Pitch correction

- Start from the BMT equation (without E field term):

$$\vec{\omega}_a = -\frac{q}{m}\left[a\vec{B} - a\frac{\gamma}{\gamma+1}\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta}\right]$$

- For now we only consider B_y (effect of B_x, B_z need to be studied with spin tracking?), we can get

$$\vec{\omega}_a = -\frac{qa}{m}\left[B_y\cos\psi\,\vec{u} + \frac{1}{\gamma}B_y\sin\psi\,\vec{v}\right] \approx -\frac{qa}{m}\left[B_y\left(1-\frac{1}{2}\psi^2\right)\right]$$

- The ψ and B_y is changing over different position, so we should average it:

$$C_p = \frac{\Delta\omega_a}{\omega_{a,0}} = \frac{1}{2}\langle\psi^2\rangle = \frac{1}{2}\frac{\int\psi^2 B_y ds}{\int B_y\,ds}$$

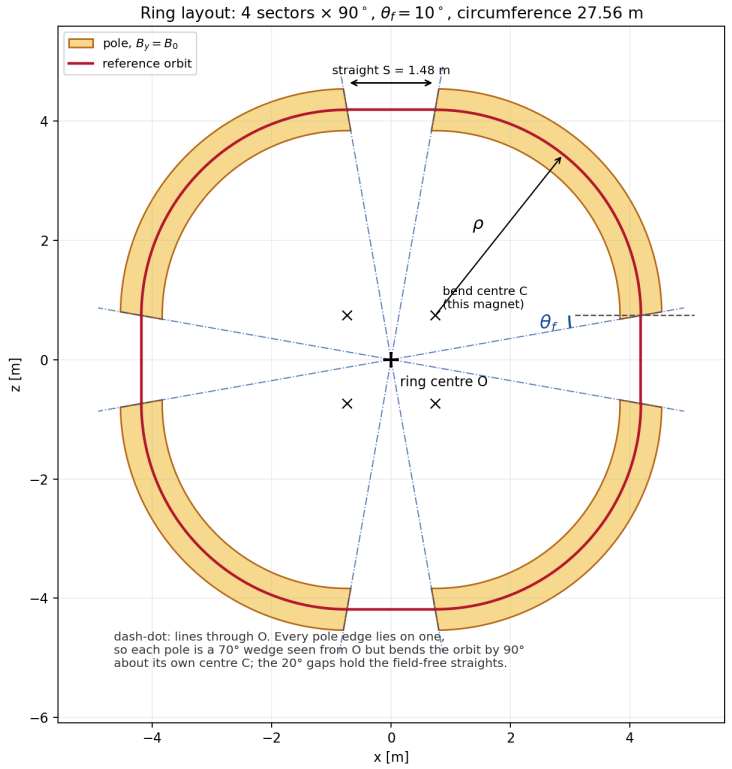
- Refer to Fermilab's result, $C_p = 170\,ppb$, correspond to $\psi_{rms} = 0.58\,mrad$.

