Muon g-2 Storage Ring (SR) Design: Kick-off Meeting

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Oct 2026

Introduction

- We are starting this group to run simulation studies for a novel storage ring for high-precision muon g-2 measurements.
- The principles and several promising concepts are summarized in arXiv:2512.11486, now accepted by EPJC.
- Two concepts were developed further: their major systematic uncertainties and a roadmap for improvement.

Stat. precision:

$$\frac{\Delta\omega_a}{\omega_a} = \frac{1}{\omega_a \gamma \tau P}\sqrt{\frac{2}{NA^2}},$$

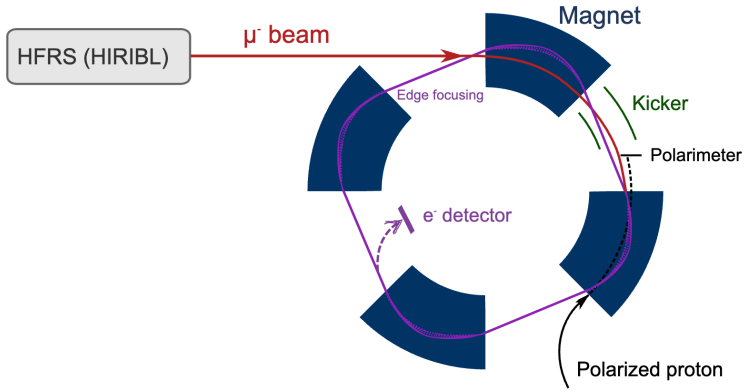


Fig. 6. Schematic design of Concept A with a polarized-proton co-magnetometer.

Farley's segmented-dipole concept

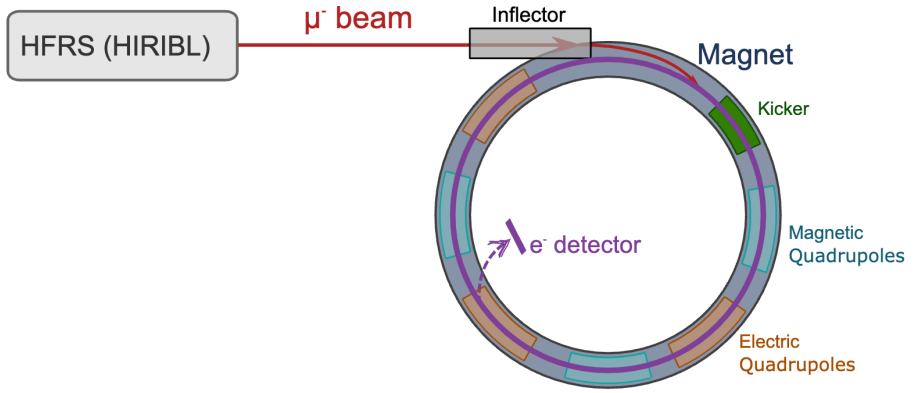


Fig. 7. Schematic design of Concept B: hybrid weak focusing using both magnetic and electric quadrupoles.

On's hybrid-quadrupole concept

Introduction

- While working on this, we identified two separate topics for the next steps:
  - Storage-ring design → this study
  - Novel magnetic-field calibration (proton co-magnetometer)
- Both concepts fit into one common study: a storage ring beyond the full-ring dipole magnet, from ‘simple’ (Farley) to ‘complex’ (On)
- Andy: the fringe-field (edge) focusing of a dipole is kind of equivalent to a B quad, so adding B quads gives a concept in between:

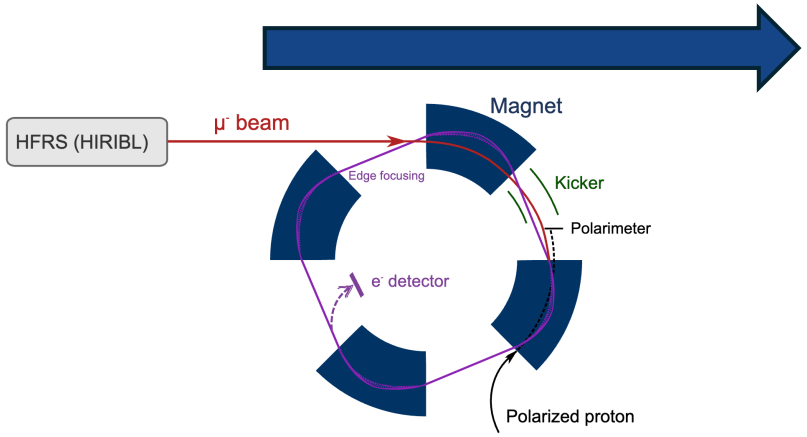


Fig. 6. Schematic design of Concept A with a polarized-proton co-magnetometer.

Farley’s segmented-dipole concept

Dipoles + B quads ?

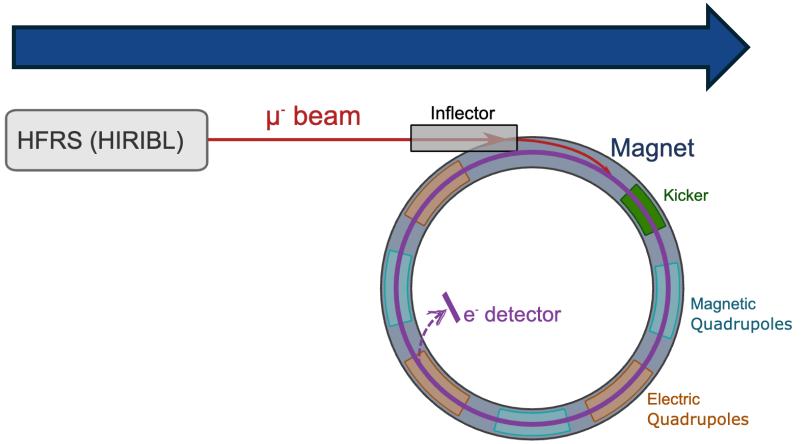


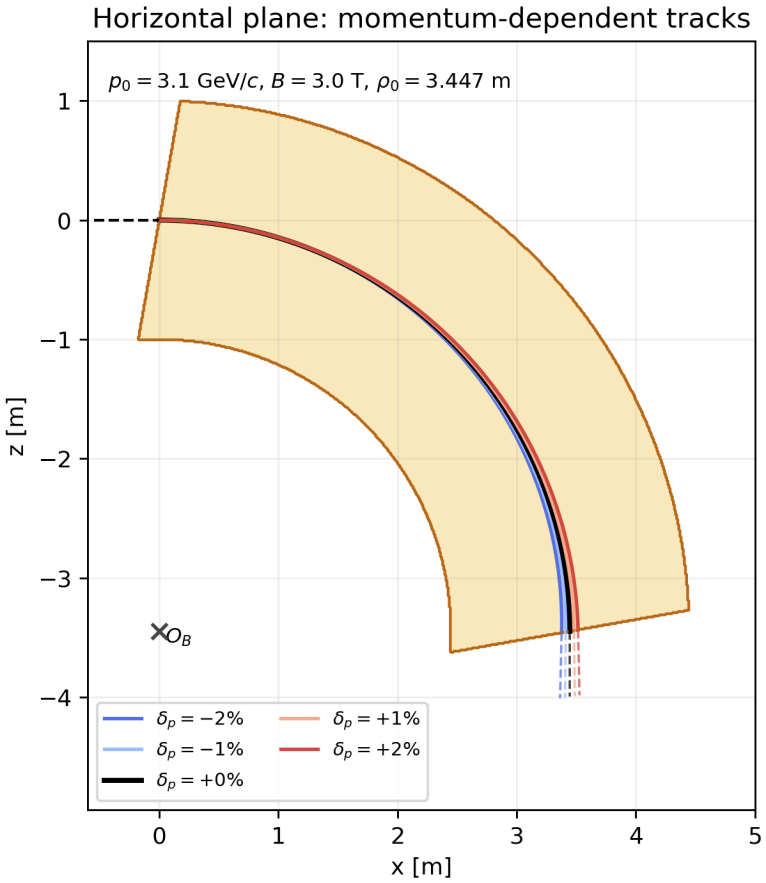
Fig. 7. Schematic design of Concept B: hybrid weak focusing using both magnetic and electric quadrupoles.

