Laser Spin Polarisation and Ionisation Studies for Ultra-Slow Muon Production at J-PARC

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Overview

1. The muon anomaly
2. The J-PARC muon $g-2$ experiment
3. The ultra-slow muon beam production
4. **This work:** Simulation-based studies of the rate-equation model of optical pumping using a Lyman α 122 nm laser and photoionisation of muonium using a 355 nm laser.
5. Summary and Outlook

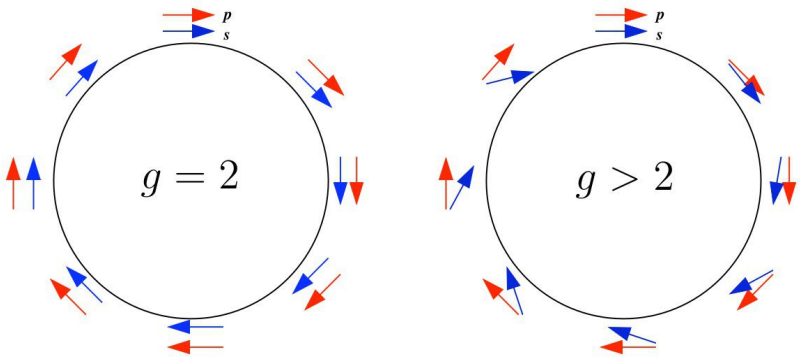
Magnetic moment and Spin precession in a magnetic field

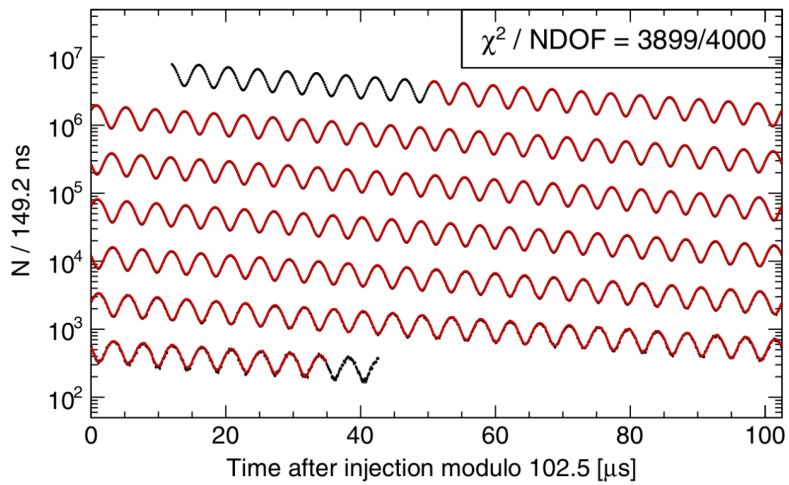
Particles with spin have an intrinsic magnetic moment

$$\vec{\mu} = g\frac{q}{2m}\vec{S}, \quad \text{g= gyromagnetic ratio}$$

Polarised μ^+ are stored in a ring with uniform magnetic field $\vec{B}$

- $g = 2$: spin locked to momentum
- $g > 2$: spin precesses around the momentum





Anomalous precession frequency

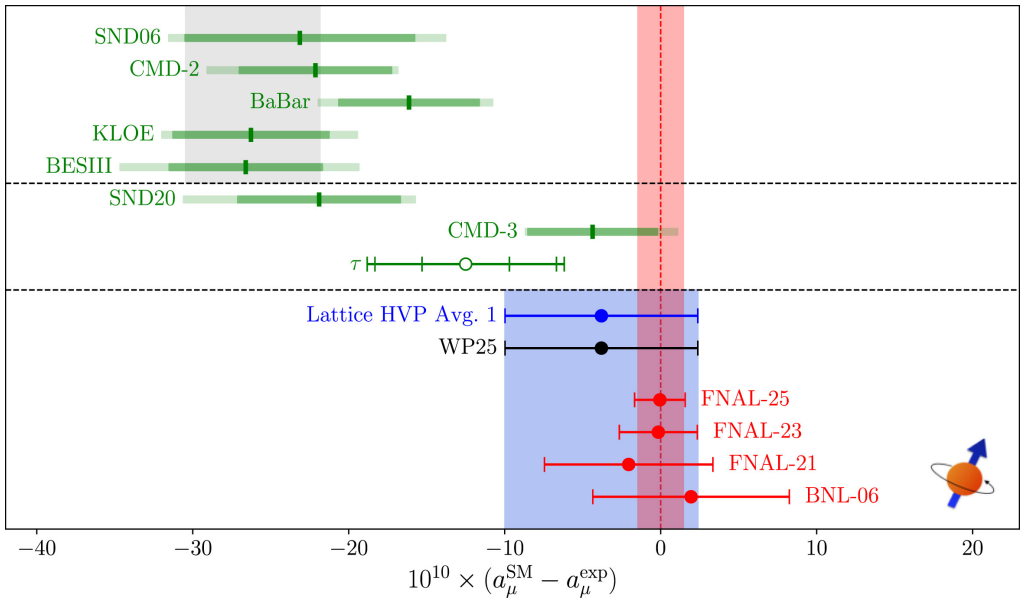
$$\vec{\omega}_a = -a_\mu \frac{e\vec{B}}{m_\mu}, \qquad a_\mu \equiv \frac{g_\mu - 2}{2}$$

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$

e^+ are emitted preferentially in the direction of the muon spin. Wiggle Plot shows the measure of the number of high-energy e^+ vs time.

Status of muon g-2

- **Experiment**: $a_\mu^{\text{exp}} = 116\,592\,071.5(14.5) \times 10^{-11}$, (124 ppb)
- **Theory**: $a_\mu^{\text{SM}} = 116\,592\,033(62) \times 10^{-11}$, (530 ppb)



