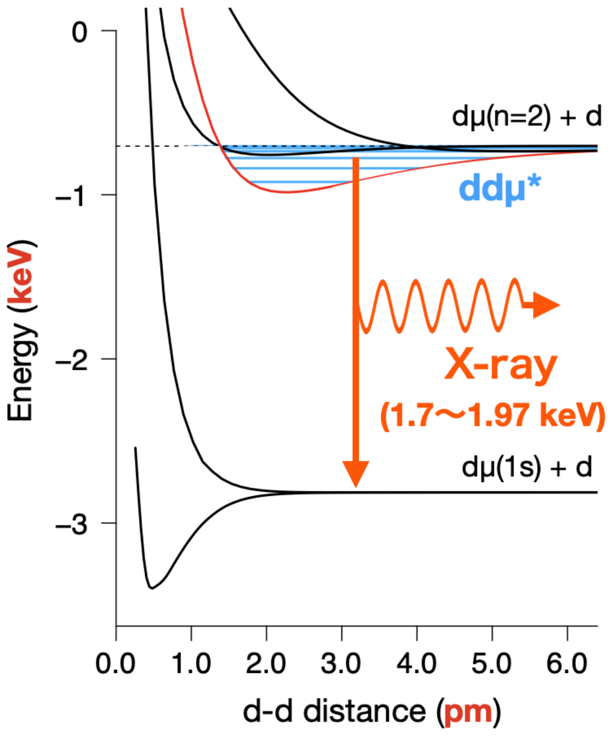
dμ+t→α+n+17.6 MeV

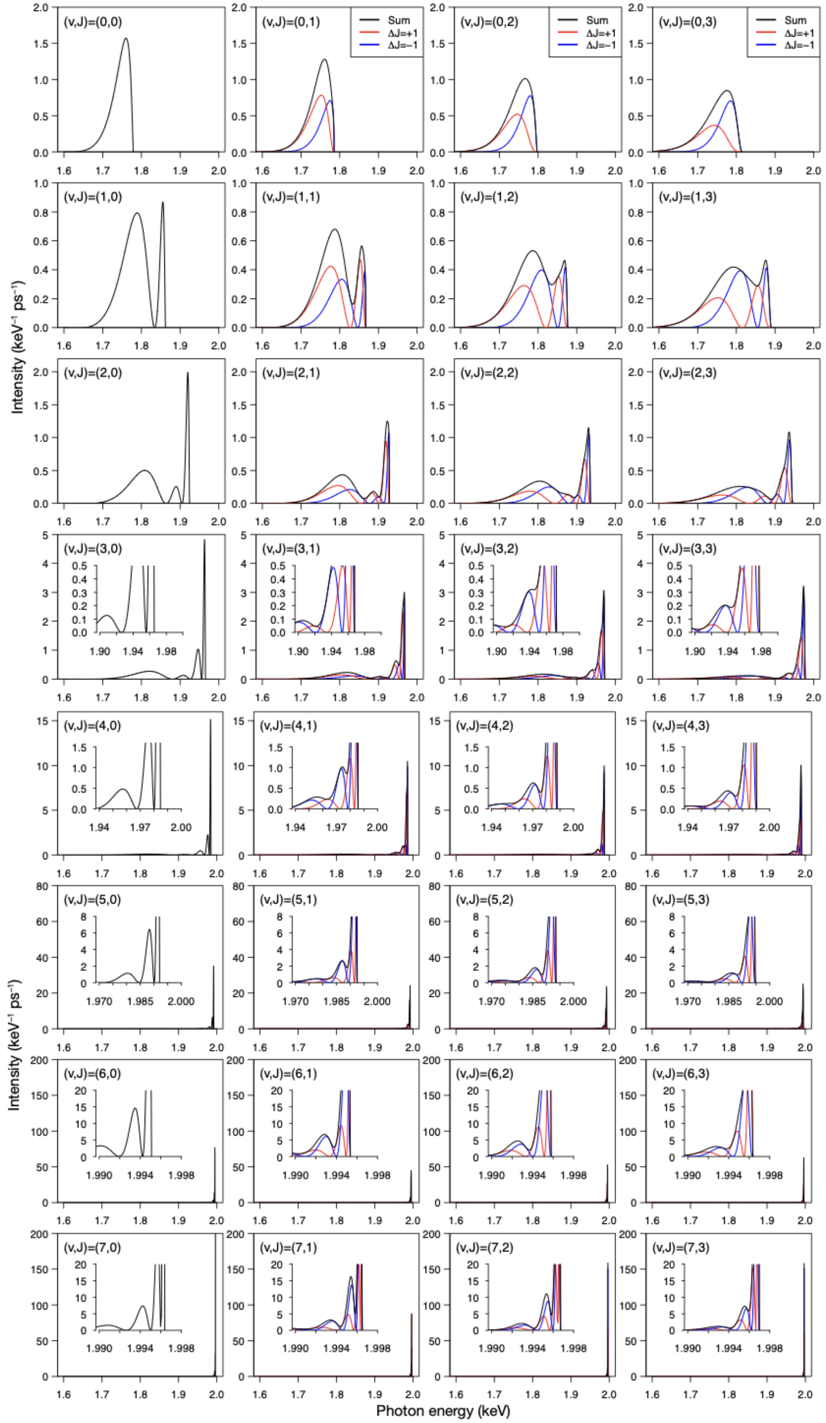




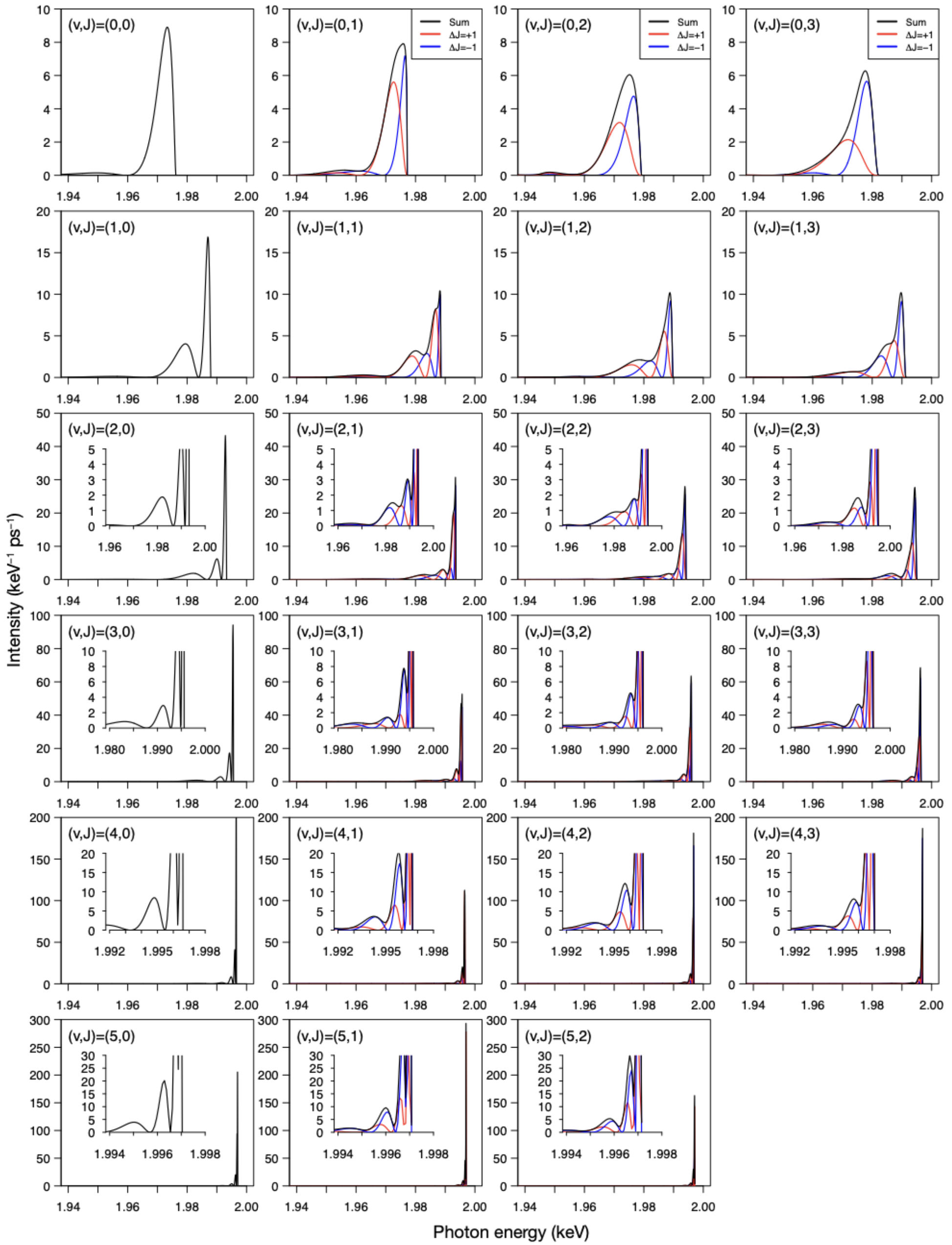
Models including dtμ* (SPM) and in-flight μCF (FIF) successfully explain past experimental results

