Cosmological Stasis

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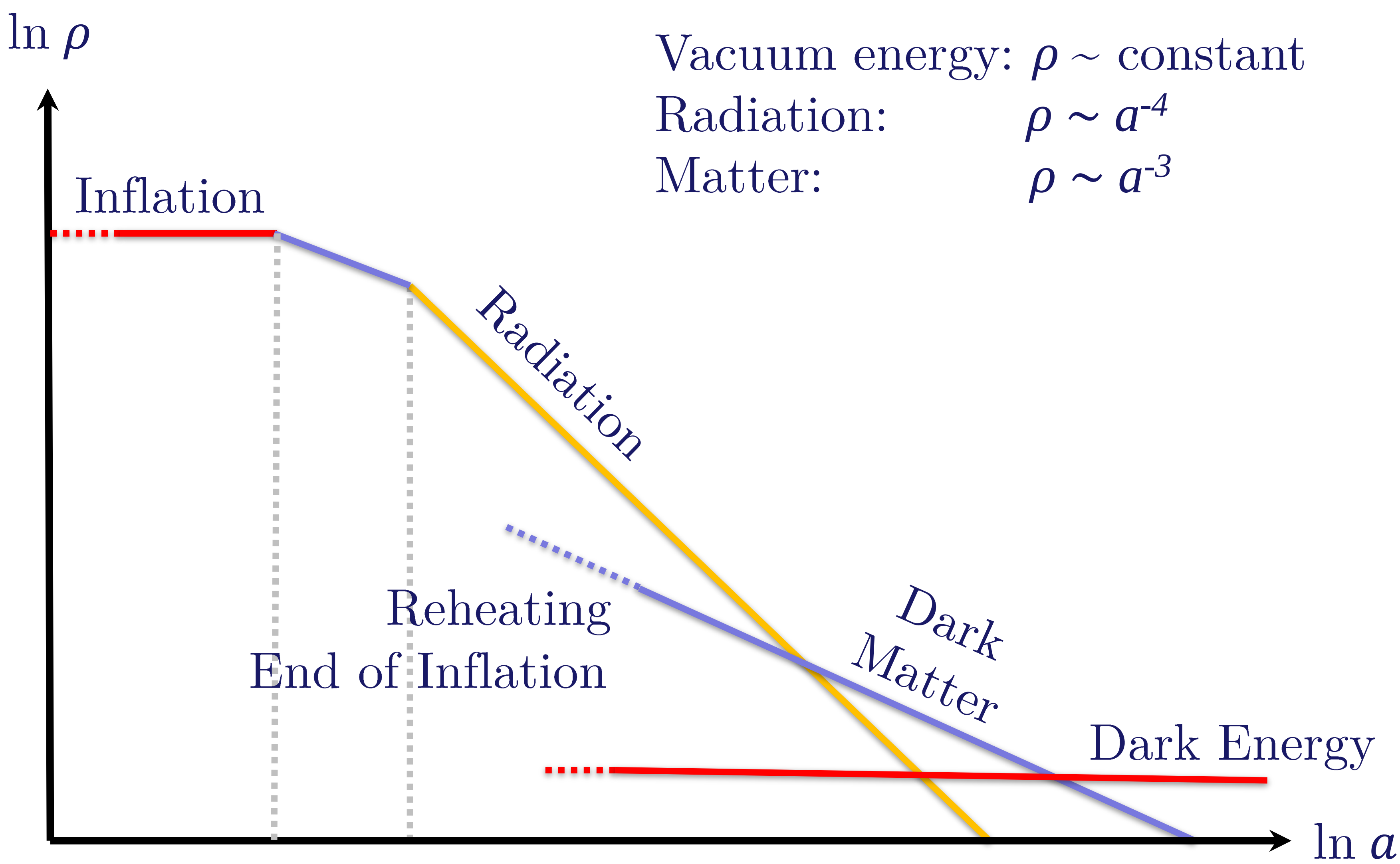
Based on arXiv:2111.04753

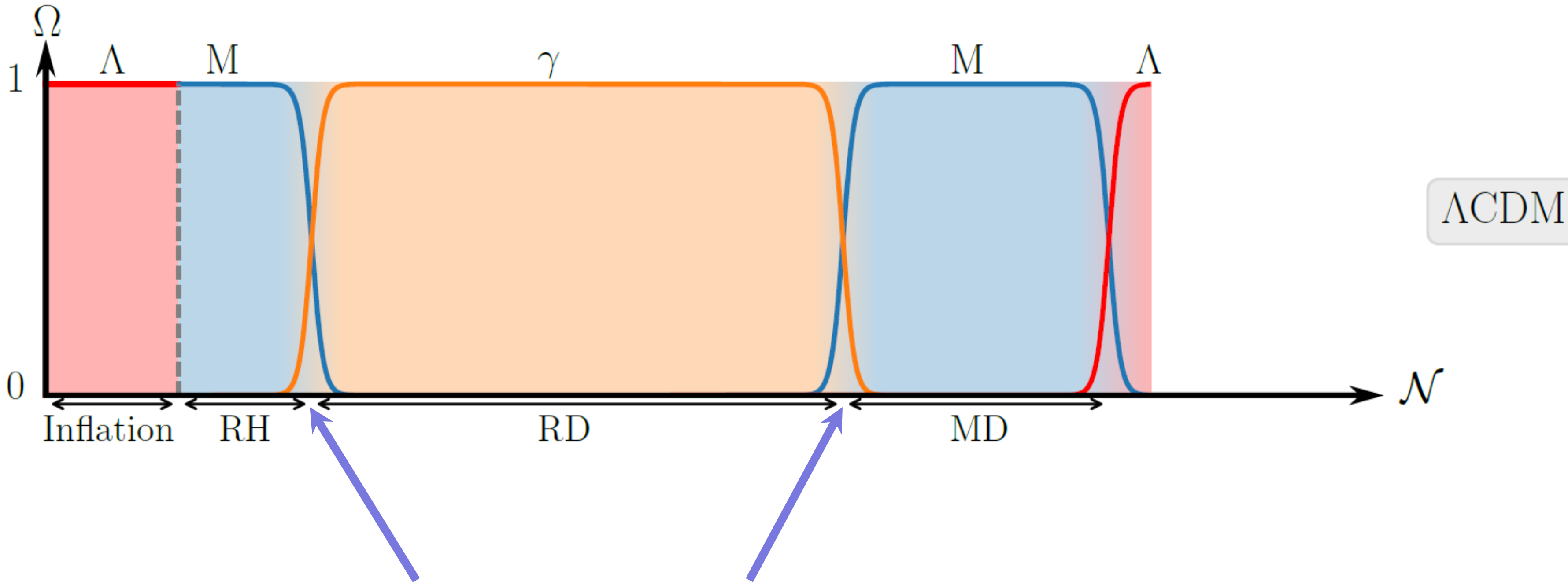
DMUK 2021 Meeting

Durham University

The ΛCDM Ideology



The ΛCDM Ideology



Matter-Radiation Equality (MRE)

