Update on DLS simulation for high energy linac

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29th June 2026

Motivation

- The statistical precision on the anomalous precession frequency ω_a is:

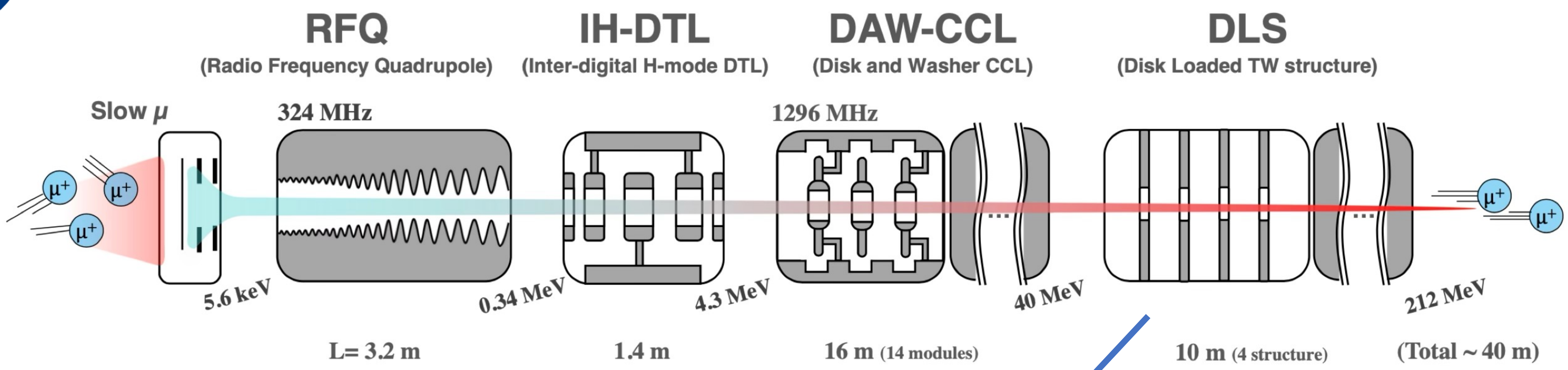
$$\boxed{\frac{\Delta\omega_a}{\omega_a} = \frac{1}{\omega_a \gamma \tau P}\sqrt{\frac{2}{NA^2}}}$$

Increasing the **momentum** improves measurement precision linearly.

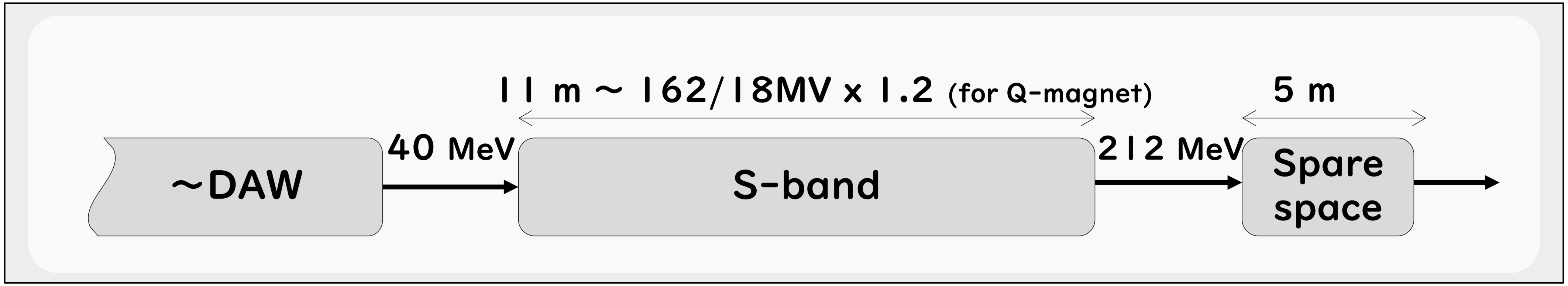
- **Magnetic Field Adjustment:**
  - Higher momentum requires a proportional increase in the magnetic field to maintain experiment size.
- **Effect on Anomalous Precession (ω_a):**
  - A **stronger magnetic** field **increases** the **anomalous precession frequency**.

→ Doubling both momentum and field ⇒ **x4** improvement in ω_a precision.

DLS structure



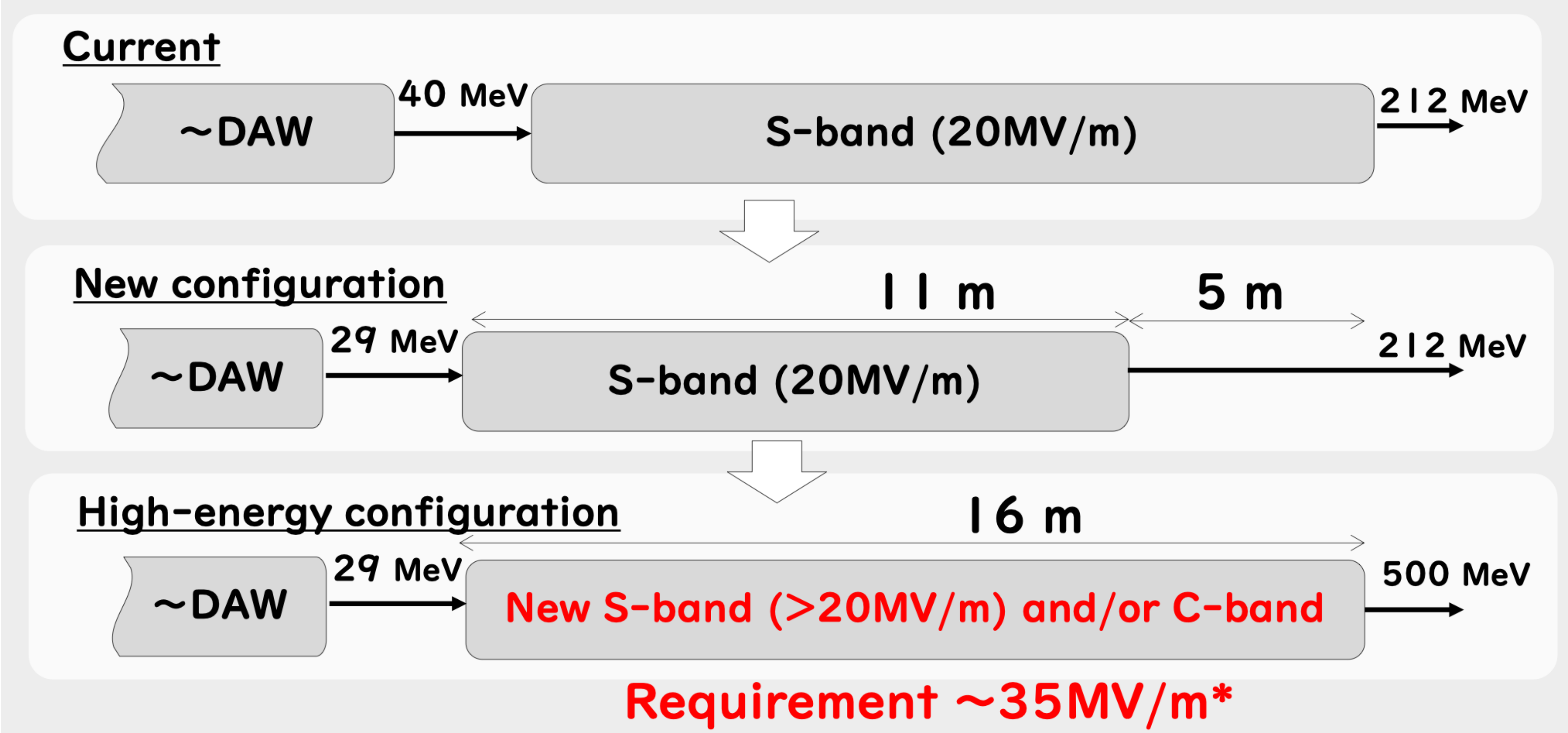
Current DLS design
Final momentum 300 MeV/*c*



DLS structure

New configuration has lower input energy 40 -> 29 MeV

High energy configuration higher final energy 212 -> 500 MeV

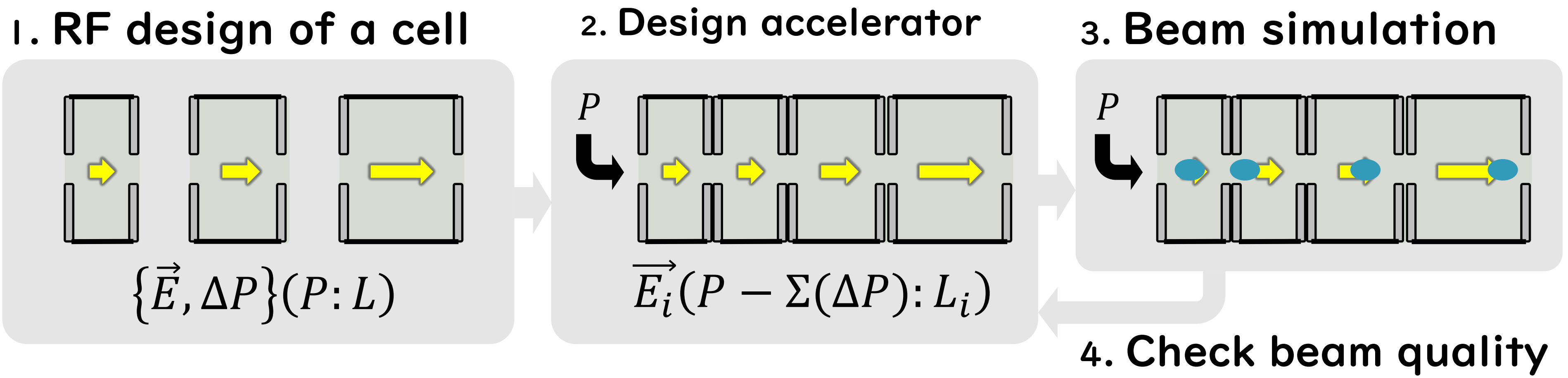


*assuming synchronous phase of 13° is used

Short prototype S-band for the FERMI FEL linac achieved higher gradient of 30 MV/m [NIMA 1055 (2023) 168543] with possibility to increase up to 40 MV/m [NIMA 979 (2020) 164473]

Work flow

- The overall flowchart is:

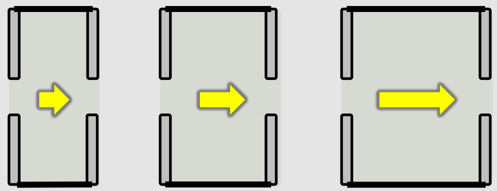


5. Design including beam matching

RF design

- As first step we redesign the cell RF for the new initial and final energy:

1. RF design of a cell

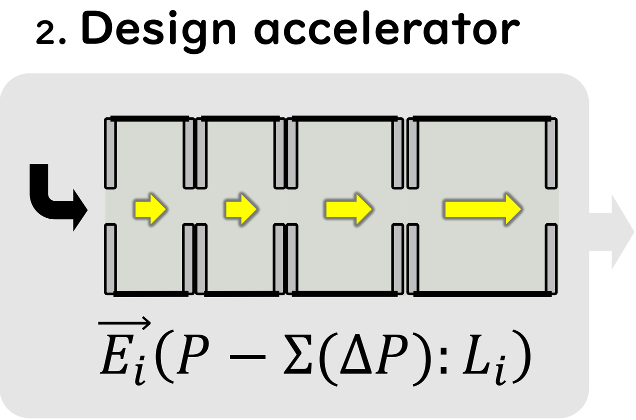


$$\{\vec{E}, \Delta P\}(P:L)$$

- We moved from:
  - Initial 41 MeV -> **29 MeV**
  - Final 212 MeV -> **500 MeV**
- This will be the input to the next step.