X-ray spectra from v=0-8, J=0-3





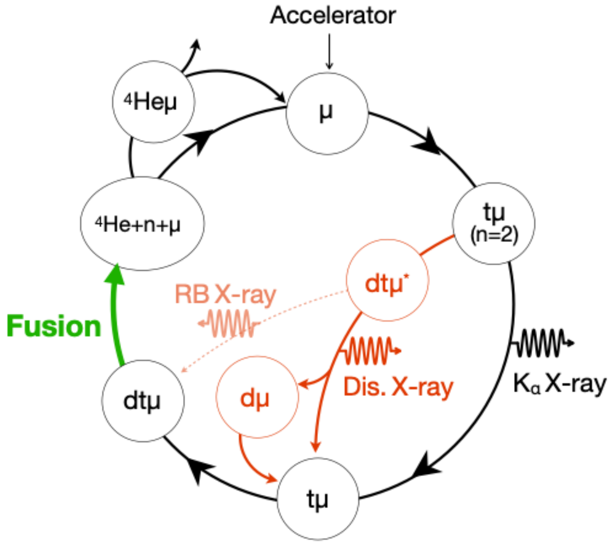
Yamashita, Yasuda, Kino, Phys. Rev. A 111, 012811 (2025).

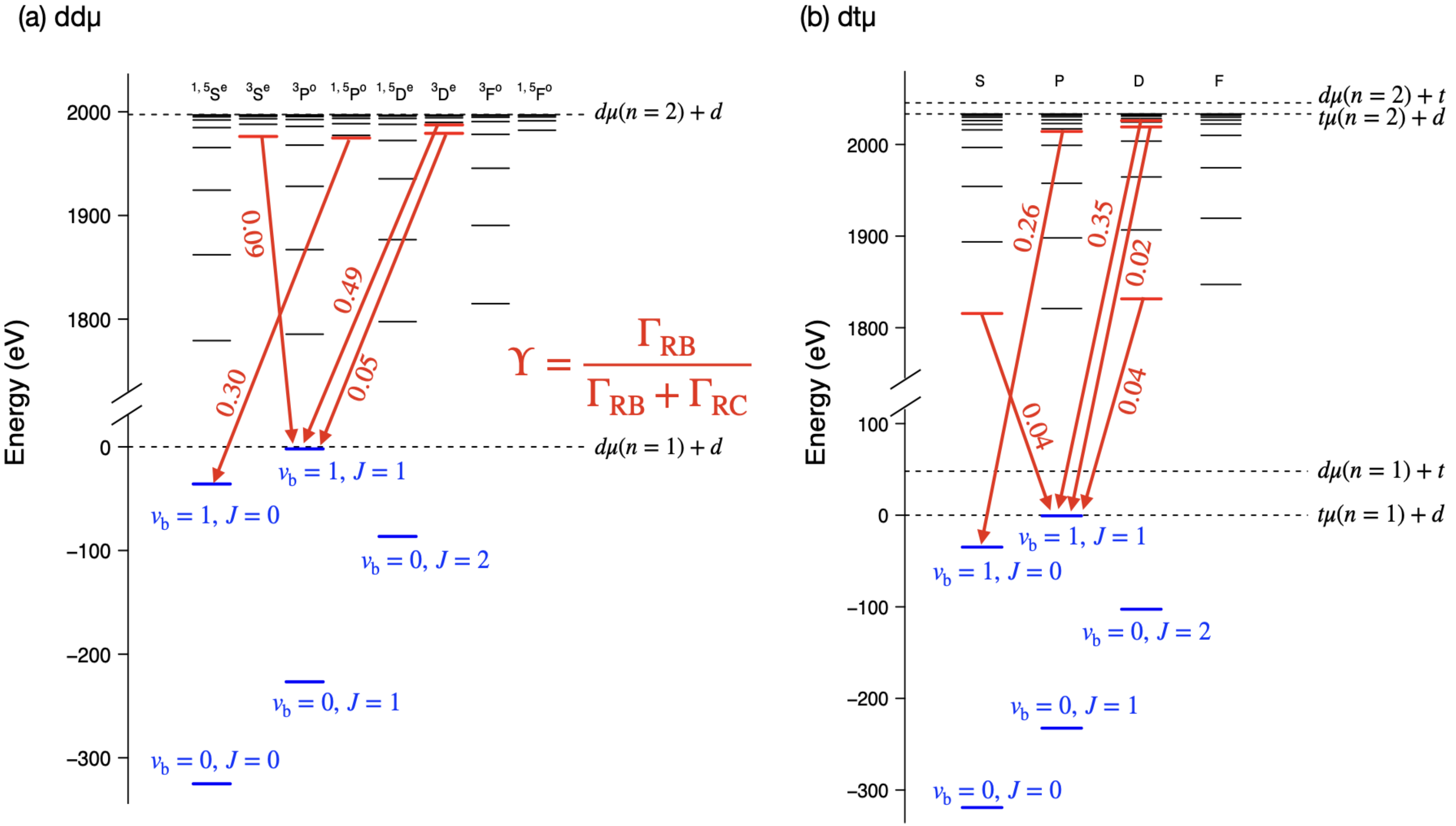


Novel transition from resonance to bound state

We theoretically discovered that some resonance states show a high branching ratio directly resulting in a bound state molecule.

(immediately undergoes fusion)