In the ΛCDM model, **MRE is localized in time**. The Universe HAS to *choose* between Matter Domination (MD) and Radiation Domination (MD).

What if...

... MRE was **not a point** in time,
but **an entire era** of the
cosmological timeline?

More Generally...

... Is it possible to maintain **the ratio $\boldsymbol{\Omega_{\mathbf{rad}}/\Omega_M}$ constant** over a sizeable number of *e*-folds?

Universe Expansion

Radiation redshifts faster than matter

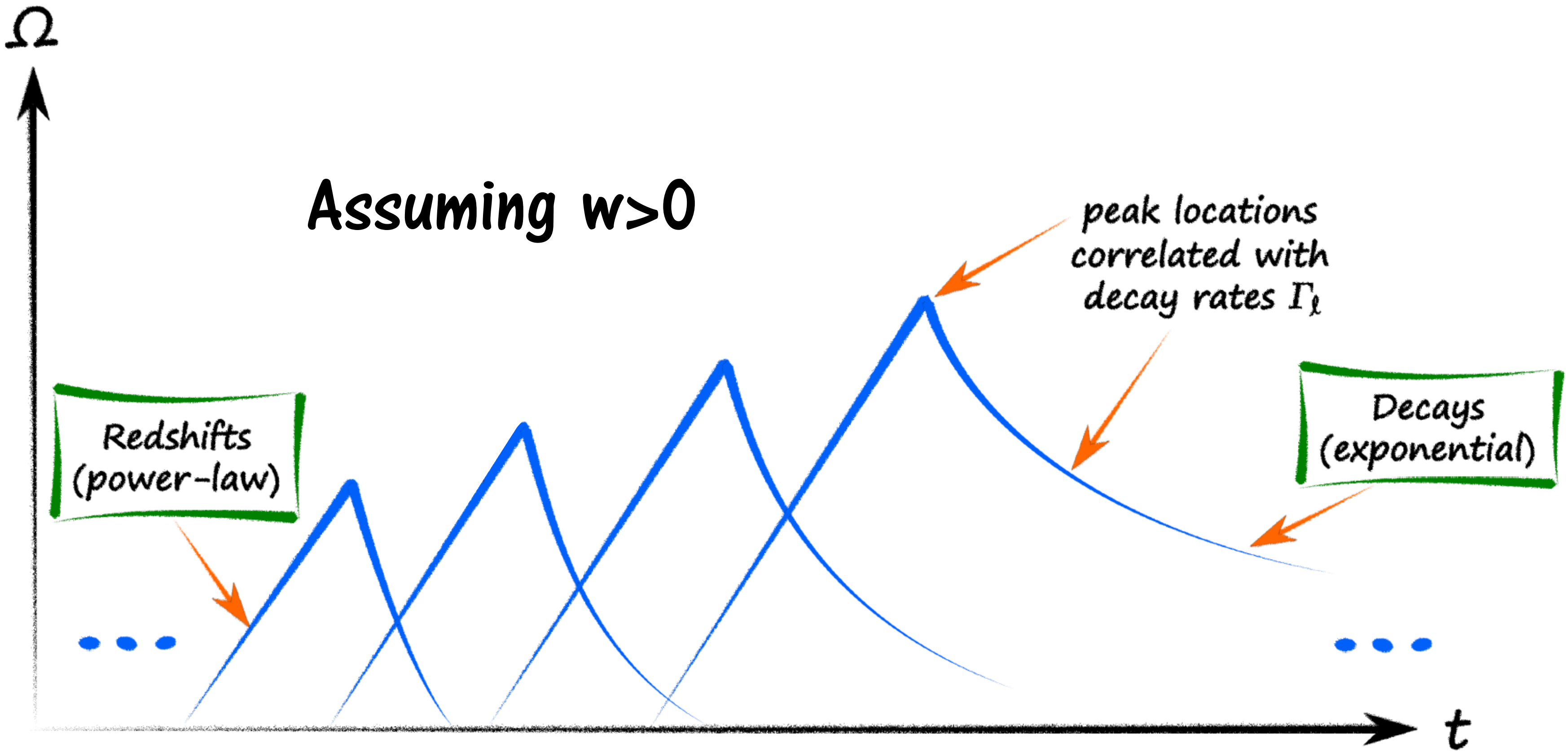
Need to transfer energy between the two sectors to compensate expansion

Can particle decays play that role?

One state decaying?

→ *Already the case during cosmic reheating...*

Many states decaying?