Accelerator design

- We used the new input RF and the new parameters to design the first part of the DLS



- The new power defined after discussion with experts is
  - **90 MW** (for operational stabilty and RF breakdown risk);

Accelerator design

- For the accelerator design we had to deal with several factors:
  - **Total length** of the building ~**16m**;
  - **Maximum power** per cell **90MW/m**;
  - **Gap length** size between DLS tanks (order of **840mm**);
  - Size of the single **DLS tank length** – **2.3m** maximum limit from the constructor.
- The best solution with the current parameters is to increase the number of tanks from **4 to 6** -> this reduces the length of each tank increasing the average gradient.
- We optimized the number of cells per tank to have the size of them quite similar.

```
Iteration Result (dls1)
a2_in  [mm] = 24.6673
a2_out [mm] = 22
E0    [MV/m] = 31.143

freq   [MHz] = 2592
phi_s  [deg] = -13
IniW   [MeV] = 29
EndW   [MeV] = 79.8751
Ncell        = 58
Qratio       = 0.97
Power   [MW] = 85.5
vg/c(58)[%] = 0.9149
tau          = 0.359008
T_f     [us] = 0.497566
```

```
Iteration Result (dls2)
a2_in  [mm] = 25.5455
a2_out [mm] = 22
E0    [MV/m] = 33.1569

freq   [MHz] = 2592
phi_s  [deg] = -13
IniW   [MeV] = 79.8751
EndW   [MeV] = 143.818
Ncell        = 58
Qratio       = 0.97
Power   [MW] = 90
vg/c(58)[%] = 0.919776
tau          = 0.34381
T_f     [us] = 0.540545
```

```
Iteration Result (dls3)
a2_in  [mm] = 25.6973
a2_out [mm] = 22
E0    [MV/m] = 33.8218

freq   [MHz] = 2592
phi_s  [deg] = -13
IniW   [MeV] = 143.818
EndW   [MeV] = 210.912
Ncell        = 56
Qratio       = 0.97
Power   [MW] = 90
vg/c(56)[%] = 0.920721
tau          = 0.332831
T_f     [us] = 0.546236
```

```
Iteration Result (dls4)
a2_in  [mm] = 25.7964
a2_out [mm] = 22
E0    [MV/m] = 33.9878

freq   [MHz] = 2592
phi_s  [deg] = -13
IniW   [MeV] = 210.912
EndW   [MeV] = 280.28
Ncell        = 56
Qratio       = 0.97
Power   [MW] = 90
vg/c(56)[%] = 0.920933
tau          = 0.332976
T_f     [us] = 0.557124
```

```
Iteration Result (dls5)
a2_in  [mm] = 25.7223
a2_out [mm] = 22
E0    [MV/m] = 34.3752

freq   [MHz] = 2592
phi_s  [deg] = -13
IniW   [MeV] = 280.28
EndW   [MeV] = 349.005
Ncell        = 54
Qratio       = 0.97
Power   [MW] = 90
vg/c(54)[%] = 0.920963
tau          = 0.323883
T_f     [us] = 0.547043
```

```
Iteration Result (dls6)
a2_in  [mm] = 25.7465
a2_out [mm] = 22
E0    [MV/m] = 34.4278

freq   [MHz] = 2592
phi_s  [deg] = -13
IniW   [MeV] = 349.005
EndW   [MeV] = 418.45
Ncell        = 54
Qratio       = 0.97
Power   [MW] = 90
vg/c(54)[%] = 0.920946
tau          = 0.324157
T_f     [us] = 0.550774
```

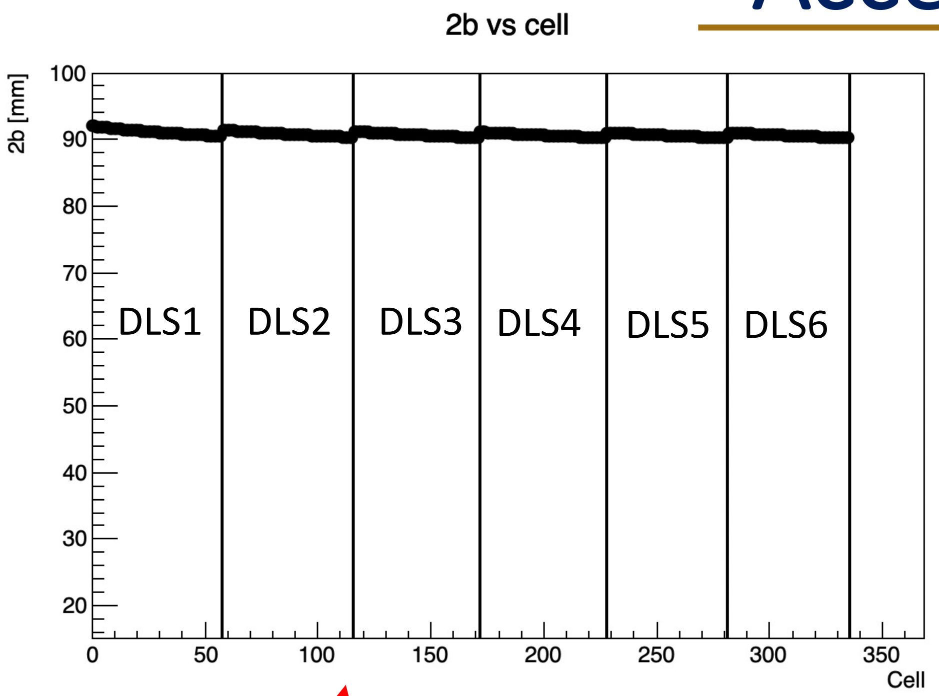
The lengths for the 6 tanks are:

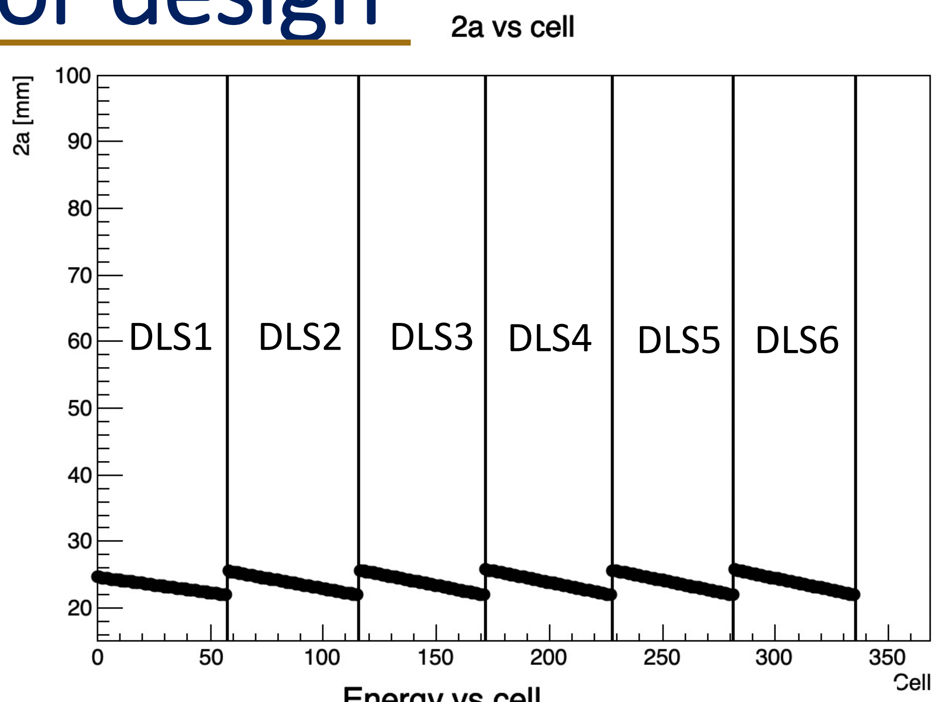
1. 1648.71 mm;
2. 1945.89 mm;
3. 2000.29 mm;
4. 2057.93 mm;
5. 2014.56 mm;
6. 2032.54 mm.

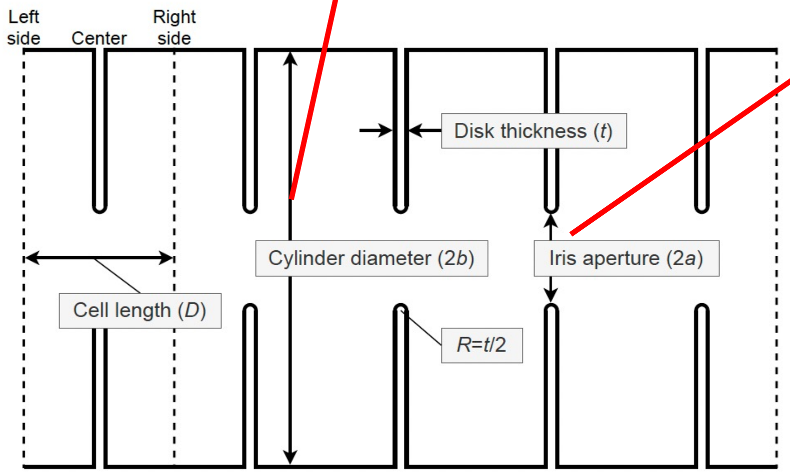
For a total length of: 11699.92 that using the 870 gap we have 11699.92+ 870·5= **16049.92 mm**