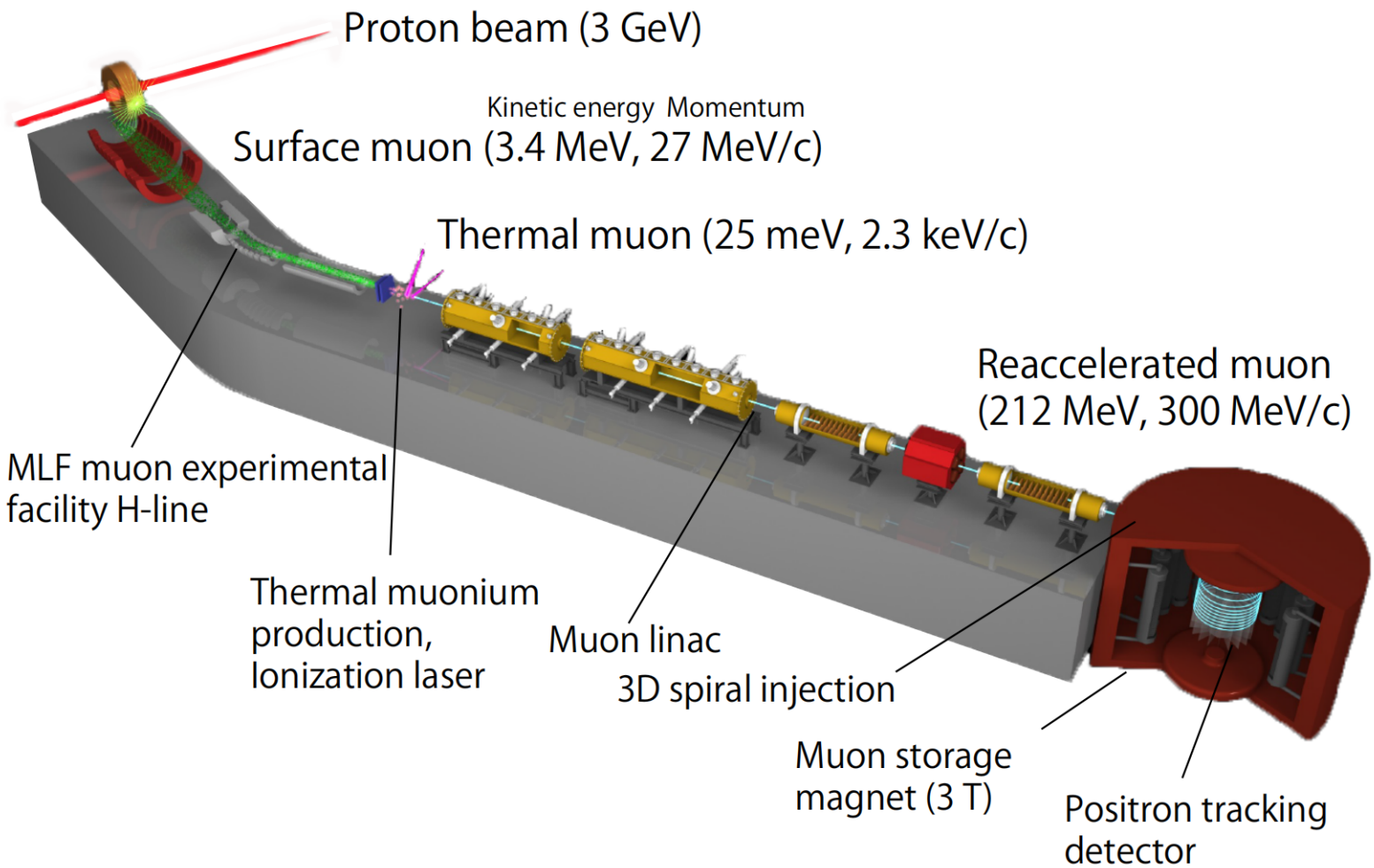
Aliberti et al. 2025

- BNL and FNAL share: the **same storage ring**, the **same magic-momentum** high emmitance beam, $p = 3.09\,\text{GeV}/c$, $B = 1.45\,T$ the **same electrostatic focusing**. A common systematic effect cannot be excluded.

An independent measurement with **entirely different systematics** ⇒ J-PARC muon $g-2$. This work aims to improve its **statistical precision** by increasing the polarisation of the ultra-slow muon beam.

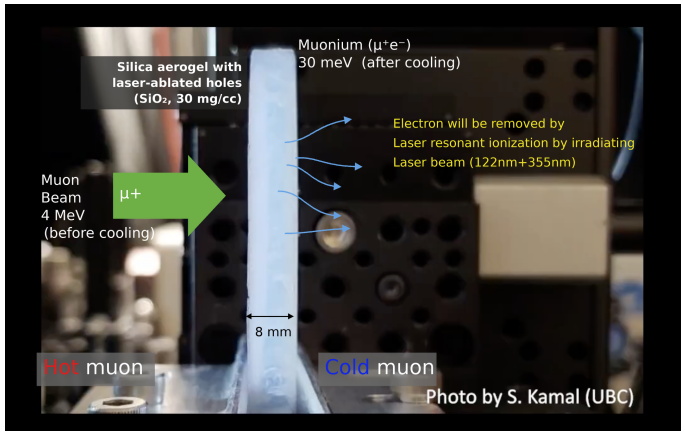
J-PARC (E34)

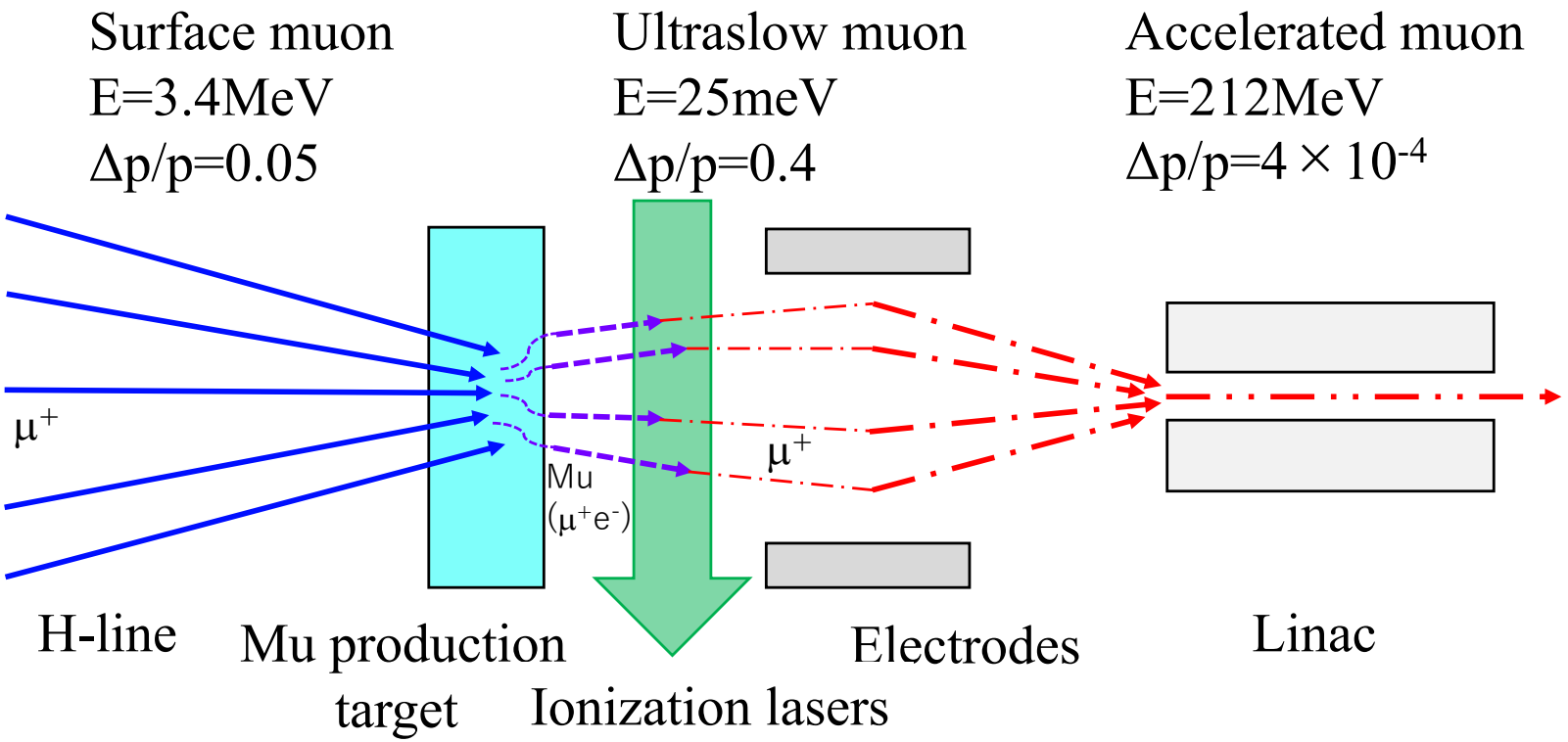
3.4 MeV surface μ^+ (high emittance) $\rightarrow$ thermal ($\sim$ 25–30 meV) $\rightarrow$ reacceleration 212 MeV (low emittance) $\rightarrow$ 3 T, 0.66m diameter storage magnet