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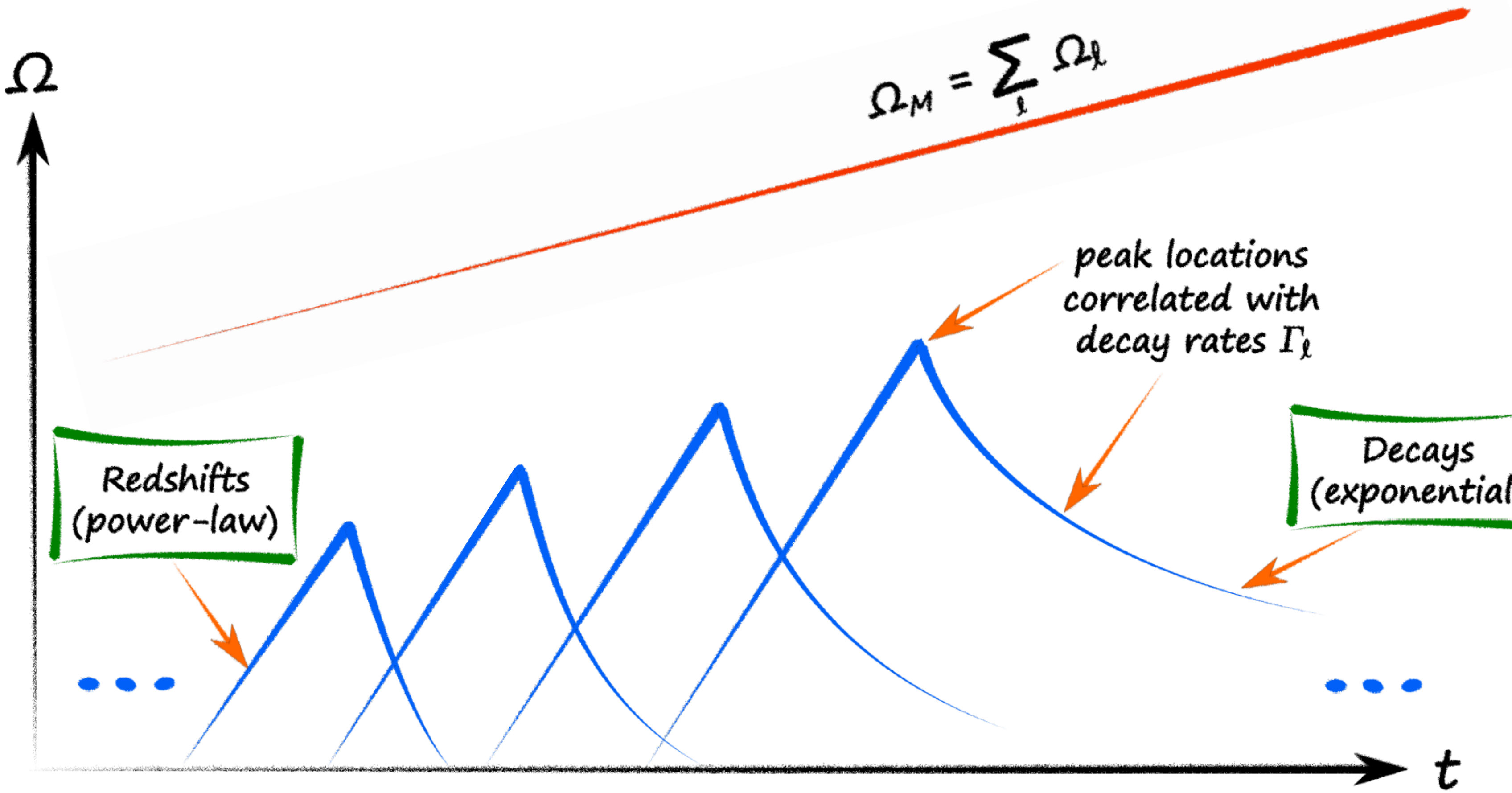

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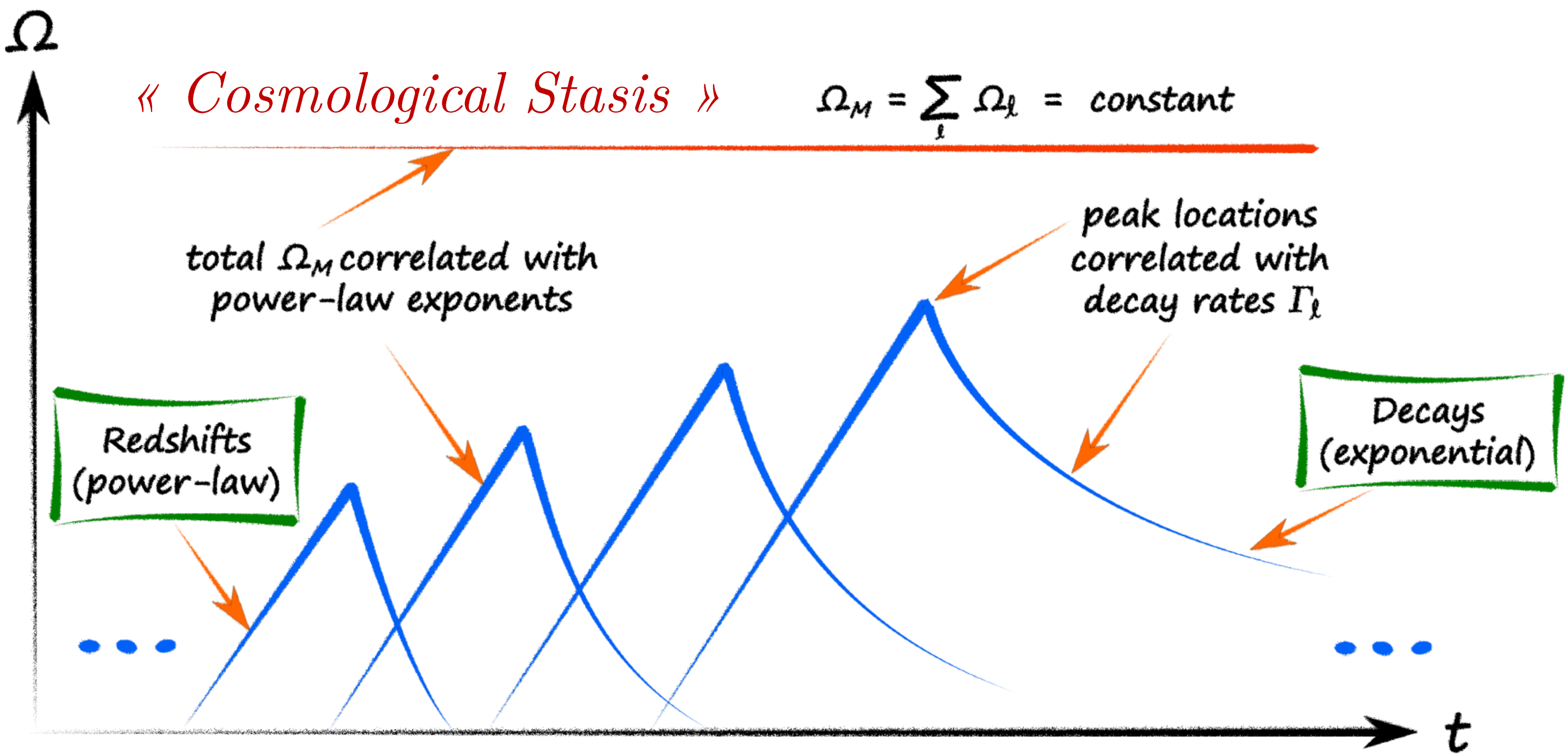