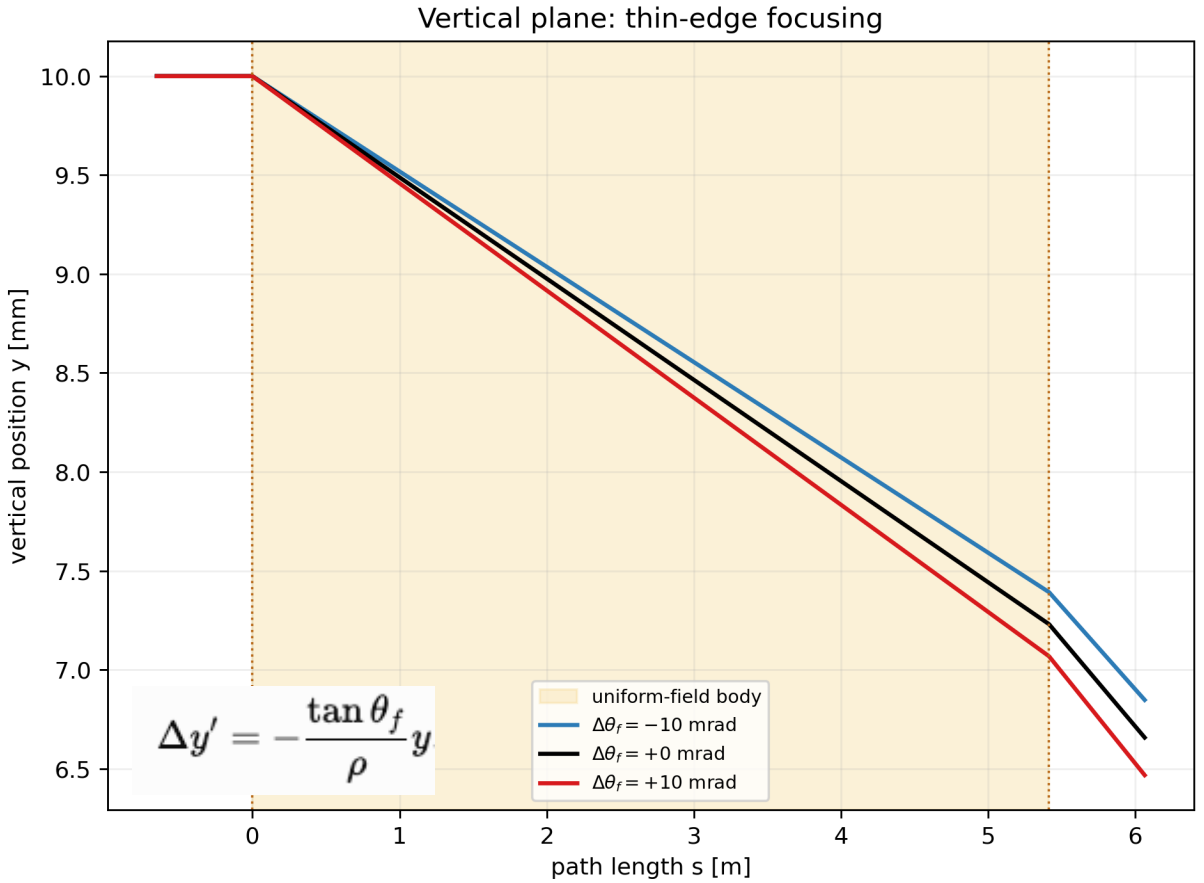
On’s hybrid-quadrupole concept

Introduction

- This is *not* a site-specific study: the conclusion of the outcoming design should apply to any muon facility worldwide interested in hosting a next-generation muon g-2 programme.
- Approach: start from the code provided by Andy
  1) Python code: tests of dipole edge focusing
  2) Simulation of muon spin precession
     - g-2 style analysis in beam dynamics: identify the major systematic sources as the function of key parameters
     - Experts in BD!
  3) Obtain optimised field maps
  4) Feed these maps into Geant4 simulations, etc.