Surface muons to Ultra-slow muons

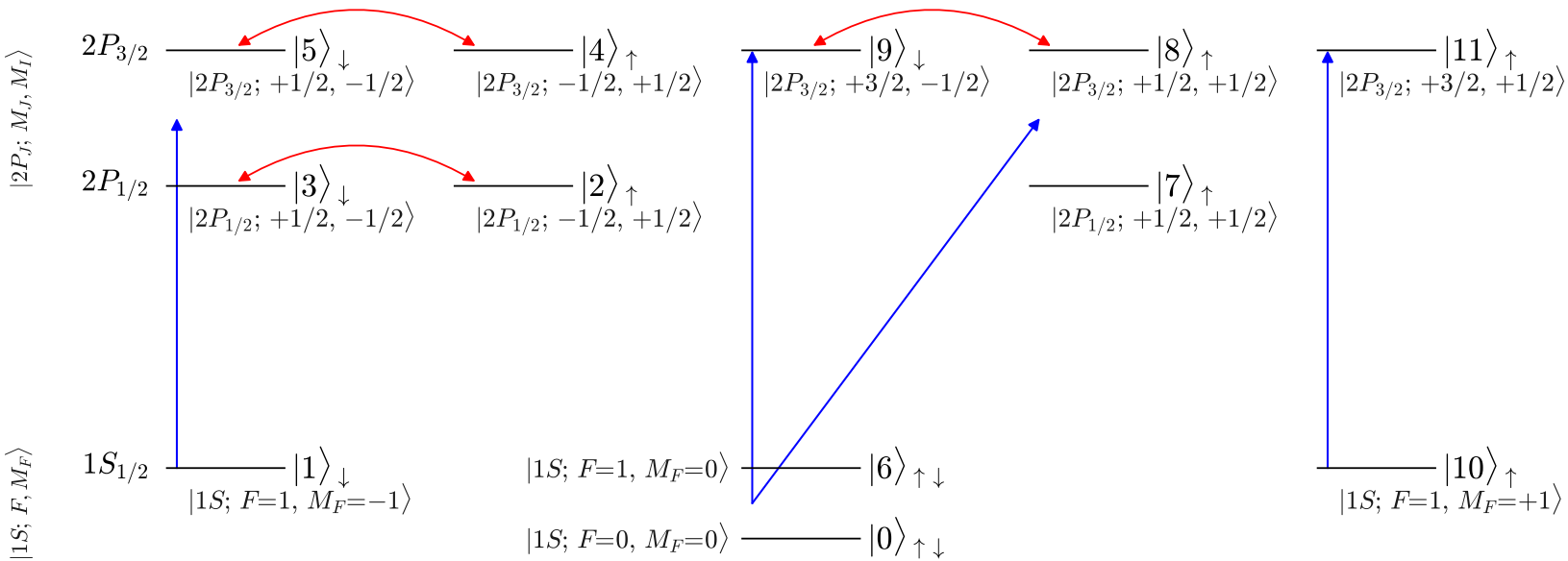
μ^+ stop in silica aerogel, and capture e^-: $\mathrm{Mu} = \mu^+ e^-$. Initially muons are 100% spin polarized $\uparrow$, but after capturing an unpolarised e^-, half the Mu formed are in a $|\uparrow\downarrow\rangle = \frac{1}{\sqrt{2}}\big(|1,0\rangle + |0,0\rangle\big)$ mixed state reducing the net polarization of the muons to $\sim 50\%$.





Muonium Level Scheme

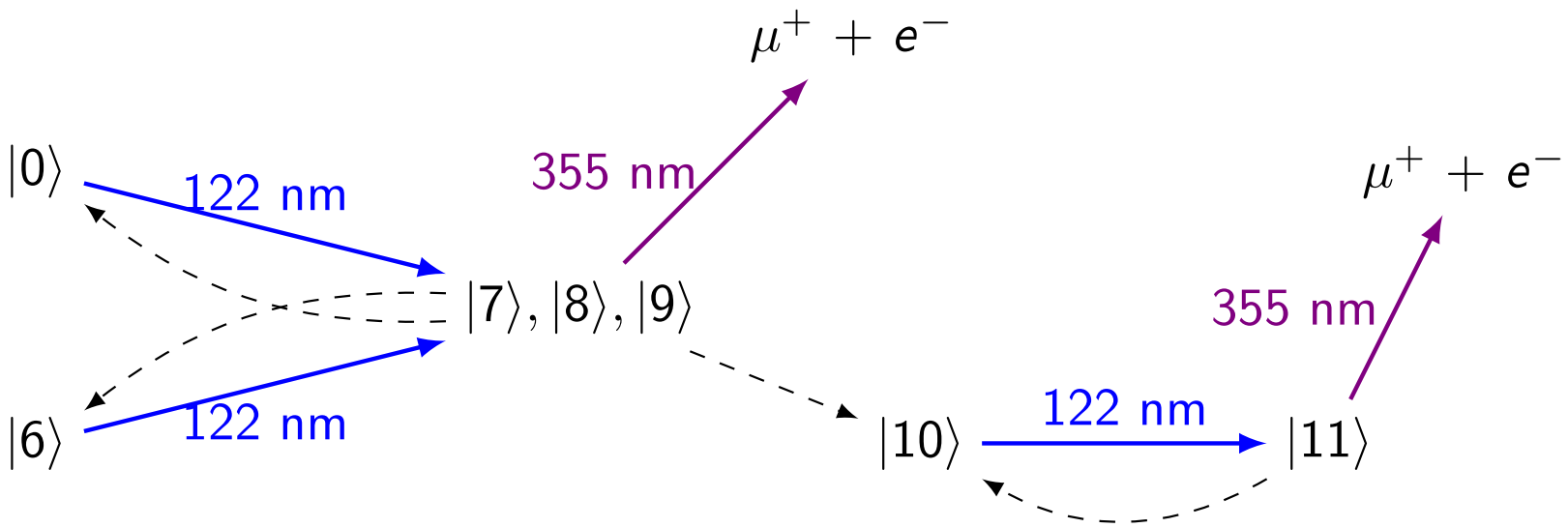
- Surface μ^+ fully polarised (spin up), captures an electron of random spin: $\sigma_{00} = \sigma_{66} = \frac{1}{4}, \quad \sigma_{10\,10} = \frac{1}{2} \Rightarrow P = 0.5$.
- The Lyman α laser pumping can only populate the states that can be achieved by $\Delta M = +1$.
- $|1\rangle$ ($M = -1$) is never populated and cannot be reached $\Rightarrow$ $|1\rangle$–$|5\rangle$ are all omitted



