Spin Polarization and Ionization of Muonium

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June 29, 2026

大強度陽子加速器施設
J-PARC
Japan Proton Accelerator Research Complex

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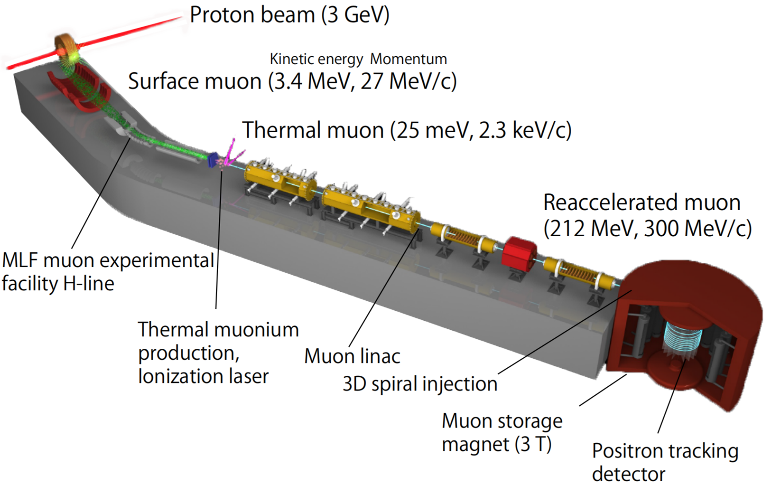
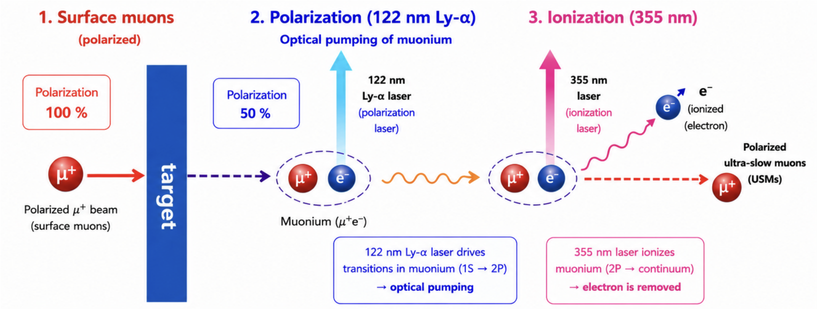
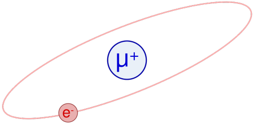


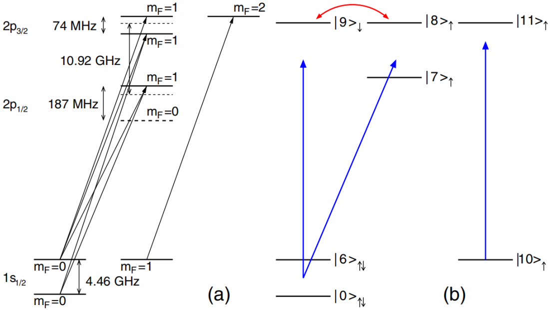
Muonium Spin Polarisation



Level structure for Muonium

- Muonium initially: $\sim 50\%$ spin polarization
- Short lifetime: $\sim 2.2\ \mu s$
- **Ground:** $|0\rangle, |6\rangle, |10\rangle$
- **Excited:** $|7\rangle, |8\rangle, |9\rangle, |11\rangle$

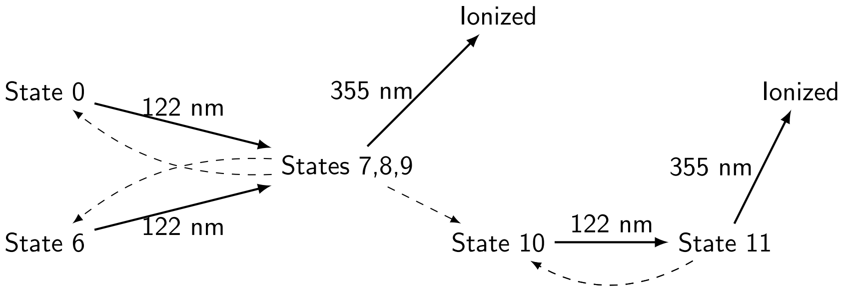




Nakajima 2012

Laser Working

- The lasers are pulsed and are assumed to have the same pulse duration (2 ns) and beam spot size (point-like for now).