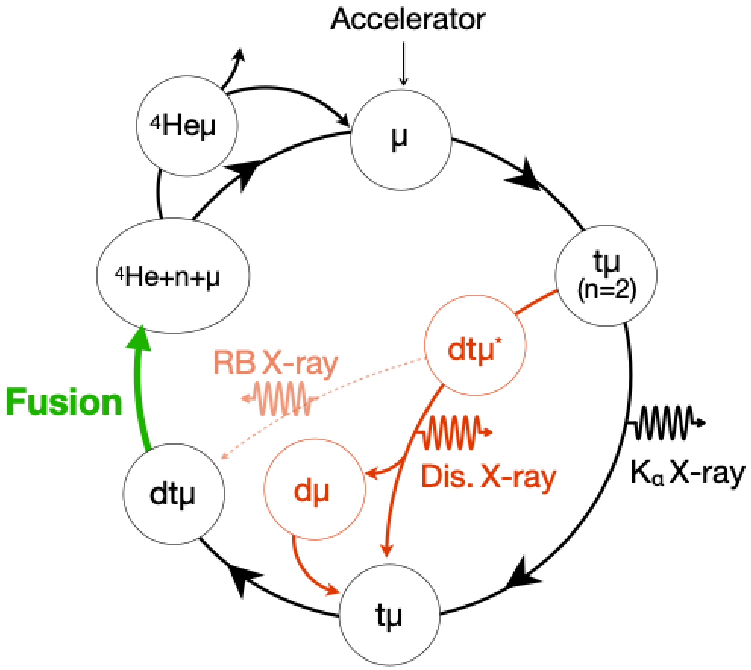




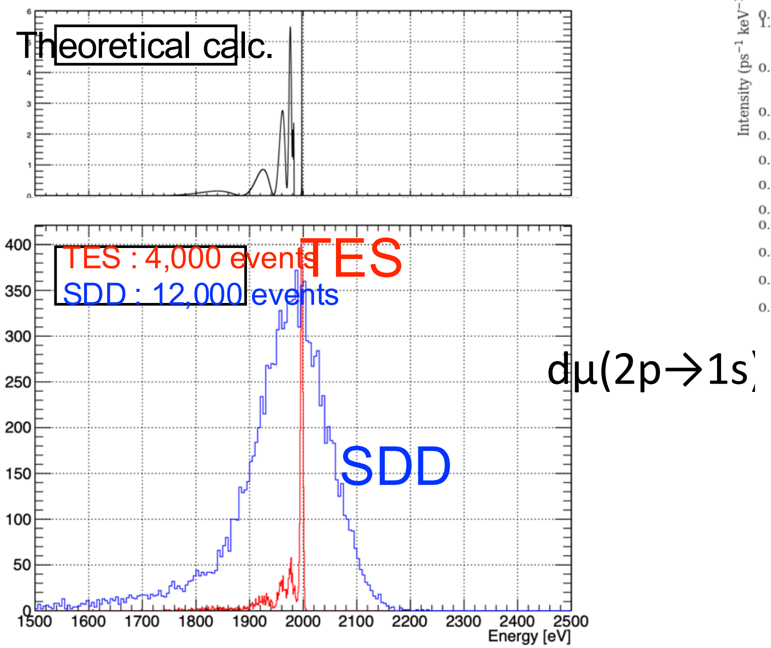
Yamashita Yasuda, Kino, Phys. Rev. A **111**, 012811 (2025).

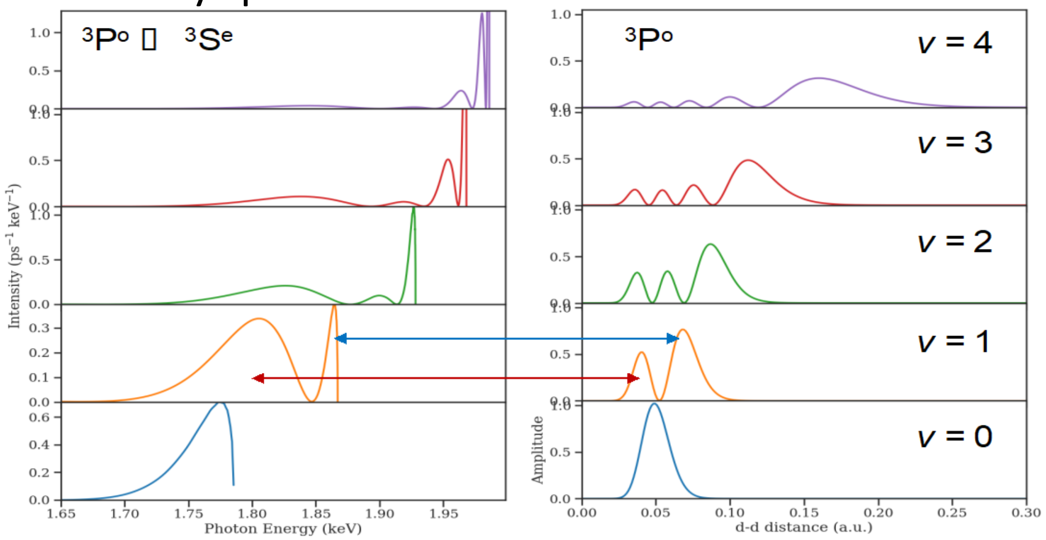
Verification of the new model→ High precision X-ray spectroscopy

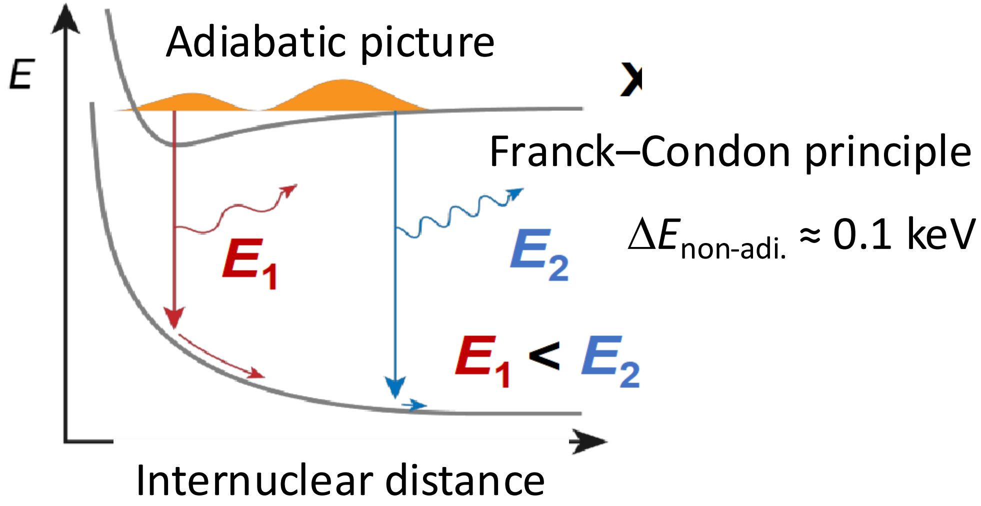
= obserbation of the X-ray from the dtμ resonance state