Adapted from Nakajima 2010

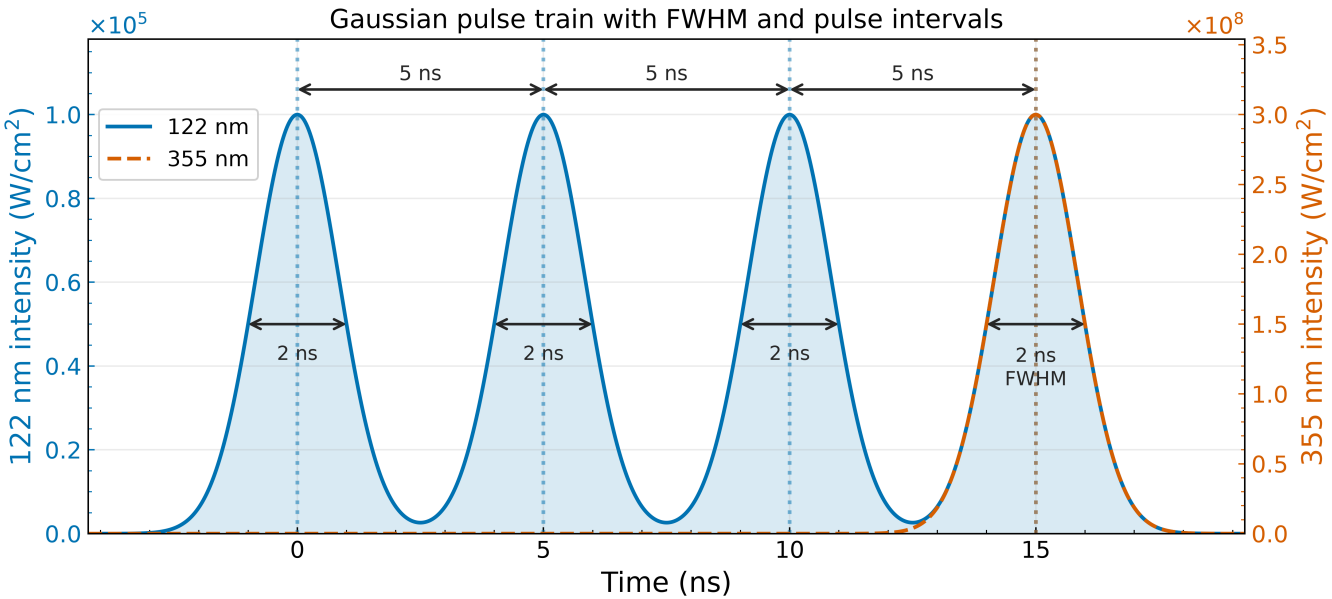
Laser polarisation and Ionisation

- **122 nm Lyman-α (polarisation laser)** Optical pumping $1S \rightarrow 2P$.
- **355 nm (ionisation laser)** Ionises from $2P$. The muon keeps the spin polarisation it had before ionisation.
- Rate-equation model of the Mu $1S$–$2P$ system (from Nakajima 2010, 2012)
- Extended with an explicit 355 nm **ionisation channel**
- Compute the polarisation of the **ionised muons**, P_{ion}.



Pulse Sequence and Parameters

$$I(t) = I_{\text{peak}} \exp\left[-\frac{(t - t_c)^2}{2\tau^2}\right], \qquad \tau = \frac{\text{FWHM}}{2\sqrt{2\ln 2}} = 0.849\,\text{ns} \;\; (\text{FWHM} = 2\,\text{ns})$$



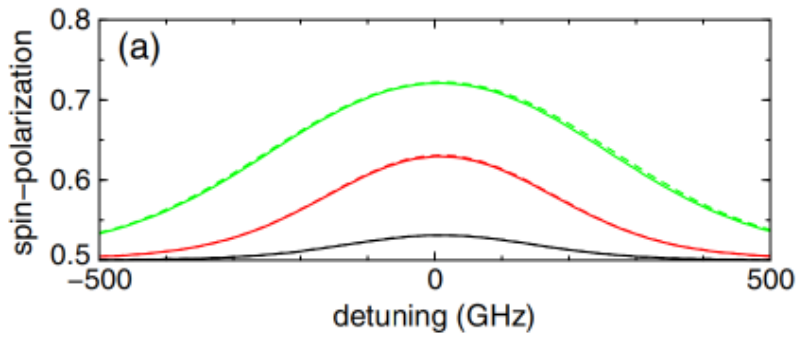
$$\vec{\sigma} = (\sigma_{00}, \sigma_{66}, \sigma_{77}, \sigma_{88}, \sigma_{99}, \sigma_{10\,10}, \sigma_{11\,11}, \sigma_{77}^{\text{ion}}, \sigma_{88}^{\text{ion}}, \sigma_{99}^{\text{ion}}, \sigma_{11\,11}^{\text{ion}})$$

$$\vec{\sigma}(t = 0) = (0.25, 0.25, 0, 0, 0, 0.5, 0, 0, 0, 0, 0)$$

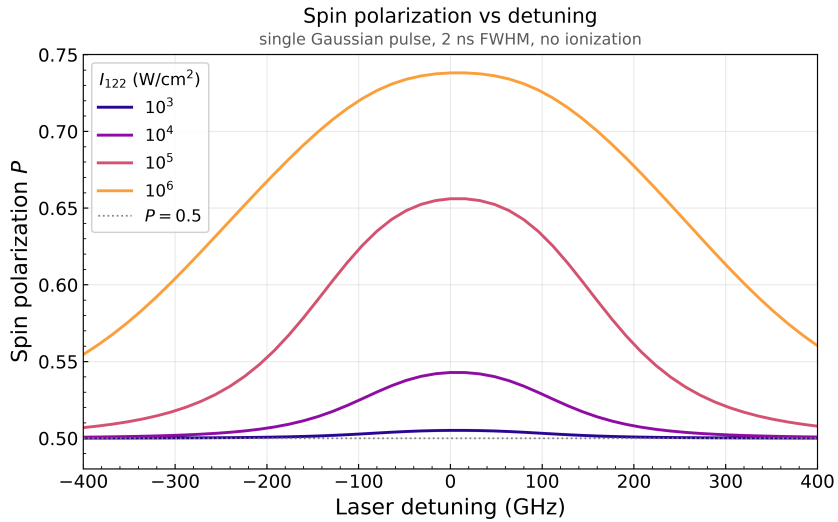
Quantity	Value
I_{122}	10^2–$10^7\,\text{W/cm}^2$
Detuning	-400 to $400\,\text{GHz}$
I_{355}	3×10^6–$3 \times 10^9\,\text{W/cm}^2$
Pulse FWHM (both)	2 ns
Interval Δt	5 ns