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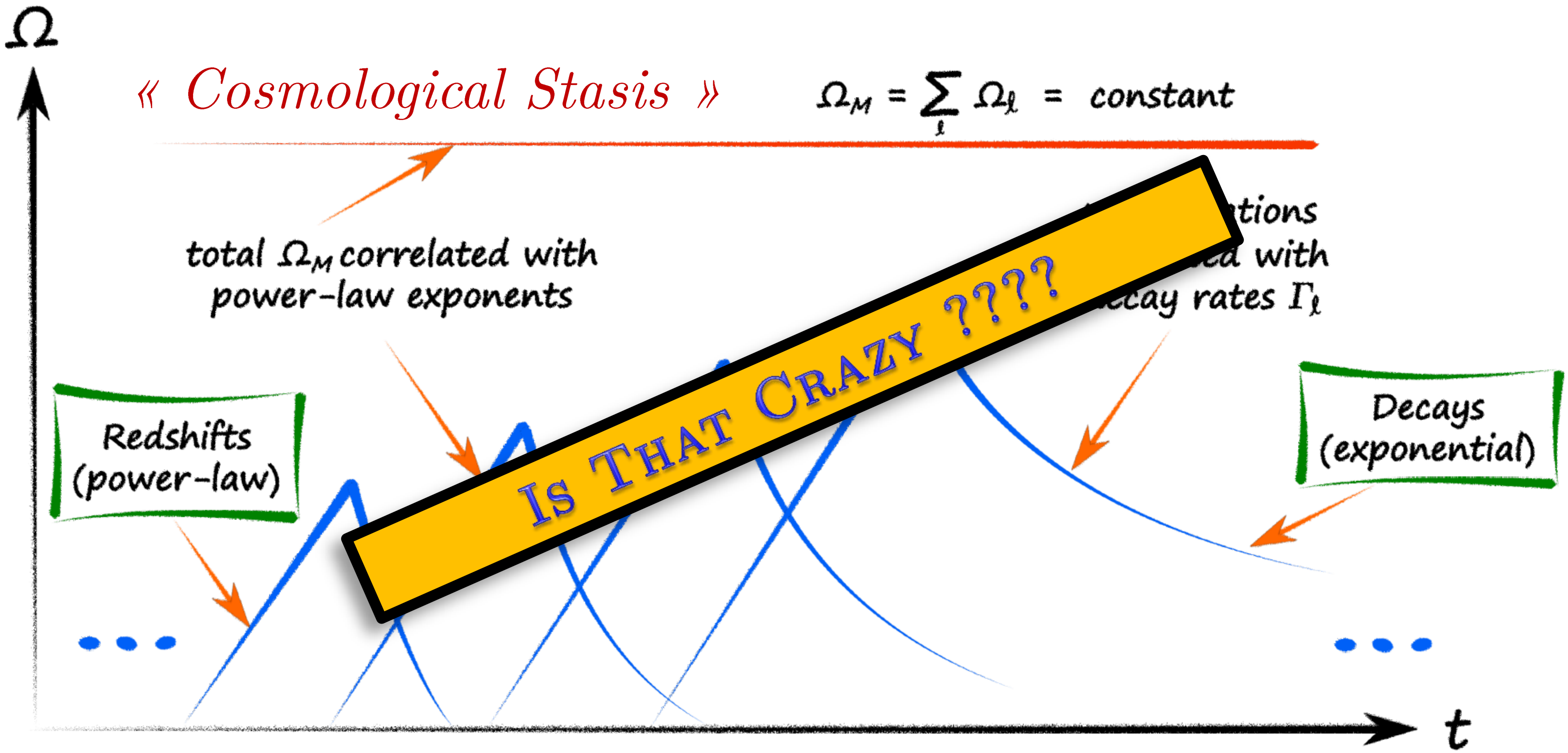
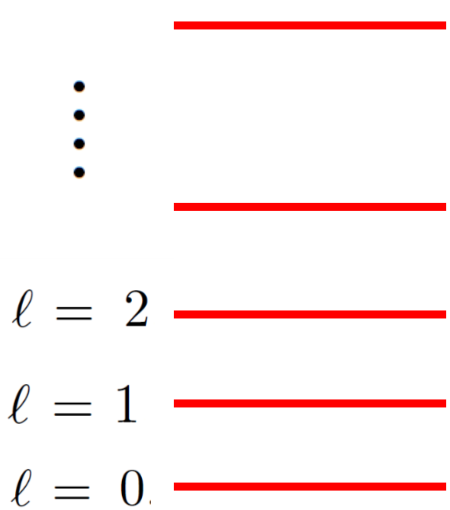
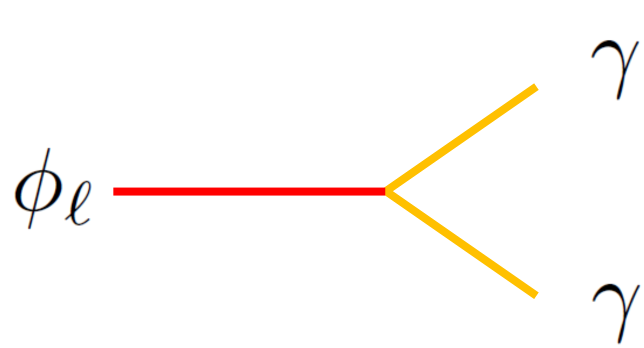


Conditions for Stasis

Mass Spectrum



Decay Processes



$$\Omega_i \equiv \frac{8\pi G}{3H^2}\rho_i$$

$$\Omega_M \equiv \sum_\ell \Omega_\ell$$

$$\Omega_M + \Omega_\gamma = 1$$

Conditions for Stasis

$$\frac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell \rho_\ell$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell \rho_\ell$$

Boltzmann Equations

+ Friedmann Equations

$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H\left(\Omega_M - \Omega_M^2\right)$$

Conditions for Stasis

$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H\left(\Omega_M - \Omega_M^2\right)$$

« *Cosmological Stasis* (I) »

$$\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2) .$$

$$d\Omega_M/dt = 0$$

« *Cosmological Stasis* (II) »

$$d^n\Omega_M/dt^n = 0$$

This equation must hold at any time...