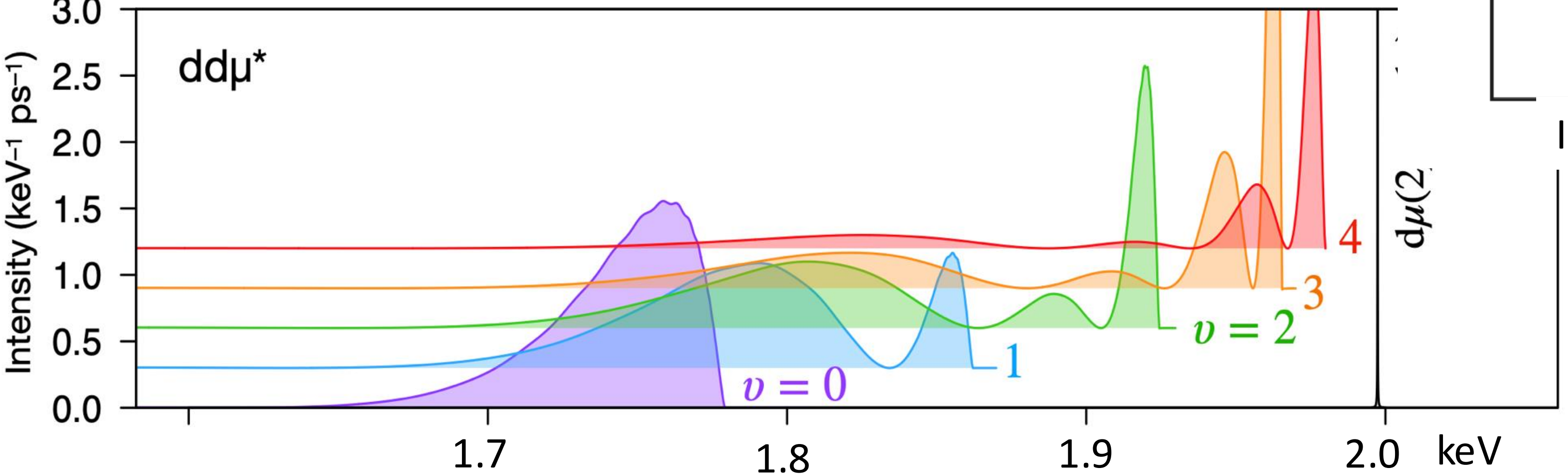


ddμ*→dμ(1s)+d+γ



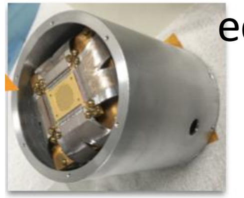




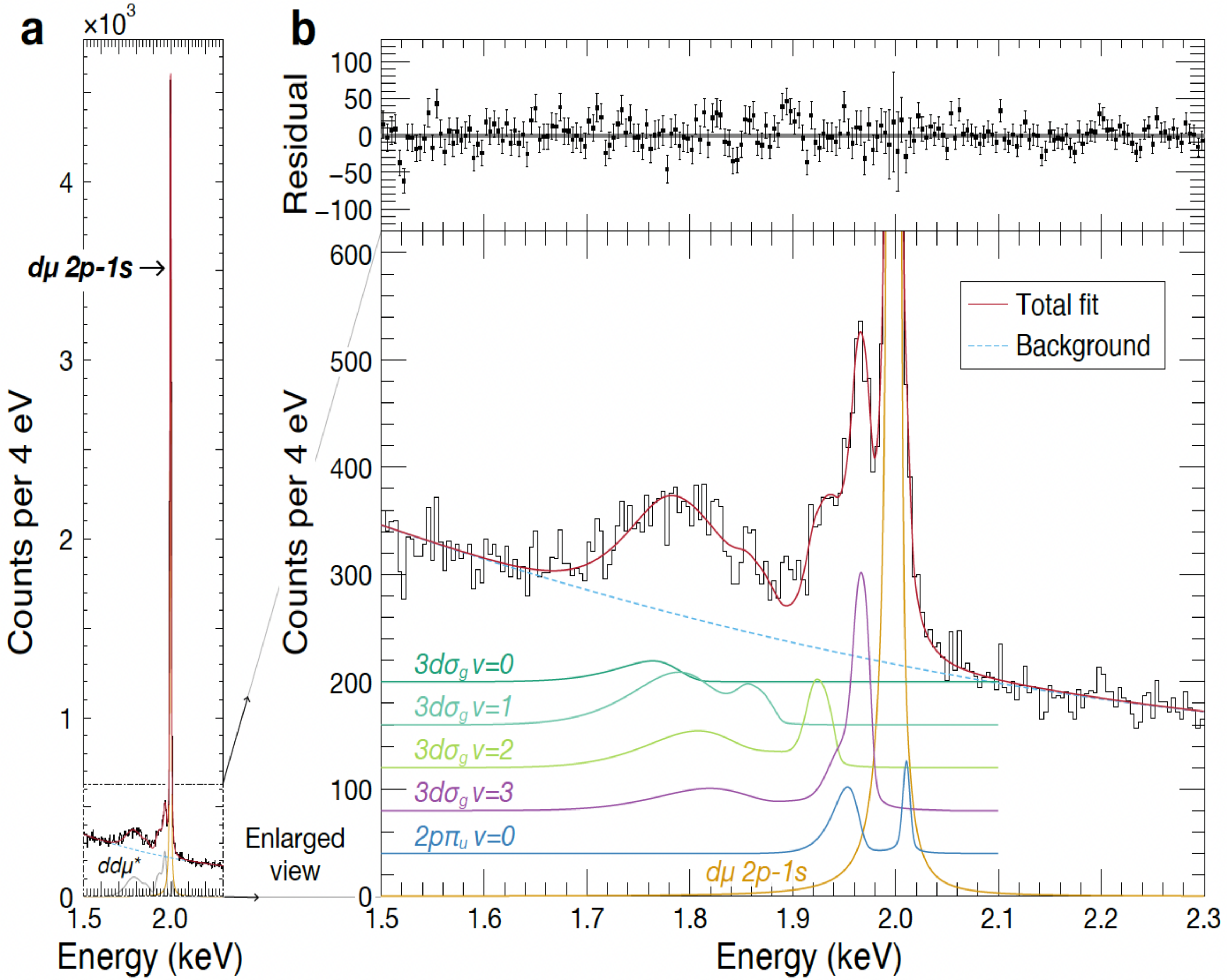


High precision X-ray spectroscopy

superconducting transition-edge sensor (TES)



5.2 eV@6 keV



Parent state of X-ray	v	Fraction
$d\mu\ 2p$	-	61.2
$dd\mu^*\ 3d\sigma_g$	0	2.5
	1	10.1
	2	10.1
	3	12.1
$dd\mu^*\ 2p\pi_u$	0	4.0

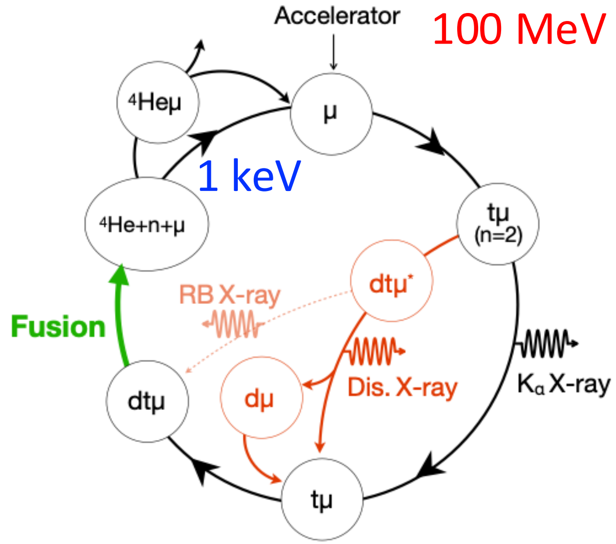
Taking nonradiative transitions into account,
ddμ* → dμ + d (particle decay)
nearly 50% of muons go through ddμ*
(the main pathway)