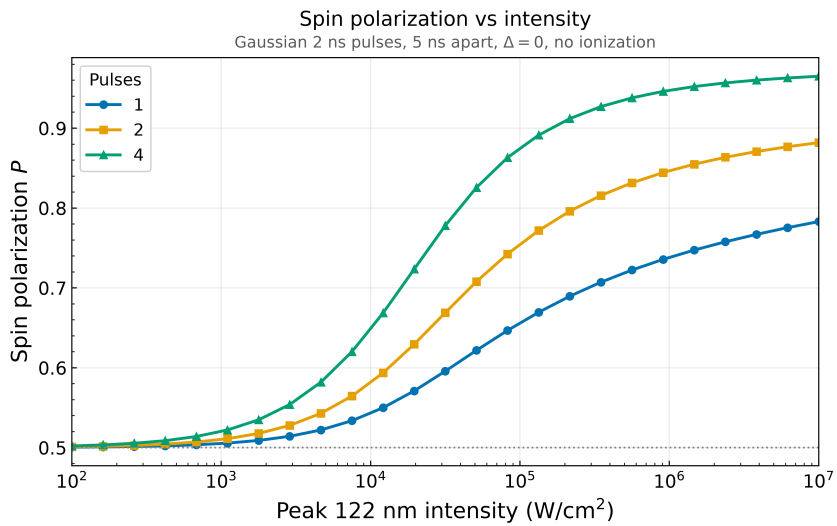
Polarisation without ionisation

Nakajima (2012)

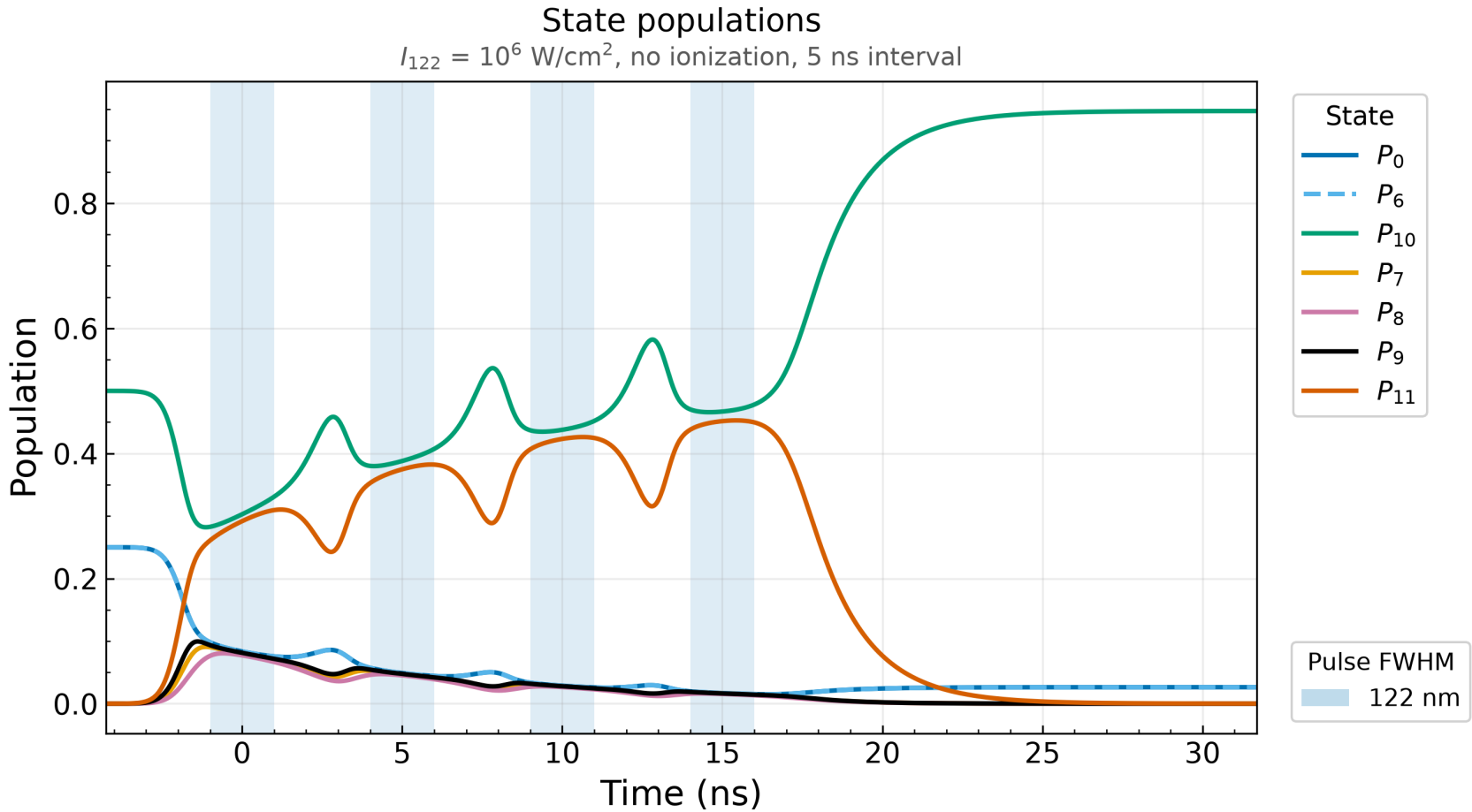


This work





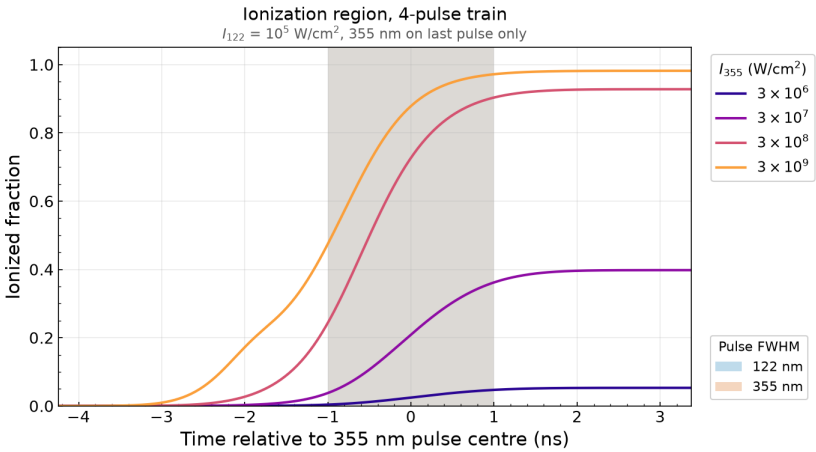
Time evolution during a four-pulse sequence



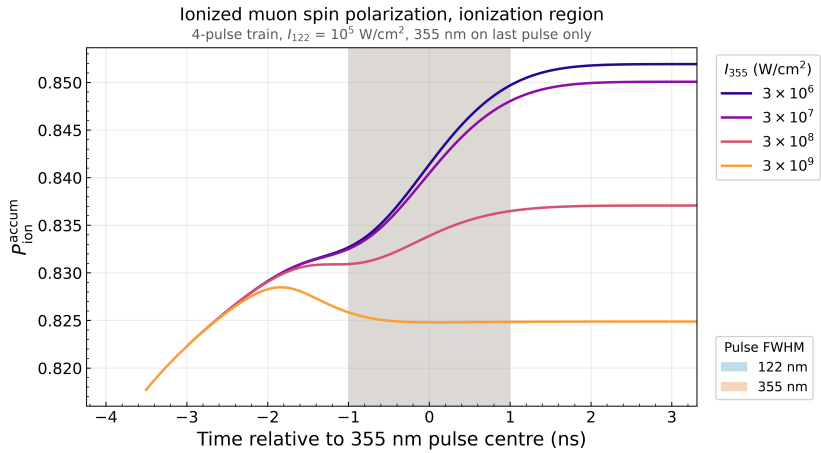
$I_{122} = 10^6\,\mathrm{W/cm^2}$, no ionisation