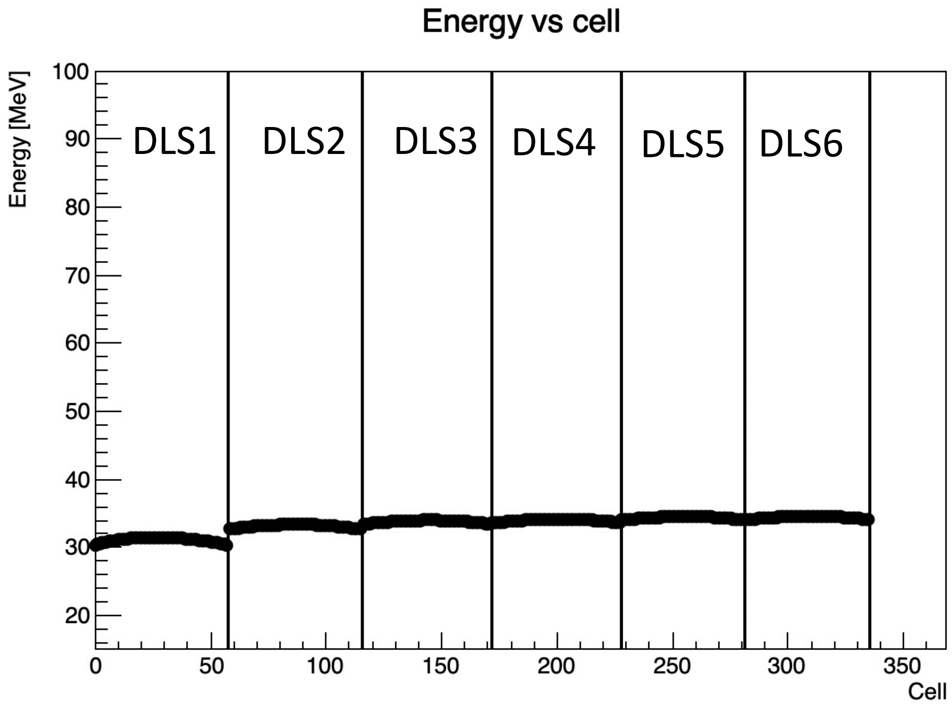
With final Momentum of ~**515 MeV**

Accelerator design

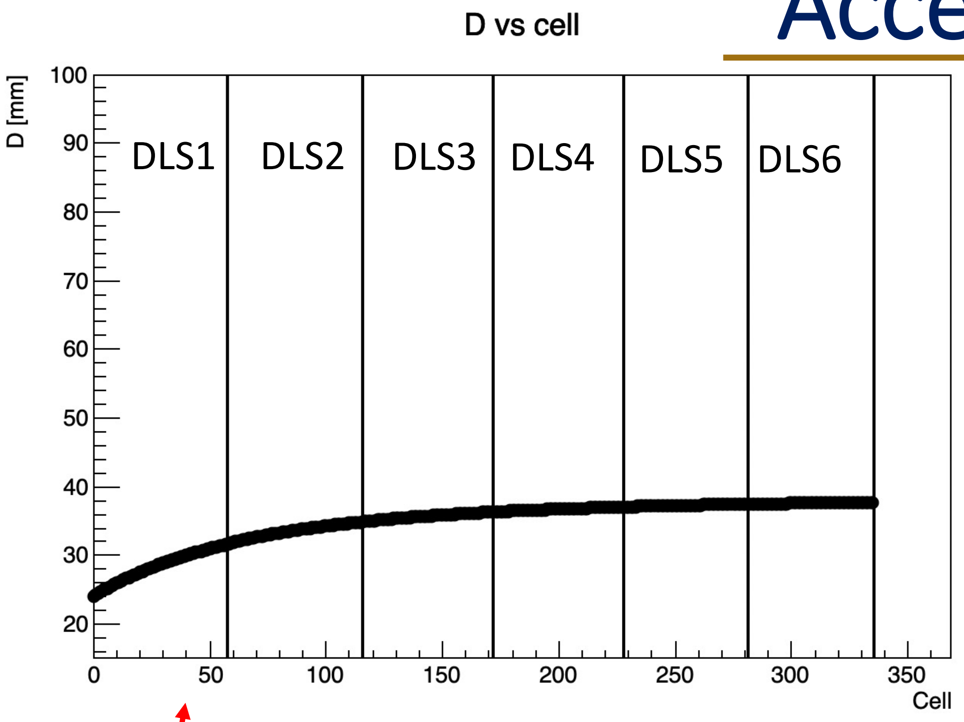


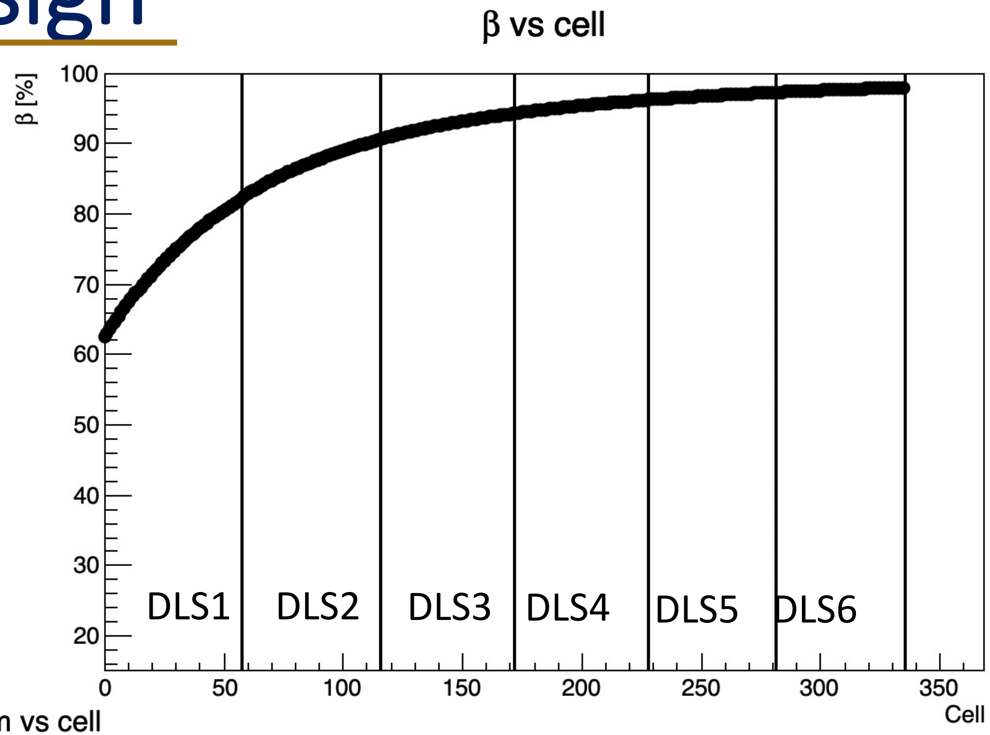


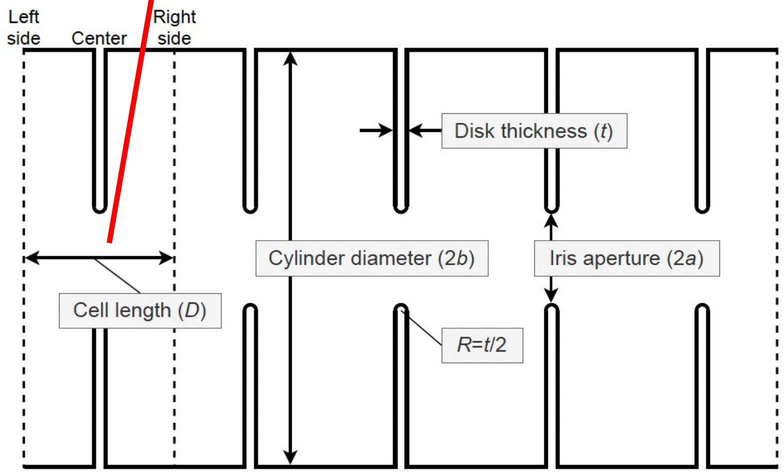


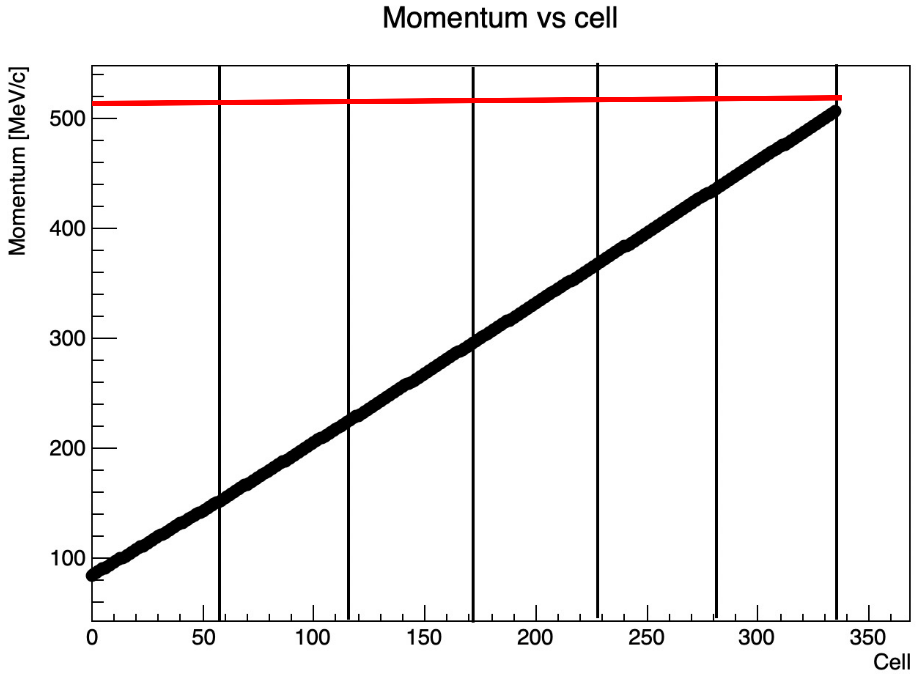


Accelerator design





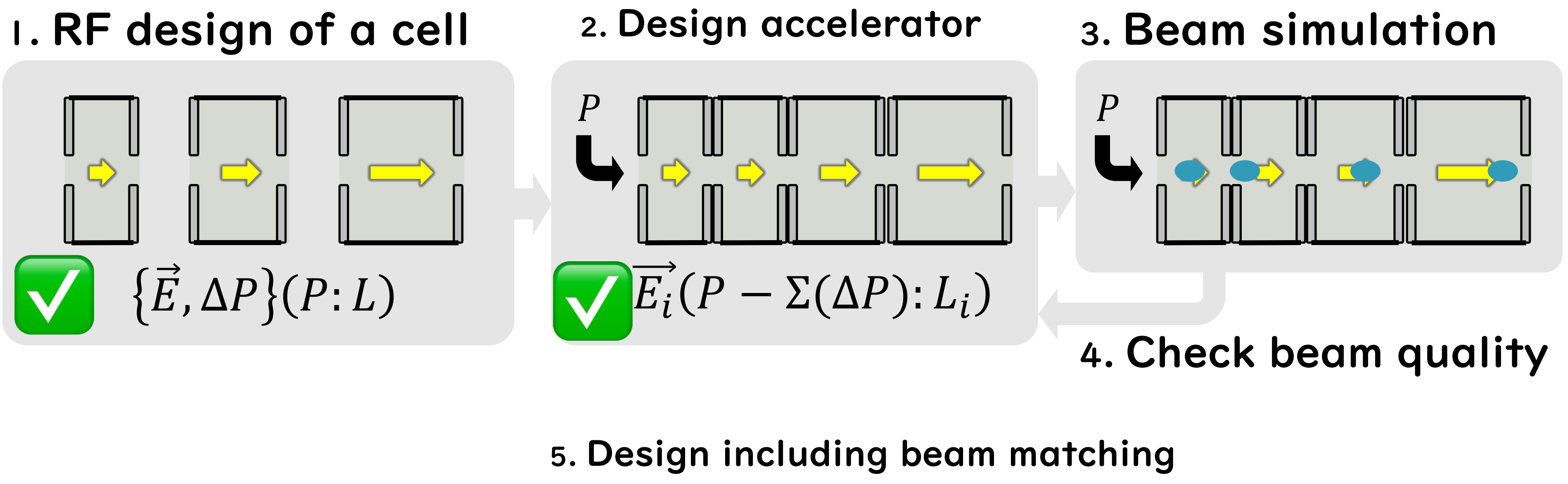




Final momentum with this design is ~515 MeV/c

Work flow at 30MV/m

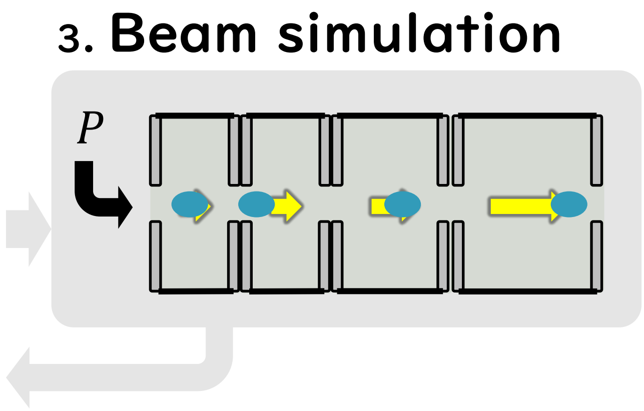
- The overall flowchart is:



- We need to do the first beam simulation;
- Check the transverse beam dynamics.

EM calculation and beam simulation

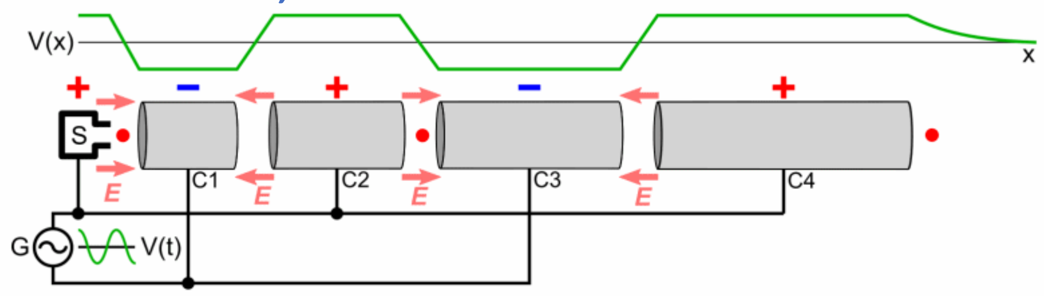
- With the input from the previous step we ran SUPER-FISH to compute the EM field on the new cavity design.



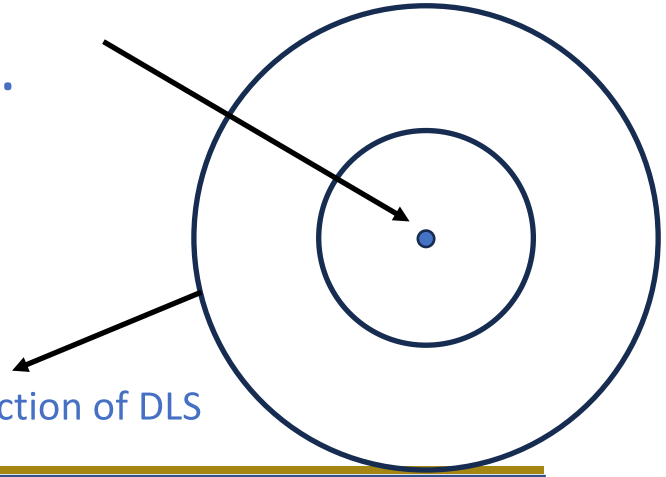
- This output will be used for track the particle and define the beam quality, covering the step 3 and 4.

Beam simulation

- Before making a full simulation of the beam inside the DLS, we need to do some initial optimization.



- The main one is the phase scan.
- The phase of the beam can be regulated by changing the distance between the DAW and the first DLS tank.
- Selecting the reference particle, a particle centered in the middle of the DLS with longitudinal momentum only, we can find the optimal phase.