Conditions for Stasis

During Stasis, $\Omega_M = \overline{\Omega}_M$ $\quad H(t) = \left(\frac{2}{4-\overline{\Omega}_M}\right)\frac{1}{t}$

$$\Omega_\ell(t) \;=\; \Omega_\ell^* \left(\frac{t}{t_*}\right)^{2-6/(4-\overline{\Omega}_M)} e^{-\Gamma_\ell(t-t_*)}$$

« Cosmological Stasis » (Eternal)

$$\sum_\ell \Omega_\ell(t) \;=\; \overline{\Omega}_M$$

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) \;=\; \frac{2\overline{\Omega}_M(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell}{\sum_\ell \Omega_\ell} \;=\; \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

A Model of Stasis

Mass Spectrum	$m_\ell = m_0 + (\Delta m)\ell^\delta$
Decay Widths	$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$
Initial Abundances	$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$

Free Parameters

$$\{\alpha, \gamma, \delta, m_0, \Delta m, \Gamma_0, \Omega_0^{(0)}, t^{(0)}\}$$

Production time of the states ϕ_ℓ

« *Cosmological Stasis* » *(Eternal)*

$$\sum_\ell \Omega_\ell(t) = \overline{\Omega}_M$$

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \frac{2\overline{\Omega}_M(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

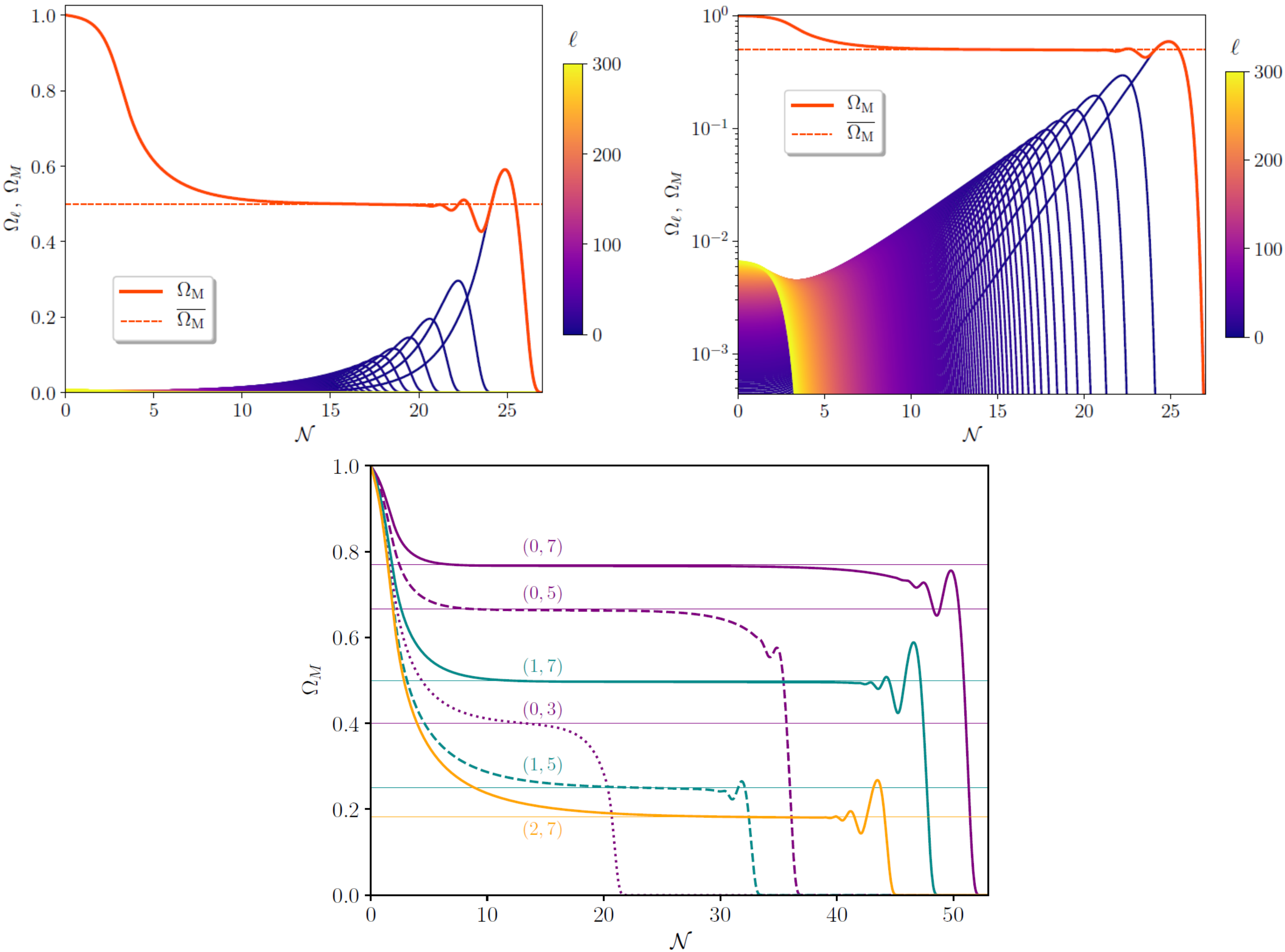
$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell}{\sum_\ell \Omega_\ell} = \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\,\frac{1}{t}$$

$$\downarrow$$

$$\frac{\sum_\ell \Gamma_\ell \Omega_\ell(t)}{\sum_\ell \Omega_\ell(t)} = \frac{\alpha + 1/\delta}{\gamma}\,\frac{1}{t-t^{(0)}}$$

$$\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}\,.$$

Let's try...