Systematics related to the sector magnets

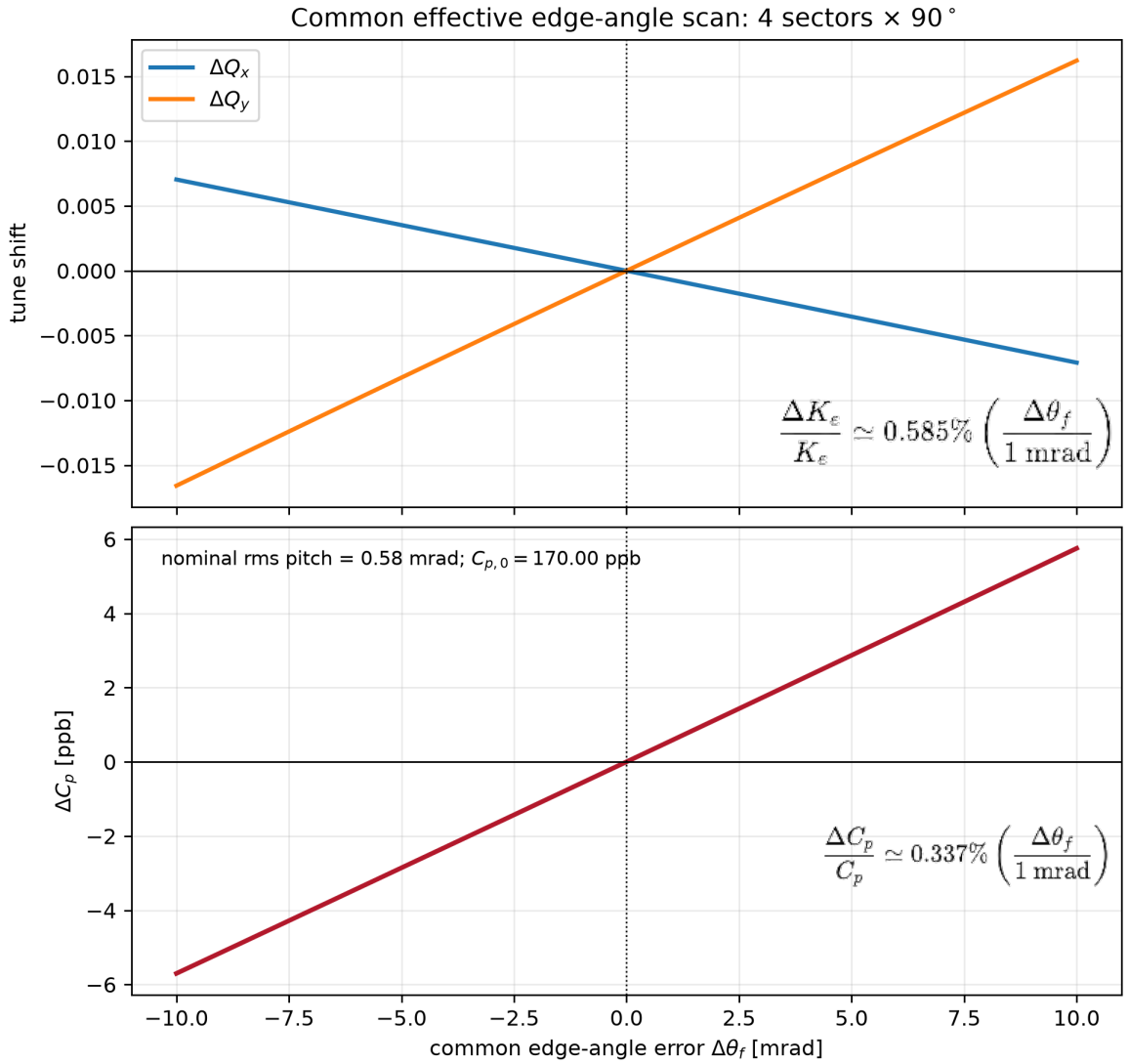




One 90° sector in the hard-edge model: muon at 3.1 GeV/c, B = 3 T, rho = 3.447 m, θ_f = 10°.

- Left: off-momentum muons bend on different radii (dispersion ≈ 4.5 m, so a 45 mm aperture accepts dp ~ +/- 1%).
- Right: the edge kick dy' = −tan θ_f · y/ρ focuses vertically; a ±10 mrad edge error changes its strength by ±5.9%.

Edge angle sensitivity



$$\Delta\theta_f \longrightarrow \frac{\tan\theta_f}{\rho} \longrightarrow \Delta Q_x, \Delta Q_y$$

edge focusing strength $K_e = \dfrac{\tan\theta_f}{\rho}$

$$\longrightarrow \frac{\Delta K_e}{K_e} \simeq \frac{\Delta\theta_f}{\sin\theta_f \cos\theta_f}$$

$$\frac{\Delta K_e}{K_e} \simeq 0.585\% \left(\frac{\Delta\theta_f}{1\ \mathrm{mrad}}\right)$$

Analytical calculation: $Q_x = 1.020$, $Q_y = 0.543$.
A common 10 mrad edge error shifts Q_x by 0.007 and Q_y by 0.017.

$$y' \longrightarrow y' - \frac{\tan\theta_f}{\rho} y$$

$$\longrightarrow C_p \simeq \frac{1}{2}\langle y'^2 \rangle_B$$ (pitch correction term in g-2 BD)

$\Delta C_p/C_p \sim 0.34\%$ per mrad: 0.57 ppb/mrad for $C_{p,0} = 170$ ppb (FNAL Run4/5/6) → so 1 ppb needs $|\Delta\theta_f| < 1.7$ mrad.
AccSimPy agrees (slides later)

The team

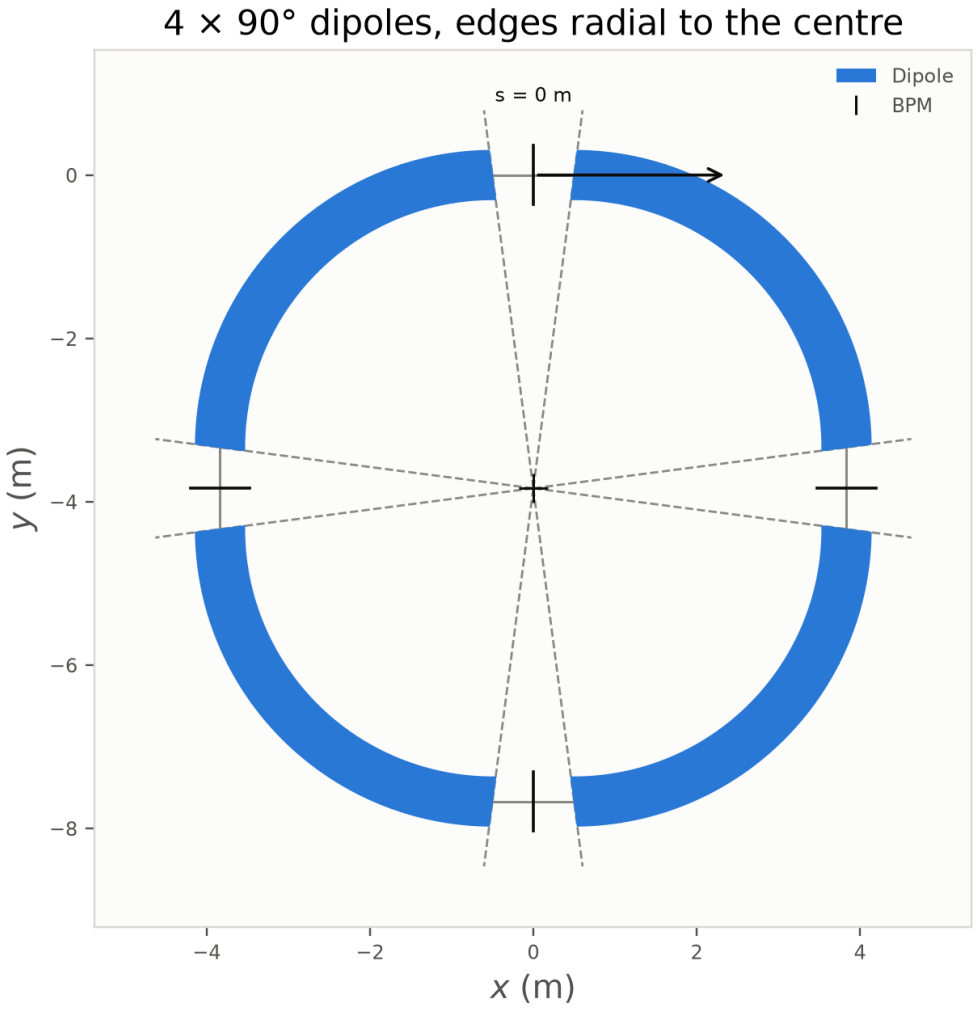
- Liverpool: Elia B., Andy W., Cedric Z.
- UW: On Kim
- SJTU: Zejia Lu, Liang Li
- SJTU→XMU: Siewyan Hoh
- SJTU→Zhejiang Lab: Bingzhi Li