122 nm Polarisation Laser

- $1s \rightarrow 2p$ pumping uses Lyman-α transition ($\lambda \approx 122$ nm).

355 nm Ionization Laser

- 355 nm laser is used to photoionize electrons excited to the $2p$ state.

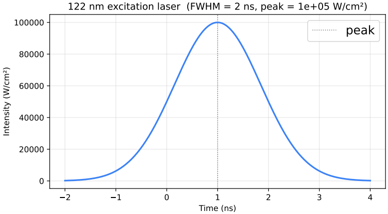
Temporal Shape of the Laser Pulses

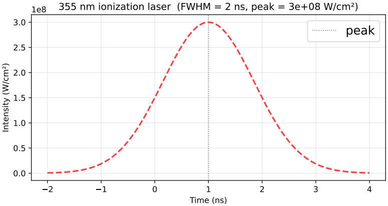
Gaussian temporal profiles,

$$I(t) = I_0 \exp\left[-\frac{(t-t_0)^2}{2\tau^2}\right]$$

I_0 : peak intensity, t_0 : pulse center, τ : Gaussian width parameter

$$\tau = \frac{\mathrm{FWHM}}{2\sqrt{2\ln 2}}$$





Rate Equations

$$\dot{\sigma}_{00} = \gamma_{\mathrm{sp}} \left(\frac{1}{3}\sigma_{77} + \frac{1}{6}\sigma_{88} + \frac{1}{2}\sigma_{99} \right) - \sum_{k=7,8,9} \frac{\gamma \Omega_{0k}^2}{\Delta_{k0}^2 + (\gamma/2)^2} (\sigma_{00} - \sigma_{kk})$$

$$\dot{\sigma}_{66} = \gamma_{\mathrm{sp}} \left(\frac{1}{3}\sigma_{77} + \frac{1}{6}\sigma_{88} + \frac{1}{2}\sigma_{99} \right) - \sum_{k=7,8,9} \frac{\gamma \Omega_{6k}^2}{\Delta_{k6}^2 + (\gamma/2)^2} (\sigma_{66} - \sigma_{kk})$$

$$\dot{\sigma}_{1010} = \gamma_{\mathrm{sp}} \left(\frac{1}{3}\sigma_{77} + \frac{2}{3}\sigma_{88} + \sigma_{1111} \right) - \frac{\gamma \Omega_{1110}^2}{\Delta_{1110}^2 + (\gamma/2)^2} (\sigma_{1010} - \sigma_{1111})$$

$$\dot{\sigma}_{77} = -\gamma_{\mathrm{sp}}\sigma_{77} + \sum_{k=0,6} \frac{\gamma \Omega_{k7}^2}{\Delta_{7k}^2 + (\gamma/2)^2} (\sigma_{kk} - \sigma_{77}) - \gamma_{\mathrm{ion}}\sigma_{77}$$

$$\dot{\sigma}_{88} = -\gamma_{\mathrm{sp}}\sigma_{88} + \sum_{k=0,6} \frac{\gamma \Omega_{k8}^2}{\Delta_{8k}^2 + (\gamma/2)^2} (\sigma_{kk} - \sigma_{88}) + \frac{2(\Omega_{89}^H)^2}{\gamma_{\mathrm{sp}}} (\sigma_{99} - \sigma_{88}) - \gamma_{\mathrm{ion}}\sigma_{88}$$