- Each pulse moves population from $|0\rangle, |6\rangle$ into the $|10\rangle \leftrightarrow |11\rangle$ cycle
- Final: $\sigma_{10\,10} = 0.947$, $\sigma_{00} = \sigma_{66} = 0.026 \Rightarrow P = 0.947$

Four polarisation pulses with ionisation on the last pulse ($I_{122} = 10^5\,\mathrm{W/cm^2}$)



(a) Ionised fraction N_{ion} vs. time



(b) Ionised-muon polarisation P_{ion} vs. time

I_{355} (W/cm^2)	$P_\uparrow$	$P_\downarrow$	N_{ion}	P_{ion}
3×10^6	0.049	0.004	0.053	0.852
3×10^7	0.368	0.030	0.398	0.850
3×10^8	0.852	0.076	0.928	0.837
3×10^9	0.896	0.086	0.982	0.825

Summary

- A 122 nm Lyman-α laser followed by a 355 nm laser can recover the polarisation lost during muonium formation.
- **Four 122 nm pulses, 5 ns apart, with a 355 nm laser on the last pulse:** $P_{\mathrm{ion}} = 0.83$–0.85, $N_{ion} = 98\%$ ionised.

Outlook

- **Realistic parameters from J-PARC:** 80 GHz bandwidth, muonium spatial distribution and beam cross-sections of the 122 and 355 nm lasers.
- **Pulse interval:** optimise interval Δt for yield and polarisation.
- **LTA:** I am preparing to go to J-PARC for the Beam Time.

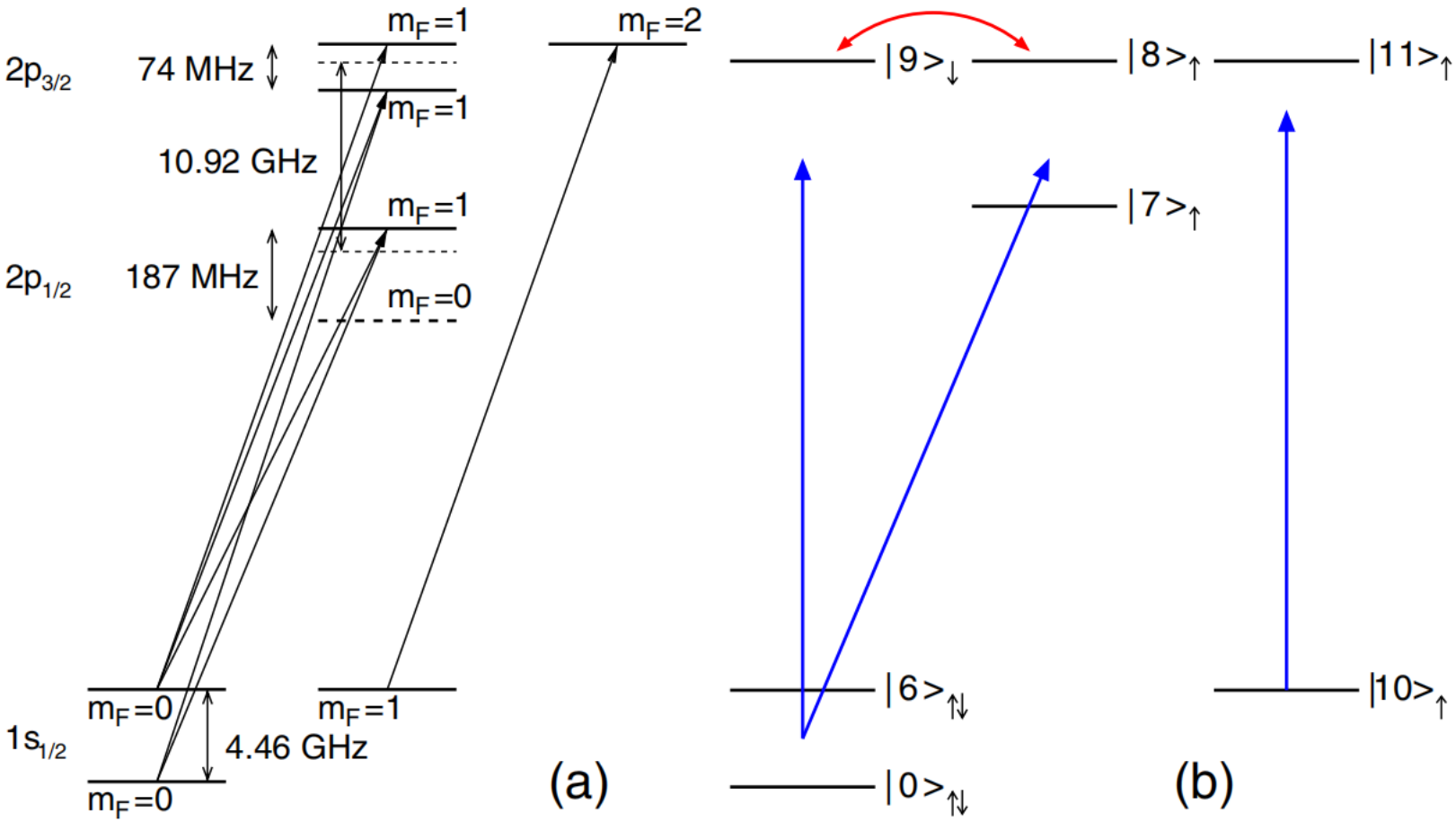
Thank You

Any Questions & Feedback?

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BACKUP SLIDES

Seven-state model: level diagram



Blue: Lyman α excitation, Red: hyperfine coupling $\Omega_{89}^{(H)}$.