Organisation

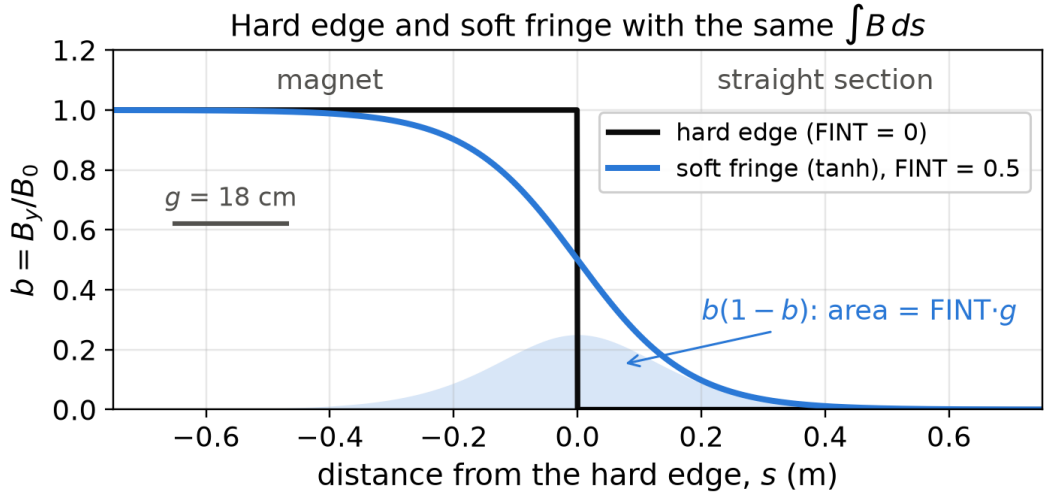
- Monthly meetings, plus smaller offline discussions as needed
- Indico page at Liverpool: https://indico.ph.liv.ac.uk/category/12/
- A Mattermost or Teams group chat?

Farley ring in AccSimPy

- Following the analysis steps of Andy's FODO example:
  - closed orbit → optics → tunes → chromaticity → tracking
- Ring: 4 × 90° uniform-field dipoles, edges on radial lines through the centre; no quad, sextupoles or RF
- muon at 3.0 GeV/c, B = 3 T (rho = 3.34 m); 1 m straight section → edge angle 7.4° (Farley eq. 5)
- Assumed fringe field: thin-edge kick with a first-order soft-fringe correction (pole gap 18 cm, FINT = 0.5; FINT = 0 is a pure hard edge)
- Dipole map: AccSimLib's C++ TrackDipole (Dipole.cpp), adapted to Python in AccSimPy (next two slides)



Edge kicks: from Dipole.cpp to AccSimPy



Entrance edge: thin kick, zero length
$\Delta p_x = k_0 \tan e_1 \cdot x / d_1^2$
$\Delta p_y = -k_0 \tan(e_1 - \psi_1) \cdot y / d_1^2$

→

Dipole body, length L
TrackDipole map: bending and focusing
for each particle's $d_1 = P/P_0$

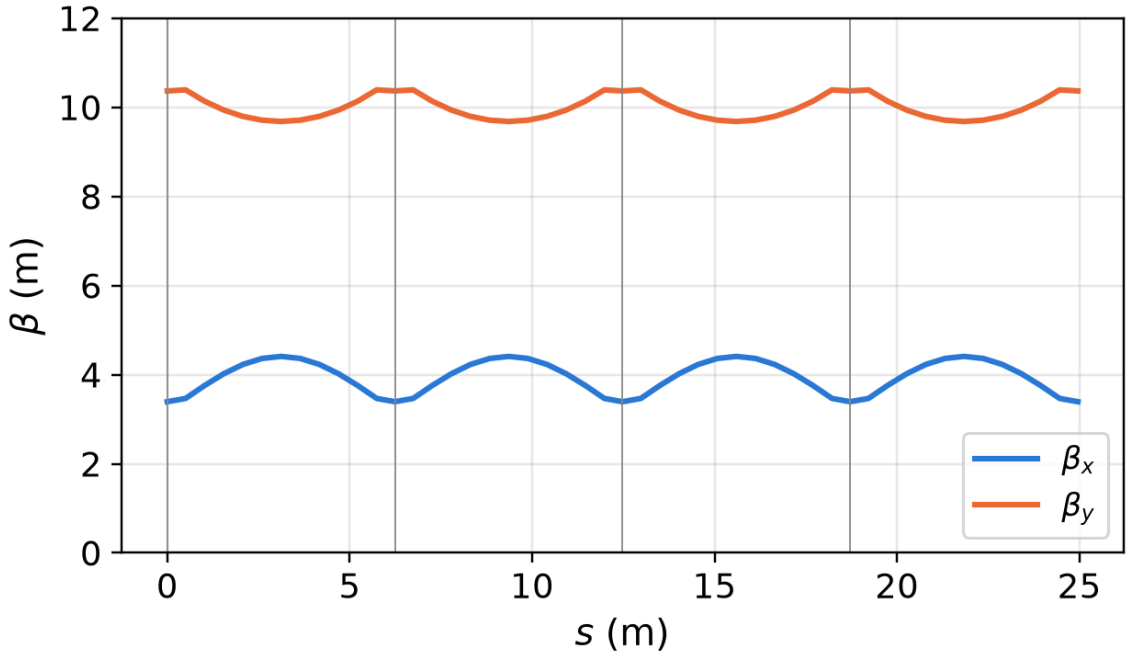
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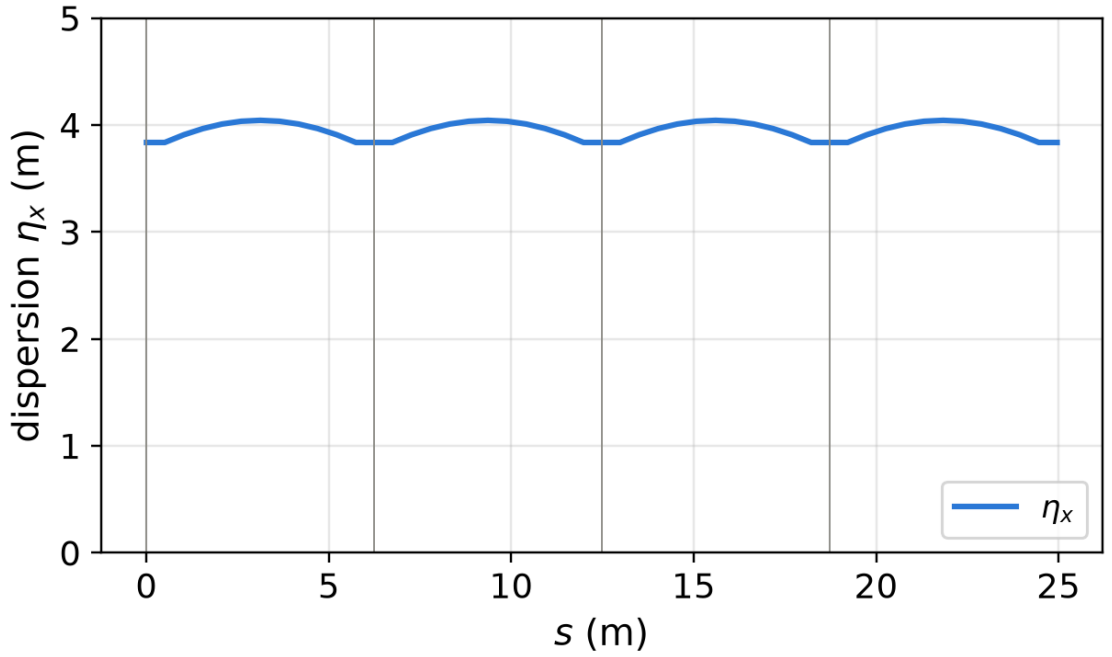
Exit edge: thin kick, zero length
same, with e_2 and ψ_2
x and y unchanged, only p_x, p_y kicked

