MUonE Theory

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Workshop on Muon Precision Physics

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Outline

- Measuring the hadronic leading order contribution to a_μ in the space-like region
- QED radiative corrections to muon-electron scattering (and their Monte Carlo implementation)
  - Muon-electron scattering at NLO
  - Muon-electron scattering at NNLO
  - NNLO virtual and real leptonic corrections to muon-electron scattering
  - Flash on $\mu e \to \mu e \pi^0$
- Conclusions and outlook

Theory work

- ⇝ Carloni Calame et al., PLB 746 (2015), 325
- ⇝ Mastrolia et al., JHEP 11 (2017) 198
- ⇝ Di Vita et al., JHEP 09 (2018) 016
- ⇝ Alacevich et al., JHEP 02 (2019) 155
- ⇝ Fael and Passera, PRL 122 (2019) 19, 192001
- ⇝ Fael, JHEP 02 (2019) 027
- ⇝ Carloni Calame et al., JHEP 11 (2020) 028
- ⇝ Banerjee et al., SciPost Phys. 9 (2020), 027
- ⇝ Banerjee et al., EPJC 80 (2020) 6, 591
- ⇝ Budassi et al., JHEP 11 (2021) 098
- ⇝ Balzani et al., PLB 834 (2022) 137462
- ⇝ Bonciani et al., PRL 128 (2022) 2, 022002
- ⇝ Budassi et al., PLB 829 (2022) 137138

- ↦ A lively theory community is active to provide state-of-the-art calculations to match the required accuracy for meaningful data analysis
- ↦ Independent numerical codes (Monte Carlo generators and/or integrators) are developed and cross-checked to validate high-precision calculations. Chiefly
  - ✓ `Mesmer` in Pavia
    `github.com/cm-cc/mesmer`
  - ✓ `McMule` at PSI/IPPP
    `gitlab.com/mule-tools/mcmule`

Theory work in this talk

- **Carloni Calame et al., PLB 746 (2015), 325**
- Mastrolia et al., JHEP 11 (2017) 198
- Di Vita et al., JHEP 09 (2018) 016
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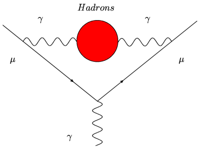
Standard approach to $a_\mu^{\rm HLO}$

⇝ In the following, focus on $a_\mu^{\rm HLO}$, which contributes (with $a_\mu^{\rm HLbL}$) to the SM uncertainty



- Using dispersion relations and the Optical Theorem

$$\boldsymbol{a_\mu^{\rm HLO}} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\; K(s)\; \sigma^0_{e^+e^-\to \text{had}}(s) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} ds\, \frac{K(s) R^{\text{had}}(s)}{s^2} =$$

$$= \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left[\int_{4m_\pi^2}^{E^2_{\text{cut}}} ds\, \frac{K(s) R^{\text{had}}_{\text{data}}(s)}{s^2} + \int_{E^2_{\text{cut}}}^{\infty} ds\, \frac{K(s) R^{\text{had}}_{\text{pQCD}}(s)}{s^2}\right]$$

$$K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}} \sim \frac{1}{s} \qquad\qquad R^{\text{had}}(s) = \frac{\sigma^0_{e^+e^-\to \text{had}}(s)}{\frac{4}{3}\pi\alpha^2/s}$$

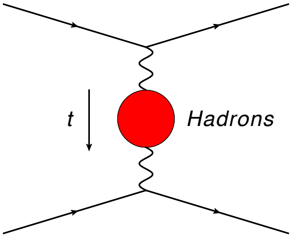
Master formula

- Alternatively (exchanging s and x integrations in $a_\mu^{\rm HLO}$)

$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

e.g. Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193



⇝ The hadronic VP correction to the running of α enters

⇝ Essentially the same formula used in lattice QCD calculation of $a_\mu^{\rm HLO}$

★ $\Delta\alpha_{\rm had}(t)$ (and $a_\mu^{\rm HLO}$) can be directly measured in a (single) experiment involving a space-like scattering process

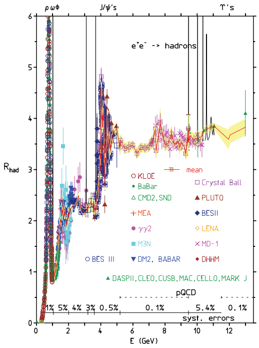
Carloni Calame, Passera, Trentadue, Venanzoni PLB 746 (2015) 325

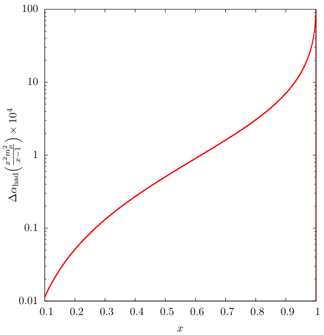
★ **Still a data-driven evaluation of $a_\mu^{\rm HLO}$, but with space-like data**

- By modifying the kernel function $\frac{\alpha}{\pi}(1-x)$, also $a_\mu^{\rm HNLO}$ and $a_\mu^{\rm HNNLO}$ can be provided!