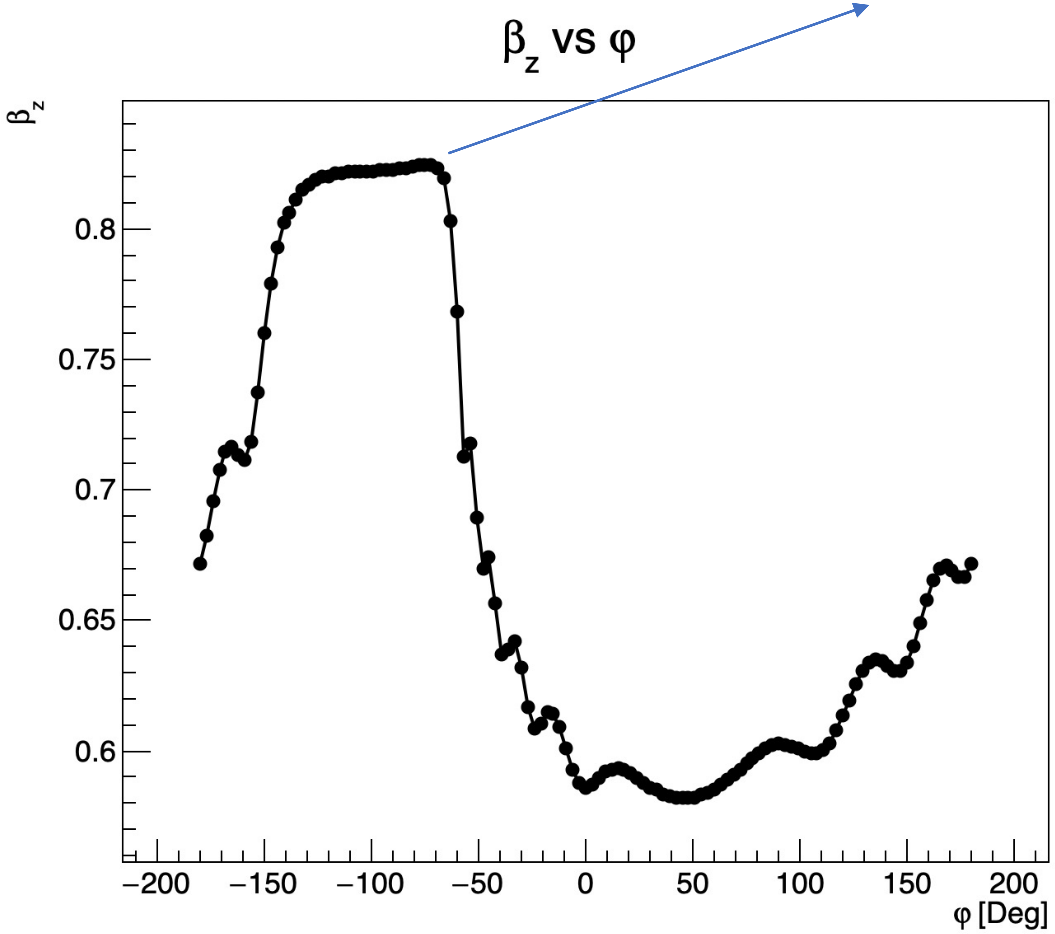
Transverse section of DLS

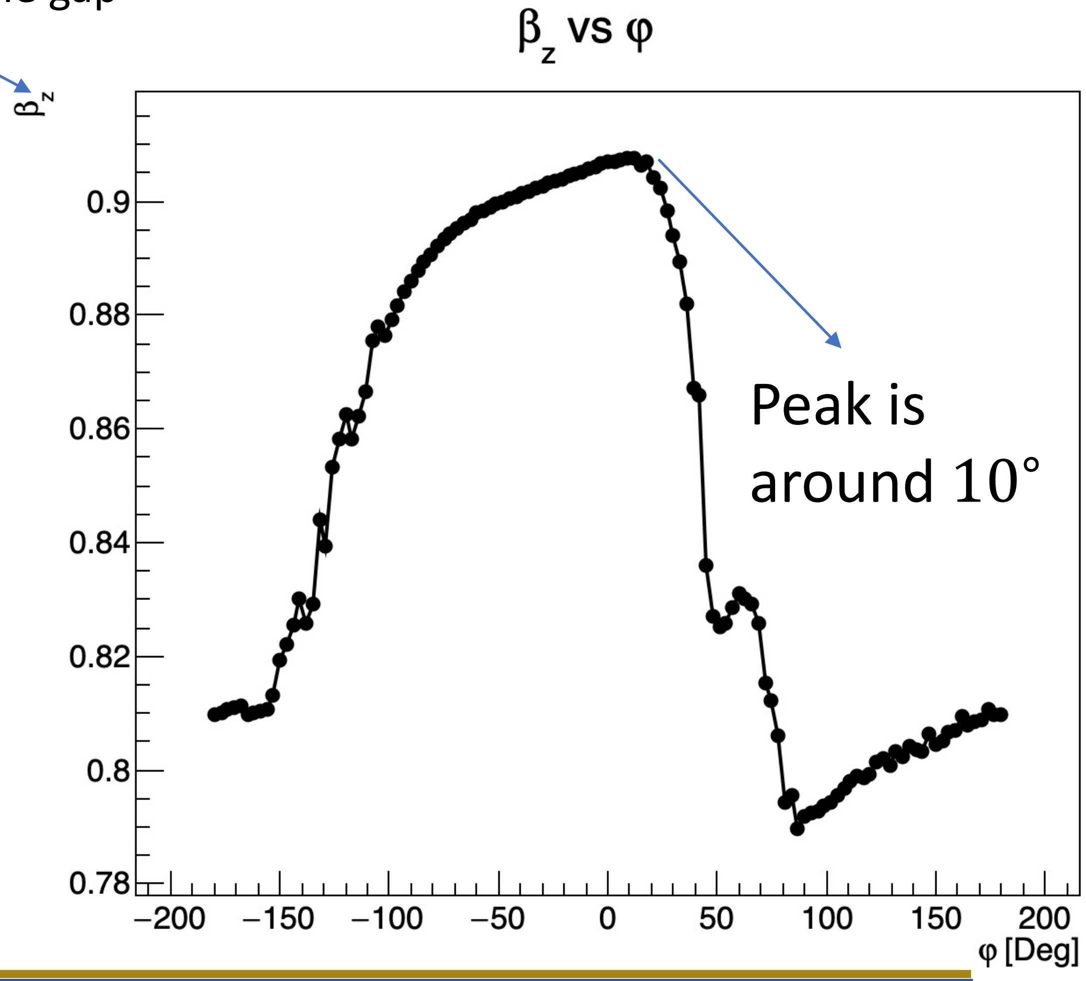
Beam simulation – phase optimization

- This is the phase optimization for the first pair of DLS tanks, this is repeated for all

φ = -72 deg

The φ which maximize the beta is used to scan the phi at the gap

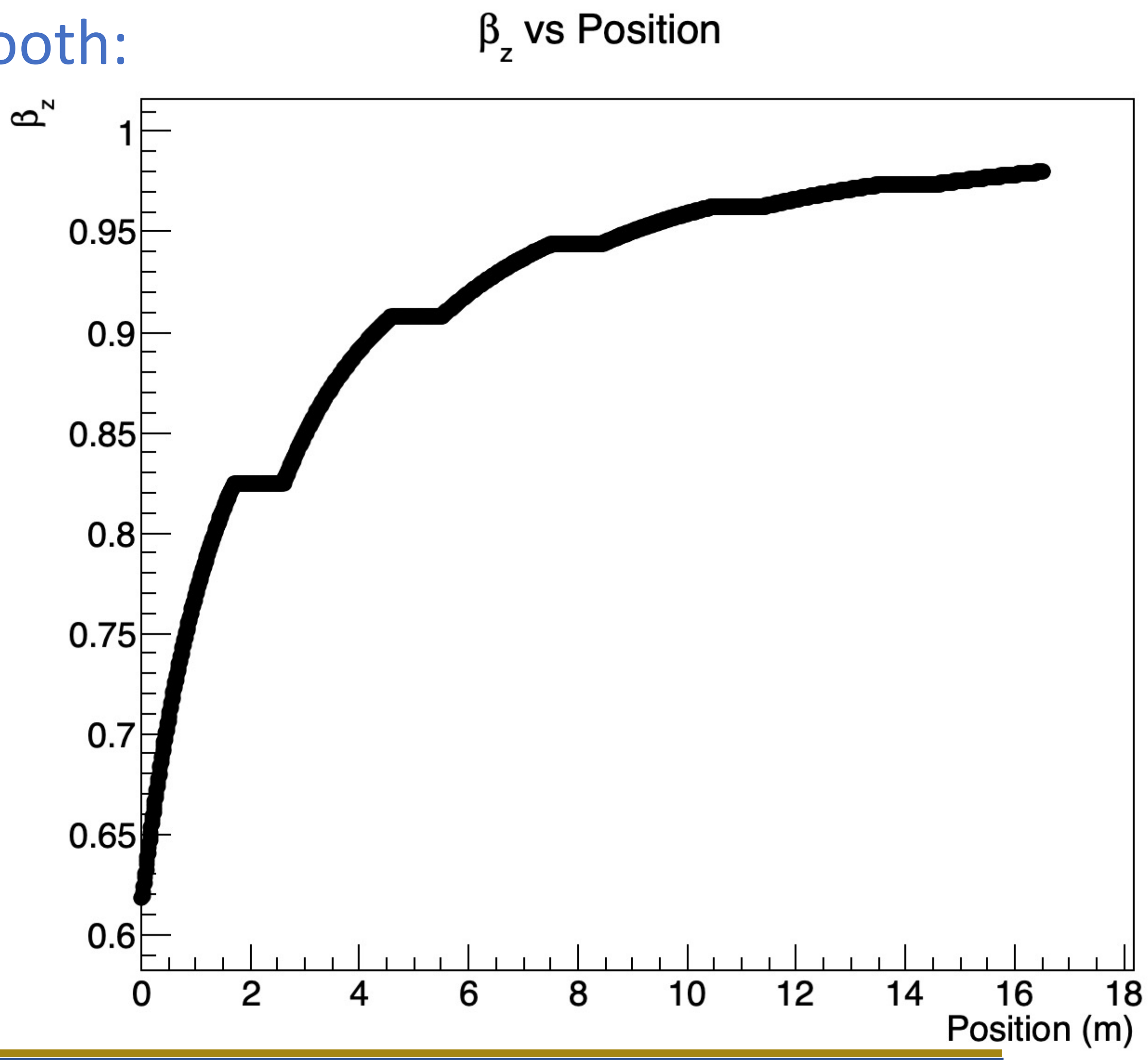




Peak is around 10°

Beam simulation – phase optimization

- After performing the optimization, plotting the β as function of the position inside the DLS we see that the acceleration is smooth:



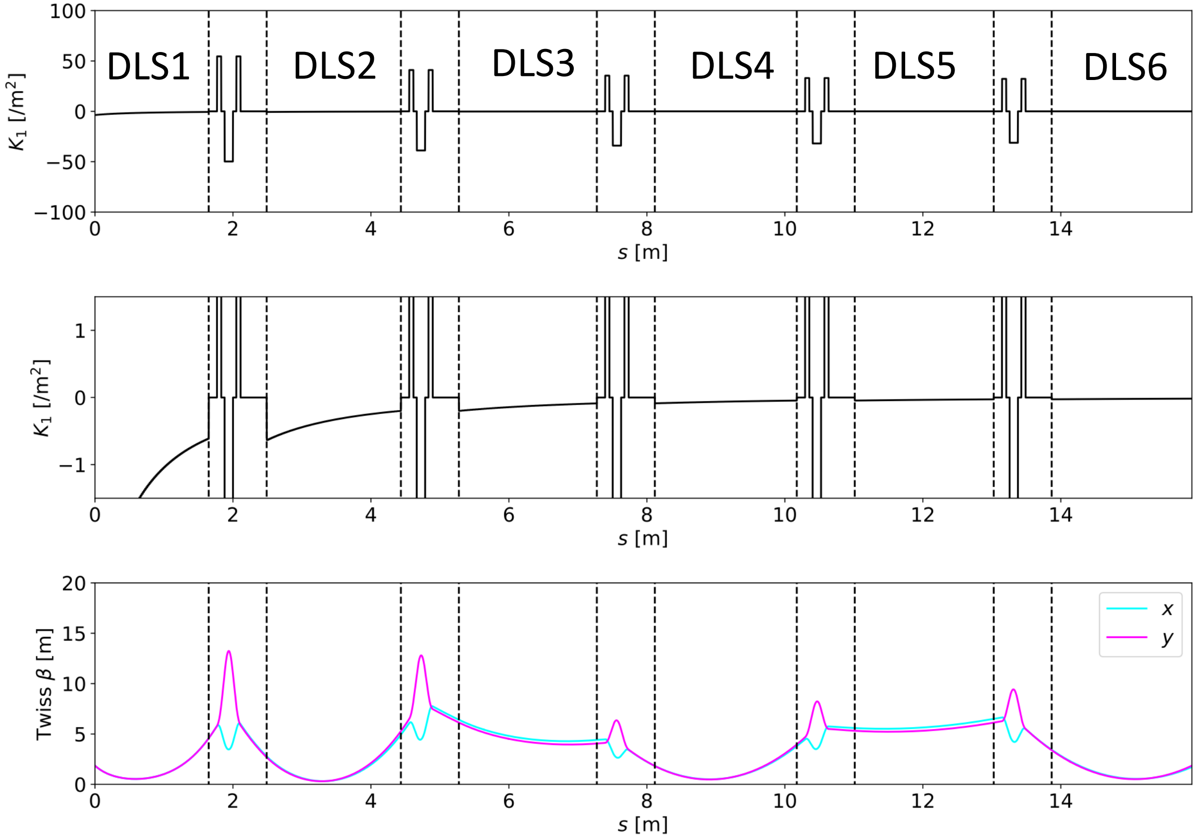
- We see a net acceleration through the **6 tanks** up to the expected final $\boldsymbol{\beta}$ **of 0.979**

Transverse Beam Dynamics Design

- Using the parameters obtained from the first step, we can start computing some beam dynamics parameters by solving the equation of motion:

$$\frac{\mathrm{d}^2\sqrt{\beta_{\mathrm{Twiss}}(s)}}{\mathrm{d}s^2} + K_1(s)\sqrt{\beta_{\mathrm{Twiss}}(s)} - \sqrt{\beta_{\mathrm{Twiss}}(s)}^{-3} = 0,$$