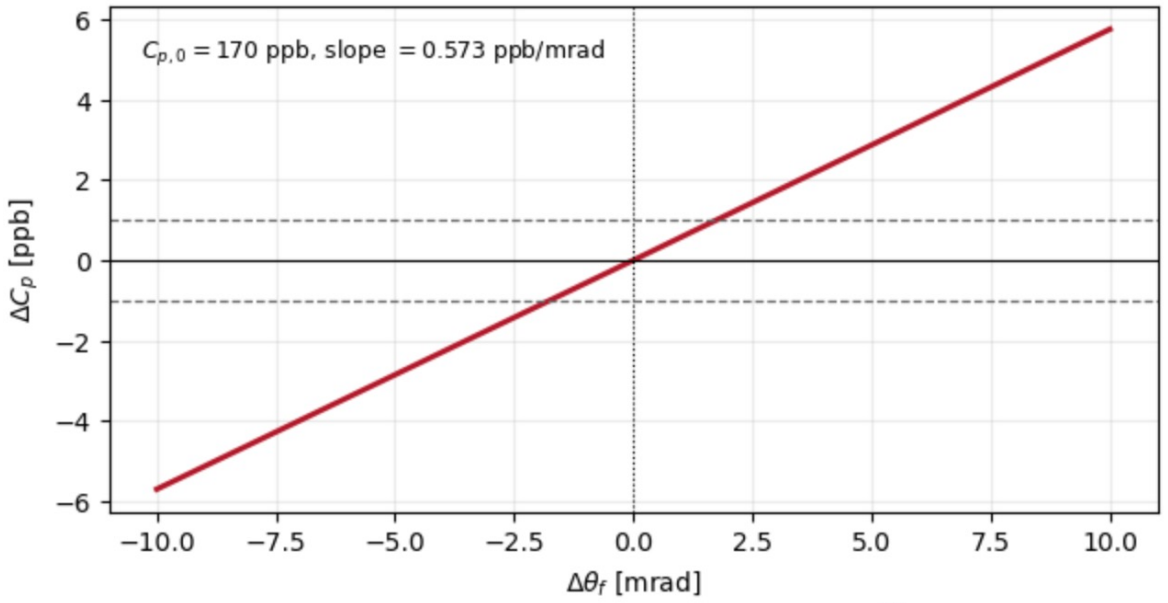
The pole-face angle effect

- The pole-face angle θ_f will affect the edge focus strength

$$K = \tan\theta_f / \rho$$

- The change in the focus strength will cause change in pitch angle;





Finite edge length

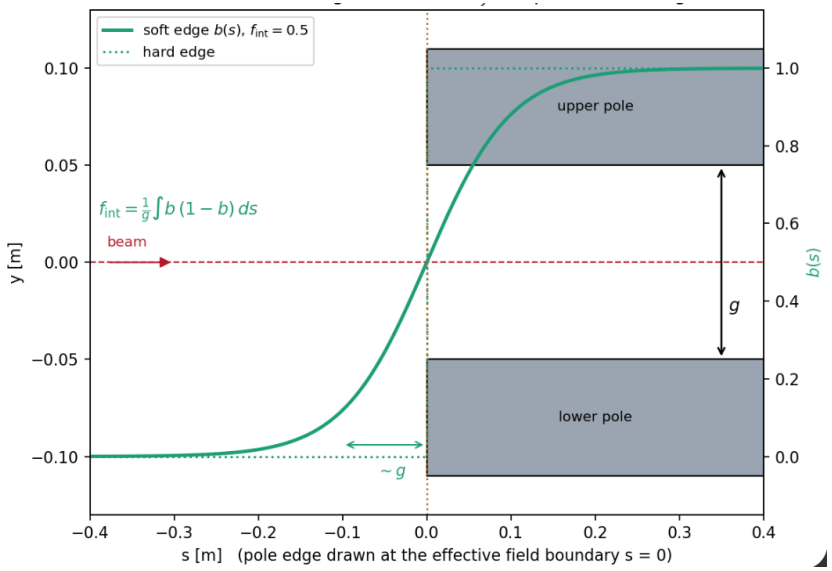
- The finite edge length will reduce the focusing strength. Defining fringe field integral:

$$f_{int} = \frac{1}{g}\int \frac{B_y}{B_0}\left(1 - \frac{B_y}{B_0}\right) ds$$

- , where g is the full gap of magnet. The effective θ_f will be reduced by

$$\theta_{eff} = \theta_f - f_{int} g \frac{1}{\rho} \frac{1 + \sin^2 \theta_f}{\cos \theta_f}$$

- and thus change the pitch correction.



Finite edge length

- According to $C_p = \frac{1}{2}\frac{\int \psi^2 B_y ds}{\int B_y ds}$, the shape of fringe field will also affect the integral.
- These two effects combine and affect C_p by 10s ppb.