Balzani, Laporta, Passera, PLB 834 (2022) 137462

Time-like $\mapsto$ **Space-like**





Smooth function

- $\mapsto$ **Time-like:** combination of many experimental data sets, control of RCs better than $\mathcal{O}(1\%)$ on hadronic channels required
- $\mapsto$ **Space-like:** in principle, one single experiment, *it's a one-loop effect, very high accuracy needed*

Abbiendi et al., EPJC 77 (2017) 3, 139

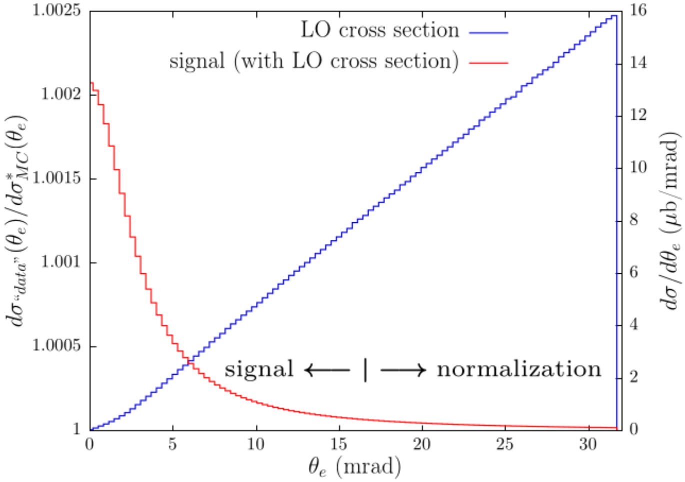
Abbiendi et al., *Letter of Intent: the MUonE project, CERN-SPSC-2019-026, SPSC-I-252 (2019)*

- Scattering μ's on e's in a low Z target looks like an ideal process (fixed target experiment)
- It is a pure t-channel process at tree level
- The M2 muon beam ($E_\mu \simeq 160$ GeV) is available at CERN
- $\sqrt{s} \simeq 0.4$ GeV and $-0.143 < t < 0$ GeV^2
- We can cover 87% of the a_μ^{HLO} space-like integral (and extrapolate to $x \to 1$)
- With ~ 3 years of data taking, a statistical accuracy of 0.3% on a_μ^{HLO} can be achieved

$$\frac{1}{2}\frac{\delta\sigma}{\sigma} \simeq \frac{\delta\alpha}{\alpha} \simeq \delta\Delta\alpha_{\text{had}}$$

$\Delta\alpha_{\text{had}}$ is a 0.1% effect in this region $\to$ to measure it at 1%, σ must be controlled at the 10^{-5} level

What we want to measure



- The μe cross section and distributions must be known as precisely as possible
  $\mapsto$ radiative corrections (RCs) are mandatory and **must** be implemented into a MC event generator for data analysis

⋆ First step are QED $\mathcal{O}(\alpha)$ (i.e. QED NLO, next-to-leading order) RCs

The NLO cross section is split into two contributions,

$$\sigma_{NLO} = \sigma_{2\to 2} + \sigma_{2\to 3} = \sigma_{\mu e\to\mu e} + \sigma_{\mu e\to\mu e\gamma}$$

$\mapsto$ IR singularities are regularized with a vanishingly small photon mass $\boldsymbol{\lambda}$

$\mapsto$ $[2\to 2]/[2\to 3]$ phase space slicing at an arbitrarily small γ-energy cutoff $\boldsymbol{\omega_s}$

- $\mu e \to \mu e$

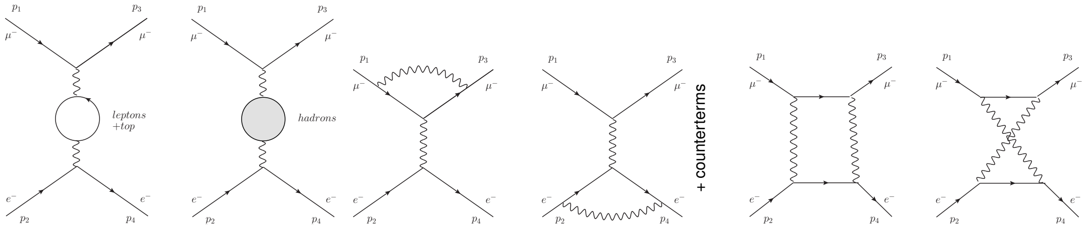
$$\sigma_{2\to 2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F}\int d\Phi_2(|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}^*_{LO}\times\mathcal{A}^{virtual}_{NLO}(\boldsymbol{\lambda})])$$