$$\dot{\sigma}_{99} = -\gamma_{\mathrm{sp}}\sigma_{99} + \sum_{k=0,6} \frac{\gamma \Omega_{k9}^2}{\Delta_{9k}^2 + (\gamma/2)^2} (\sigma_{kk} - \sigma_{99}) - \frac{2(\Omega_{89}^H)^2}{\gamma_{\mathrm{sp}}} (\sigma_{99} - \sigma_{88}) - \gamma_{\mathrm{ion}}\sigma_{99}$$

$$\dot{\sigma}_{1111} = -\gamma_{\mathrm{sp}}\sigma_{1111} + \frac{\gamma \Omega_{1110}^2}{\Delta_{1110}^2 + (\gamma/2)^2} (\sigma_{1010} - \sigma_{1111}) - \gamma_{\mathrm{ion}}\sigma_{1111}$$

Numerical Solution and Spin Polarization

- Integrated using Runge–Kutta RK45

Pulse sequence:

- Each pulse: evolve for $\tau = 2ns$ with (Δ, Ω)
- Between pulses: free evolution $(\Omega = 0)$ for interval $= 2ns$

Spin-up population:

$$P_{up} = \frac{1}{2}P_0 + \frac{1}{2}P_6 + P_{10}$$

Spin-down population:

$$P_{down} = \frac{1}{2}P_0 + \frac{1}{2}P_6$$

Final polarization:

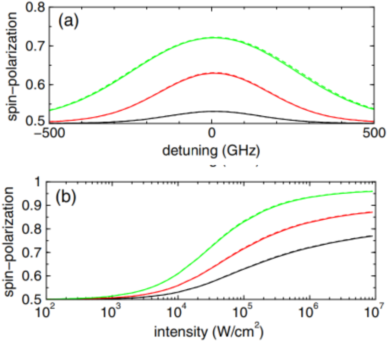
$$P = \frac{P_{up} - P_{down}}{P_{up} + P_{down}}$$

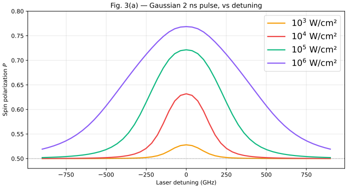
Polarisation of Muonium with Gaussian Pulse(s)

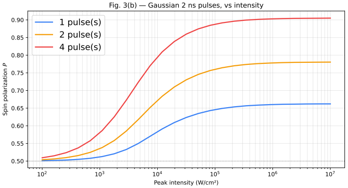
Ionisation laser: OFF

- Spin Polarization vs Detuning at Different Intensities
- Spin Polarization vs Intensity at Different Number of Pulses

Nakajima (2012)