→

- **The model in AccSimLib:** Hard edge: B is a step at the pole face;
- Each edge acts as a thin quadrupole (for e > 0: defocusing in x, focusing in y), the TRANSPORT/MAD edge matrix. Particles are not tracked through the fringe; FINT only changes the vertical kick (phi1 = ψ, hgap = g/2) (Zero length kick)
- **This study: g = 18 cm, FINT = 0.5**
- Imported the body map from Dipole.cpp: energy-dependent bending and focusing for each particle

Farley ring: optics and tunes





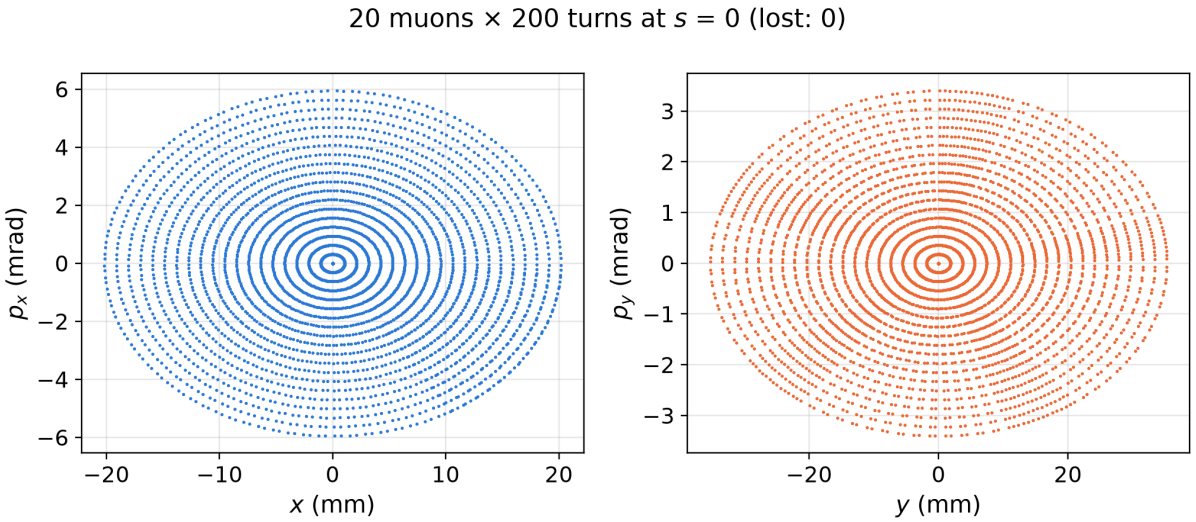
	Q_x	Q_y
Farley's matrices, ideal hard edge (FINT = 0)	1.0108	0.4502
AccSimPy, ideal hard edge (FINT = 0)	1.0108	0.4502
AccSimPy, soft-fringe correction, 18 cm gap (FINT = 0.5)	1.0108	**0.3978**

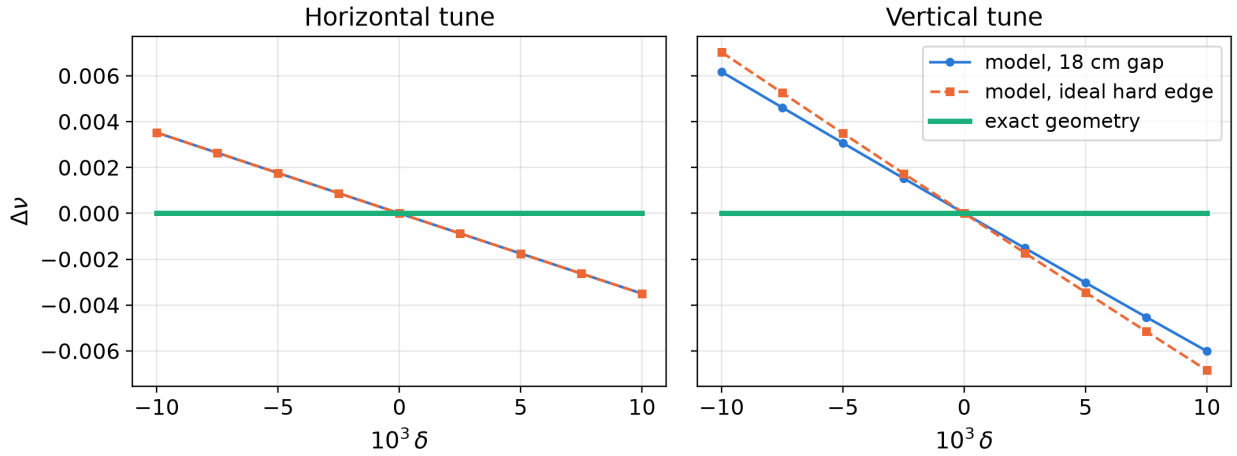
Stable in both planes; closed orbit zero.

β_x 3.4–4.4 m, β_y 9.7–10.4 m, η_x 3.8–4.0 m, $\eta_y = 0$ (no vertical bend)