- $\mu e \to \mu e\gamma$

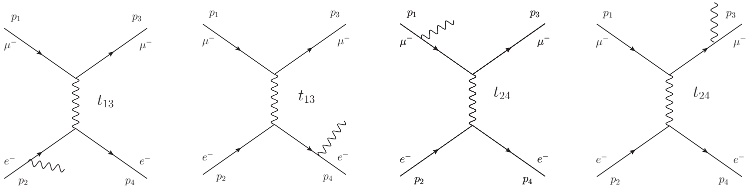
$$\sigma_{2\to 3} = \frac{1}{F}\int_{\omega>\boldsymbol{\lambda}} d\Phi_3|\mathcal{A}^{1\gamma}_{NLO}|^2 = \frac{1}{F}\left(\int_{\boldsymbol{\lambda}<\omega<\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}^{1\gamma}_{NLO}|^2 + \int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}^{1\gamma}_{NLO}|^2\right)$$
$$= \Delta_s(\boldsymbol{\lambda},\boldsymbol{\omega_s})\int d\sigma_{LO} + \frac{1}{F}\int_{\omega>\boldsymbol{\omega_s}} d\Phi_3|\mathcal{A}^{1\gamma}_{NLO}|^2$$

- the integration over the 2/3-particles phase space is performed with MC techniques and fully-exclusive events are generated

- interference of LO $\mu e \to \mu e$ amplitude with



- squared absolute value of

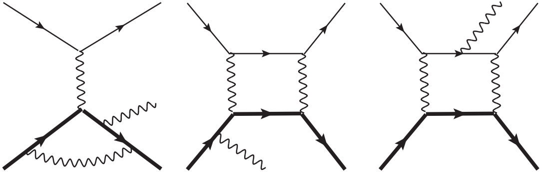


Second step, *photonic* radiative corrections at **NNLO**

JHEP 11 (2020) 028

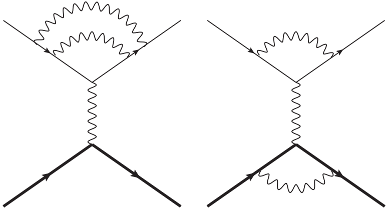
calculated exactly

- | NLO virtual diagrams $|^2$
- interference of LO $\mu e \to \mu e \gamma$ amplitude with



$+$ many others

- interference of LO $\mu e \to \mu e$ amplitude with

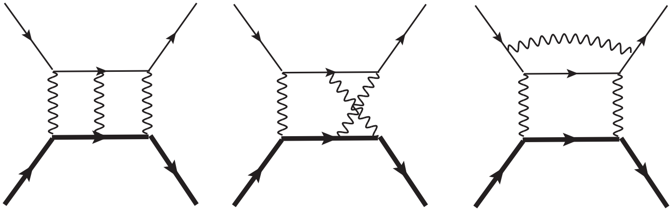


$+$ many others

2-loop QED vertex form factors borrowed from **Mastrolia and Remiddi, NPB 664 (2003) 341**

- interference of LO $\mu e \to \mu e$ amplitude with

approximated à la YFS



$+$ many others

⇝ NNLO double-virtual amplitudes where at least 2 photons connect the e and μ lines are approximated according to the Yennie-Frautschi-Suura ('61) formalism to catch the infra-red divergent structure

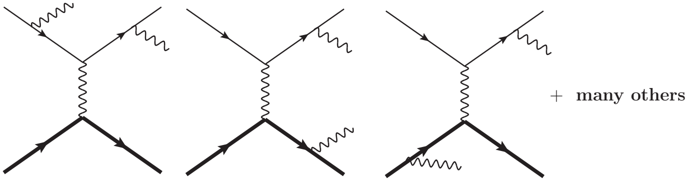
$$\widetilde{\mathcal{A}}^{\alpha^2} = \underbrace{\mathcal{A}_e^{\alpha^2} + \mathcal{A}_\mu^{\alpha^2} + \mathcal{A}_{e\mu,\,\mathrm{1L\times 1L}}^{\alpha^2}}_{\text{exact}} + \underbrace{\frac{1}{2} Y_{e\mu}^2 \mathcal{T} + Y_{e\mu}\left(Y_e + Y_\mu\right)\mathcal{T} + (Y_e + Y_\mu)\mathcal{A}_{e\mu}^{\alpha^1,\mathrm{R}} + Y_{e\mu}\mathcal{A}^{\alpha^1,\mathrm{R}}}_{\text{YFS approximated}}$$