Y. Toyama et al., Sci. Adv. 12, eaed3321 (2026).

Full few-body study of nuclear reactions in muon-catalyzed fusion

Phys. Rev. C 107, (2023) 03467

M. Kamimura, Y. Kino and T. Yamashita

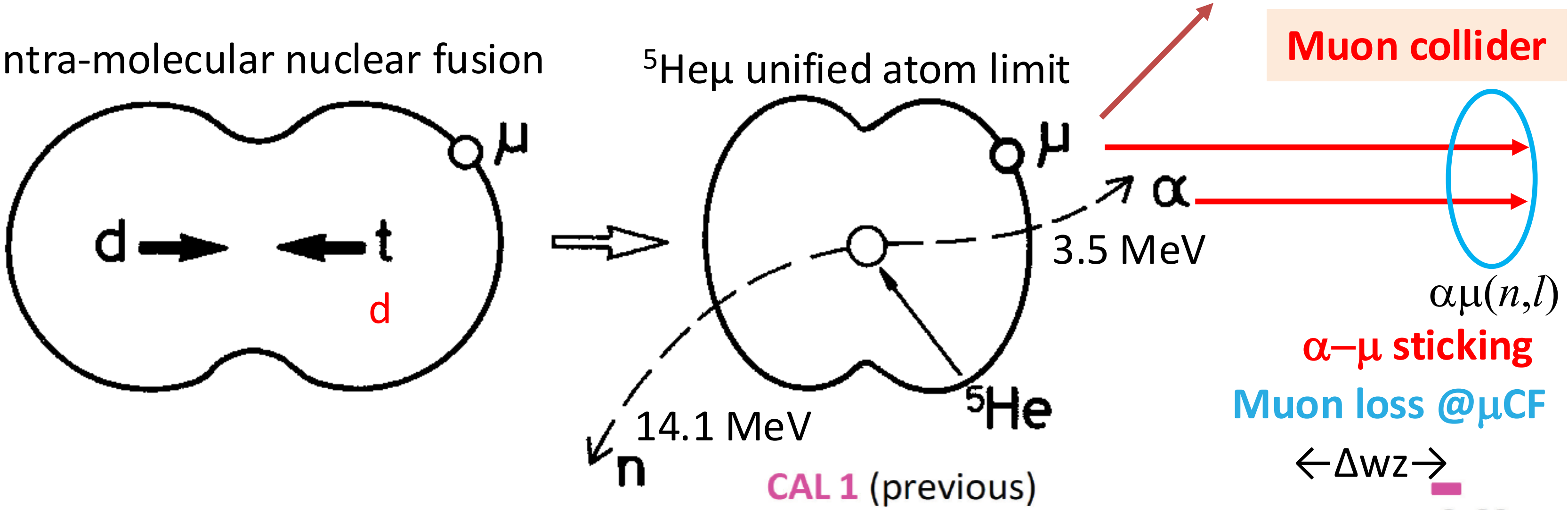


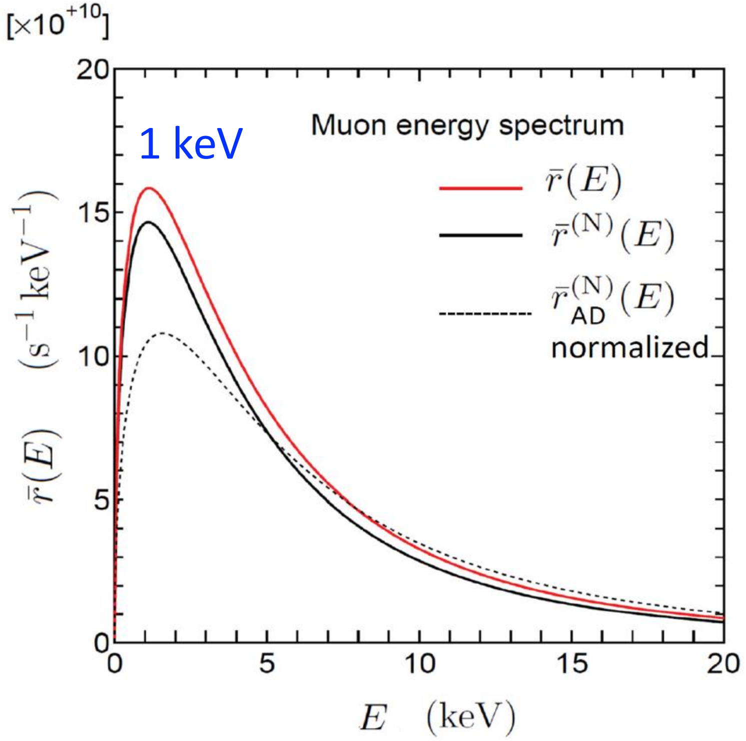
slow muon 1 keV

Muon collider

Intra-molecular nuclear fusion

$^5He\mu$ unified atom limit

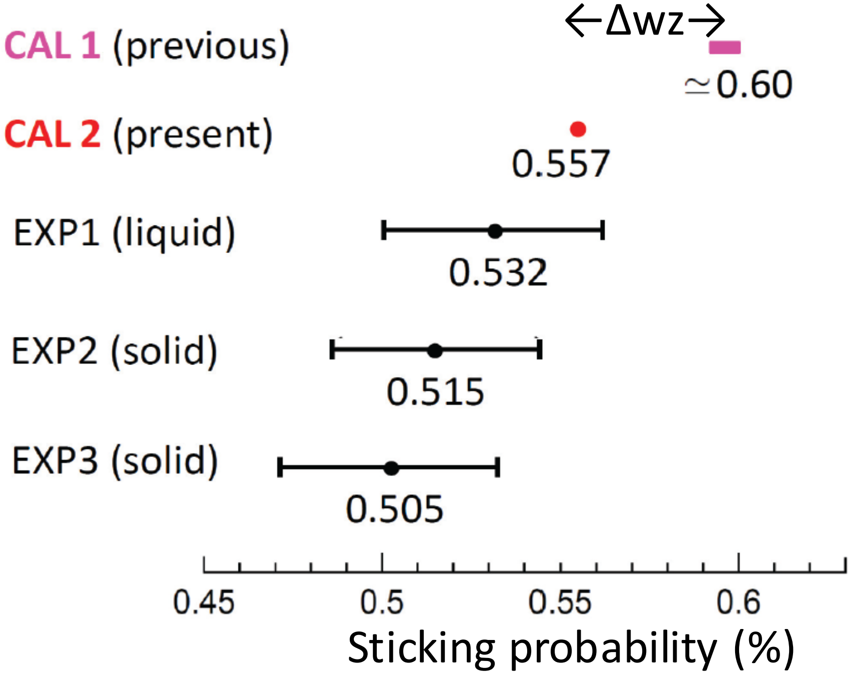




Precise nuclear interaction
exact boundary condition
Full few body calculation

Fusion rate : $1.15 \times 10^{12}\ s^{-1}$

Δw=0.05 %
#(fusion) 200→220



Future applications

compact neutron source
energy production

New model vs. experimental value

shadow : ambiguities in atomic physics

$0.1 \le \Upsilon_d \le 0.9$

__Cycle rate__ $\lambda_c = \lambda_\mu \left[\varphi\left(Y_f^{-1} - W\right)\right]^{-1}$

__Total sticking probability__

$$W = \sum_F P_{d\mu(1s,F)} \frac{0.58 c_d \lambda_{dd\mu(F)} \omega_d}{c_d \lambda_{dd\mu(F)} + c_t \lambda_{dt}} + \sum_F P_{t\mu(1s,F)} \frac{c_d \lambda_{dt\mu(F)} \omega_s + c_t \lambda_{tt\mu} \omega_t}{c_d \lambda_{dt\mu(F)} + c_t \lambda_{tt\mu}}$$

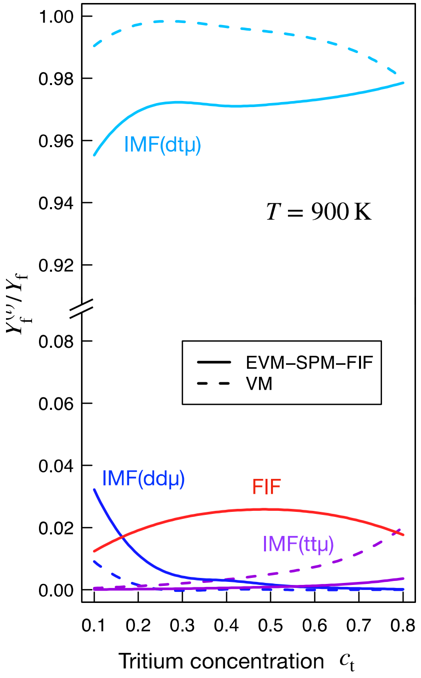
#(fusion)