FINT acts only on the vertical edge kick: Q_y drops by 0.05, Q_x is unchanged

Farley ring: tracking and chromaticity

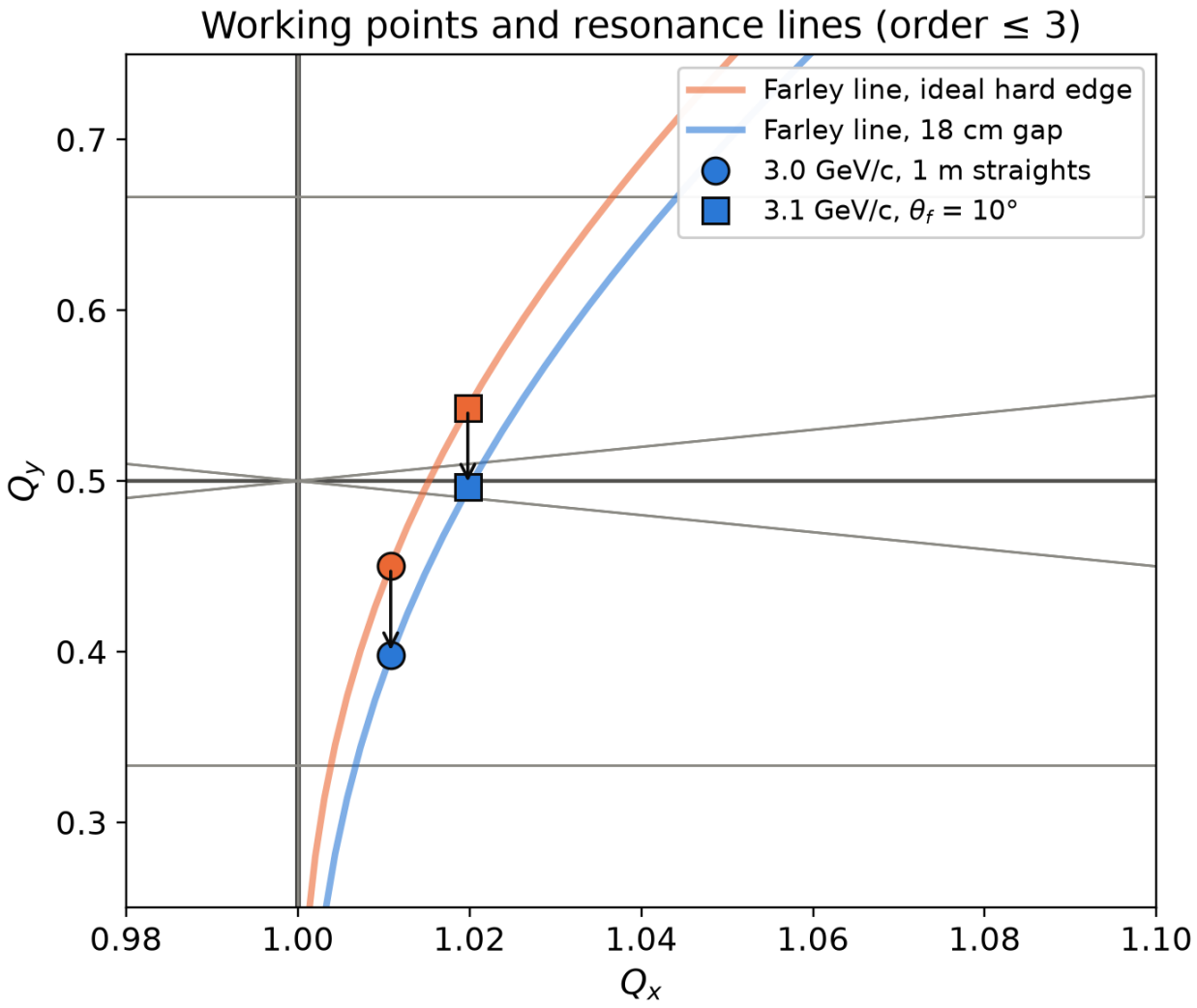




- 20 muons (actions up to 60 mm·mrad), 200 turns: no losses in the 45 mm aperture
- Clean ellipses: the body map and the edge kicks are linear in x and y (no nonlinear fringe terms), so no amplitude dependence yet
- BPM signal agrees with the linear optics

- Model chromaticity ~ −0.35 (x), −0.61 (y), but exact geometric tracking gives 0 (as Farley's scaling also predicts)
- So the expanded dipole map (body and edge kicks together) is not accurate off momentum
- To improve: an exact hard-edge + body map, or tracking through a field map?

Farley ring: is the working point good?



Stable, but the working point is fragile

- Q_x = 1.01–1.02, **just above the integer**
- Q_y is set by the edges and **very sensitive to the edge model**: FINT = 0.5 alone lowers it by ~0.05. With θ_f = 10° (3.1 GeV/c), Q_y moves 0.543 → 0.497, onto the half-integer; (C_p 170 → 154 ppb)

Missing in the current edge focusing?

- A realistic fringe-field profile instead of a thin edge kick with a FINT correction?
- Nonlinear and off-momentum terms of the dipole map (model chromaticity ≠ 0)

Conclusion

- The ring model (Farley's) has been set up with AccSimPy, giving the same Qx, Qy as in Farley's matrix;
- Dipole code: adapt the code from AccSimLib's Dipole.cpp to AccSimPy with the edge model (the zero length kick).
- The ring seems stable in both X and Y planes.
  - Qx ~ 1.01–1.02 sits just above the integer resonance.
  - Qy depends strongly on the edge model.

- Chromaticity does not give 0 – more realistic field or tracking?

Backup

Beyond Fermilab's magic-γ Paradigm

$$\frac{\Delta\omega_a}{\omega_a} \propto \frac{1}{\gamma B P \sqrt{N}}$$

Reaching the precision limit
(stat ≈ syst ~100 ppb)

$$\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}$$

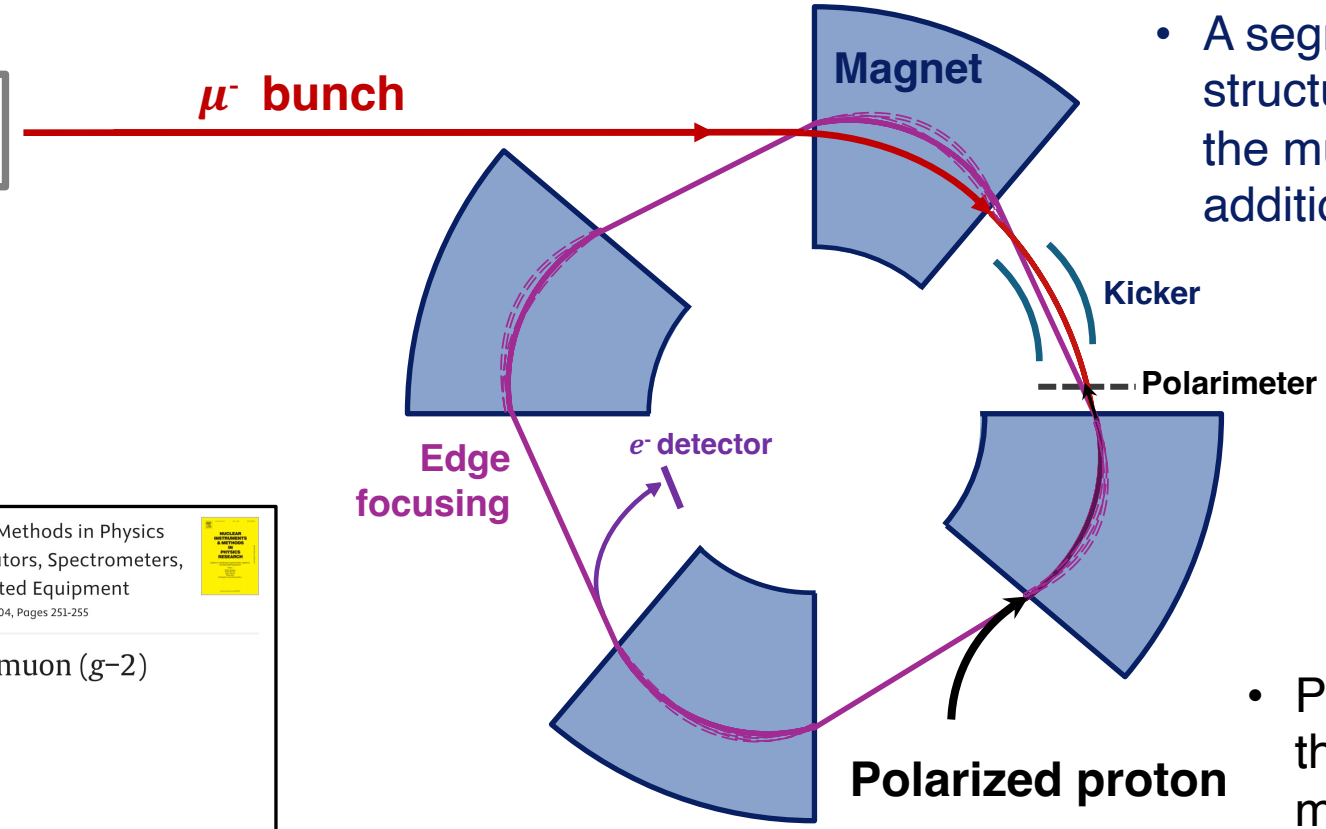
E field 'issue → **'Magic γ'** → **muon at 3.1 GeV/c, B = 1.45 T** → **CERN/BNL/FNAL Muon g-2**

remove E-field? → **How to focus (store) muons?**

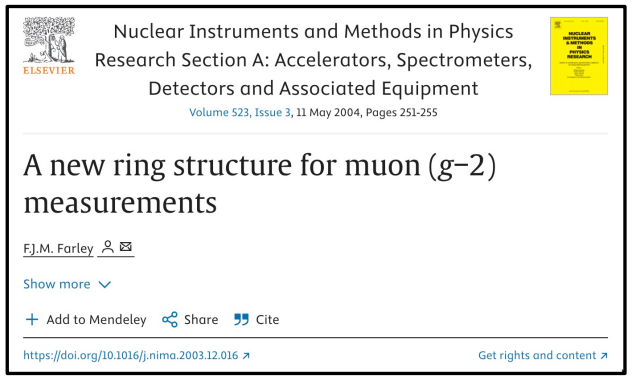
- **novel muons** → **J-PARC g-2**
  (even storable w/o an E field)
- **novel focusing fields** → **New ideas...**
  (capable of focusing conventional

Concept A) Proton beam comagnetometer

HFRS (HIRIBL)



- A segmented sector magnet structure with fringe fields to focus the muons, removing the need for additional E field focusing.