- going beyond this requires the full two-loop virtual amplitudes

R. Bonciani *et al.*, PRL 128 (2022) 2

calculated exactly

- squared absolute value of



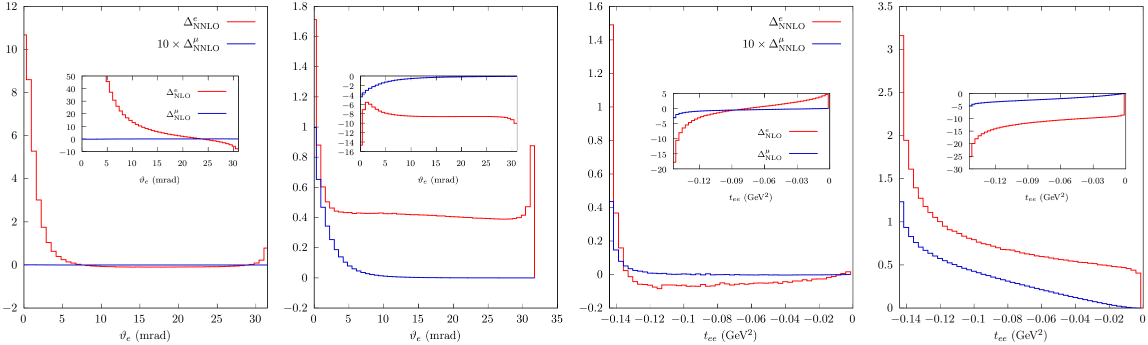
⇝ also at NNLO we use a vanishingly small photon mass λ and the "slicing method" to deal with IR divergences

⇝ phase space integration and event generation is again performed with MC techniques allowing for fully exclusive event generation

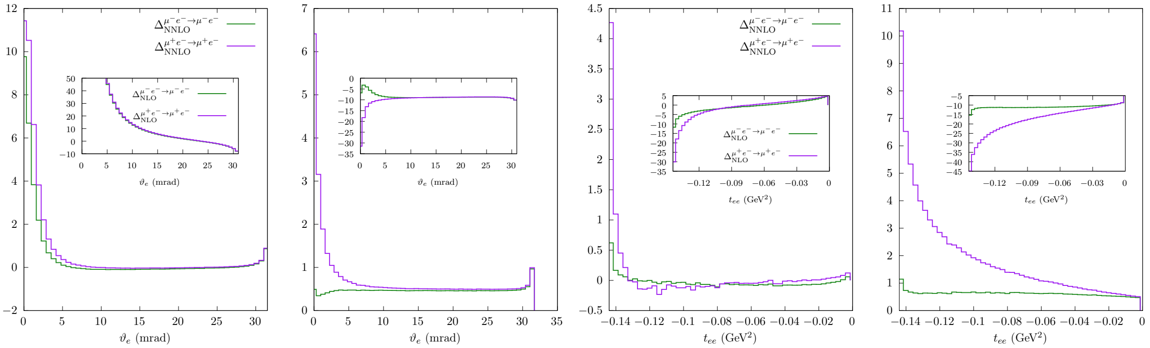
- Showing

$$\Delta^i_{\text{NNLO}} \equiv 100 \times \frac{d\sigma^i_{\text{NNLO}} - d\sigma^i_{\text{NLO}}}{d\sigma_{\text{LO}}}$$

⇝ exact NNLO radiation from electron or muon leg, with or without acoplanarity cut



- full NNLO[1] radiation for incoming μ^+ or μ^-, with or without acoplanarity cut



- we estimate the subset of amplitudes in YFS approximation to miss terms of order

$$\left(\frac{\alpha}{\pi}\right)^2 \ln^2\left(m_\mu^2/m_e^2\right) \simeq 5\times 10^{-4}$$

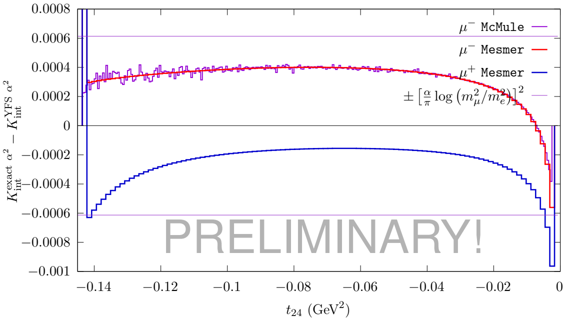
[1] of course with "double boxes" in YFS approximation