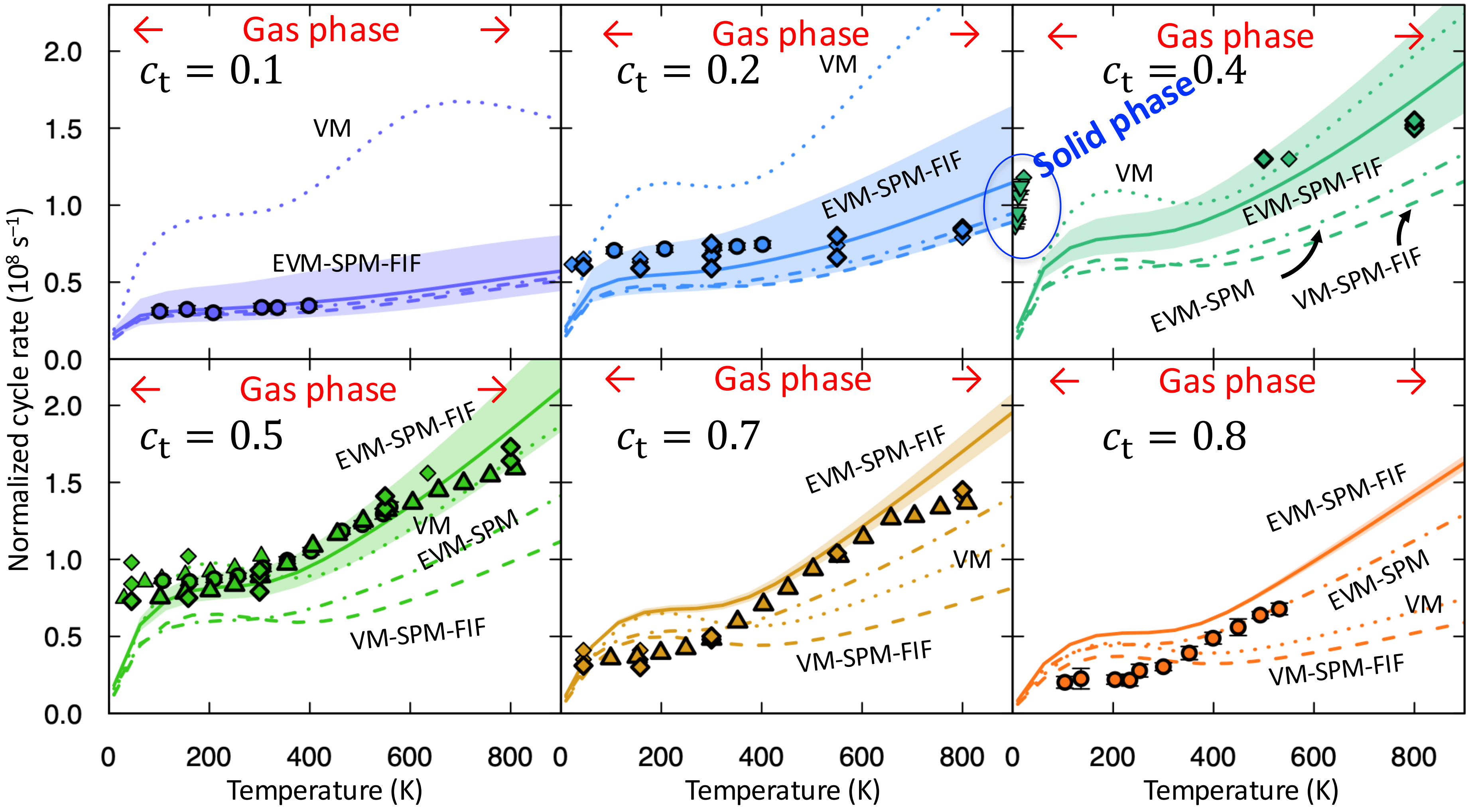
$$Y_f = \left[\frac{\lambda_\mu}{\varphi \lambda_c} + W\right]^{-1}$$

10^{-2} 10^{-2}

In-flight μCF

dμ+t→α+n+17.6 MeV





Models including dtμ* (SPM) and in-flight μCF (FIF) successfully explain past experimental results

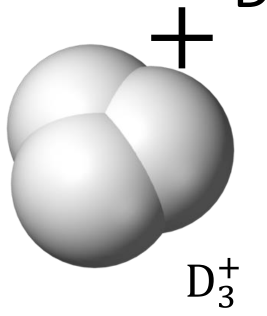
・Radiation Chemical Effects

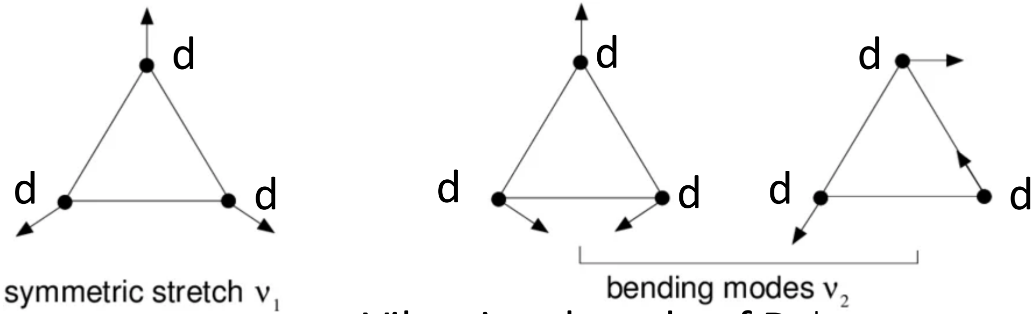
α, μ, n, ^3H β-decay, external radiation

Radiation → D_2 → D_2^+ + e, $D_2^+ + D_2 \rightarrow D_3^+$ + D (exothermic)

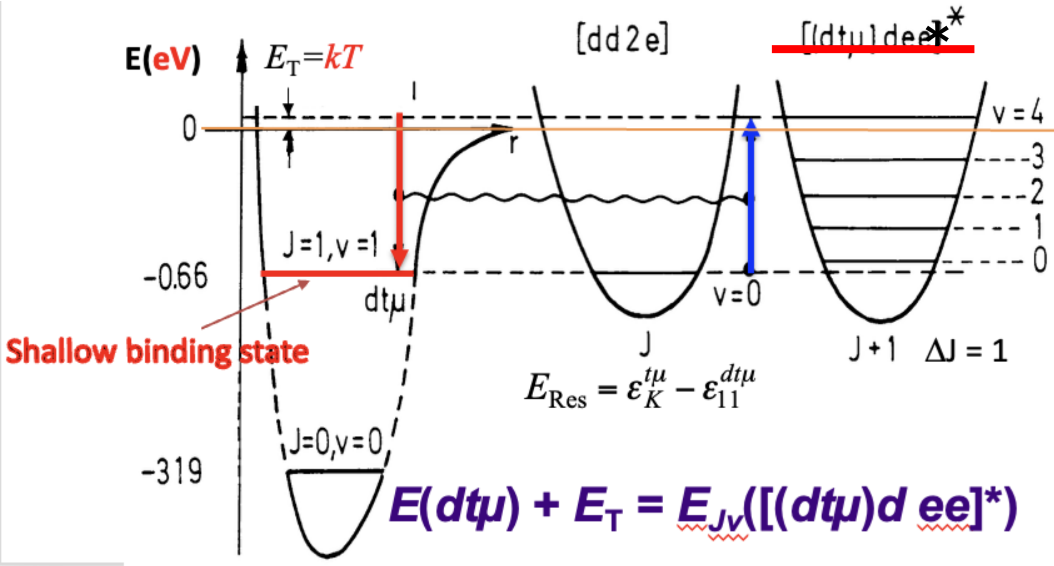
(long lifetime) Radiation Induced Species (RIS) :