Where K_1 is the focusing force

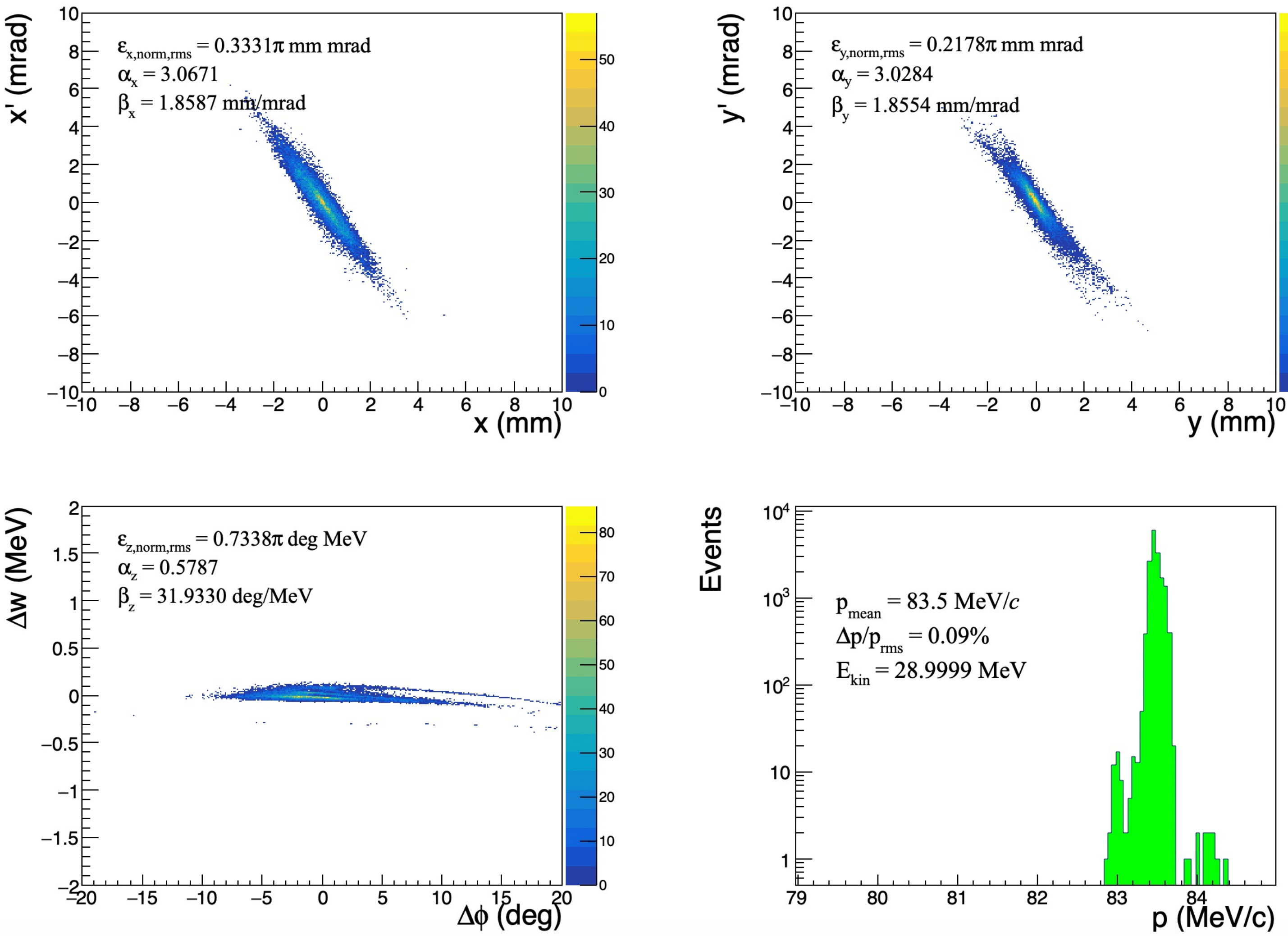


Transverse Beam Dynamics Design

- From the transverse beam calculation we get some important parameters:
  - Twiss parameters $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$;
  - **Quadrupole strenght** between DLSs;
- We use the realistic distribution of the beam at the exit of the DAW to simulate also particle that doesn't lie on the ideal orbit.
- For now we use the input distribution produced in previous DAW simulation, with final energy at 41 MeV and rescaled to much our current parameters.

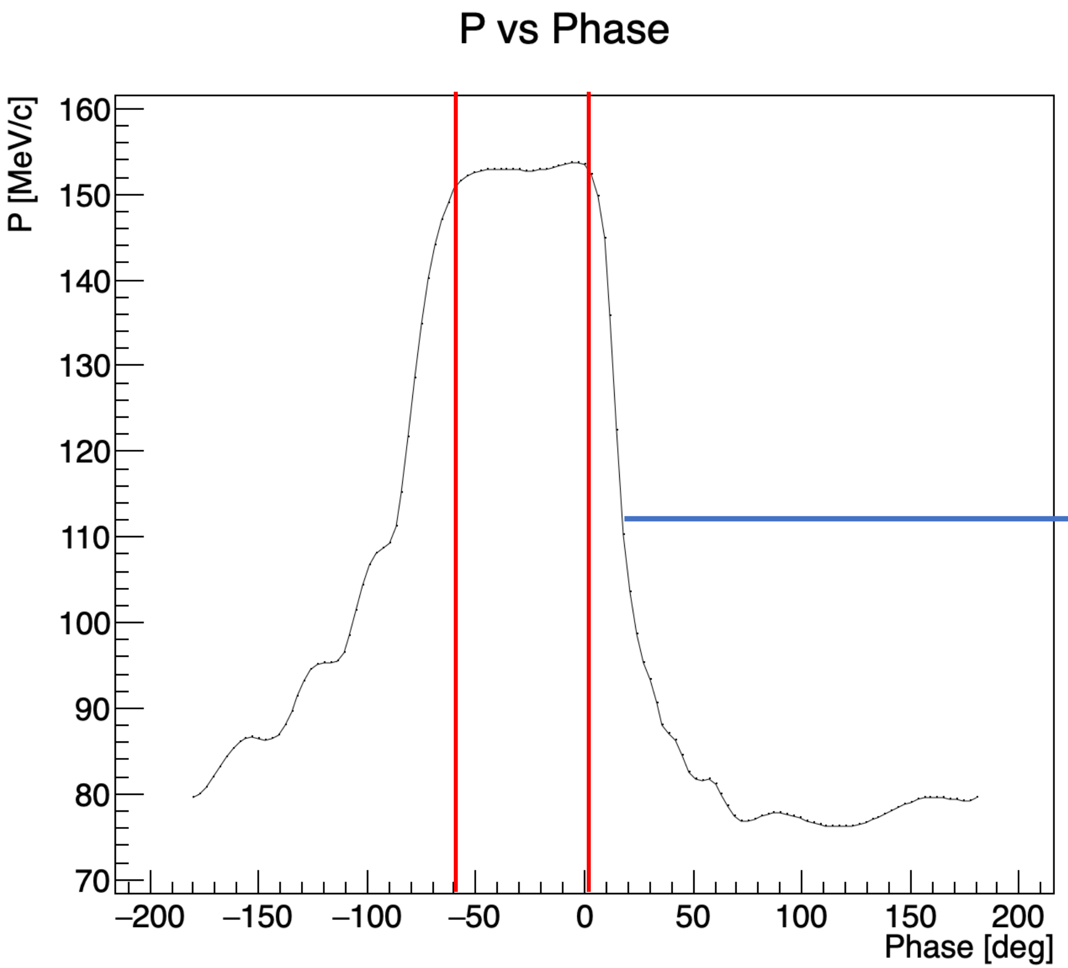
Full beam simulation

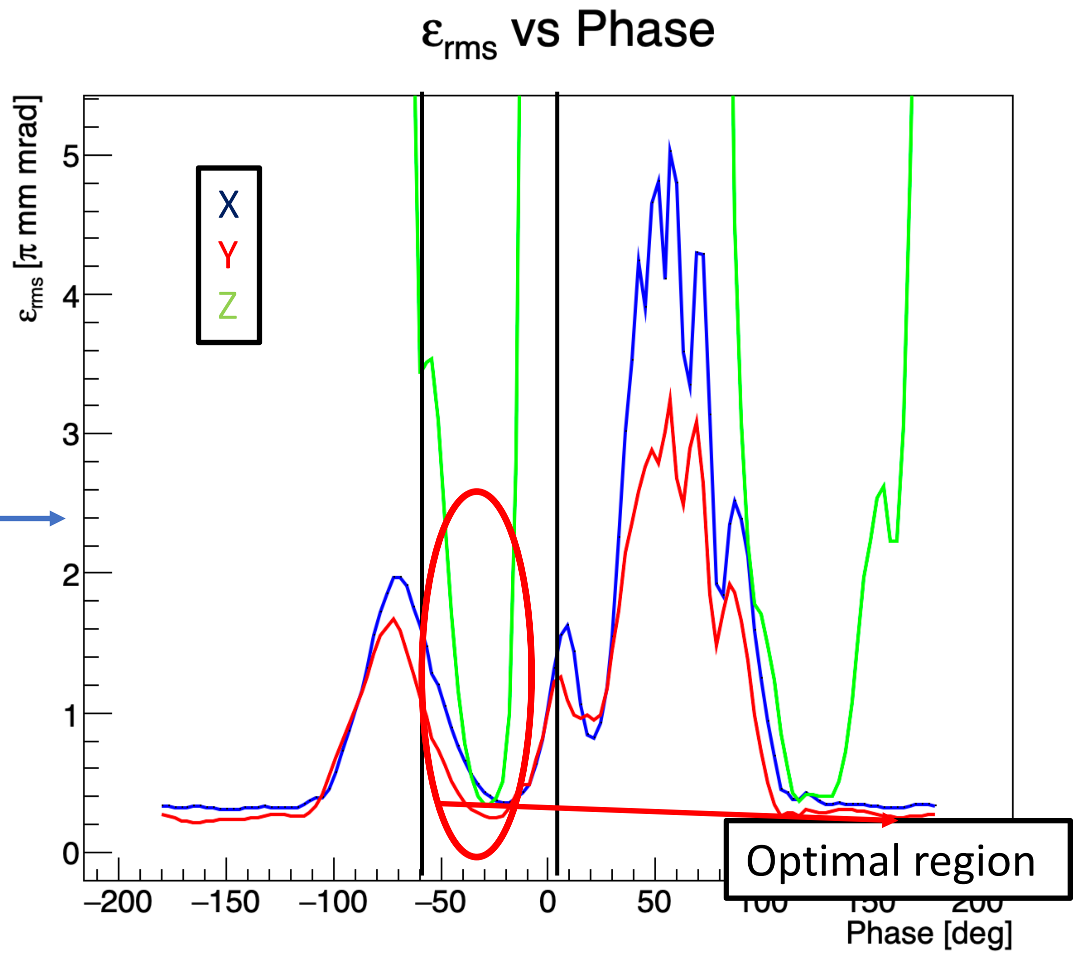
- From the existing beam distribution we generated the required beam:
- We used the $\alpha_{x,y}, \beta_{x,y}$ and $\varepsilon_{x,y}$ twiss parameters from previous step to generate the distribution.
- The twiss parameters are:
  - $\alpha_{x,y} = 3.076$
  - $\beta_{x,y} = 1.856$
  - $\varepsilon_x = 0.33\,\pi \cdot mm \cdot mrad$
  - $\varepsilon_y = 0.22\,\pi \cdot mm \cdot mrad$



Full beam simulation – DLS1

- As we did in the preliminary analysis with reference particle with need to scan over the