Exact NNLO photonic corrections

- R. Bonciani *et al.* in PRL 128 (2022) 2 calculated the complete two-loop photonic corrections to $f\bar{f} \to F\bar{F}$, with $m_f = 0$. It can be used for $\mu^\pm e^- \to \mu^\pm e^-$ via crossing symmetry
- The massless amplitude needs to go through an elegant and complex procedure known as ***massification*** to recover collinear divergencies in terms of $\log(Q^2/m_e^2)$

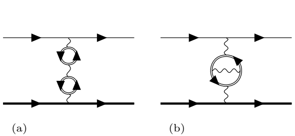
Engel *et al.*, JHEP 02 (2019) 118

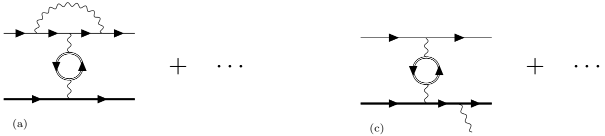
⇝ Difference between YFS-approximated and exact NNLO (photonic) K factor (preliminary!)



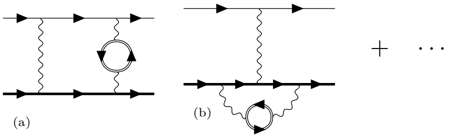
- the evaluation of NNLO “double-boxes” is very CPU expensive, > 1 s/event (on a sigle core)

- any lepton (and hadron) in the VP blobs
- interfered with $\mu e \to \mu e$ or $\mu e \to \mu e \gamma$ amplitudes





- interfered with $\mu e \to \mu e$ amplitude



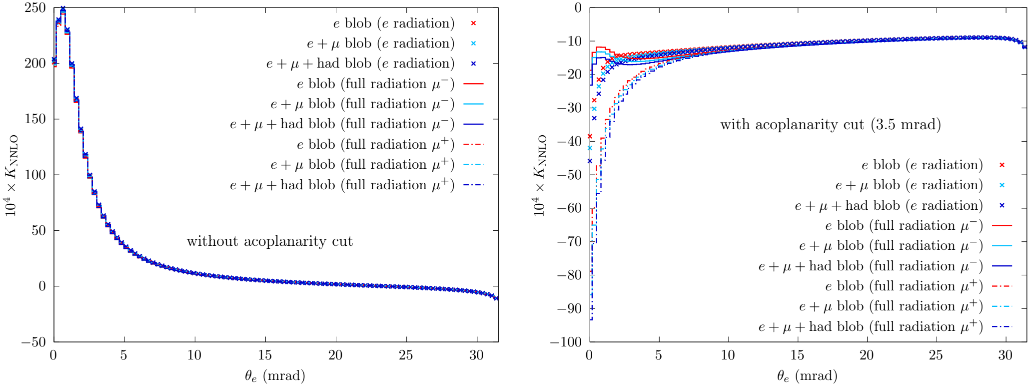
Here the 2-loop integral is evaluated with dispersion relation techniques

see also Fael & Passera, PRL 122 (2019) 19

$$\frac{g_{\mu\nu}}{q^2+i\epsilon} \to g_{\mu\nu}\frac{\alpha}{3\pi}\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{R_\ell(z)}{q^2-z+i\epsilon} = g_{\mu\nu}\frac{\alpha}{3\pi}\int_{4m_\ell^2}^{\infty}\frac{dz}{z}\frac{1}{q^2-z+i\epsilon}\left(1+\frac{4m_\ell^2}{2z}\right)\sqrt{1-\frac{4m_\ell^2}{z}}$$

- Showing NNLO differential K-factors $\times 10^4$

$$K_{\text{NNLO}} \equiv \frac{d\sigma_i^{\alpha^2,\text{ virtual pairs}}}{d\sigma_{\text{LO}}}$$