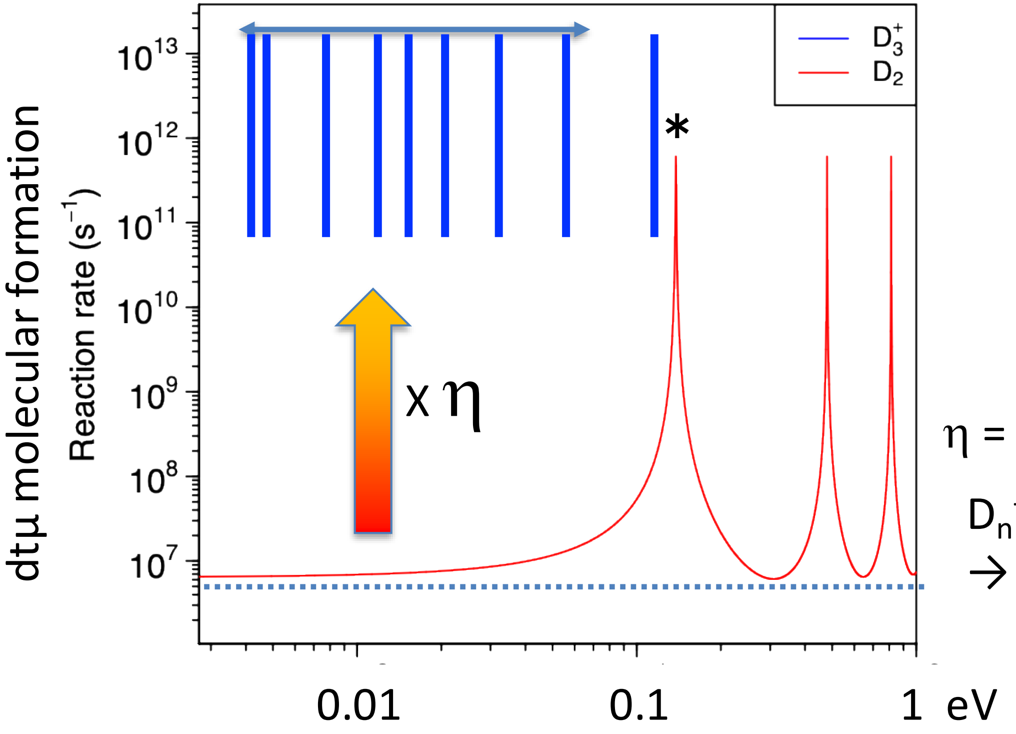
D_3^+, D_n^+ Clusters with D_3^+, Trapped electron, D_2^-





Vibrational mode of D_3^+





D_3^+

→ high formation rate for dtμ and dtμ*

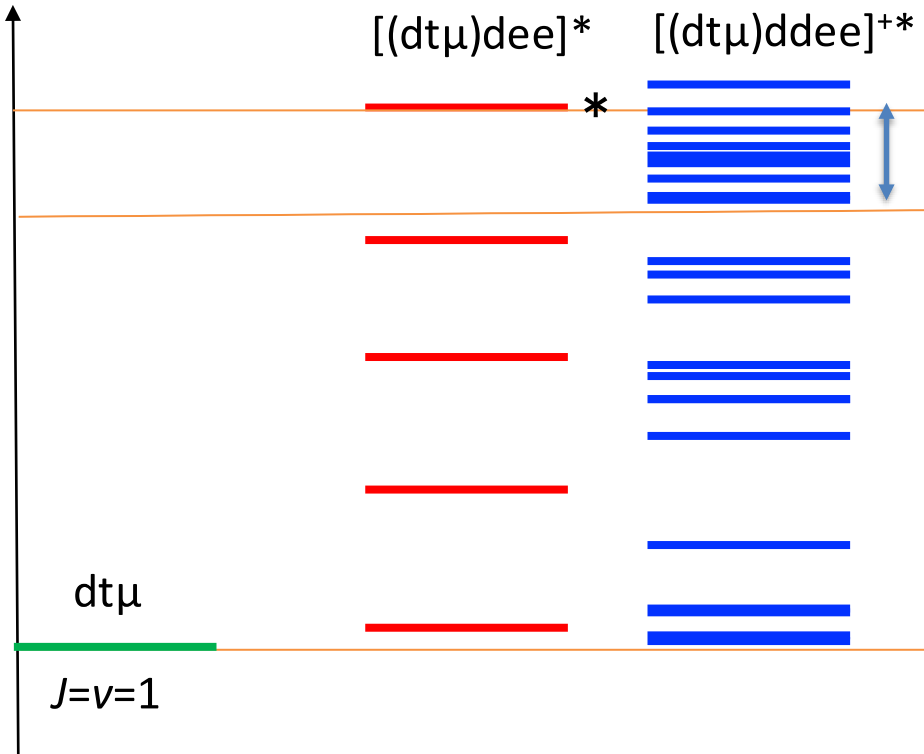
If ηC_{RIS} = 1, $\lambda_{dt\mu} \rightarrow 2\lambda_{dt\mu}$

η = 1000, $C_{RIS} = D_3^+/D_2$ = 0.1%

D_n^+ Clusters

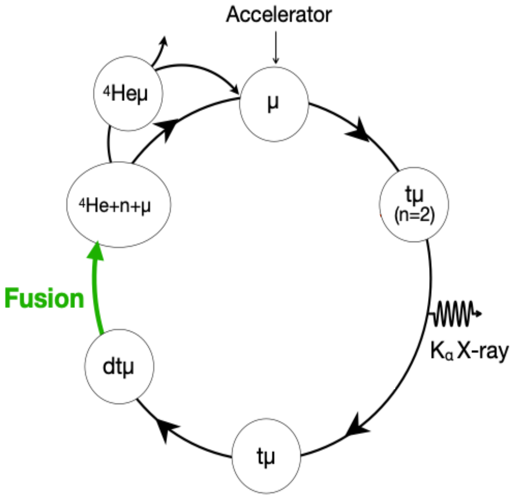
→ high density near μ (<1 μm)