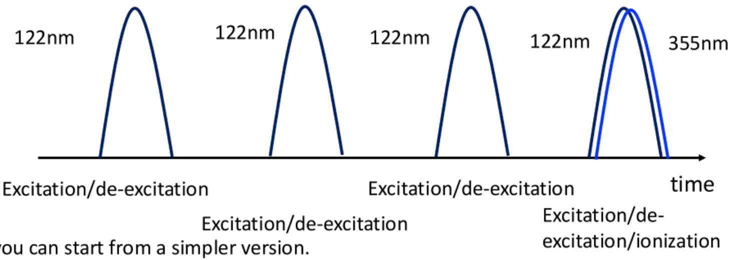




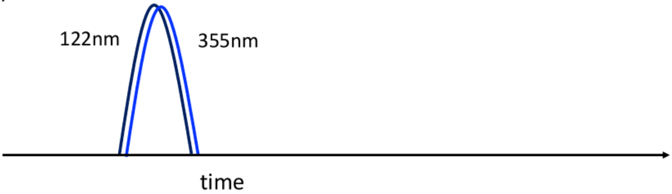


Pulse Sequence

Finally, we will use several 122nm pulses and shoot only one 355nm pulse



But you can start from a simpler version.
Only one 122nm & 355nm



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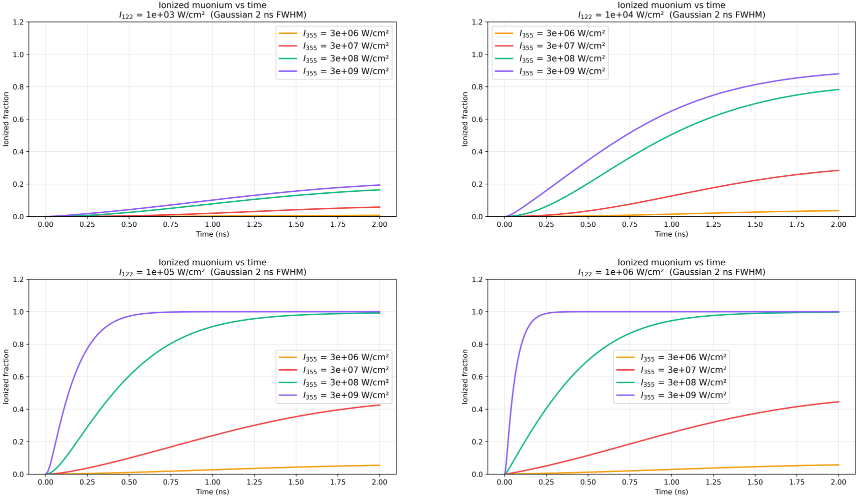
Slide courtesy: Kamioka-san

Ionized Fraction vs Time

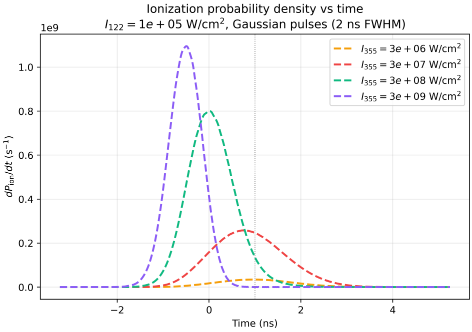


Ionization Probability Density

Comparison of the temporal ionization rate for different 355 nm laser intensities. It gives the time at which muons are produced most rapidly.

- The area under each curve equals the final ionized fraction:

$$P_{\text{ion}}(t) = \int_0^t \frac{dP_{\text{ion}}}{dt'}\, dt'.$$



Muon Spin Polarization after Ionisation

Ionization removes the electron while preserving the muon-spin assignment

State	Muon spin
$\|7\rangle$	$\uparrow_\mu$
$\|8\rangle$	$\uparrow_\mu$
$\|9\rangle$	$\downarrow_\mu$
$\|11\rangle$	$\uparrow_\mu$

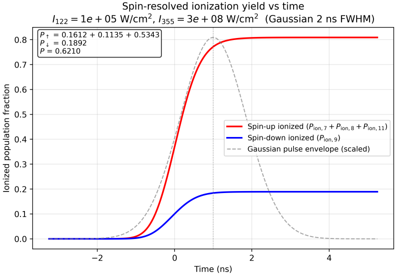
$$P_\uparrow^{\text{ion}} = P_{\text{ion7}} + P_{\text{ion8}} + P_{\text{ion11}}$$

$$P_\downarrow^{\text{ion}} = P_{\text{ion9}}$$

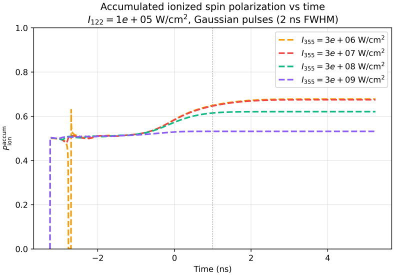
$$P_{\text{ion}} = \frac{P_\uparrow^{\text{ion}} - P_\downarrow^{\text{ion}}}{P_\uparrow^{\text{ion}} + P_\downarrow^{\text{ion}}}$$