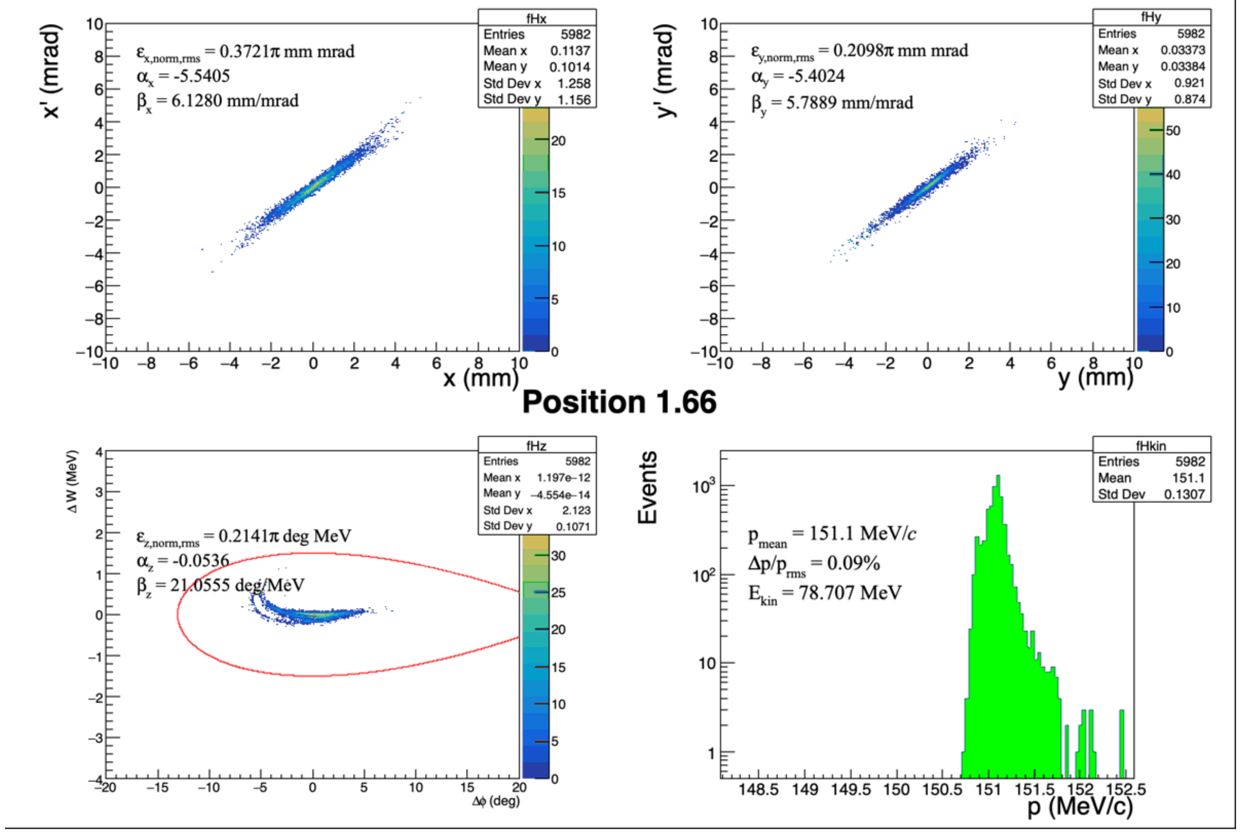




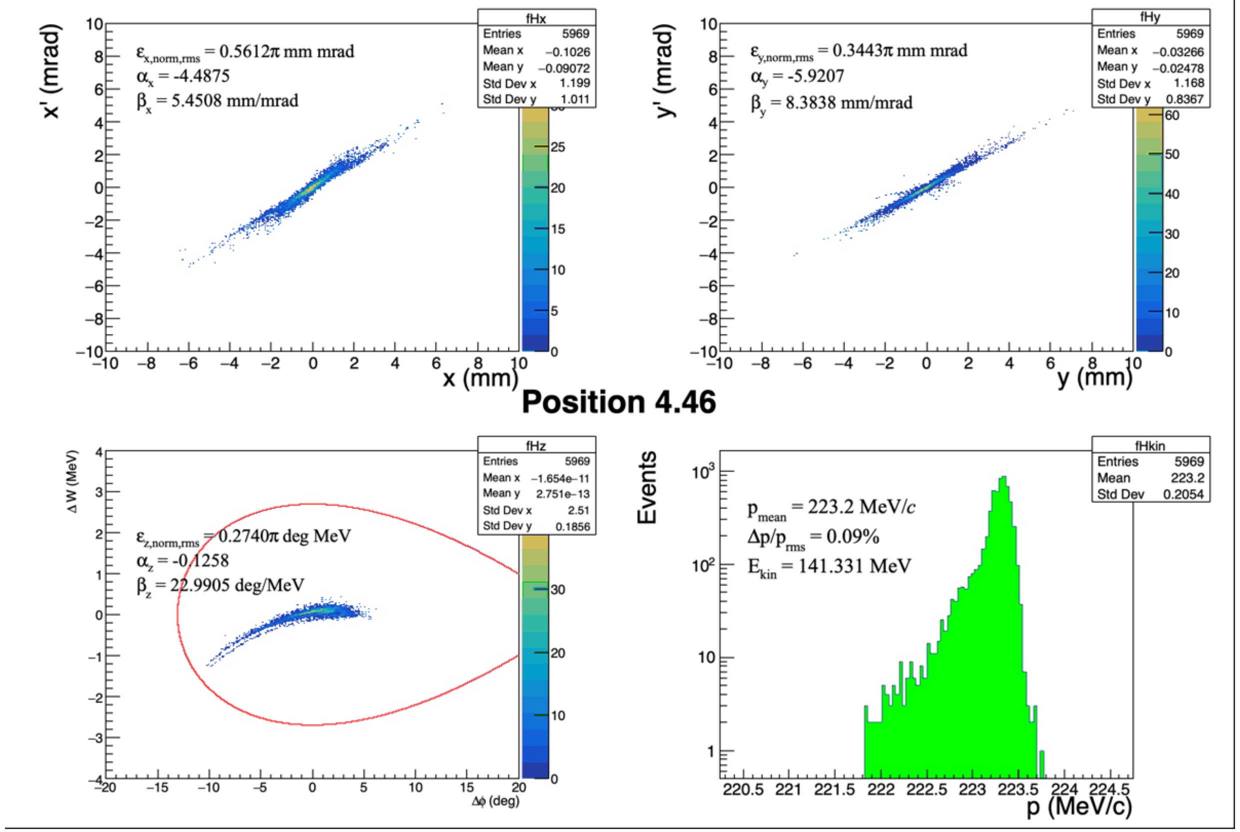
Full beam simulation – phase scan

- The phase scan is done for all the tanks.

DLS1 exit



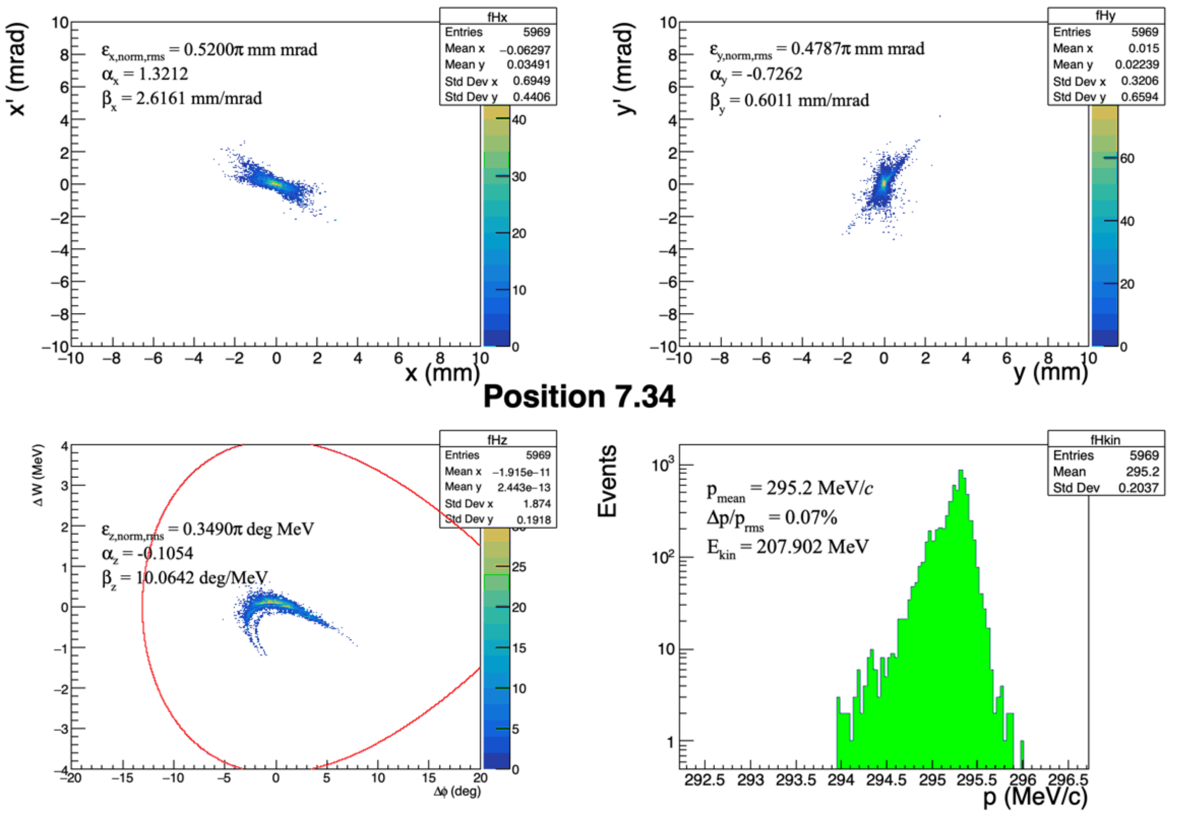
DLS2 exit



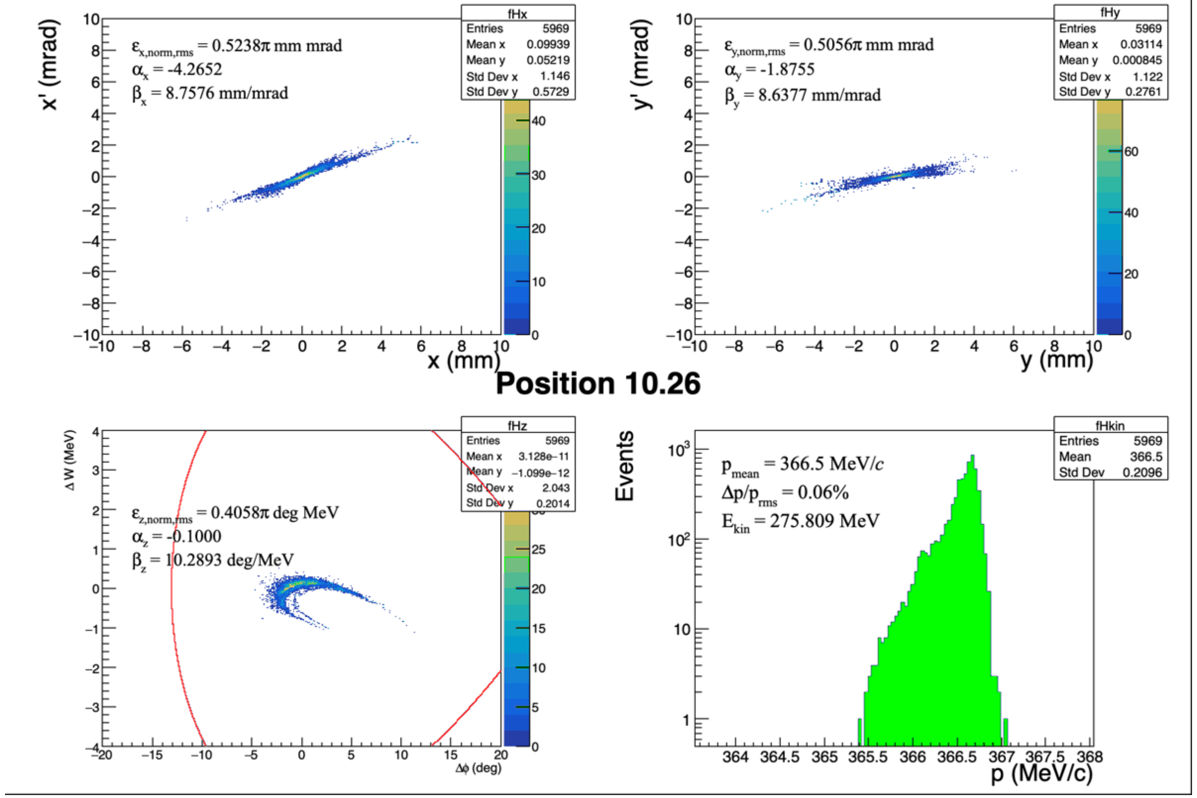
Full beam simulation – phase scan

- The phase scan is done for all the tanks.