$\Omega_\ell\,,\ \Omega_M$
$\mathcal{N}$
$\Omega_{\rm M}$
$\overline{\Omega_{\rm M}}$
ℓ
0
100
200
300
10^0
10^{-1}
10^{-2}
10^{-3}
Ω_M
$(0,7)$
$(0,5)$
$(1,7)$
$(0,3)$
$(1,5)$
$(2,7)$

Stasis as a Global Attractor

Friedmann Equation

$$\frac{dH}{dt} = -\tfrac{1}{2}H^2\,(4-\Omega_M)$$

$$\frac{1}{H} - \frac{1}{H^{(0)}} = (t-t^{(0)})\left[\frac{4-\langle\Omega_M\rangle}{2}\right]$$

$$\langle\Omega_M\rangle \equiv \frac{1}{t-t^{(0)}}\int_{t^{(0)}}^{t} dt'\,\Omega_M(t')\ .$$

$$\frac{d\Omega_M}{dt} = \frac{\Omega_M}{t-t^{(0)}}\left[\frac{2(1-\Omega_M)}{4-\langle\Omega_M\rangle} - \left(\frac{\alpha+1/\delta}{\gamma}\right)\right]$$

$$\frac{d\langle\Omega_M\rangle}{dt} = \frac{1}{t-t^{(0)}}\left[\Omega_M - \langle\Omega_M\rangle\right]$$

Equilibrium: $\Omega_M = \langle\Omega_M\rangle = \boxed{\overline{\Omega}_M = \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta-(1+\alpha\delta)}\ .}$

Stasis as a Global Attractor

$$\begin{cases} \dfrac{d\Omega_M}{dt} = \dfrac{1}{t - t^{(0)}} f(\Omega_M, \langle\Omega_M\rangle) \\ \dfrac{d\langle\Omega_M\rangle}{dt} = \dfrac{1}{t - t^{(0)}} g(\Omega_M, \langle\Omega_M\rangle) \end{cases},$$

where

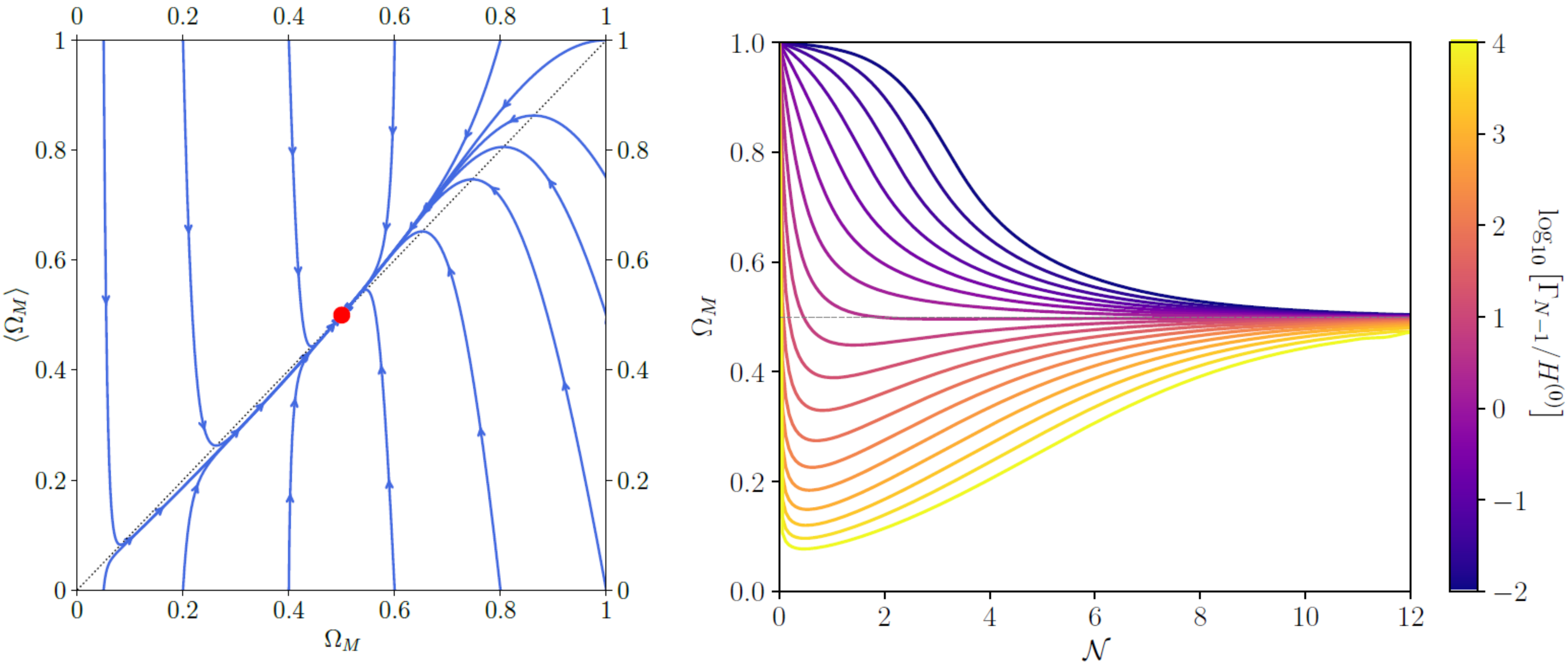
$$f(\Omega_M, \langle\Omega_M\rangle) \equiv \Omega_M \left[\frac{2(1-\Omega_M)}{4-\langle\Omega_M\rangle} - \frac{2(1-\overline{\Omega}_M)}{4-\overline{\Omega}_M}\right]$$

$$g(\Omega_M, \langle\Omega_M\rangle) \equiv \Omega_M - \langle\Omega_M\rangle \, .$$

$$\widehat{J} = \begin{pmatrix} \partial_{\Omega_M} f & \partial_{\langle\Omega_M\rangle} f \\ \partial_{\Omega_M} g & \partial_{\langle\Omega_M\rangle} g \end{pmatrix} \qquad \lambda_\pm = \frac{-(4+\overline{\Omega}_M) \pm \sqrt{\overline{\Omega}_M^2 - 16\overline{\Omega}_M + 16}}{2(4-\overline{\Omega}_M)}$$

$$\lambda_\pm < 0 \quad \text{for all} \quad 0 \le \overline{\Omega}_M \le 1$$

Stasis as a Global Attractor



The attractor is GLOBAL!!!