Ionized Spin Composition and Polarization

I355 (W/cm²)	Pion7	Pion8	Pion9	Pion11	Pup	Pdown	P
3.0e+06	0.010839	0.009712	0.011296	0.038291	0.058842	0.011296	0.677892
3.0e+07	0.078859	0.068508	0.083239	0.281350	0.428717	0.083239	0.674821
3.0e+08	0.161176	0.113479	0.189163	0.534292	0.808946	0.189163	0.620957
3.0e+09	0.166500	0.090787	0.234111	0.508603	0.765889	0.234111	0.531779



Spin-resolved ionized populations
$I_{122} = 10^5\ \mathrm{W/cm^2}$
$I_{355} = 3 \times 10^8\ \mathrm{W/cm^2}$

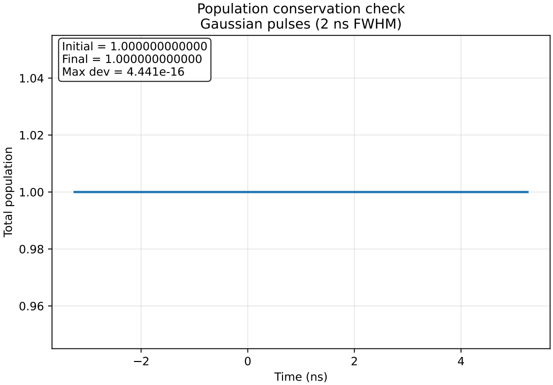


Ionized-spin polarization for different 355 nm ionization intensities.

Population Conservation

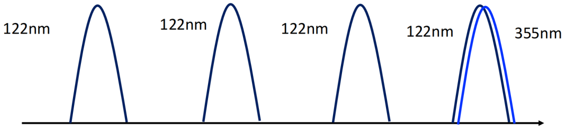
$$P_{\mathrm{tot}}(t) = \sum_i \sigma_{ii}(t)$$

$$P_{\mathrm{tot}}(t) = \sigma_{00} + \sigma_{66} + \sigma_{77} + \sigma_{88} + \sigma_{99} + \sigma_{1010} + \sigma_{1111} \\ + \sigma_{\mathrm{ion}7} + \sigma_{\mathrm{ion}8} + \sigma_{\mathrm{ion}9} + \sigma_{\mathrm{ion}11}$$

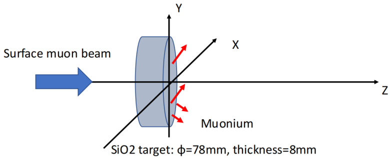


Outlook

- Including the pulse sequence : 3 pumping and 1 (pumping + ionization) pulse



- Realistic muonium samples



- Beam with some cross-section area, not a point like cross-section.

THANK YOU
QUESTIONS & FEEDBACK?

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BACKUP SLIDES

Ionized-State Populations

The ionization process transfers population from the excited states to dedicated ionized population states:

$$\begin{aligned}
\dot{\sigma}_{\text{ion},77} &= \gamma_{\text{ion}}\sigma_{77}, \\
\dot{\sigma}_{\text{ion},88} &= \gamma_{\text{ion}}\sigma_{88}, \\
\dot{\sigma}_{\text{ion},99} &= \gamma_{\text{ion}}\sigma_{99}, \\
\dot{\sigma}_{\text{ion},11} &= \gamma_{\text{ion}}\sigma_{1111}.
\end{aligned}$$

The total ionized fraction is

$$\sigma_{\text{ion}}^{\text{total}} = \sigma_{\text{ion},77} + \sigma_{\text{ion},88} + \sigma_{\text{ion},99} + \sigma_{\text{ion},11}.$$

The 355 nm laser removes population from the excited $2p$ states at rate γ_{ion}, while conserving the total population when ionized states are included explicitly.

Photoionization Rate $\gamma_{\rm ion}$

$$\gamma_{\rm ion} = \sigma_{\rm ion} \frac{I_{355}}{h\nu_{355}}$$

where

- $\sigma_{\rm ion}$: photoionization cross section from the excited $2p$ state
- I_{355} : 355 nm laser intensity
- $h\nu_{355}$: photon energy at 355 nm

Increasing the 355nm intensity increases $\gamma_{\rm ion}$ linearly, leading to faster ionization and a larger final ionized fraction.