Rate equations

$$\dot{\sigma}_{00} = \gamma_{\rm sp}\left(\tfrac{1}{3}\sigma_{77} + \tfrac{1}{6}\sigma_{88} + \tfrac{1}{2}\sigma_{99}\right) - \sum_{k=7,8,9} W_{0k}\left(\sigma_{00} - \sigma_{kk}\right)$$

$$\dot{\sigma}_{66} = \gamma_{\rm sp}\left(\tfrac{1}{3}\sigma_{77} + \tfrac{1}{6}\sigma_{88} + \tfrac{1}{2}\sigma_{99}\right) - \sum_{k=7,8,9} W_{6k}\left(\sigma_{66} - \sigma_{kk}\right)$$

$$\dot{\sigma}_{10\,10} = \gamma_{\rm sp}\left(\tfrac{1}{3}\sigma_{77} + \tfrac{2}{3}\sigma_{88} + \sigma_{11\,11}\right) - W_{10\,11}\left(\sigma_{10\,10} - \sigma_{11\,11}\right)$$

$$\dot{\sigma}_{77} = -\gamma_{\rm sp}\sigma_{77} + \sum_{k=0,6} W_{k7}(\sigma_{kk} - \sigma_{77}) - \gamma_{\rm ion}\sigma_{77}$$

$$\dot{\sigma}_{88} = -\gamma_{\rm sp}\sigma_{88} + \sum_{k=0,6} W_{k8}(\sigma_{kk} - \sigma_{88}) + \frac{2(\Omega_{89}^{(H)})^2}{\gamma_{\rm sp}}(\sigma_{99} - \sigma_{88}) - \gamma_{\rm ion}\sigma_{88}$$

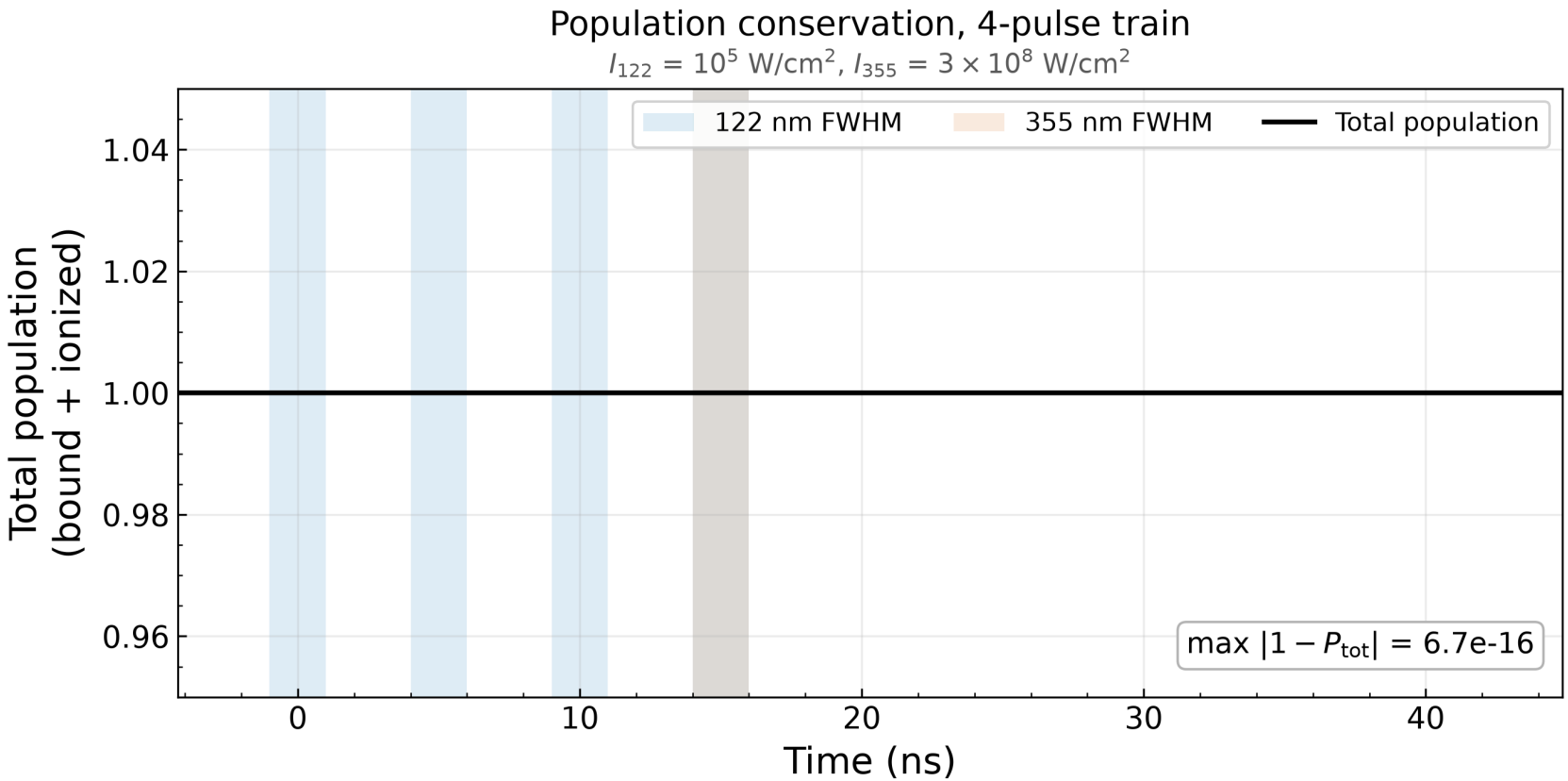
$$\dot{\sigma}_{99} = -\gamma_{\rm sp}\sigma_{99} + \sum_{k=0,6} W_{k9}(\sigma_{kk} - \sigma_{99}) - \frac{2(\Omega_{89}^{(H)})^2}{\gamma_{\rm sp}}(\sigma_{99} - \sigma_{88}) - \gamma_{\rm ion}\sigma_{99}$$

$$\dot{\sigma}_{11\,11} = -\gamma_{\rm sp}\sigma_{11\,11} + W_{10\,11}(\sigma_{10\,10} - \sigma_{11\,11}) - \gamma_{\rm ion}\sigma_{11\,11}$$

$$W_{ij}(t) = \frac{\gamma\,\Omega_{ij}(t)^2}{\Delta_{ij}^2 + (\gamma/2)^2}$$

Population conservation

$$P_{\mathrm{tot}}(t) = \sigma_{00} + \sigma_{66} + \sigma_{77} + \sigma_{88} + \sigma_{99} + \sigma_{10\,10} + \sigma_{11\,11} + \sigma^{ion}_{7\,7} + \sigma^{ion}_{8\,8} + \sigma^{ion}_{9\,9} + \sigma^{ion}_{11\,11}$$



4 pulses, $I_{122} = 10^5\,\mathrm{W/cm^2}$, $I_{355} = 3 \times 10^8\,\mathrm{W/cm^2}$ on the last pulse

BNL/FNAL vs. J-PARC

	BNL E821 / FNAL E989	**J-PARC E34**
Muon momentum	3.09 GeV/c	300 MeV/c
Polarisation	$\sim$ 100%	50%**
Storage field	1.45 T	3.0 T
Ring diameter	14 m	0.66 m
Focusing	Electric quadrupoles	no focussing
Positron detection	PbF_2 calorimeters	Silicon tracker
Sensitivity goal (a_μ)	140 ppb	450 ppb**

** Without optical pumping; reduced by muonium formation.

This work: can laser optical pumping recover the polarisation and in turn improve the precision?