Implication for Cosmology

Stasis:

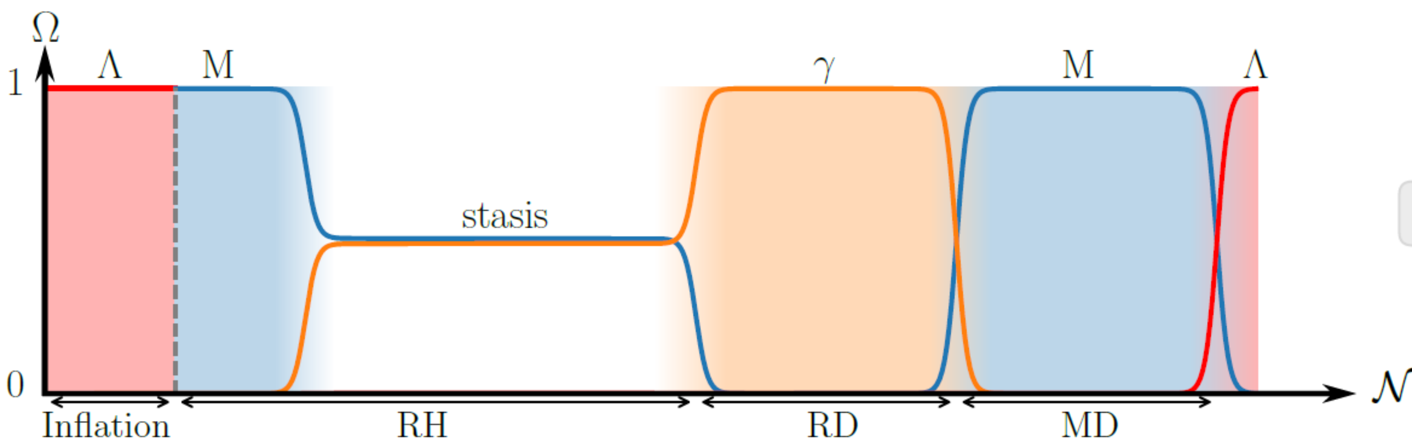
Matter Domination (MD) → Radiation Domination (RD)

Let's splice it in the cosmological timeline!

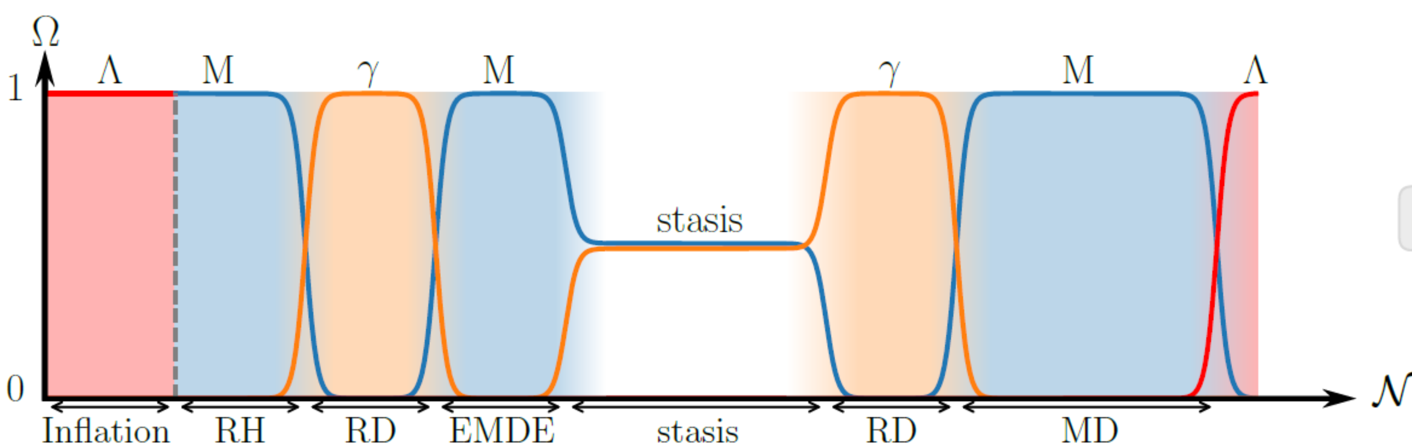
Implication for Cosmology

Stasis:

Matter Domination (MD) → Radiation Domination (RD)