Photoionization Cross Section σ_{ion}

σ_{ion} is the effective area that quantifies the probability of a photon ionizing an atom from an excited state. $\sigma_{2P \to \text{cont}}$ is the photoionization cross section of muonium from the 2P state. It is the effective probability (area units) that a 2P muonium atom absorbs a 355 nm photon and becomes ionized.

The photoionization cross section is calculated within the electric-dipole approximation using QED.

The resulting cross section (from Meng's thesis) is

$$\sigma_{\text{ion}} = \sigma(2P \to \text{unbound}) = 1.26 \times 10^{-21}\ \text{m}^2$$

A larger photoionization cross section means a higher probability of ionization for the same laser intensity.

Ionized Fraction vs Time

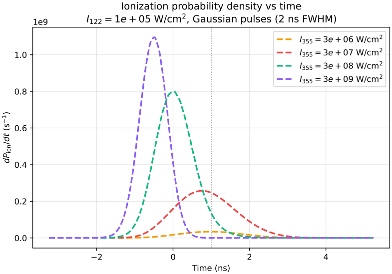
- The time evolution of the total ionized muonium fraction during a 2 ns laser pulse.

Simulation Parameters

- Pulse duration: $\tau = 2\ \mathrm{ns}$, Detuning: $\Delta = 0$
- 122 ns intensities: $10^3,\ 10^4,\ 10^5,\ 10^6\ \mathrm{W/cm^2}$
- 355 ns intensities: $3x10^3,\ 3x10^4,\ 3x10^5,\ 3x10^9\ \mathrm{W/cm^2}$
- Total ionized fraction: $P_{\mathrm{ion}} = P_{\mathrm{ion7}} + P_{\mathrm{ion8}} + P_{\mathrm{ion9}} + P_{\mathrm{ion11}}$

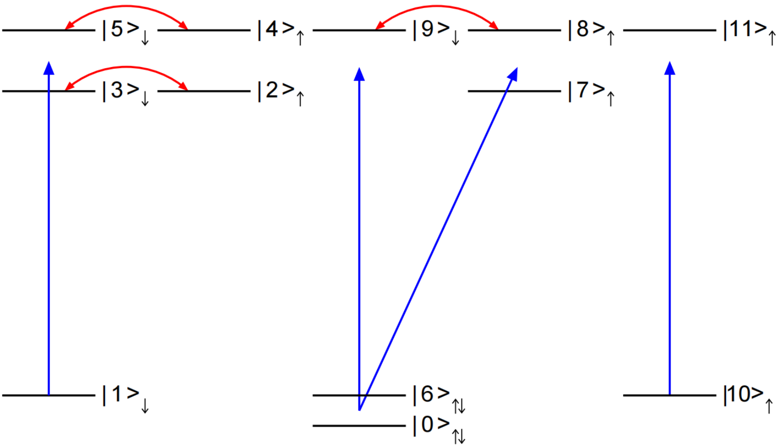
Ionization Probability Density

- Increasing I_{355} increases the peak ionization rate and shifts ionization to earlier times.
- For large I_{355}, ionization occurs almost immediately after population reaches the excited states.
- At low I_{355}, ionization is slow and spread throughout the pulse; at high I_{355}, ionization becomes nearly instantaneous.