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Volume 523, Issue 3, 11 May 2004, Pages 251-255

A new ring structure for muon (g−2) measurements

F.J.M. Farley

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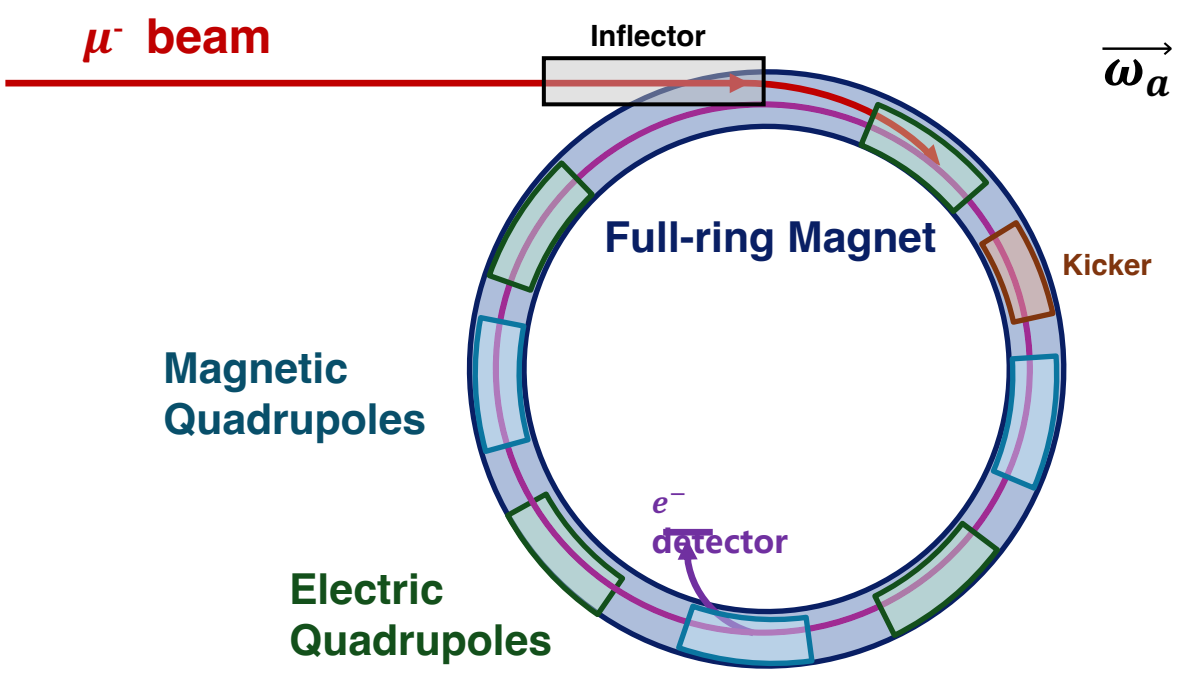
https://doi.org/10.1016/j.nima.2003.12.016

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- Polarized protons experience the same magnetic field as the muons for a direct calibration of the magnetic field.

Concept B) Hybrid weak focusing

- A hybrid focusing system with E-quadrupoles and B-quadrupoles, using higher-order B fields to compensate for higher-order E-fields:



$$\overrightarrow{\omega_a} = -a_\mu \frac{q}{m_\mu}(\vec{B}_0 + \cdots) + \frac{q}{m_\mu}\left[\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

PHYSICAL REVIEW ACCELERATORS AND BEAMS **25,** 024001 (2022)

Analytical estimations of the chromaticity and corrections to the spin precession frequency in weak focusing magnetic storage rings

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Final a_mu formular

For FNAL (fixed probe calibration):

$$\widetilde{B} = \frac{\hbar\,\widetilde{\omega}'_p(T_r)}{2\mu'_p(T_r)}, \tag{3}$$

where $\widetilde{B} = \langle B \times M \rangle$ and M is the reconstructed muon distribution,

Combining the experimentally determined frequency ratio with the definition of the Bohr magneton, $\mu_{\mathrm{B}} = e\hbar/2m_e$, the muon anomalous magnetic moment can be expressed as

$$a_\mu = \frac{\omega_a}{\widetilde{\omega}'_p(T_r)} \frac{\mu'_{\mathrm{p}}(T_r)}{\mu_{\mathrm{B}}} \frac{m_\mu}{m_{\mathrm{e}}}. \tag{4}$$

where the ratios $\mu'_{\mathrm{p}}(T_r)/\mu_{\mathrm{B}}$ and m_μ/m_e are precisely known from other experiments [83].

Final a_mu formular

In the proton co-magnetometer case, we need to consider BMT eq for proton spin precession:

$$\vec{\omega}_{a,p} = -\frac{q}{m_p}\left[a_p \vec{B} - a_p\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta} - \left(a_p - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]. \tag{10}$$

Assuming no E field and ideally $\vec{\beta}\cdot\vec{B} = 0$, the later terms cancel and a_p is simplified as

$$a_p = \frac{\omega_{a,p}}{B} m_p\,, \tag{11}$$

we get $B = \frac{\omega_{a,p}}{a_p} m_p$, gives final a_mu: $a_\mu = \frac{\omega_{a,\mu}}{\langle \omega_{a,p} \times M \rangle} a_p \frac{m_\mu}{m_p}$,

Final a_mu formular

FNAL (J-PARC)

$$a_\mu = \frac{\omega_a}{\widetilde{\omega}'_p(T_r)} \frac{\mu'_{\rm p}(T_r)}{\mu_{\rm B}} \frac{m_\mu}{m_{\rm e}}.$$

$$a_\mu = \frac{\omega^m_{a,\mu}(1 + C_e + C_p + C_{ml} + C_{dd} + C_{pa})}{\langle \omega^m_p(T_r) \times M \rangle (1 + B_k + B_q)} \frac{\mu'_{\rm p}(T_r)}{\mu_{\rm B}} \frac{m_\mu}{m_{\rm e}}$$

Proton co-magnetometer

$$a_\mu = \frac{\omega_{a,\mu}}{\langle \omega_{a,p} \times M \rangle} a_p \frac{m_\mu}{m_p},$$

$$a_\mu = \frac{\omega^m_{a,\mu}(1 + C_p + C_{ml} + C_{dd} + C_{pa})}{C_{\mu/p}\langle \omega^m_{a,p} \times M \rangle (1 + B_k + B_p + B_{pl} + B_{pa})} a_p \frac{m_\mu}{m_p}$$

a total calibration factor?