DLS3 exit



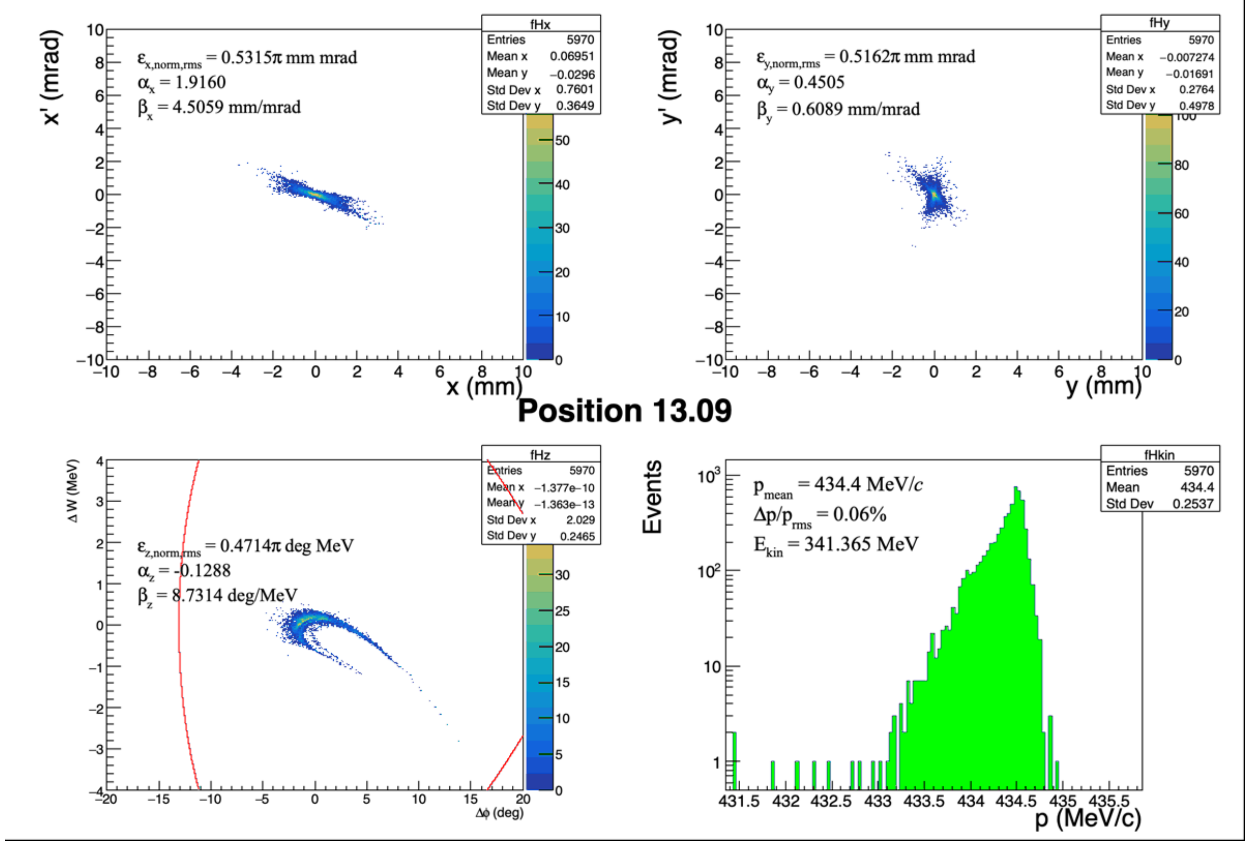
DLS4 exit



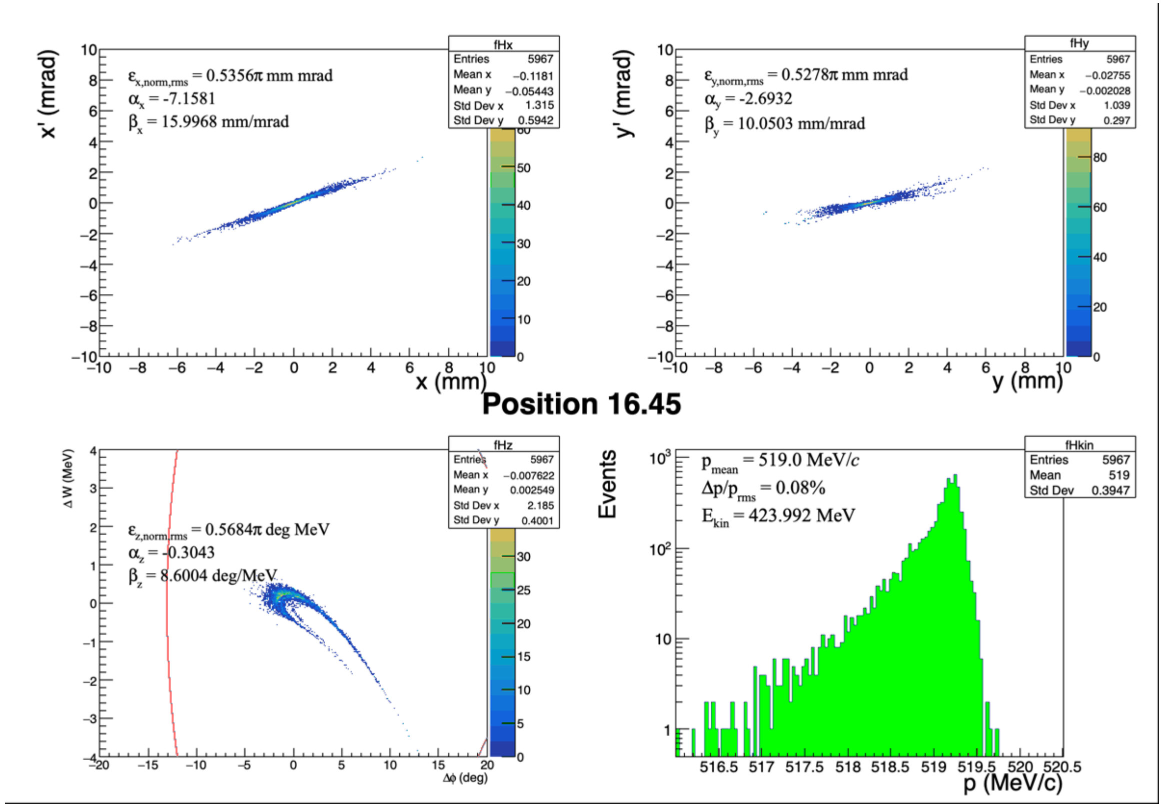
Full beam simulation – phase scan

- The phase scan is done for all the tanks.

DLS5 exit

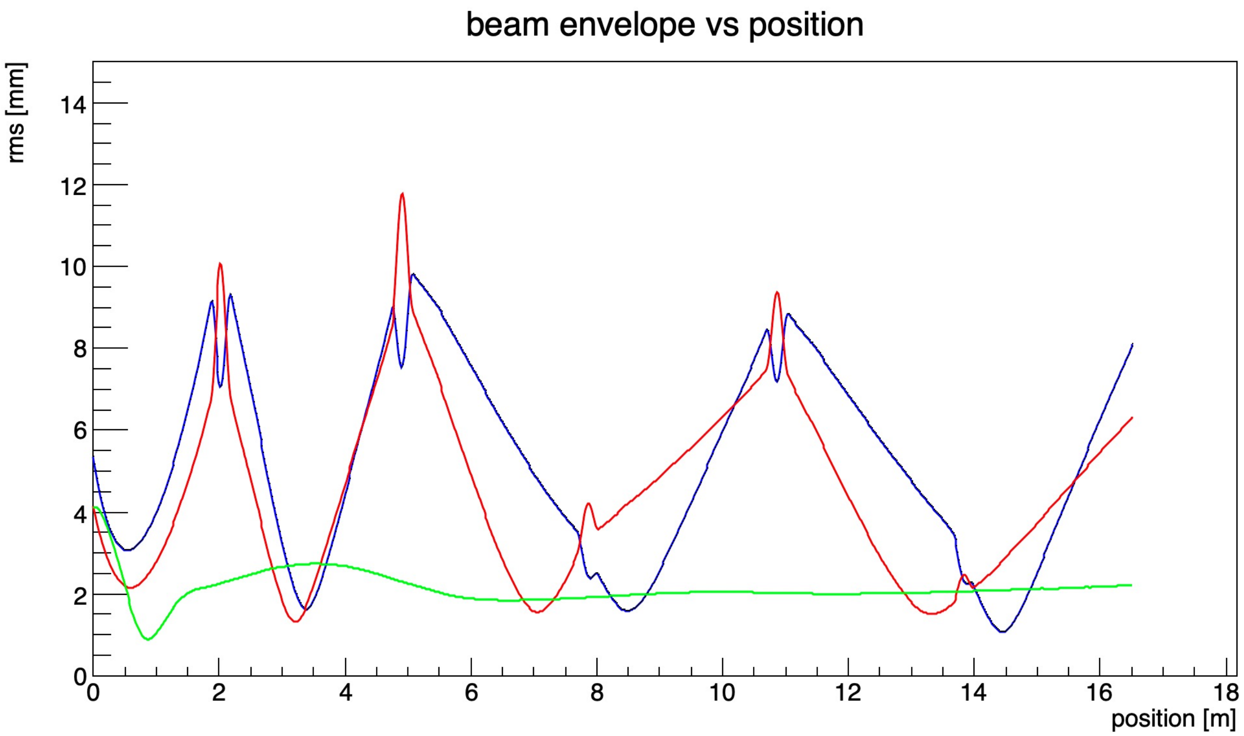


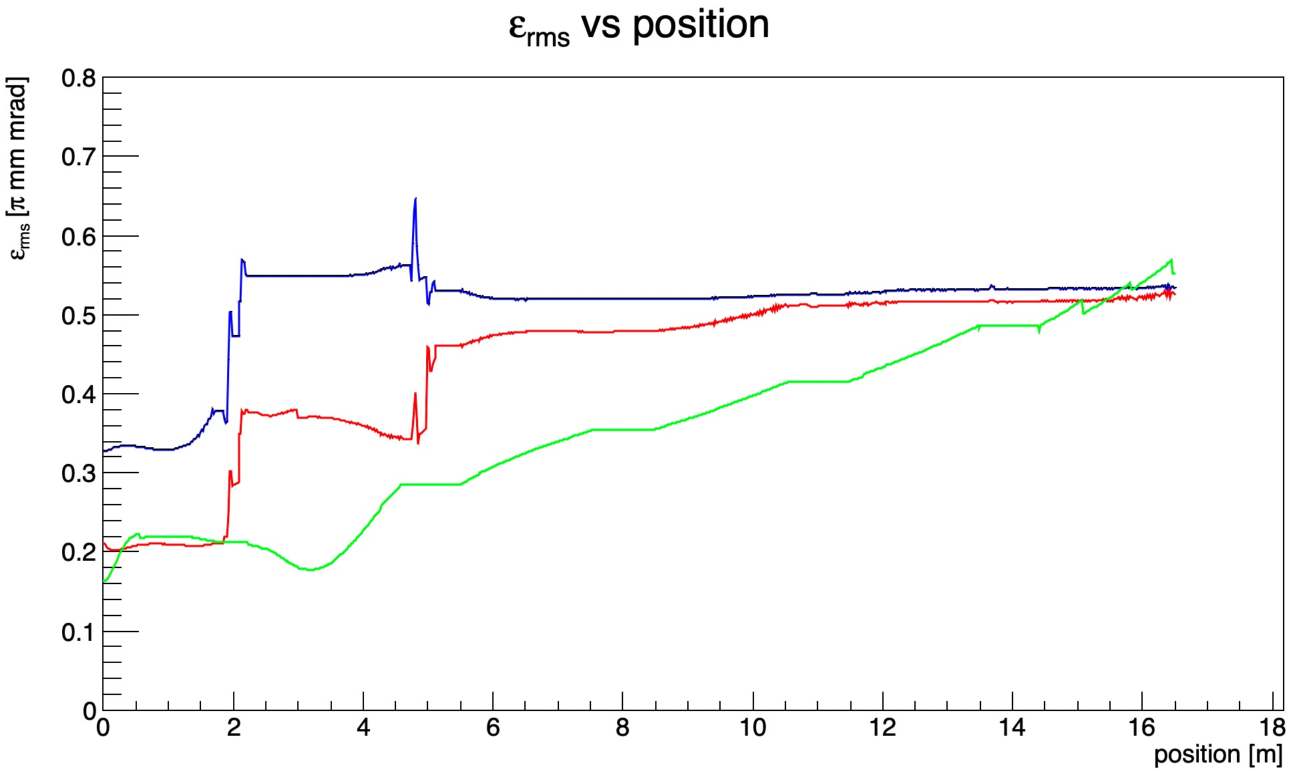
DLS6 exit



Full beam simulation

- Here is the evolution of the beam as function of position:

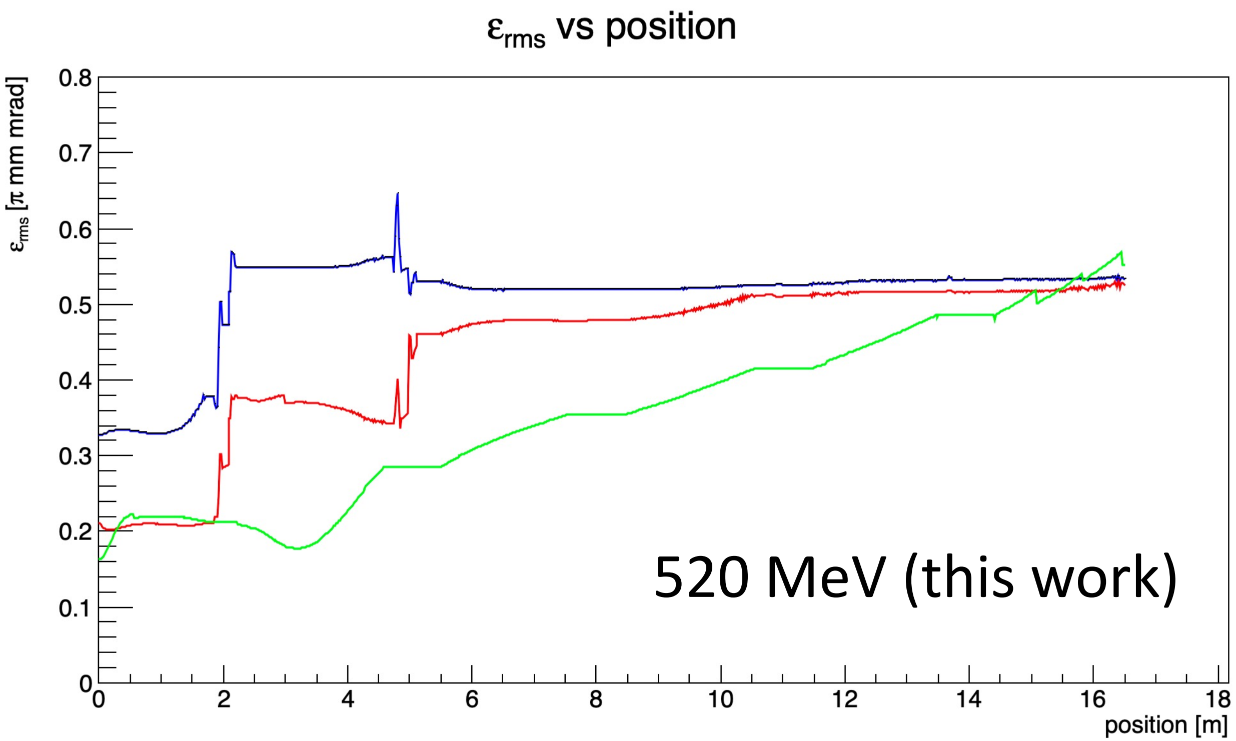


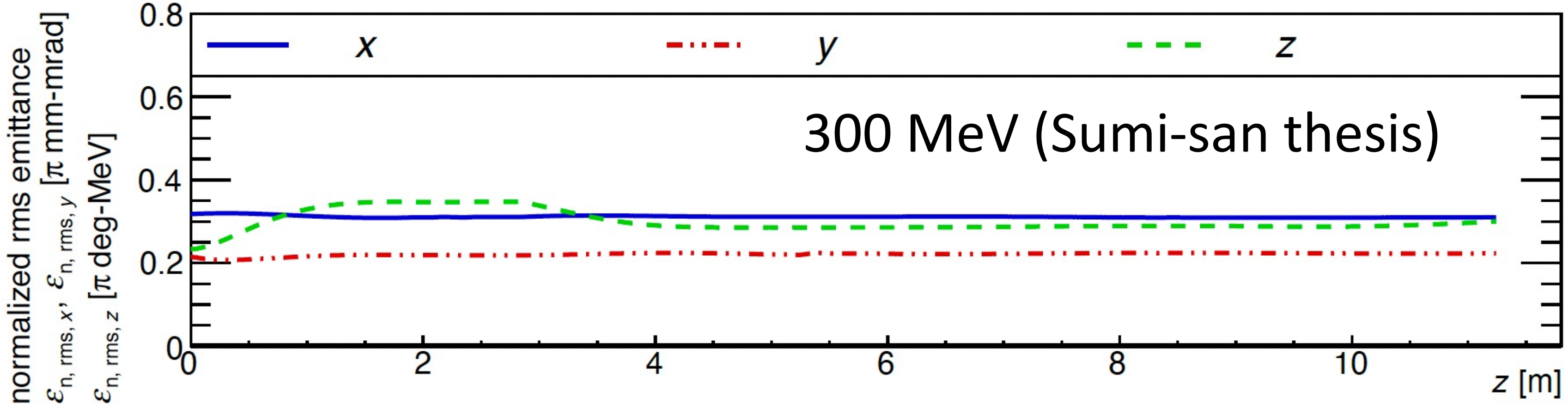


- The beam envelope shows mismatch show some mismatch that needs to be correct with a further tuning. This will also reduce the emittance growth.

Full beam simulation

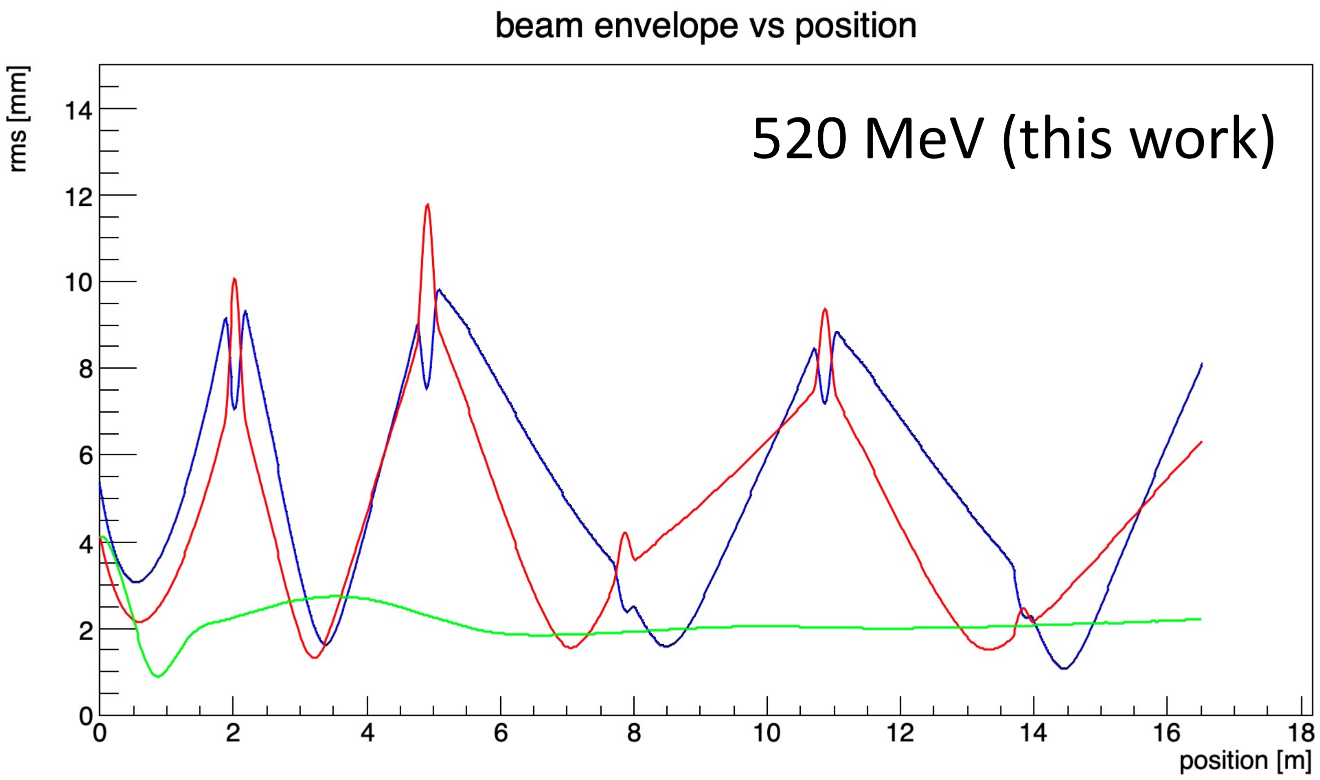
- Here is the comparison of ε_{rms} vs position wrt the 300 MeV case:

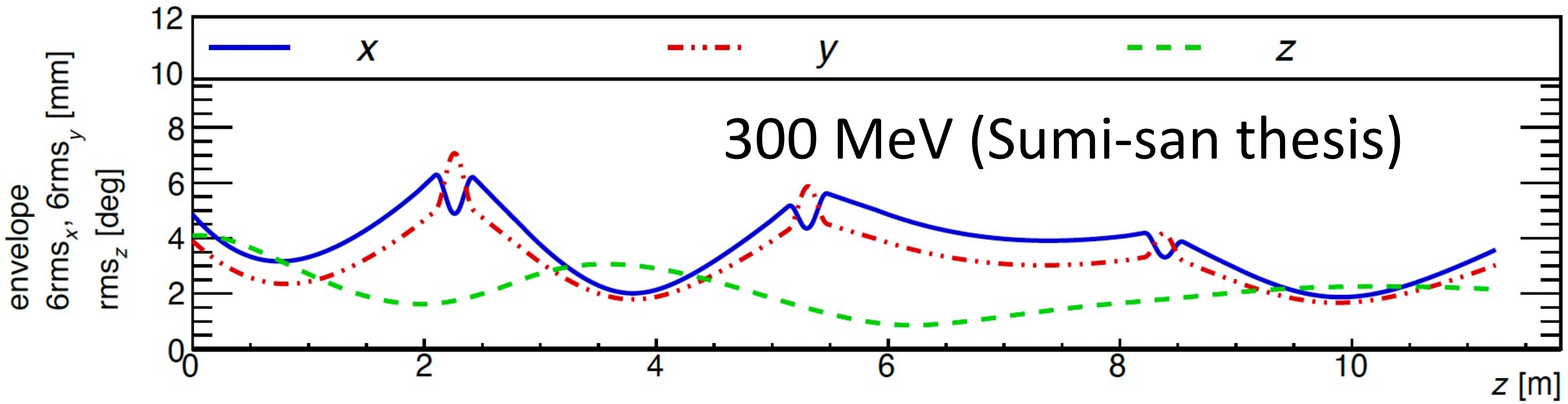




Full beam simulation

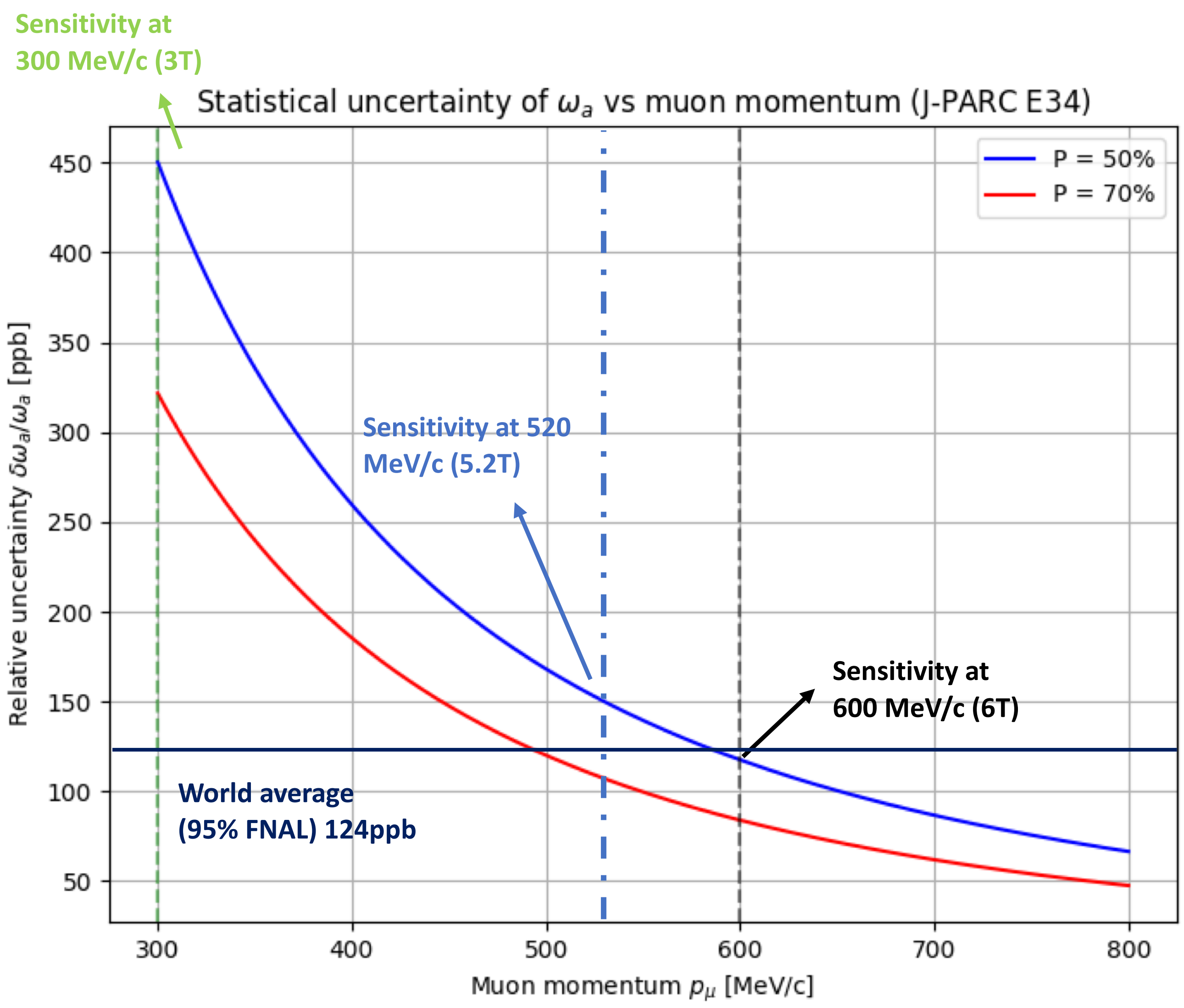
- Here is the comparison of beam envelope wrt the 300 MeV case:





New momentum

- The aimed momentum for this study was 600 MeV/c;
- With this configuration we reached about **520 MeV/c**;
- Current energy already allow us to reach **150 ppb** and improve by **factor 3** the current design;



[https://arxiv.org/pdf/2512.20335]

Next steps

- A **finer tuning** is needed to have a better beam match;
  - This will **fix** the **increase of emittance**.
- We need to further **study** the **transverse beam dynamics**;
- Given that we are a bit limited by space/power, we are thinking about other configuration that can bring us to a higher energy;
- Once the best configuration is finalized Takeuchi-san and Nakazawa-san will join the effort for detailed design such as the 3D cavity design etc

Conclusion

- A first configuration for the DLS at higher energy has been produced.
- We managed to accelerate muons up to **520 MeV**!
- With current design we could aim to improve the current design by 3 times, projected statistical error of ~**150 ppb**.
  - Investigation is still ongoing to improve the current value towards the final precision
- Finally, we want to refine and finalize the current design to have a smooth acceleration.