Virtual pair effects

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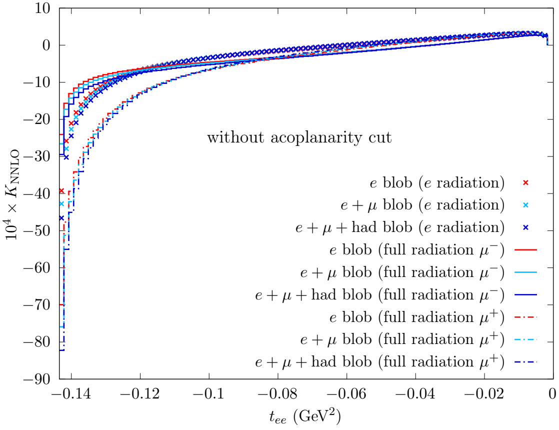
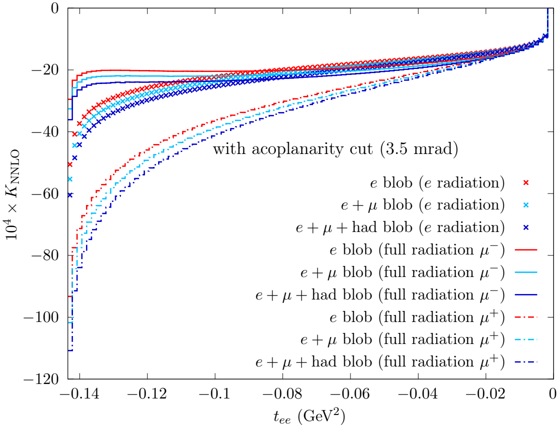
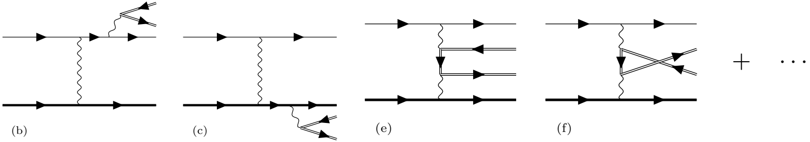




- they also contribute at NNLO
- squared absolute vaule of



- the emission of an extra electron pair $\mu e \to \mu e\, e^+ e^-$ is potentially a dramatically large (reducible) background, because of the presence of “peripheral” diagrams

⇝ A set of experimental cuts is needed to get rid of it.
In addition to basic cuts (exactly one muon-like and one electron-like, with $E \geq 1$ GeV, particle in the detector), we consider

1. $\theta_{\mu\text{-like}}, \theta_{e\text{-like}} \geq \theta_c = 0.2$ mrad
2. acoplanarity ≤ 3.5 mrad
3. geometric distance from the elastic curve in the $[\theta_\mu, \theta_e]$ plane < 0.2 mrad

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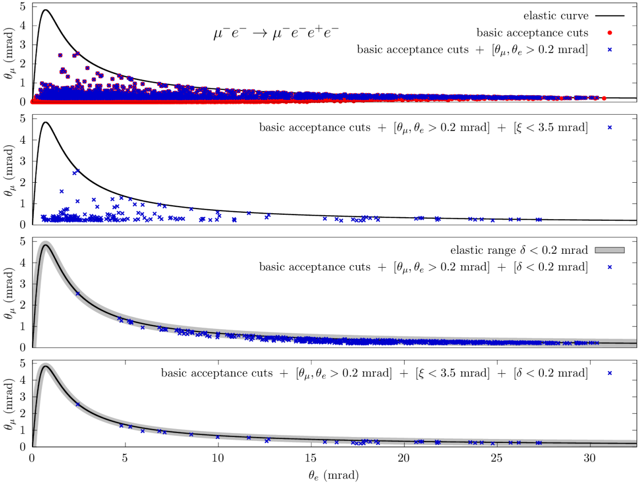
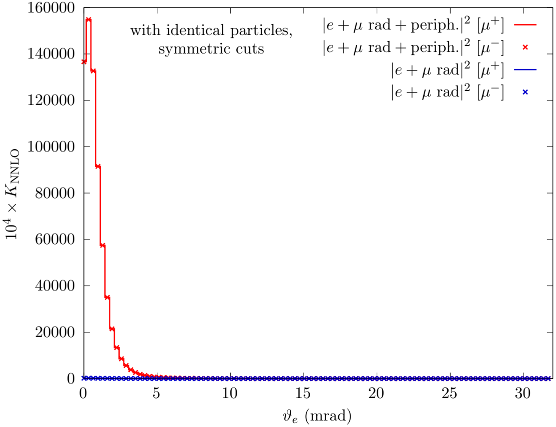
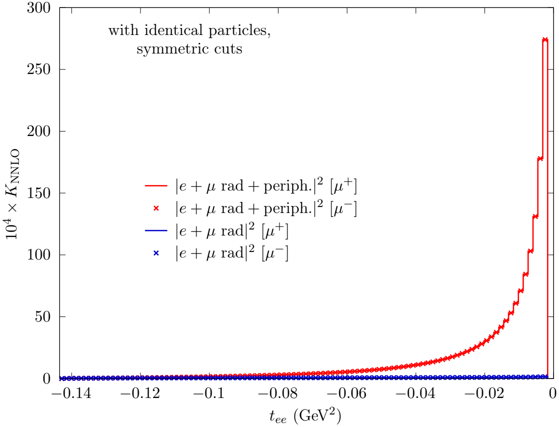