Proton precession analysis (CBO, pileup, gain)

B_p: pitch
B_pl: proton loss
B_pa: proton phase acc

external parameter here is also ~22ppb!

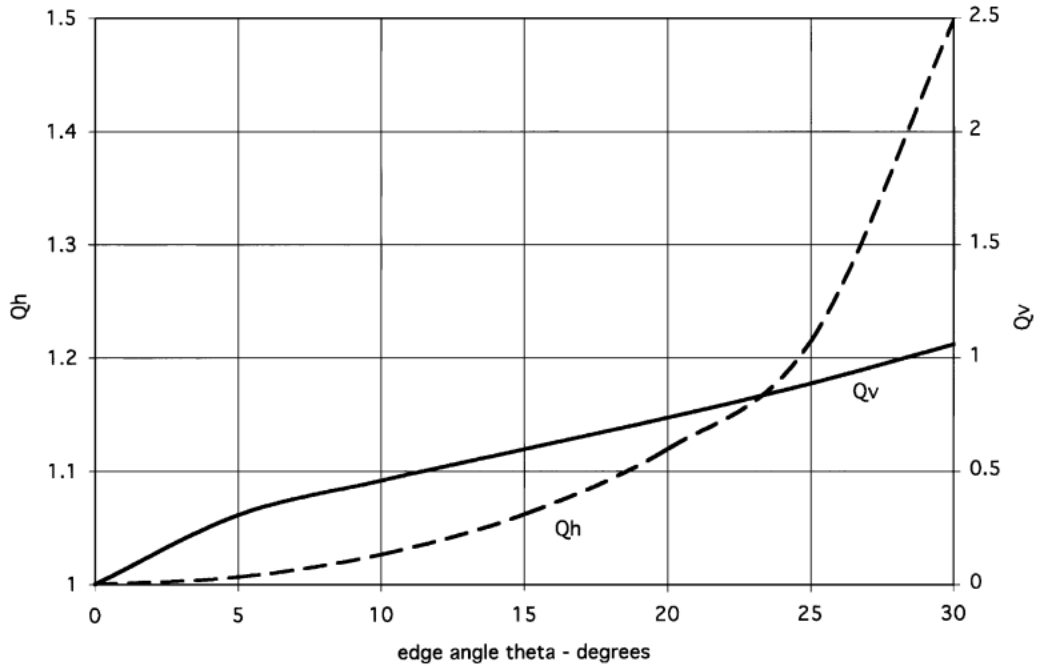
Table 3. Comparison of the dominant systematic uncertainties between the Fermilab Muon g–2 experiment and the Concept A and Concept B.

Systematic source	FNAL Runs 4-6	Concept A	Concept B	Comments
CBO	19-24 ppb	≤ 15 ppb	≤ 15 ppb	
Detector Gain effect	15-25 ppb	≤ 10 ppb	≤ 10 ppb	
Pileup	4-5 ppb	≤ 15 ppb	≤ 15 ppb	
Total $\omega_{a,\mu}^{m}$ systematic	30 ppb	≤ 21 ppb	≤ 21 ppb	
Muon electric field correction C_e	27 ppb	≤ 1 ppb	≤ 5 ppb	Concept B: higher order term?
Muon pitch correction C_p	9 ppb	≤ 9 ppb	≤ 9 ppb	A good modeling simu dev
Muon loss correction C_{ml}	2 ppb	≤ 3 ppb	≤ 3 ppb	
Muon phase acceptance correction C_{pa}	15 ppb	≤ 10 ppb	≤ 10 ppb	
Muon differential Decay C_{dd}	27 ppb	≤ 20 ppb	≤ 20 ppb	DH: 10 ppb
Absolute/trolley calibration	34 ppb	-	34 ppb	
Field mapping and tracking	34 ppb	≤ 12 ppb	34 ppb	
Quad transient correction B_q	20 ppb	≤ 1 ppb	≤ 10 ppb	
Kicker transient correction B_q	22 ppb	≤ 15 ppb	≤ 15 ppb	
Proton pitch correction B_p	-	≤ 6 ppb	-	
Proton loss correction B_{pl}	-	≤ 1 ppb	-	
Proton phase acceptance correction B_{pa}	-	≤ 1 ppb	-	
$\omega_{a,p}^{m}$ extraction	-	≤ 10 ppb	-	
$C_{\mu/p}$	-	≤ 15 ppb	-	
External $\mu'_{\mathrm{p}}(T_r)/\mu_{\mathrm{B}}$ and m_μ/m_e	22 ppb	-	10 ppb	
External $a_p \frac{m_\mu}{m_p}$	-	22 ppb	-	
Total systematic	78 ppb	≤ 52 ppb		

Systematic buget

Quantity	Uncert at Fermilab (ppb)	Projection for new exp	
Wa (sys)	30	<20	fitting start time CBO: no good ways to reduce its uncert?
Ce	27	~0	
Cp	9	<10	The dominant systematic uncertainty is from the straw-tracker alignment and reconstruction. There are smaller contributions from the acceptance and resolution corrections. Other possible sources of uncertainty come from the estimation procedure for Cp described above, as well as possible errors in alignment and calibration of the ESQ system that are described in detail in Appendix B.
Cpa	15	<10	better detector performance
Cdd	27	<20	better bunching layered momentum distributions? $\propto 1/p$? non-ideal kicker?
Cml	2	<5	more stored muons per fill → more pileup? a better detection system?
<wp x M> (mapping, tracking)	34	<20	
<wp x M> (calibration)	34	<20	
<wp x M> total	48	<30	Proton co-magnetometer

3 sectors --> 4 sectors



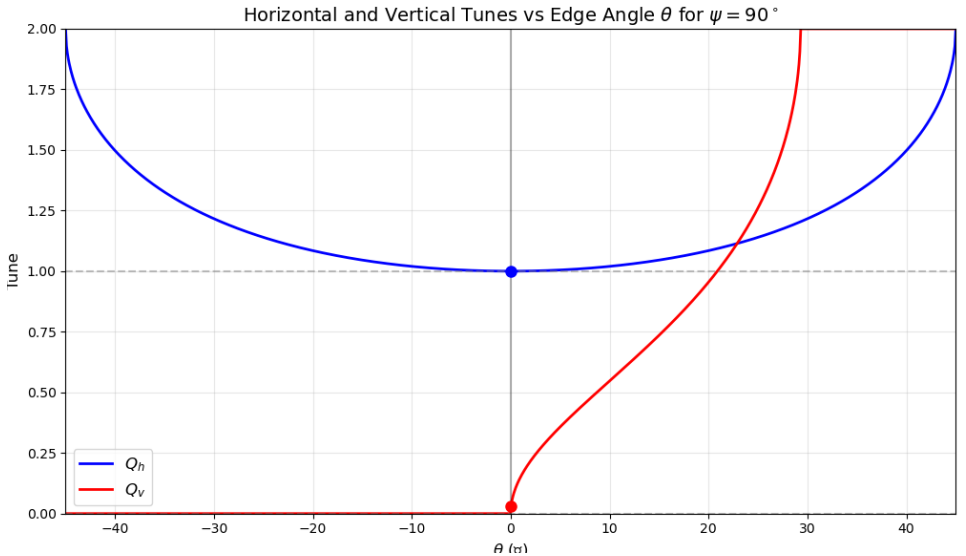


Fig. 2. Tune of three-magnet ring. The bend in each magnet is 120°. The edge angle is half the opening angle between magnets.