The BMT equation and the magic momentum

Spin precession relative to momentum, with an EDM $\vec{d}_\mu = \eta \frac{e}{2m_\mu c}\vec{s}$:

$$\vec{\omega} = -\frac{e}{m_\mu}\left[\underbrace{a_\mu \vec{B}}_{\text{(i)}} \underbrace{- \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}}_{\text{(ii)}} + \underbrace{\frac{\eta}{2}\left(\vec{\beta}\times\vec{B} + \frac{\vec{E}}{c}\right)}_{\text{(iii)}}\right]$$

- (i) anomalous precession, (ii) motional field from E focusing, (iii) EDM
- Vertical betatron motion ($\vec{\beta}\cdot\vec{B} \neq 0$) $\Rightarrow$ pitch correction

CERN III, BNL, FNAL: cancel (ii) at the **magic** $\gamma = \sqrt{1 + 1/a_\mu} \simeq 29.3$

- $p = 3.09\,\text{GeV}/c$, $B = 1.45\,\text{T}$ $\Rightarrow$ ring radius $\approx 7\,\text{m}$
- Residual corrections for off-magic muons

The alternative: no electric field

With $\vec{E} = 0$, for **any** momentum:

$$\vec{\omega} = -\frac{e}{m_\mu}\left[a_\mu \vec{B} + \frac{\eta}{2}\vec{\beta} \times \vec{B}\right]$$

- $\vec{\omega}_a \parallel \vec{B}$ (vertical), $\vec{\omega}_e \parallel \vec{\beta} \times \vec{B}$ (radial): orthogonal
- An EDM tilts the precession plane:

$$\tan\delta = \frac{\omega_e}{\omega_a} = \frac{\eta\beta}{2a_\mu}$$

- Seen as an up–down e^+ asymmetry oscillating at ω_a
- a_μ and EDM from the **same data**, no E-field correction, free choice of momentum

Price

Only very weak magnetic focusing $\Rightarrow$ the beam must have an **extremely small emittance**

Terms of the rate equations

Linewidth: $\gamma = \gamma_{\rm sp} + \gamma_L(\Delta_{ij}), \quad \gamma_{\rm sp} = (1.6\,\mathrm{ns})^{-1} = 6.25\times10^8\,\mathrm{s}^{-1}$

$$\gamma_L(\Delta) = \gamma_{L0}\,\frac{b^2}{\Delta^2 + b^2}, \qquad \gamma_{L0} = 2\pi\times 230\,\mathrm{GHz}, \quad b = 0.8\,\gamma_{L0}$$

Rabi frequency: $\Omega_{ij}(t) = d_{ij}\,\Omega_0(t)$

$$\Omega_0(t) = \frac{d_0 E_0(t)}{2\hbar}, \quad E_0 = \sqrt{\frac{2I_{122}(t)}{c\varepsilon_0}}, \quad d_0 = \frac{2^7\sqrt{2}}{3^5}\,e\,a_0^* \simeq 0.745\,e\,a_0^*$$

$a_0^* \simeq 1.005\,a_0$ (reduced mass); at $10^5\,\mathrm{W/cm^2}$: $\Omega_0 \simeq 2\pi\times 4.2\,\mathrm{GHz}$

$$d_{07} = d_{67} = \sqrt{\tfrac{1}{3}}, \quad d_{08} = d_{68} = \sqrt{\tfrac{1}{6}}, \quad d_{09} = d_{69} = \sqrt{\tfrac{1}{2}}, \quad d_{10\,11} = 1$$

Detuning: $\Delta_{ij} = \delta - (\omega_j - \omega_i)$, δ measured from the centre of gravity of $1S$–$2P_{1/2}$

Hyperfine coupling in $2P_{3/2}$: $\Omega_{89}^{(H)} = 2\pi\times 74\,\mathrm{MHz}/2$, incoherent rate $2(\Omega_{89}^{(H)})^2/\gamma_{\rm sp}$

Laser bandwidth model and level energies

Broadband 2 ns pulse: Lorentzian linewidth with cutoff

$$\gamma_L(\Delta) = \gamma_{L0}\frac{b^2}{\Delta^2 + b^2}, \qquad \gamma = \gamma_{\mathrm{sp}} + \gamma_L(\Delta_{ij})$$

- $\gamma_{L0} = 2\pi \times 230\,\mathrm{GHz}$, $b = 0.8\,\gamma_{L0}$
- Cutoff b prevents unphysical excitation far off resonance
- γ evaluated at each transition's own detuning

Level energies (from $2P_{1/2}$):

$$\omega_0 = -\tfrac{3}{4}\cdot 2\pi \cdot 4.46\,\mathrm{GHz}, \quad \omega_6 = \omega_{10} = +\tfrac{1}{4}\cdot 2\pi \cdot 4.46\,\mathrm{GHz}$$

$|8\rangle, |9\rangle, |11\rangle$ ($2P_{3/2}$) at $2\pi \times 10.92\,\mathrm{GHz}$; $\quad \Delta_{ij} = \delta - (\omega_j - \omega_i)$

Spontaneous decay and hyperfine mixing

Spontaneous decay: $\gamma_{\mathrm{sp}} = (1.6\,\mathrm{ns})^{-1}$, agrees with $A_{2P\to 1S} = 6.265\times 10^8\,\mathrm{s}^{-1}$ to $< 0.3\%$

From	to $\lvert 0\rangle$	to $\lvert 6\rangle$	to $\lvert 10\rangle$
$\lvert 7\rangle$	1/3	1/3	1/3
$\lvert 8\rangle$	1/6	1/6	2/3
$\lvert 9\rangle$	1/2	1/2	0
$\lvert 11\rangle$	0	0	1

Hyperfine mixing $\lvert 8\rangle \leftrightarrow \lvert 9\rangle$**:**

$$\Omega_{89}^{(H)} = \frac{2\pi\times 74\,\mathrm{MHz}}{2}, \qquad \text{rate } \frac{2(\Omega_{89}^{(H)})^2}{\gamma_{\mathrm{sp}}}(\sigma_{99}-\sigma_{88})$$

Photoionisation cross section

- σ_{ion}: effective area for a $2P$ muonium atom to absorb a 355 nm photon and be ionised
- Calculated in the electric-dipole approximation (bound $2P \rightarrow$ continuum)

$$\sigma_{\mathrm{ion}} = \sigma(2P \rightarrow \mathrm{unbound}) = 1.26 \times 10^{-21}\ \mathrm{m}^2$$

$$\gamma_{\mathrm{ion}} = \sigma_{\mathrm{ion}} \frac{I_{355}}{h\nu_{355}} \quad \Rightarrow \quad \gamma_{\mathrm{ion}} \propto I_{355}$$

- $\gamma_{\mathrm{ion}} = \gamma_{\mathrm{sp}}$ at $I_{355} \simeq 2.8 \times 10^7\ \mathrm{W/cm^2}$

Why the muon?

- $m_\mu = 105.658\,\mathrm{MeV}/c^2 \approx 207\,m_e, \quad \tau_\mu \approx 2.2\,\mu\mathrm{s}$
- Contribution of a heavy state of mass M (Berestetskii 1956):

$$\frac{\delta a_\ell}{a_\ell} \propto \left(\frac{m_\ell}{M}\right)^2 \quad \Rightarrow \quad \left(\frac{m_\mu}{m_e}\right)^2 \simeq 4 \times 10^4$$

- τ: more sensitive still, but lifetime $\sim 290\,\mathrm{fs}$ and many decay modes $\Rightarrow$ out of reach
- Muon: heavy, long-lived enough to store, one dominant decay that reveals its spin

a_e tests QED; a_μ tests the **full** SM (QED, weak, hadronic) and probes new physics