⇝ only 0.007% of $\mu e \to \mu e\ e^+e^-$ events survives the combination of the three cuts


with identical particles,
symmetric cuts
$|e+\mu\ \mathrm{rad}+\mathrm{periph.}|^2\ [\mu^+]$
$|e+\mu\ \mathrm{rad}+\mathrm{periph.}|^2\ [\mu^-]$
$|e+\mu\ \mathrm{rad}|^2\ [\mu^+]$
$|e+\mu\ \mathrm{rad}|^2\ [\mu^-]$
$10^4 \times K_{\mathrm{NNLO}}$
ϑ_e (mrad)
0
20000
40000
60000
80000
100000
120000
140000
160000
0
5
10
15
20
25
30


with identical particles,
symmetric cuts
$|e+\mu\ \mathrm{rad}+\mathrm{periph.}|^2\ [\mu^+]$
$|e+\mu\ \mathrm{rad}+\mathrm{periph.}|^2\ [\mu^-]$
$|e+\mu\ \mathrm{rad}|^2\ [\mu^+]$
$|e+\mu\ \mathrm{rad}|^2\ [\mu^-]$
$10^4 \times K_{\mathrm{NNLO}}$
t_{ee} (GeV^2)
0
50
100
150
200
250
300
−0.14
−0.12
−0.1
−0.08
−0.06
−0.04
−0.02
0

Real e^+e^- pairs (applying extra cuts)

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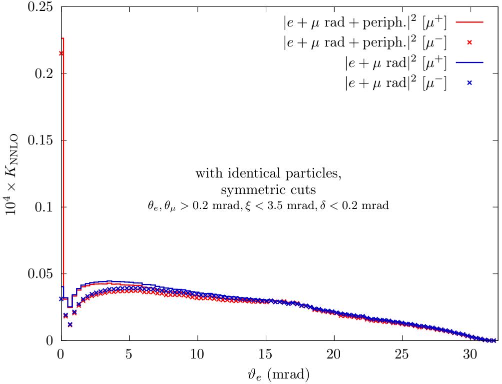
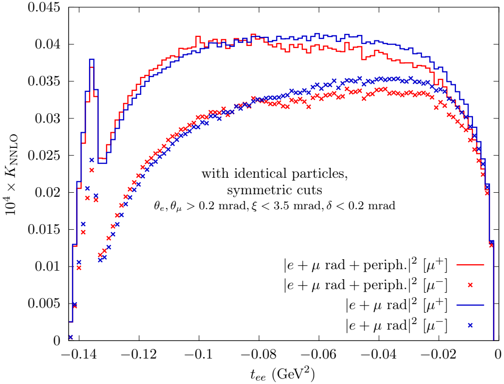
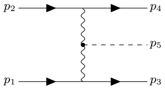


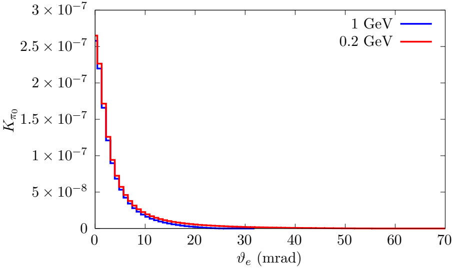


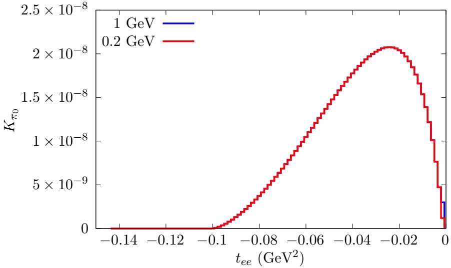
⇝ $\mu e \to \mu e\, \mu^+\mu^-$ is always tiny, because of tiny available phase space



- We studied also the process $\mu e \to \mu e \pi^0$ with $\pi^0 \to \gamma\gamma$ as possible background, using a phenomenological model for the $\gamma^\star\gamma^\star\pi^0$ effective vertex