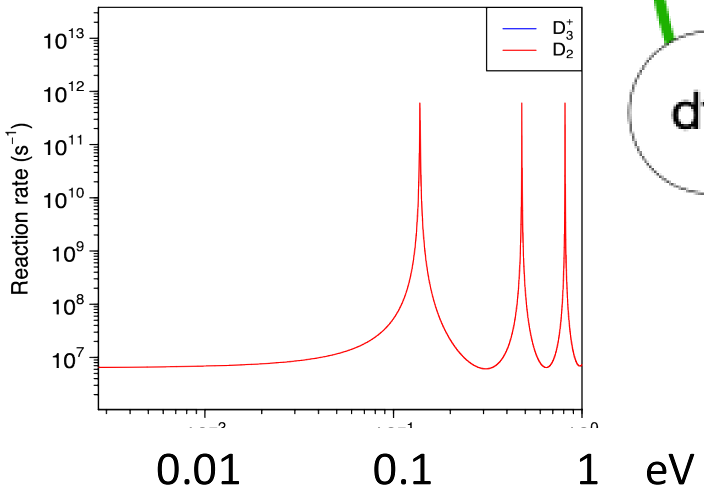
λ_c increases

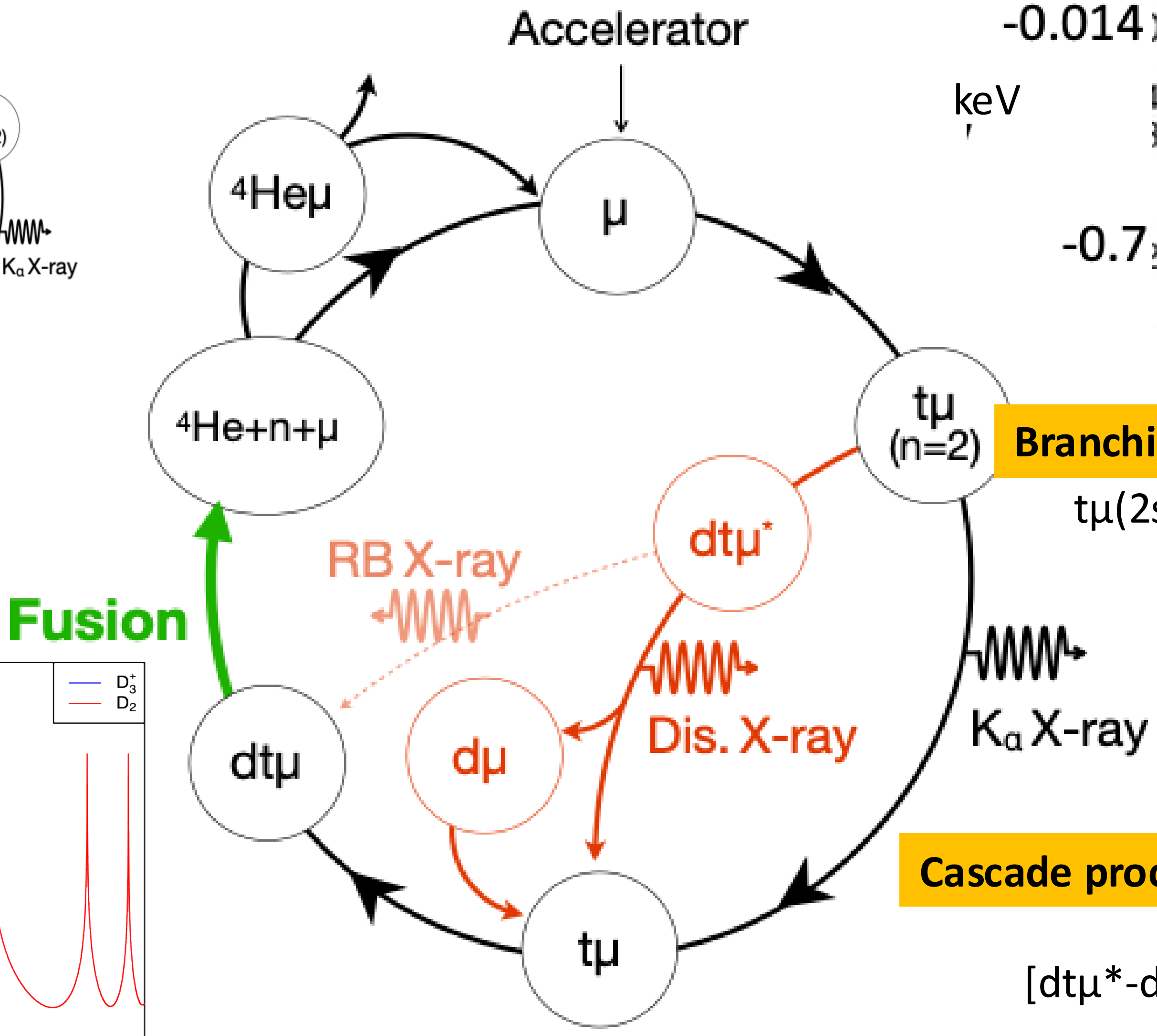


Future challenges

Conventional μCF cycle

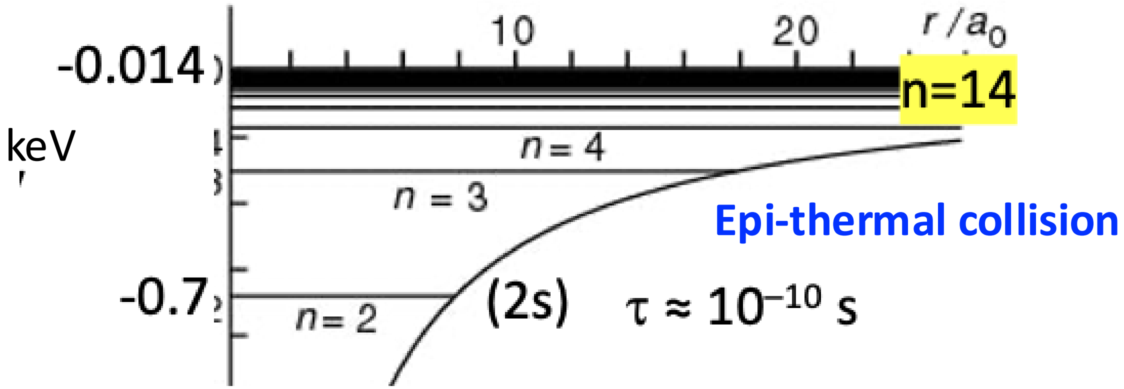






New μCF cycle

Cascade and collision processes of tμ and dμ atoms



tμ(n) + D_2 → tμ(n') + D_2^+ etc.

Branching ratio of dtμ* and thermalization of hot tμ

tμ(2s)→dtμ* or tμ(2s)→tμ(2p)→tμ(1s)

dtμ* gives hot tμ(1s) or dμ(1s)

High dtμ formation rate $>10^9$ s^{-1}

Few-body collision

tμ + D_2(Jv)

Cascade process of dtμ*

[$dt\mu_{Jv}$*-d]ee→ [$dt\mu_{J'v'}$*-d]e + e, etc.

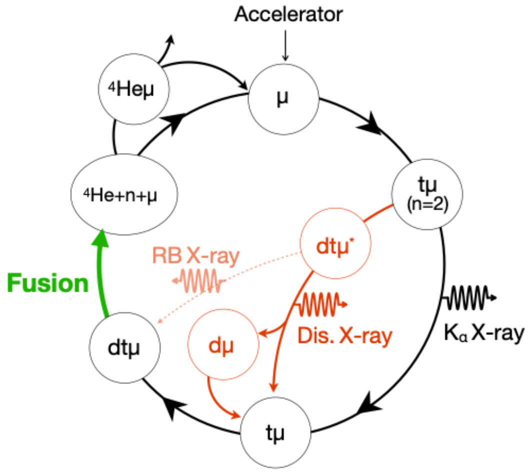
[dtμ*-d]ee

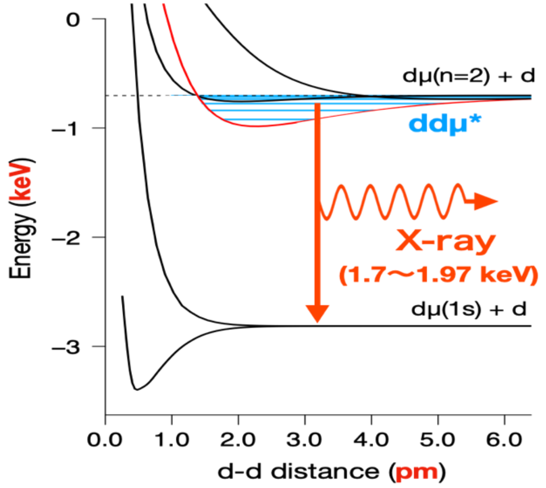
dtμ*-e and [d*-d]ee

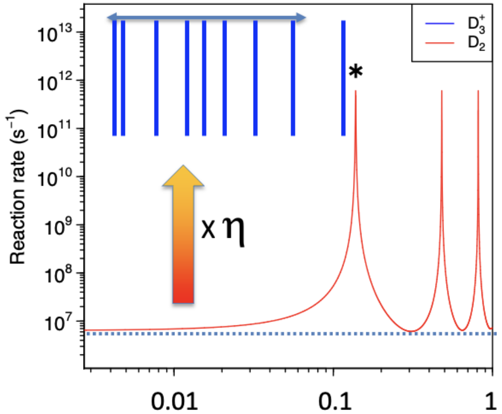
4-body problem

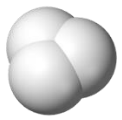
Cross sections (meV-eV)→ particle transportation calculation

Summary









D_3^+

- We have revived the muon catalyzed fusion (μCF) with precise few-body calculations and new μCF cycle.
- A new μCF cycle involving the metastable state (resonance state) dtμ*(ddμ*) of the muon molecule is verified both theoretically and experimentally.
- We found that D_3^+ produced in solid hydrogen by the radiochemical effect of alpha particles increases the muonic molecule production rate, the rate-limiting process of μCF.
- The μCf will open new research field for theoretical and experimental physics and chemistry.

Muon collider (slow muon source)
Nuclear transmutation for LLFP (Long Lived Fission Products)
Power station

Thank you for your attention

Acknowledgement

MOONSHOT
RESEARCH & DEVELOPMENT PROGRAM

科研費
KAKENHI