Stasis during Reheating

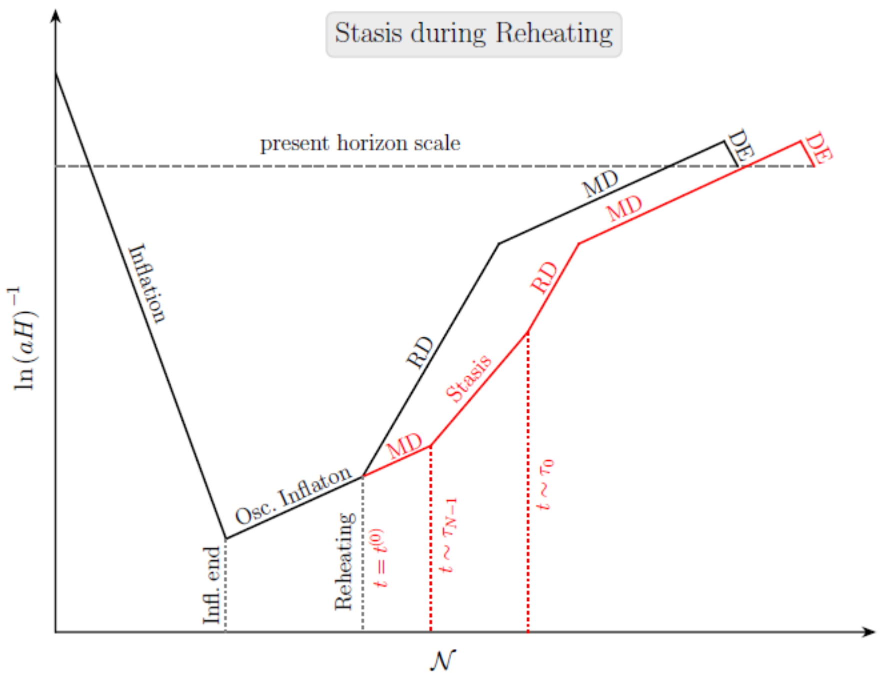


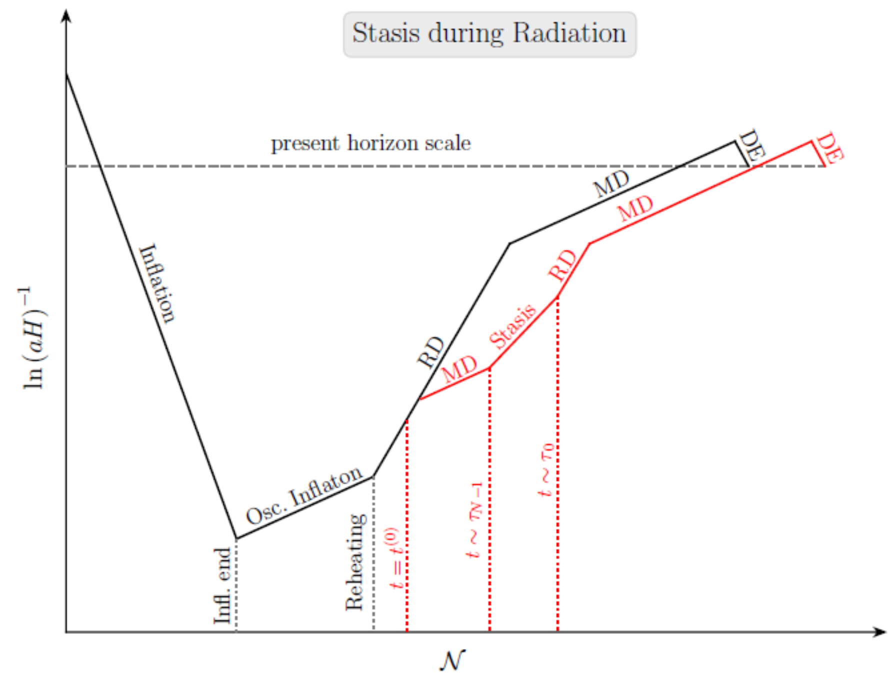
Stasis during Radiation

Implication for Cosmology

Stasis:

Matter Domination (MD) → Radiation Domination (RD)





Implication for Cosmology

- Stasis **modifies the cosmological timeline**
- It **increases the number of *e*-folds** since horizon exit
- It introduces an **era of non-standard cosmology** different than early MD

→ Dark Matter Production

→ Axion Cosmology

→ Baryo/Leptogenesis

→ Growth of Primordial Perturbations

Conclusion

- Decaying Kaluza-Klein states can lead to (very) long periods of stasis;
- The stasis regime is insensitive to initial conditions, it is a global attractor;
- Stasis can co-exist with other fluids as long as they preserve the Matter-Radiation mixture EoS;
- Applications are numerous, in particular regarding the reheating mechanism, constraints on inflation, thermal particle production in the early universe, etc.
- This stasis scenario can also be explored in many different situations, including the production of massive states instead of photons, accounting for thermal blocking of the decays, thermalization of part of the spectrum, etc.

Back Up

Durham University

Conditions for Stasis

Assume that stasis is established at time t

$$\Omega_\ell(t) \;=\; \Omega_\ell^{(0)}\, h(t^{(0)}, t)\, e^{-\Gamma_\ell (t-t^{(0)})}$$

Non-trivial redshift

$$\sum_\ell \Omega_\ell(t) \;=\; \Omega_0^{(0)}\, h(t^{(0)}, t) \sum_\ell \left(\frac{m_\ell}{m_0}\right)^\alpha e^{-\Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma (t-t^{(0)})}$$

Continuous Limit

$$= \;\frac{\Omega_0^{(0)}}{\delta(\Delta m)^{1/\delta}}\, h(t^{(0)}, t) \int_0^\infty dm\, m^{1/\delta-1} \left(\frac{m}{m_0}\right)^\alpha e^{-\Gamma_0 \left(\frac{m}{m_0}\right)^\gamma (t-t^{(0)})}$$

$$= \;\frac{\Omega_0^{(0)}}{\gamma\delta} \left(\frac{m_0}{\Delta m}\right)^{1/\delta} \Gamma\left(\frac{\alpha+1/\delta}{\gamma}\right) h(t^{(0)}, t) \left[\Gamma_0 (t-t^{(0)})\right]^{-(\alpha+1/\delta)/\gamma}$$

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Mass Spectrum	$m_\ell \;=\; m_0 + (\Delta m)\ell^\delta$
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KK spectrum scalar field compactified on a circle of radius R

[Dienes & Thomas, Phys.Rev.D 85, 083523 / 85, 083524 / 86, 055013]

$mR \ll 1$ or $mR \gg 1$ $\longrightarrow$ $\{m_0, \Delta m, \delta\} \;=\; \{m, 1/R, 1\}$ or $\{m_0, \Delta m, \delta\} \;=\; \{m, 1/(2mR^2), 2\}$

Bound states of some strongly coupled theory $\longrightarrow$ $\delta \;=\; 1/2$ [Dienes, Huang, Su, and Thomas, PRD 95, 043526 (2017)]

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Decay Scaling

$$\mathcal{O}_\ell \sim c_n \phi_\ell \mathcal{F} / \Lambda^{d-4}$$

Depends on the microscopic theory $\longrightarrow$

$$\gamma = 2d - 7$$

$$\gamma = \{3, 5, 7\}$$

A Model of Stasis

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Abundances

Depends on the production mechanism...

Universal Inflaton Decay $\longrightarrow$ $\alpha = 1$

Stasis with an Extra Component

Ω_X in addition to Ω_M and Ω_γ

$$p_X = w_X \rho_X$$

Stasis requires $d\Omega_X/dt = 0$

$$\begin{cases} \overline{\Omega}_M = (1-3w_X)(1-\overline{\Omega}_X) \\ \overline{\Omega}_\gamma = 3w_X(1-\overline{\Omega}_X)\ . \end{cases}$$

$$\boxed{w_X = \frac{\overline{\Omega}_\gamma}{3(\overline{\Omega}_M + \overline{\Omega}_\gamma)}\ .}$$

Line of Attractors...