Czyż, Kisza, Tracz, PRD 97 (1) (2018) 016006



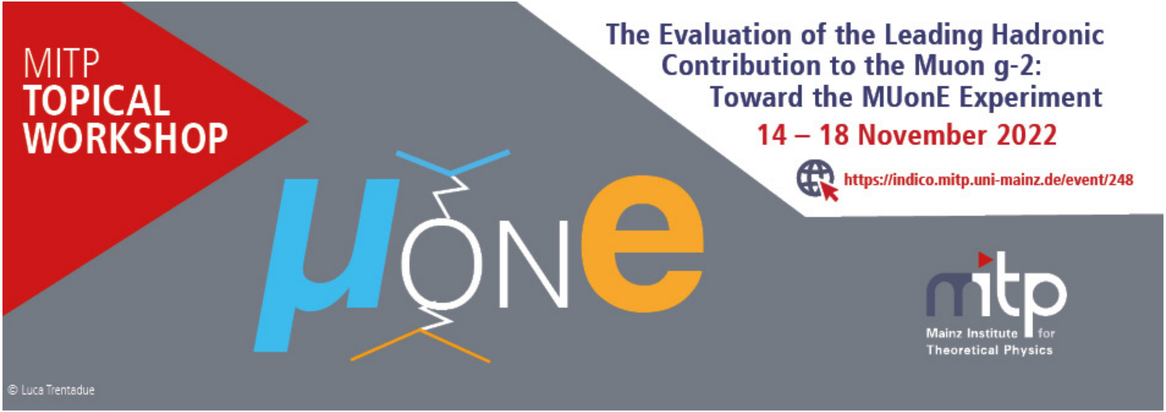


⇝ not an issue in the signal region

⇝ perhaps to be considered for NP searches in phase space region outside the signal one

Conclusions and outlook

- A lively theory community is collaborating to provide MUonE experiment with high-precision calculations for meaningful data analysis
- For instance, the **Mesmer** MC generator includes
  - exact NLO corrections
  - almost exact NNLO photonic corrections
  - exact NNLO virtual leptonic (and hadronic) corrections
  - $\mu e \to \mu e\, \ell^+\ell^-$ $(\ell = \mu, e)$ and $\mu e \to \mu e\, \pi^0$ (with $\pi^0 \to \gamma\gamma$), which also contribute at NNLO
- Many cross checks/tuned comparisons have been (and are being) carried out among independent calculations
  - ✓ **All successful!**
- Wish-list for the (next) future
  - ↪ include complete QED NNLO corrections in **Mesmer**
  - ↪ implement resummation of higher-order QED corrections, matched to NNLO exact contributions


MITP
TOPICAL
WORKSHOP
The Evaluation of the Leading Hadronic
Contribution to the Muon g-2:
Toward the MUonE Experiment
14 – 18 November 2022
https://indico.mitp.uni-mainz.de/event/248
μONe
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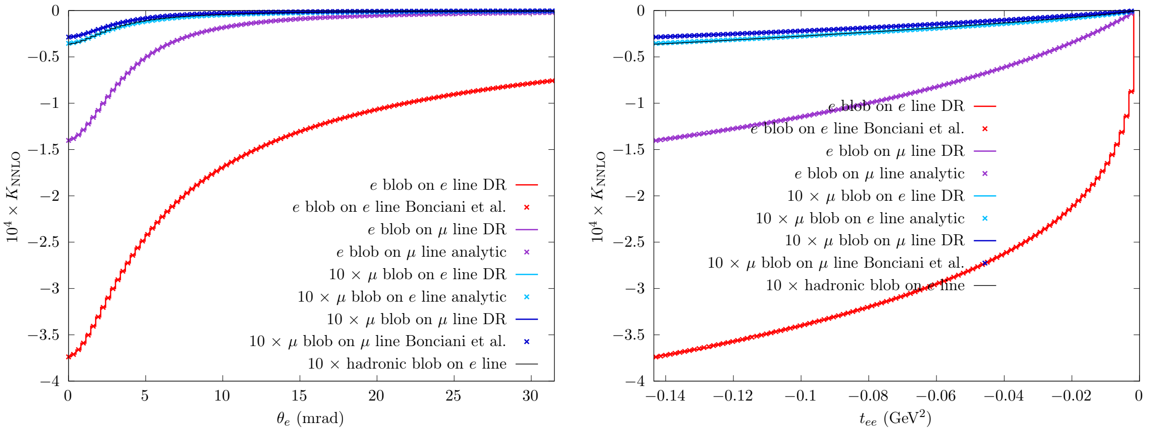
SPARES

Virtual leptonic (and hadronic NNLO) vertex corrections



Virtual leptonic (and hadronic) NNLO VP corrections