- The area under each curve equals the final ionized fraction:

$$P_{\mathrm{ion}}(t) = \int_0^t \frac{dP_{\mathrm{ion}}}{dt'}\, dt'.$$



$|5>_\downarrow$
$|4>_\uparrow$
$|9>_\downarrow$
$|8>_\uparrow$
$|11>_\uparrow$
$|3>_\downarrow$
$|2>_\uparrow$
$|7>_\uparrow$
$|1>_\downarrow$
$|6>_{\uparrow\downarrow}$
$|0>_{\uparrow\downarrow}$
$|10>_\uparrow$

Rabi Frequency

Rabi frequency for transition $i \to j$

$$\Omega_{ij} = \frac{d_{ij} E_0}{\hbar}$$

Electric field and Dipole moment:

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}}, \quad d_{ij} = C_{ij} \cdot d_0$$

- I: Laser Intensity
- C_{ij}: Clebsch–Gordan coefficients
- $d_0 = ea_0$ (electron charge * Bohr radius)

$$\Omega_{ij} = C_{ij} \cdot \Omega_0, \quad \Omega_0 = \frac{d_0}{\hbar}\sqrt{\frac{2I}{c\epsilon_0}}$$

- Laser intensity sets overall coupling strength
- Relative transition strengths set by C_{ij}

Numerical Solution (WITH IONISATION)

Solving the rate equations:

- Integrated using the Runge–Kutta RK45 method.
- Initial population vector:

$$(P_0, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{\mathrm{ion7}}, P_{\mathrm{ion8}}, P_{\mathrm{ion9}}, P_{\mathrm{ion11}})$$

$$= (0.25, 0.25, 0, 0, 0, 0.5, 0, 0, 0, 0, 0)$$

- Population is tracked in both bound and ionized states.
- The 122 nm laser drives optical pumping through the rates

$$W_{ij}(\Delta, \Omega)$$

- The 355 nm laser introduces photoionization with rate

$$\gamma_{\mathrm{ion}} = \frac{I_{355}}{h\nu_{355}} \sigma_{\mathrm{ion}}$$

Numerical Solution (NO IONISATION)

Solving the rate equations:

- Integrated using Runge–Kutta RK45
- Initial state:

$$(P_0, P_6, P_7, P_8, P_9, P_{10}, P_{11}) = (0.25, 0.25, 0, 0, 0, 0.5, 0)$$

- For ns pulse cutoff (b) is included in γ calculation, which changes the W_{ij} at different Δ.
- For ps pulse $\gamma = \gamma_{sp}$

Pulse sequence:

- Each pulse: evolve for $\tau = 2ns/2ps$ with (Δ, Ω)
- Between pulses: free evolution $(\Omega = 0)$ for interval $= 5ns$

Spin polarization:

$$P = \frac{P_\uparrow - P_\downarrow}{P_\uparrow + P_\downarrow}$$
$$P_\uparrow = \tfrac{1}{2}(P_0 + P_6) + P_{10}, \quad P_\downarrow = \tfrac{1}{2}(P_0 + P_6)$$

Spontaneous Decay and Hyperfine Mixing

Spontaneous decay (γ_{sp}):

- Excited states ($|7\rangle, |8\rangle, |9\rangle$) decay to ground states
- Lifetime: 1.6 ns $\Rightarrow$ $\gamma_{sp} = \frac{1}{1.6 \text{ ns}}$
- Branching ratios are the squares of Clebsch–Gordan coefficients

Hyperfine mixing ($|8\rangle \leftrightarrow |9\rangle$):

- Coupling within $2p_{3/2}$
- Hyperfine splitting: $\omega_{hf} = 74$ MHz

$$\Omega_{89} = \frac{\Delta\omega_{hf}}{2}$$

Dynamics:

- Decay redistributes population to ground states
- Hyperfine mixing transfers population between $|8\rangle$ and $|9\rangle$

Laser Bandwidth Model: 2 ns

- **Broadband ns pulse:**
  - Lorentzian linewidth with cutoff:

$$P_{ns}(\Delta) = \frac{1}{\pi}\frac{\gamma}{\Delta^2 + (\gamma/2)^2}, \quad \gamma_L = \gamma_{L0}\frac{b^2}{\Delta^2 + b^2}$$

- $\gamma_{L0} \approx 230$ GHz, $\quad b = 0.8\,\gamma_{L0}, \quad \gamma = \gamma_{sp} + \gamma_L$
- Cutoff b prevents unphysical long Lorentzian tails